PASCOS 2025, Durham University

In Search of Cosmic Topology with AI

Andrius Tamošiūnas

IFT UAM-CSIC

Case Western Reserve University

COMPACT Collaboration

COMPACT Collaboration


Glenn STARKMAN


Craig COPI


Arthur KOSOWSKY


Andrew JAFFE


Yashar AKRAMI


Thiago PEREIRA


Stefano ANSELMI


Fernando CORNET-GÓMEZ


Javier DUQUE


Deyan MIHAYLOV


Anna NEGRO


Andrius TAMOŠIŪNAS


Mikel MARTIN


Joline NOLTMANN


Ananda SMITH

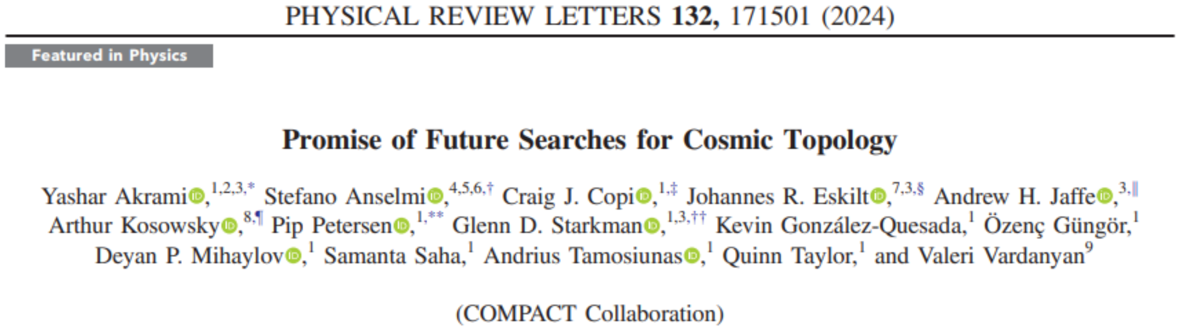

Amirhossein SAMANDAR

Introduction: Cosmic Topology

- Cosmic topology
- The observational signatures of cosmic topology
- Detectability of cosmic topology

Cosmic Topology

- Cosmic topology: an **old problem**.
- A key goal of cosmic topology: to measure the **shape** of the Universe.
- If we model the Universe as a **manifold**, what is the topology of that manifold?
- I.e. is the Universe:
  - Finite or infinite?
  - Open or closed?
  - Simply or multiply-connected?
  - Orientable or not?
- If the Universe is **flat**, 18 allowed topology classes: $\boldsymbol{E_1}$-$\boldsymbol{E_{18}}$ (**Riazuelo et al. 2004** and **Akrami et al. 2024** for a review).



PHYSICAL REVIEW LETTERS **132,** 171501 (2024)

Featured in Physics

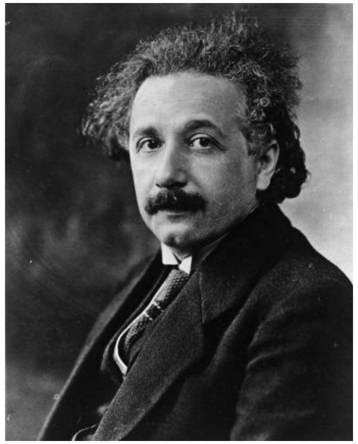
Promise of Future Searches for Cosmic Topology

Yashar Akrami,[1,2,3,*] Stefano Anselmi,[4,5,6,†] Craig J. Copi,[1,‡] Johannes R. Eskilt,[7,3,§] Andrew H. Jaffe,[3,‖] Arthur Kosowsky,[8,¶] Pip Petersen,[1,**] Glenn D. Starkman,[1,3,††] Kevin González-Quesada,[1] Özenç Güngör,[1] Deyan P. Mihaylov,[1] Samanta Saha,[1] Andrius Tamosiunas,[1] Quinn Taylor,[1] and Valeri Vardanyan[9]

(COMPACT Collaboration)

Henri Poincaré : pioneer of modern topology

Karl Schwarzschild: multiple images of astronomical sources in a Universe with non-trivial topology (1900)

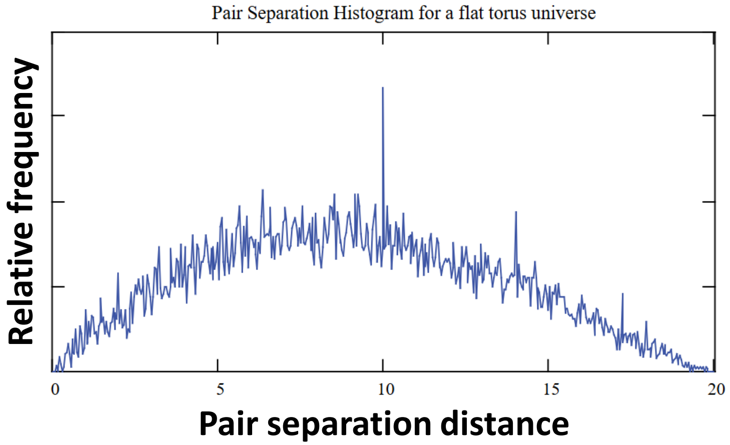


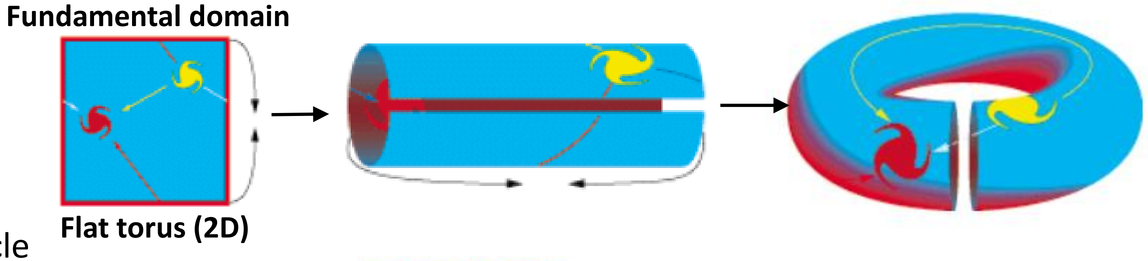
Albert Einstein: 1917 -- Universe as a simply-connected positively curved hypersphere ($\boldsymbol{S^3}$)

