The Shape of Space in Polarized Light

Mikel Martin Barandiaran
PHD at IFT, Madrid

PASCOS 2025, Durham
July 24th 2025

Instituto de Física Teórica UAM-CSIC

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CSIC Consejo Superior de Investigaciones Científicas

UAM Universidad Autónoma de Madrid

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Cosmic Topology and its implications for CMB Polarization

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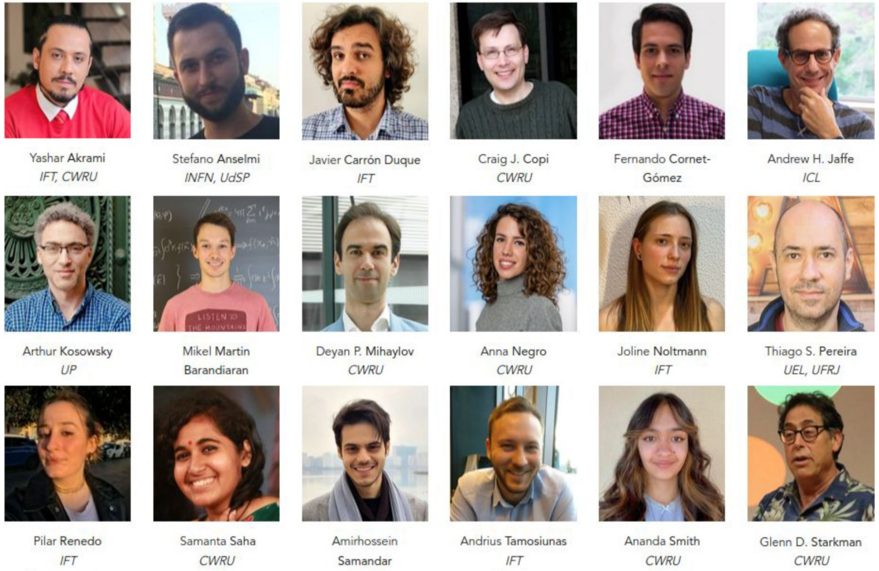

... and growing

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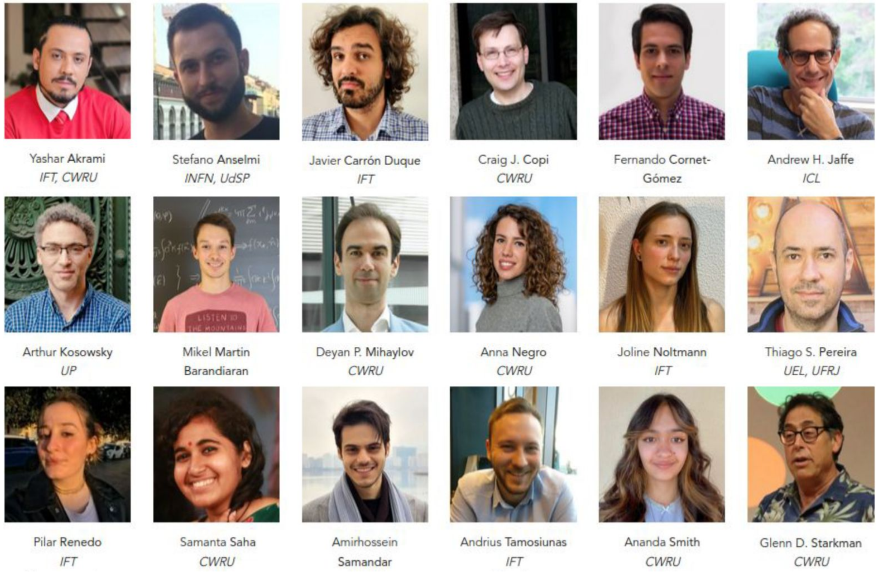

... and growing

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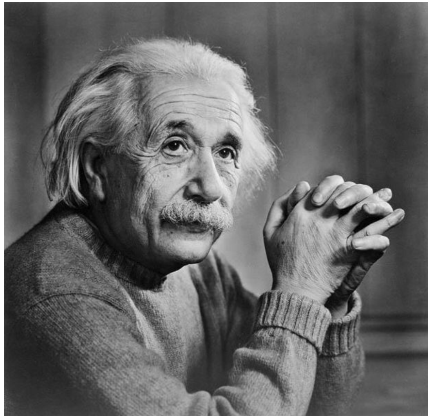