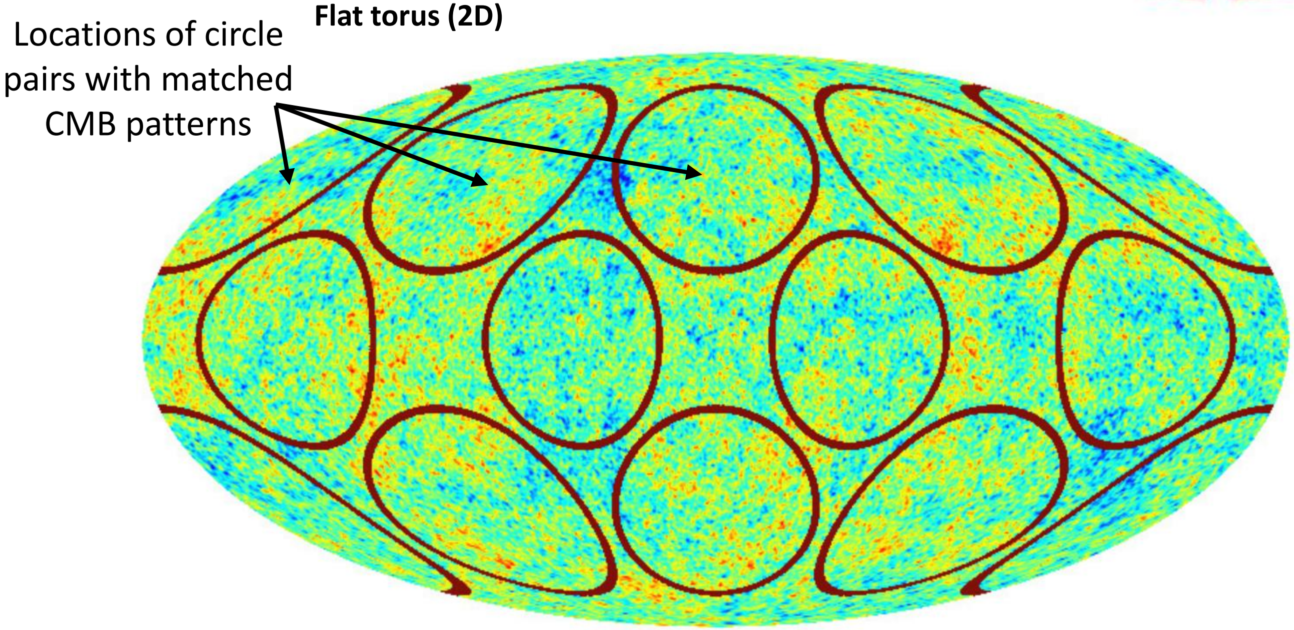
William Thurston: Fields medal for the study of 3-manifolds (1982)

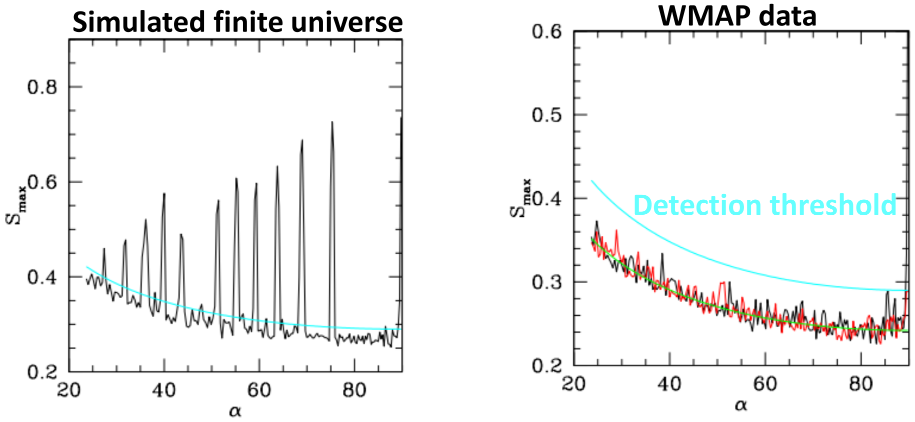
Cosmic Topology: Observational Signatures

Clone images of astronomical sources









Cosmic Topology: Detectability

- **Non-diagonal elements** in the covariance matrix encode information about topology:

$$C^{XY}_{\ell m \ell' m'} \equiv \left\langle a^{X}_{\ell m} a^{Y*}_{\ell' m'} \right\rangle \qquad XY \in \{TT, TE, TB, EE, EB, BB\}$$

- E.g., temperature covariance matrix (E_1):

Transfer functions; Primordial power spectrum

$$C_{\ell m \ell' m'} = \frac{(4\pi)^2}{L^3} \sum_{\boldsymbol{n} \in \mathcal{N}^{E_1}} \Delta_\ell(k_{\boldsymbol{n}}) \Delta^*_{\ell'}(k_{\boldsymbol{n}}) \frac{2\pi^2 \mathcal{P}^{\mathcal{R}}(k_{\boldsymbol{n}})}{k^3_{\boldsymbol{n}}} \xi^{E_1;\hat{\boldsymbol{k}}_{\boldsymbol{n}}}_{k_{\boldsymbol{n}} \ell m} \xi^{E_1;\hat{\boldsymbol{k}}_{\boldsymbol{n}}}_{k_{\boldsymbol{n}} \ell' m'}$$

Wave vectors

$$\xi^{E_1;\hat{\boldsymbol{k}}_n}_{k_n \ell m} \equiv e^{-i\boldsymbol{k}_n \cdot \boldsymbol{x}_0} i^\ell Y^*_{\ell m}\left(\hat{\boldsymbol{k}}_n\right)$$

- **Kullback–Leibler (KL) divergence** measures the detectability of these features:

$$D_{\mathrm{KL}}(p\|q) = \int \mathrm{d}\{a_{\ell m}\} p(\{a_{\ell m}\}) \ln\left[\frac{p(\{a_{\ell m}\})}{q(\{a_{\ell m}\})}\right]$$