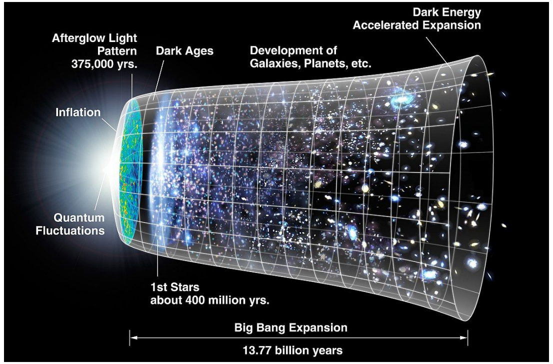

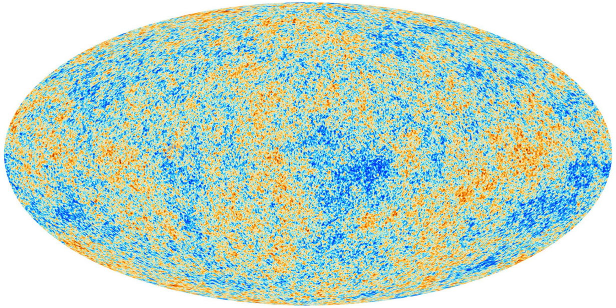

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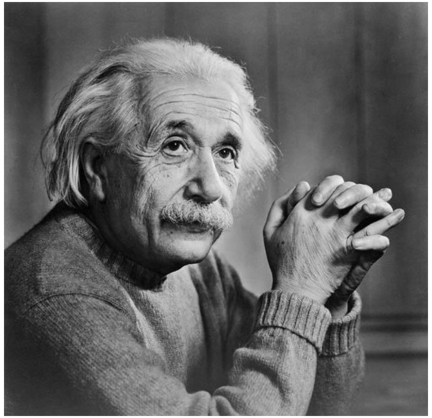
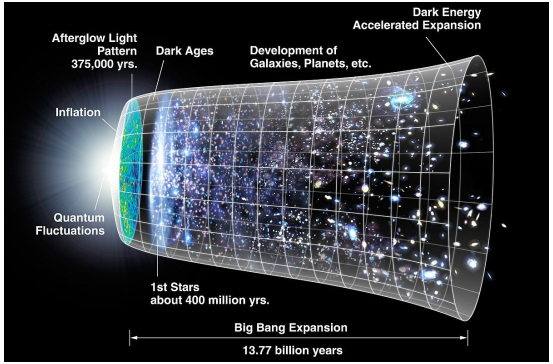
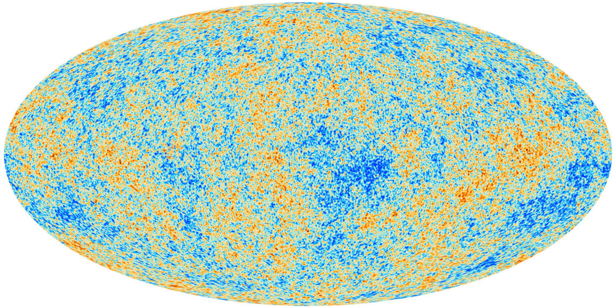
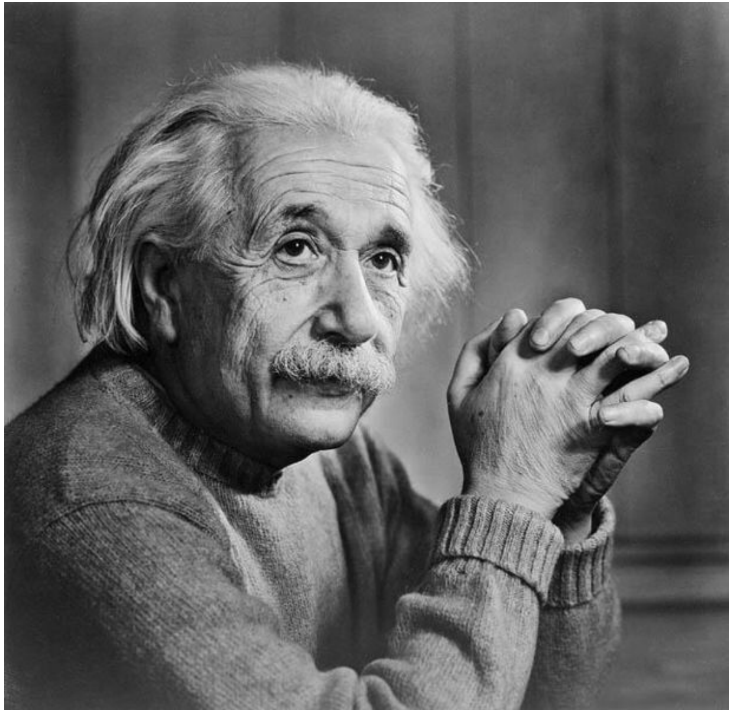
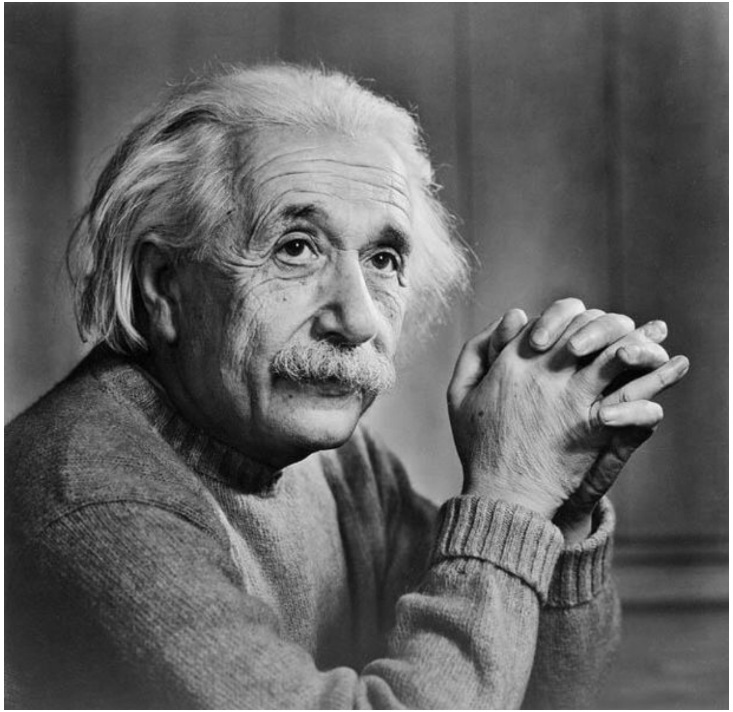