$p(\{a_{\ell m}\})$ **- probability of non-trivial topology**

$q(\{a_{\ell m}\})$ **- probability of trivial topology**

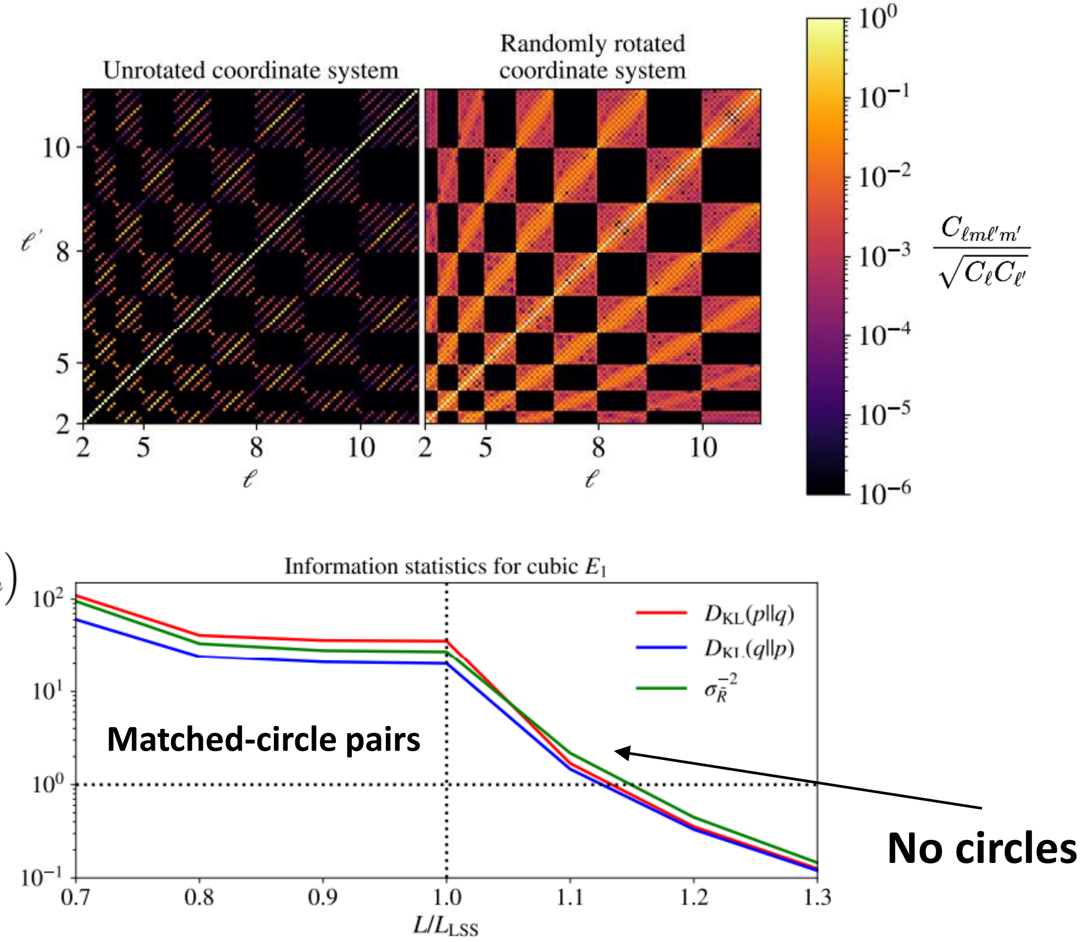


Figure 1: temperature covariance matrix (**top**).
KL divergence in E_1 topology (**bottom**).

Part 1: In Search of Cosmic Topology with Artificial Intelligence

- The problem
- The dataset
- The algorithms

Detecting Cosmic Topology with AI

- The goal: an algorithm to classify **harmonic space** realizations and **CMB maps**.
- Start with a single topology: 3-torus (***E*₁**) of different sizes (**T + E data**).
- Two dataset classes: **rotated** and **non-rotated**.
- Algorithms to try:
  - Random forests and XGBoost;
  - 1D convolutional neural networks;
  - 2D convolutional neural networks;
  - Complex neural networks;
  - GCNNs trained on **spherical map data**.

4 classes: 40,000 – 200,000 realizations

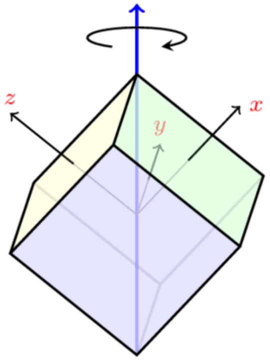
- E1 with $L_x = L_y = L_z = 0.05 \times L_{LSS}$
- E1 with $L_x = L_y = L_z = 0.1 \times L_{LSS}$
- E1 with $L_x = L_y = L_z = 0.5 \times L_{LSS}$
- Trivial topology $L_x = L_y = L_z = L_\infty$

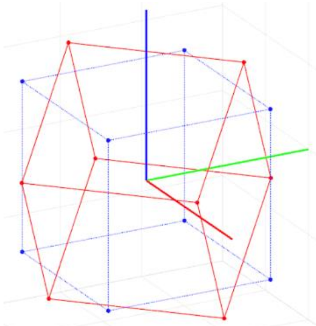
$a_{\ell m}$ realizations → Algorithm 1 → **Class 1** or **Class 2** … **Class N**

$C_{\ell m \ell' m'} = a_{\ell m} a^*_{\ell' m'}$ realizations → Algorithm 2 → **Class 1** or **Class 2** … **Class N**