$$S = \frac{1}{2\kappa}\int_{\mathcal{M}} d^4x\,\sqrt{-g}\,R$$



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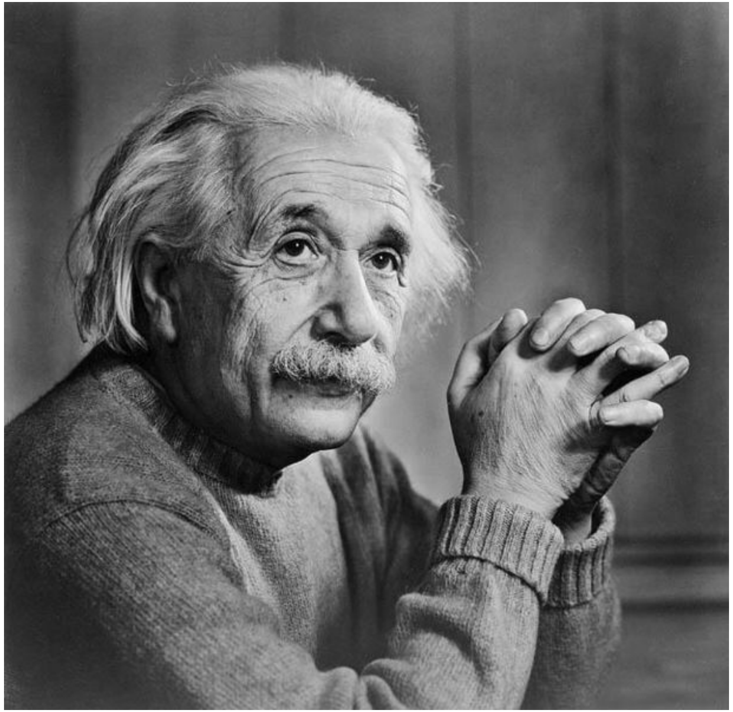
???