CMB healpix maps → Algorithm 3 → **Class 1** or **Class 2** … **Class N**

Fundamental domain for a given topology



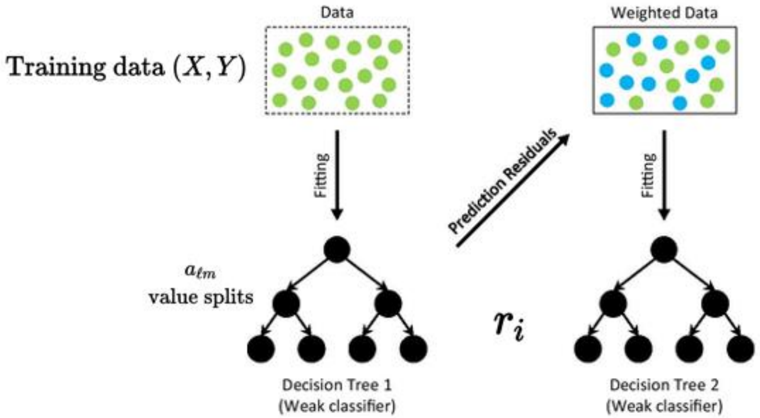


Rotating the coordinate system (multiplying by Wigner D-matrix)

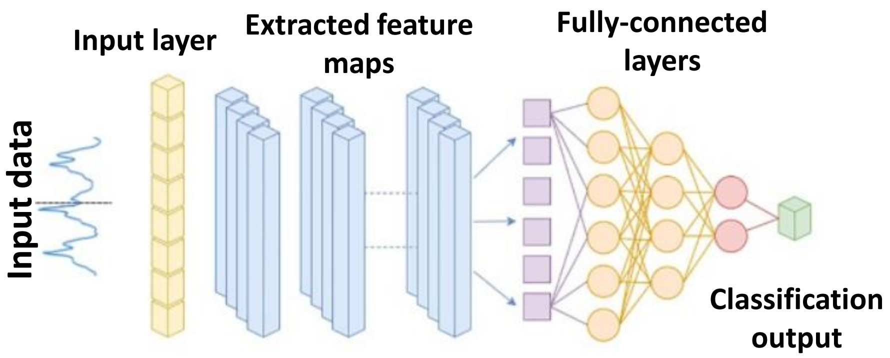
$$a^{E1}_{\ell m} = \frac{4\pi}{\sqrt{V_{E1}}} i^l \sum_{\vec{n}} \delta_{\vec{k}_{\vec{n}}} e^{-i\vec{k}_{\vec{n}}\cdot\vec{x}_0} Y^\star_{\ell m}(\hat{k}) \Delta_\ell(k)$$

The Algorithms

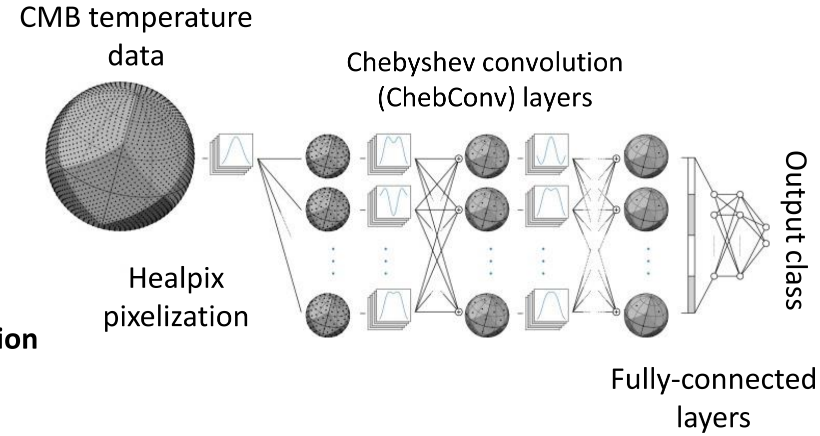
Algorithm 1: random forests and extreme gradient boosting



Algorithm 2: 1D and 2D convolutional neural networks (CNNs)



Algorithm 3: Spherical graph convolutional neural networks



- Trained on $a_{\ell m}$ data.
- Simple, yet powerful algorithms.
- Allow calculating **feature importance** statistics.
- Random forest implementation: **scikit-learn**.
- Extreme gradient boosting implementation: **XGBoost**

- Trained on $a_{\ell m}$ and $C_{\ell m \ell' m'}$ data.
- Very powerful algorithms, but can be difficult to train.
- 1D and 2D CNN implementations with **TensorFlow.**
- Complex neural network implementation with **CVNN**.

- Trained on T and E map data.
- Implementation: based on **DeepSphere** (spherical graph CNNs).
- Extracted features are rotationally-**equivariant**.
- Results depend on map resolution, and the details of the graph CNN.

Part 2: Classifying Topologies in Harmonic Space with AI

- The results: XGBoost, 1D, 2D CNNs and CVNNs
- Feature importance analysis
- Results for large topologies