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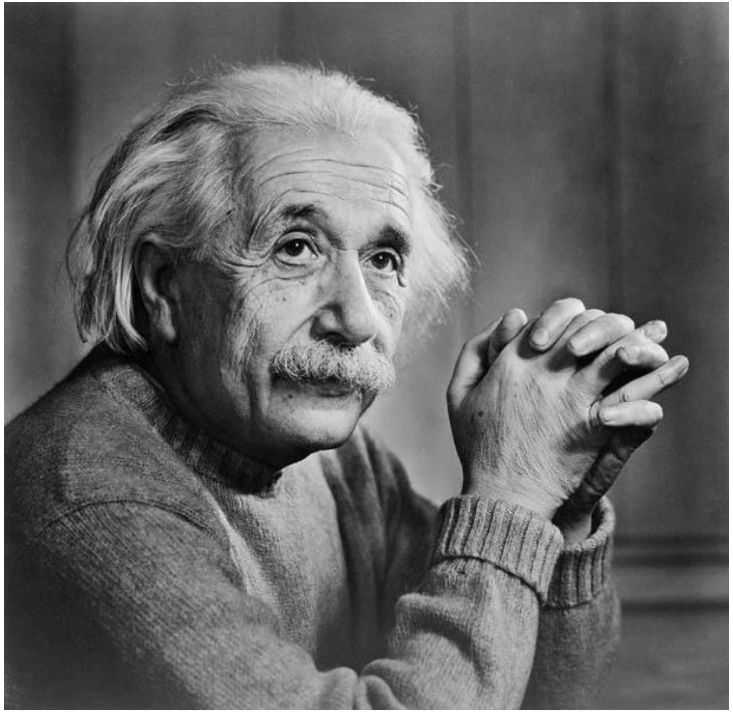
???

Causality enforces

$$\mathcal{M}_4 \cong \underbrace{\mathcal{M}_3}_{Space} \times \underbrace{\mathbb{R}}_{Time}$$



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What can we say about this?

Cosmology Recipe Book: How to Build your Universe

Cosmological Principle

Maximally Symmetric

FLRW metric

$$ds^2 = -\,dt^2 + a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right)$$

$k = 0$ $\quad$ $k > 0$ $\quad$ $k < 0$

$$\mathcal{M}_3 = \begin{cases} \mathbb{R}^3\,(k=0) \\ \mathbb{S}^3\,(k>0) \\ \mathbb{H}^3\,(k<0) \end{cases}$$

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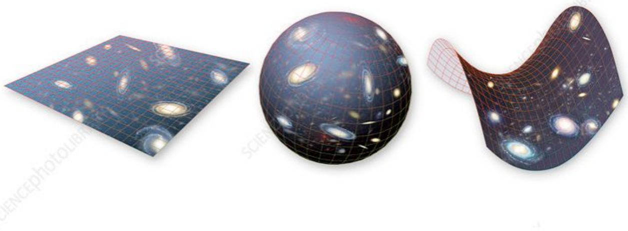

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Euclidean Topologies: A Recap

➔ Manifolds that admit the flat FLRW metric. (E1-E18)

$X \cong \widetilde{X}/G$

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➔ Retain local geometry, but are not maximally symmetric

  ➔ *Homogeneity and Isotropy are generally <u>broken</u>* !

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Euclidean Topologies: A Recap

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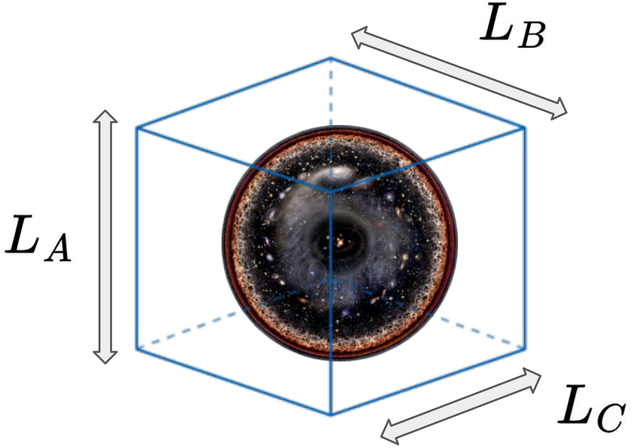
➔ Retain local geometry, but are not maximally symmetric

  ➔ *Homogeneity and Isotropy are generally <u>broken</u>* !

➔ They are compact along at least 1 direction

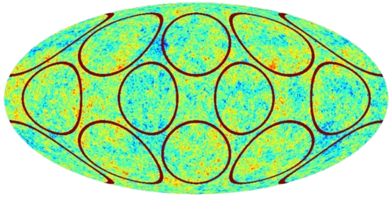
  ◆ Topology lengthscales L_i

$X \cong \widetilde{X}/G$



Cosmological Imprints of Non-Trivial Topologies

➔ If manifolds are "too small", obvious signals in the CMB



"Circles in the Sky"
(See Deyan's talk)

[Cornish, Spergel, Starkman 98, 04]

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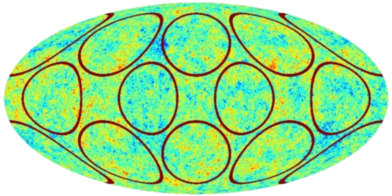

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➔ When the topology scale is larger than the diameter of the Last Scattering Surface, the imprint of the topology is more subtle

Correlation Structure of Perturbations

Expand Perturbations into eigenmodes of Laplacian

Cosmology Recipe Book

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Cosmology Recipe Book

1 $$\triangle\Psi(\mathbf{x}) = -k^2\,\Psi(\mathbf{x}) \qquad\qquad \Psi_{\mathbf{k}}(\mathbf{x}) = \exp(i\mathbf{k}\mathbf{x})$$

Expand Perturbations into eigenmodes of Laplacian

Cosmology Recipe Book

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$$\triangle\Psi(\mathbf{x}) = -k^2\,\Psi(\mathbf{x})$$

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$$\Psi_{\mathbf{k}}(g\,\mathbf{x}) = \Psi_{\mathbf{k}}(\mathbf{x}) \;\; \forall g \in G$$

$$\exp(i\mathbf{k}(\mathbf{x}+\mathbf{T})) = \exp(i\mathbf{k}\mathbf{x})$$

$$\mathbf{kT} = 2\pi\mathbf{n} \implies \mathbf{k}_{\mathbf{n}} = \left(\frac{2\pi n_x}{L_x}, \frac{2\pi n_y}{L_y}, \frac{2\pi n_z}{L_z}\right)$$

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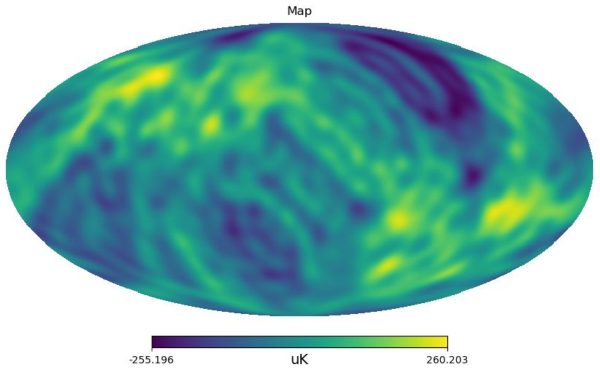
3 $$\phi(\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\phi(\mathbf{k})\exp(i\mathbf{k}\mathbf{x}) \longrightarrow \phi(\mathbf{x}) = \frac{1}{V}\sum_{k_n}\phi(\mathbf{k}_{\mathbf{n}})\exp(i\mathbf{k}_{\mathbf{n}}\mathbf{x})$$

Expand Perturbations into eigenmodes of Laplacian

Cosmology Recipe Book

$$\phi(\mathbf{k_n}) \longrightarrow a_{\ell m} \longrightarrow C_{\ell m \ell' m'} = \langle a_{\ell m} a^*_{\ell' m'} \rangle$$