The Results: XGBoost, 1D and 2D CNNs

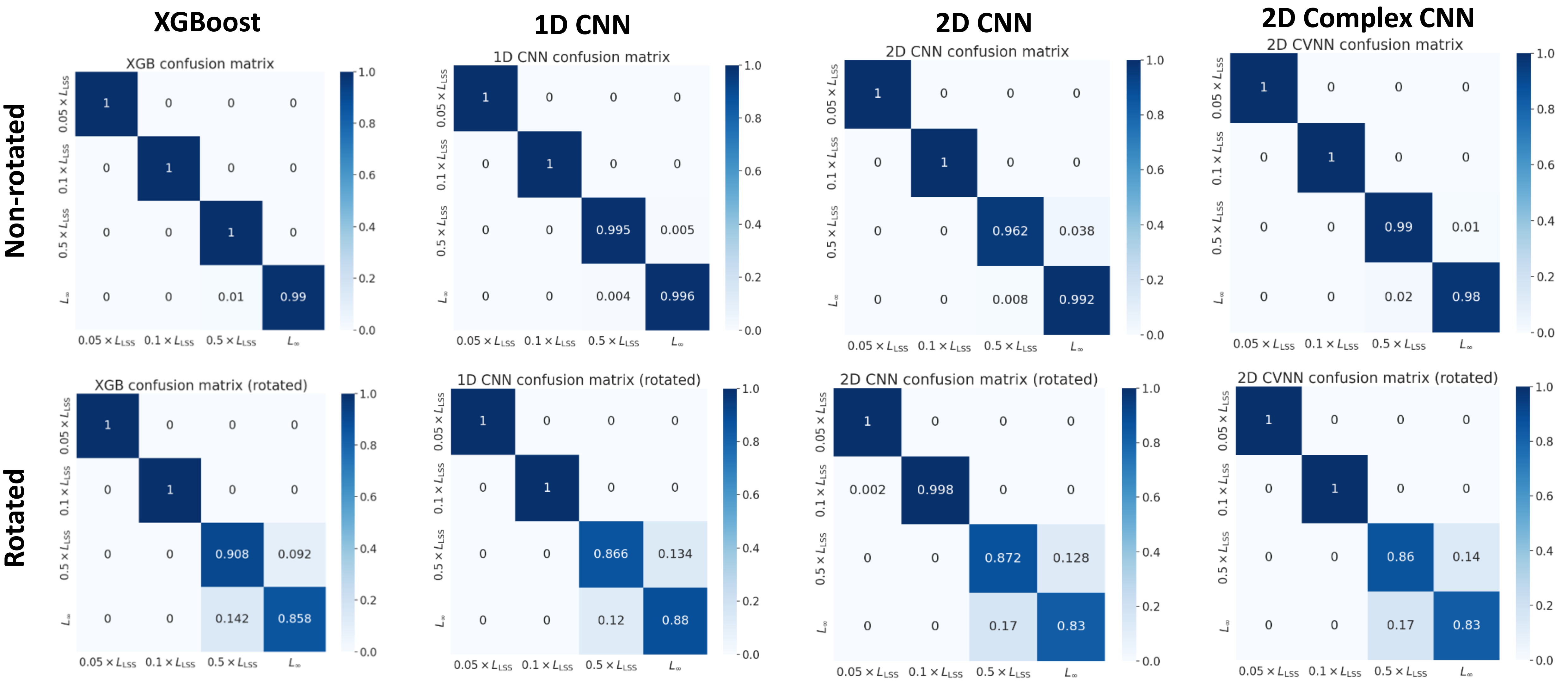


Figure 2: Harmonic space realization classification results.

The Results: Feature Importance

Random forests:

XGBoost classifier:

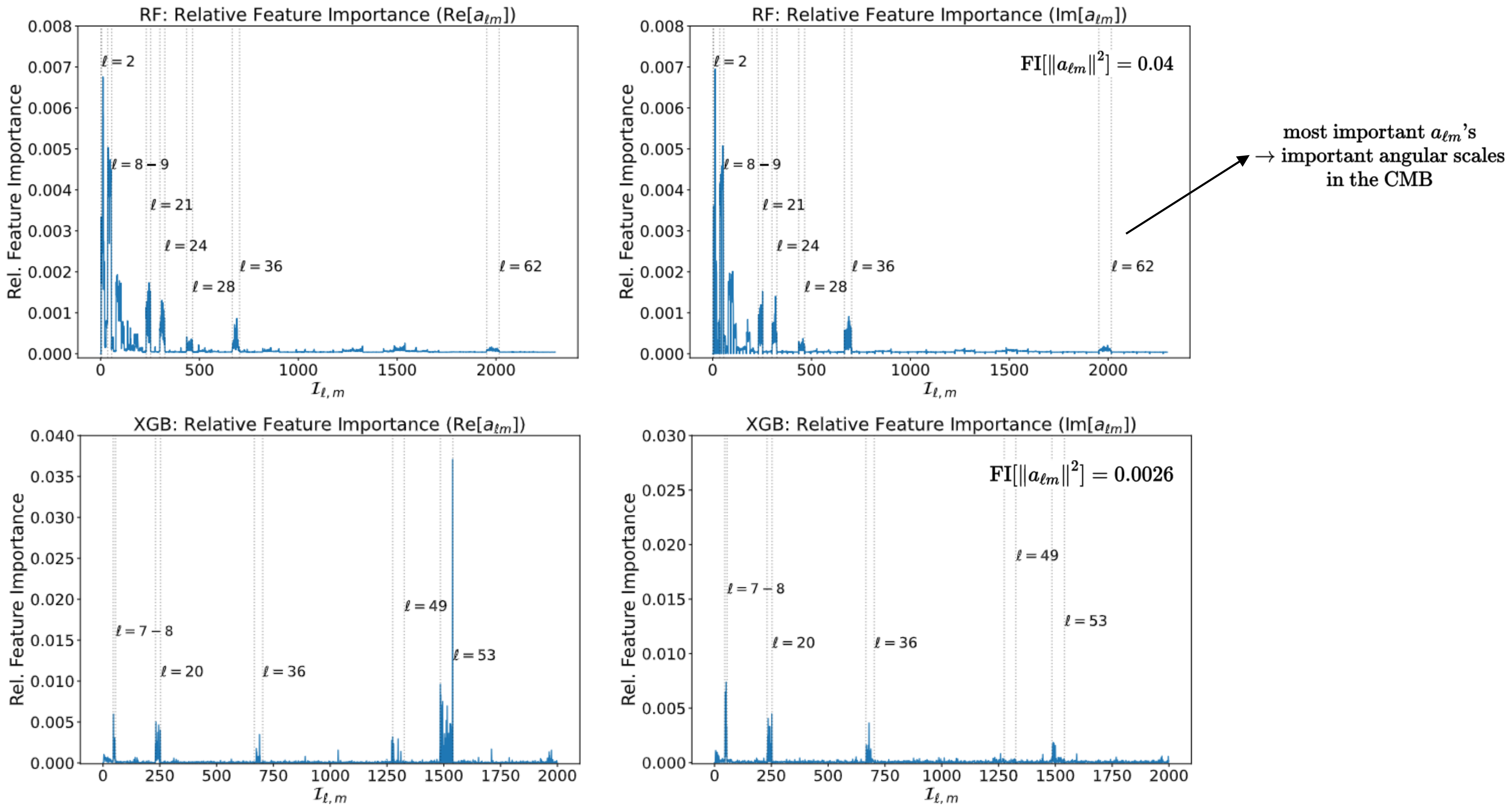
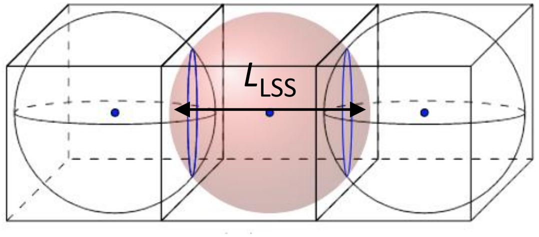


Figure 3: Feature importance analysis for the random forest and XGBoost classifiers.