Quick Review: Harmonic Coefficients and Symmetries

Harmonic expansion of a random field

$$\phi^X(\hat{\boldsymbol{\Omega}}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \phi^X_{\ell m} Y_{\ell m}(\hat{\boldsymbol{\Omega}})\,.$$

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Statistical Isotropy implies

$$\langle \phi^X_{\ell m} \rangle \propto \delta_{\ell\, 0}\, \delta_{m\, 0} \,.$$

$$C^{XY}_{\ell m \ell' m'} \equiv \langle \phi^X_{\ell m} \phi^{Y*}_{\ell' m'} \rangle = C^{XY}_{\ell}\, \delta_{\ell \ell'} \delta_{m m'} \,.$$

Parity Conservation implies

$$\langle \phi^{X+}_{\ell m}\, \phi^{Y+*}_{\ell' m'} \rangle = \langle \phi^{X-}_{\ell m}\, \phi^{Y-*}_{\ell' m'} \rangle = 0, \quad \ell + \ell' \text{ odd.}$$

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$\langle \phi_{\ell}$ [illegible] $0\,.$

$C^{XY}_{\ell m \ell' m'} \equiv \langle$ [illegible] $C_\ell^{XY}\,\delta_{\ell\ell'}\delta_{mm'}\,.$

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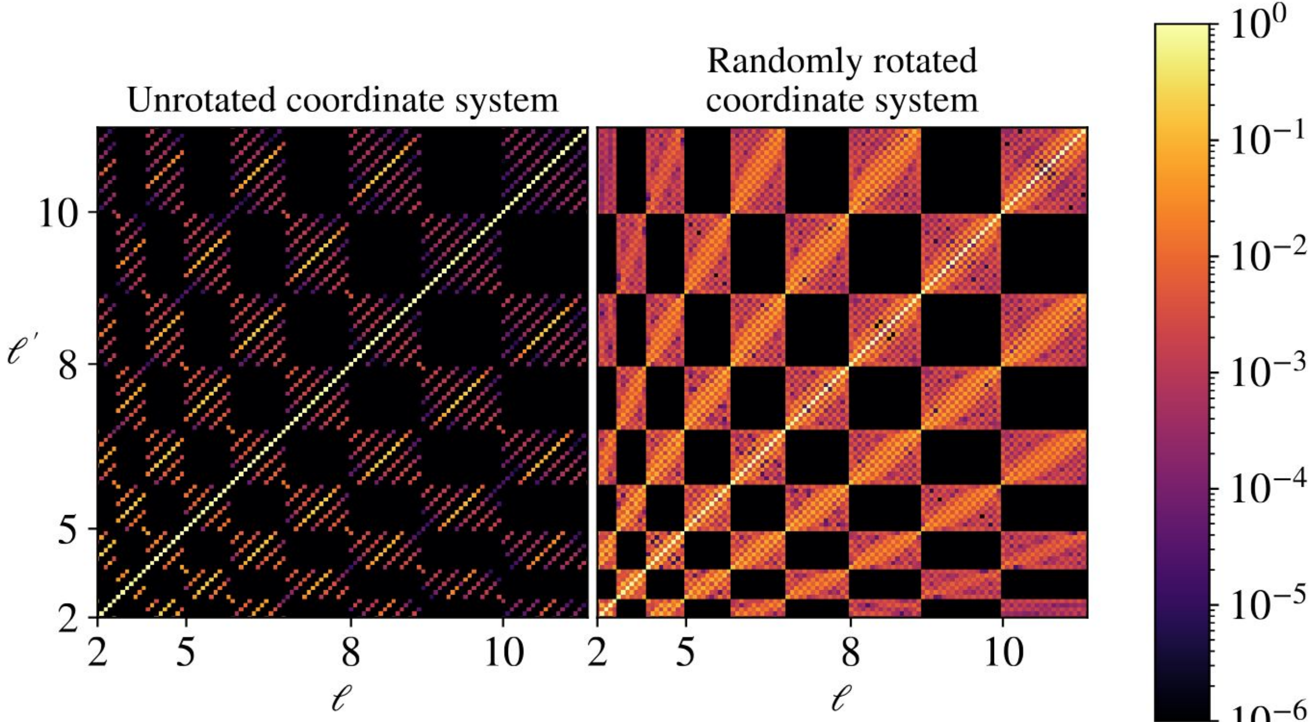
$C^{TT}_{\ell m \ell' m'}$ for E_2



How “different” are the correlations in the non-trivial topology compared to the fiducial isotropic case?