The Results: $L \approx L_{\text{LSS}}$



- Next challenge: classify realizations with $L > L_{\text{LSS}}$.
- We expect this to be more challenging (**smaller KL divergence, no circles**).
- Our techniques work well on **non-rotated** data.
- Key challenge: classifying **randomly rotated** harmonic space realizations and CMB maps.

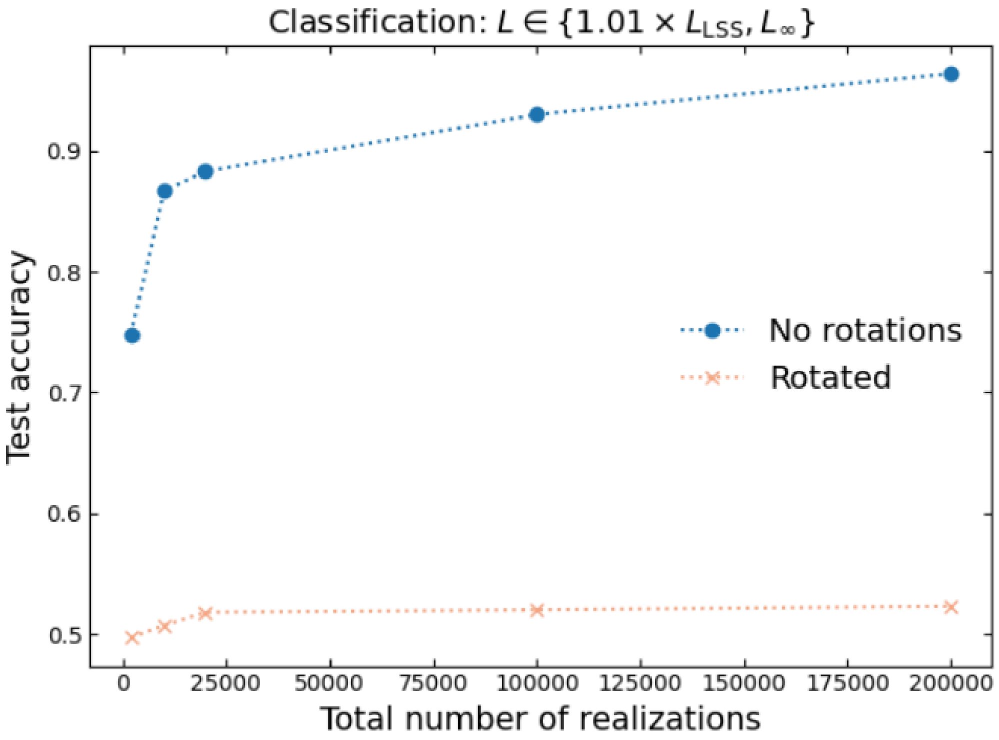
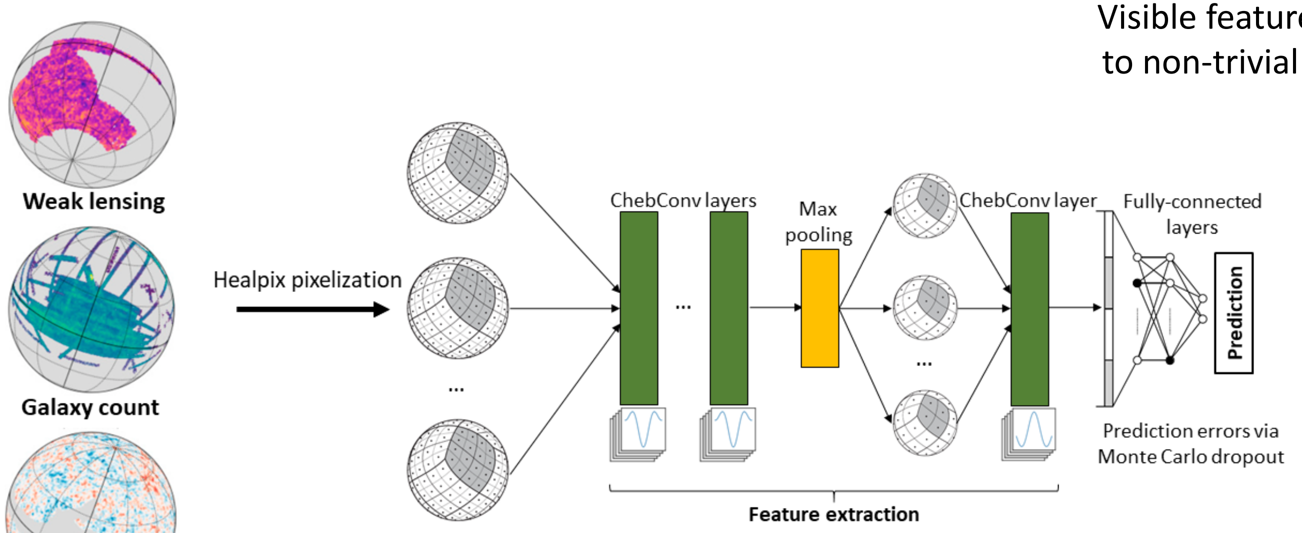


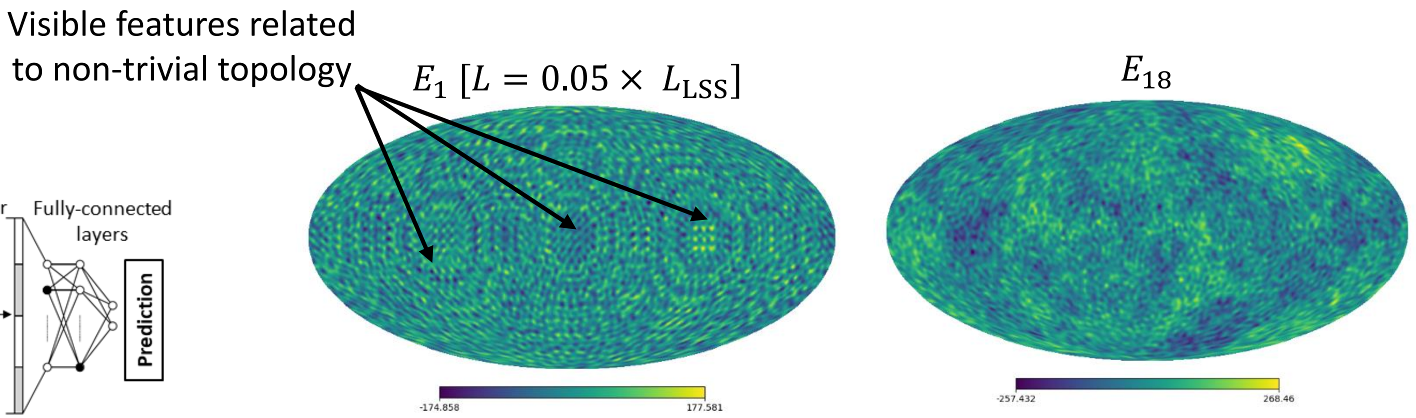
Figure 4: Classification results for realizations with $L \approx L_{\text{LSS}}$.

Part 3: CMB Map Classification with Spherical GCNNs

- Training a CMB map classifier
- DeepSphere results
- Future avenues of research

E_1 vs E_{18} in Pixel Space





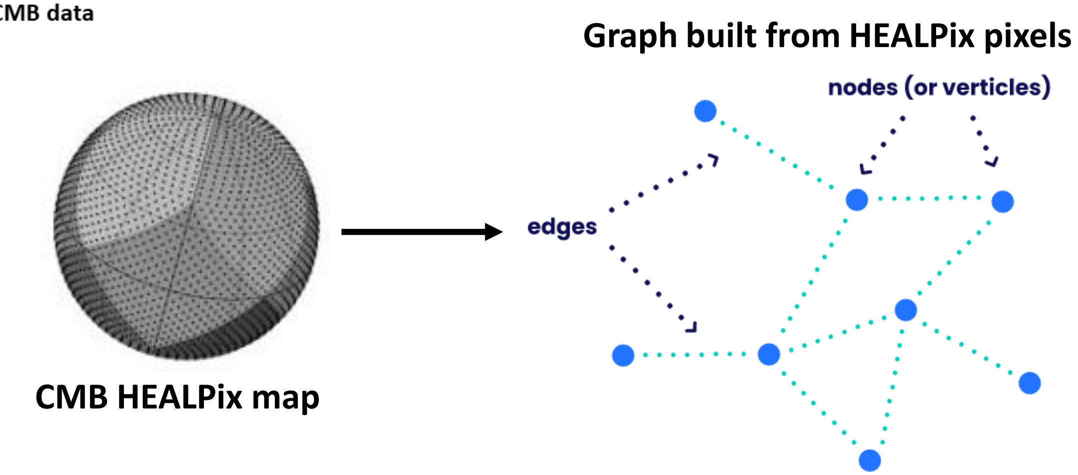


Figure 5. **Top:** DeepSphere: a graph-based neural network that allows applying convolutions on spherical data (Defferrard et al. 2020). **Top right:** E_1 topology features in a CMB map.

Our approach:

- Use temperature and E-mode **polarisation** maps as input.
- **Derivative** maps (w.r.t. angular coordinates) as extra input channels.
- Use **statistical layers** (that apply mean/max/histogram operations).
- Explore different resolutions, Chebyshev polynomial degrees, pooling operations etc.

DeepSphere Classification Results

- Large topology classification results: 98-99% (**non-rotated**), 63-64% (**rotated**).
- As before, E-mode data is crucial.
- Listed classification accuracy requires a training dataset with **100-200k maps**.
- Results depend on the map **resolution** and **pixel-ordering**.
- Training procedure is generally difficult – fine-tuning of model parameters are needed.
- A key question: how to make the architecture of DeepSphere **rotationally invariant**?

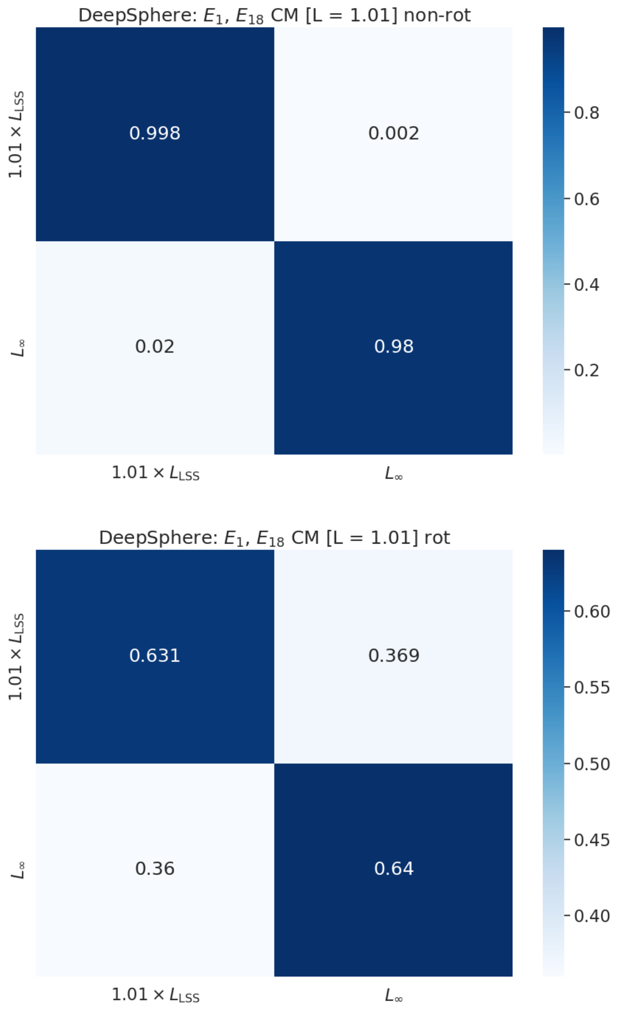


Figure 6: DeepSphere classification results for E_1 $[L = 1.01 \times L_{\rm LSS}]$ vs. E_{18} non-rotated (**top**) and rotated data (**bottom**).