If we have data D and models M1, M2, we use the Bayes Factor to distinguish among them

$$\log \frac{p(D|M_1)}{p(D|M_2)}$$

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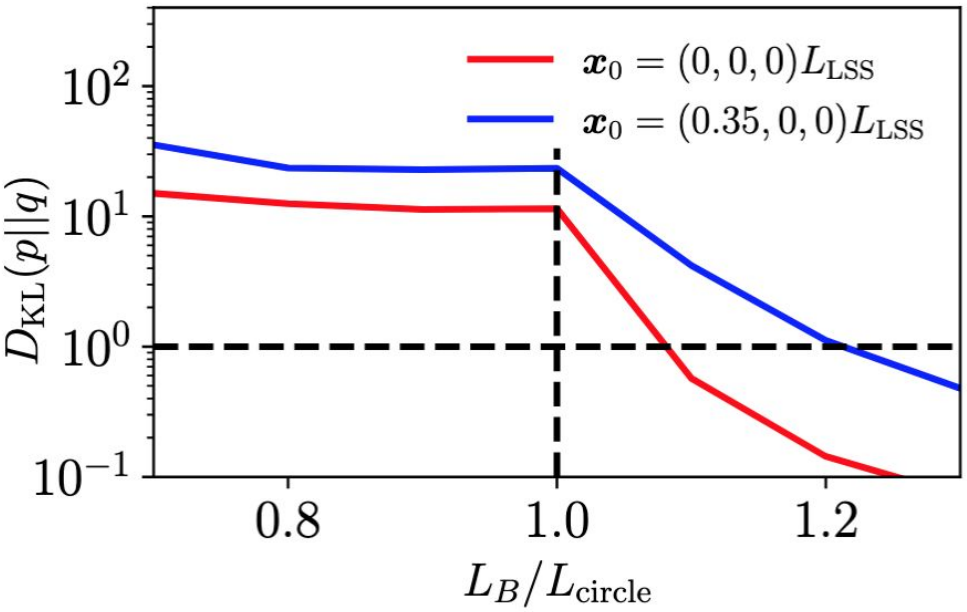
If we have data D and models M1, M2, we use the Bayes Factor to distinguish among them

$$\log \frac{p(D|M_1)}{p(D|M_2)}$$

We can compute the **expected Bayes Factor**, to quantify how different the observables produced by both models are.

$$\mathbb{E}_{D\sim M_1}\left[\log \frac{p(D|M_1)}{p(D|M_2)}\right] \equiv \mathrm{KL}(M_1||M_2) \qquad \mathrm{KL} = \begin{cases} \ll 1 \text{ undistinguishable} \\ \sim 1 \text{ threshold convention} \\ \gg 1 \text{ clearly distinguishable} \end{cases}$$

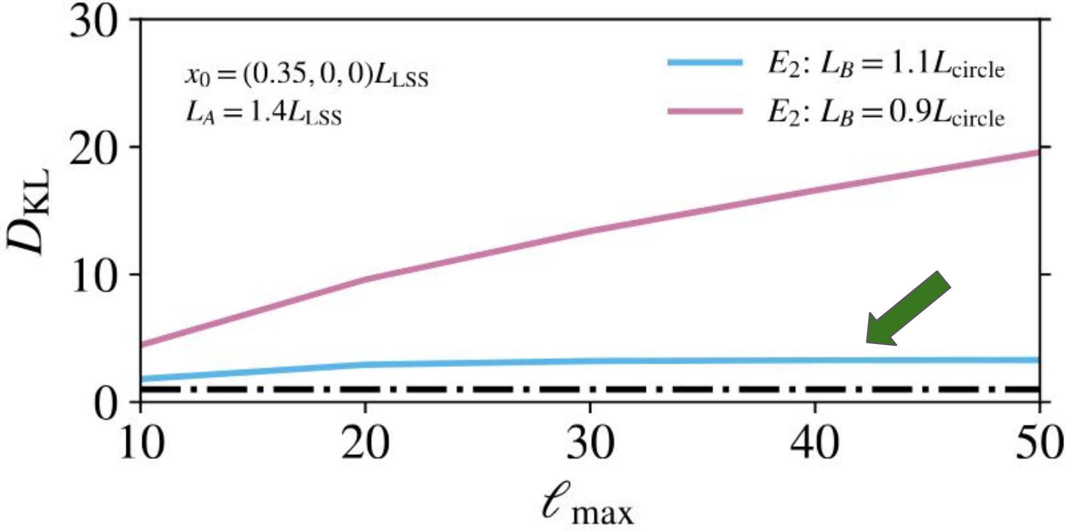
KL for E_2



As expected, when the topology scale becomes larger that the observable Universe, the KL drops significantly