- **Current efforts**: getting to the bottom of the rotation issue.
- A high-dimensional problem – results depend on:
  - Architecture of the spherical graph CNN;
  - Map resolution and pixel ordering;
  - Hyperparameters;
  - Statistical layers.
- A unique approach: employ the **AI Cosmologist** (arXiv:2504.03424).

The AI Cosmologist I: An Agentic System for Automated Data Analysis

Adam Moss [1*]

[1*]School of Physics and Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, UK.

Corresponding author(s). E-mail(s): adam.moss@nottingham.ac.uk;

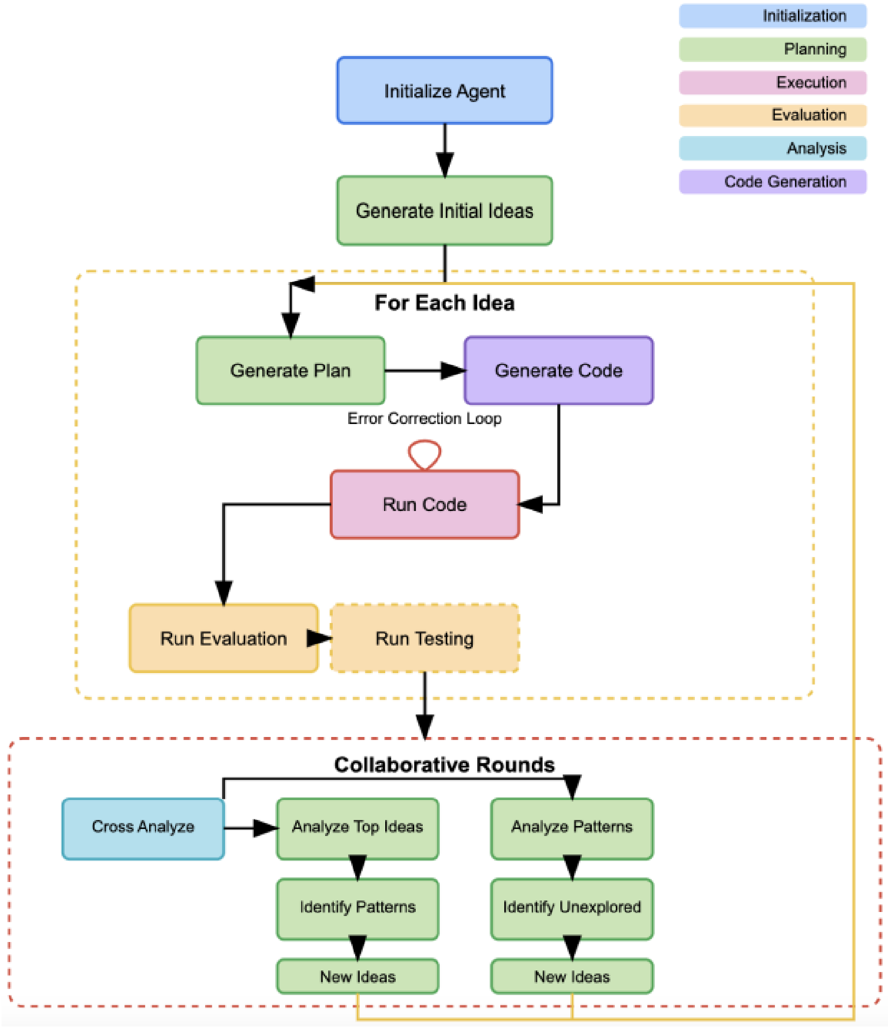
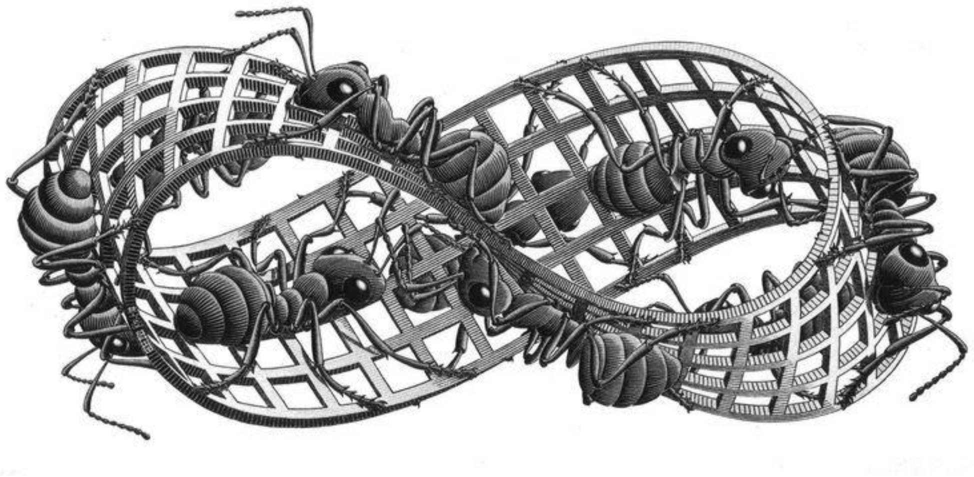


Figure 7: Pipeline of the AI Cosmologist, that employs large language model agents interacting in order to generate and test new scientific ideas and approaches.

Thank you for listening!

Summary:

- AI offers a set of valuable tools to detect signatures of non-trivial topology.
- ML can correctly classify **small** and **medium**-sized maps and $a_{\ell m}$'s.
- Classifying large **randomly rotated** $a_{\ell m}$'s and maps – the principle challenge.
- A promising approach: DeepSphere.
- Further avenues to explore: AI Cosmologist.



Papers:

- PRL: Promise of Future Searches for Cosmic Topology (arXiv:2210.11426)
- Limits on orientable Euclidean manifolds from circle searches (arXiv:2211.02603)
- Classification of manifolds using machine learning: a case study with small toroidal universes (arXiv:2404.01236)
- Microwave background parity violation without parity-violating microphysics (arXiv:2407.09400)

Papers

Website