Also, the location matters! In some locations detecting that you are in a non-trivial topology is “easier” than in others!

KL for E_2



Universe is larger than the interior of the Last Scattering Surface

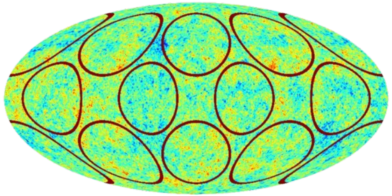
$$\Downarrow$$

It seems that all information in TT is contained in the largest scales!

$$\ell_{max} \sim 30$$

Cosmological Imprints of Non-Trivial Topologies

➔ If manifolds are "too small", obvious signals in the CMB



"Circles in the Sky"
(See Deyan's talk)

[Cornish, Spergel, Starkman 98, 04]

➔ When the topology scale is larger than the diameter of the Last Scattering Surface, the imprint of the topology is more subtle

⇨ **Correlation Structure of Perturbations** ⇦

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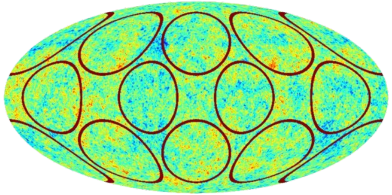

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⇨ **Correlation Structure of Perturbations** ⇦

↳ So far, only scalar sourced T,E had been considered

Cosmological Imprints of Non-Trivial Topologies

➔ If manif

What about tensor perturbations in these topologies?

➔ When ... st Scattering Surfac

So far, only scalar sourced T,E had been considered

Implications for CMB Polarization

From CMB maps we compute temperature (T) and polarization (E , B) coefficients.

$$a^T_{\ell m}\,,\; a^E_{\ell m}\,,\; a^B_{\ell m} \implies C_\ell^{TT}, C_\ell^{BB}, C_\ell^{TE} \ldots$$

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$$a^T_{\ell m}\,,\ a^E_{\ell m}\,,\ a^B_{\ell m} \implies C_\ell^{TT}, C_\ell^{BB}, C_\ell^{TE}\dots$$

Statistical Isotropy + Parity Conservation implies

$$\boxed{C^{EB}_{\ell m \ell' m'} \equiv 0 \qquad C^{TB}_{\ell m \ell' m'} \equiv 0}$$

Implications for CMB Polarization

Non-trivial topologies break isotropy $\Longrightarrow$ EB and TB correlations are expected!

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$$\begin{aligned} C^{E_i;XY}_{\ell m \ell' m'} &\equiv \langle a^{E_i,X}_{\ell m} a^{E_i,Y*}_{\ell' m'} \rangle \\ &= \frac{\pi^2}{2V_{E_i}} \sum_{\lambda=\pm 2} \sum_{\boldsymbol{n}\in\mathcal{N}^{E_i}} \frac{\mathcal{P}^{\mathrm{t}}(k_n)}{k_{\boldsymbol{n}}^3} \Delta^X_\ell(k_{\boldsymbol{n}}) \Delta^{Y*}_{\ell'}(k_{\boldsymbol{n}}) \, \xi^{E_i;X,\hat{\boldsymbol{k}}_{\boldsymbol{n}}}_{k_{\boldsymbol{n}},\ell m,\lambda} \, \xi^{E_i;Y,\hat{\boldsymbol{k}}_{\boldsymbol{n}}*}_{k_{\boldsymbol{n}},\ell' m',\lambda}. \end{aligned}$$

Implications for CMB Polarization

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$$C^{E_i;XY}_{\ell m \ell' m'} \equiv \langle a^{E_i,X}_{\ell m} a^{E_i,Y*}_{\ell' m'} \rangle$$

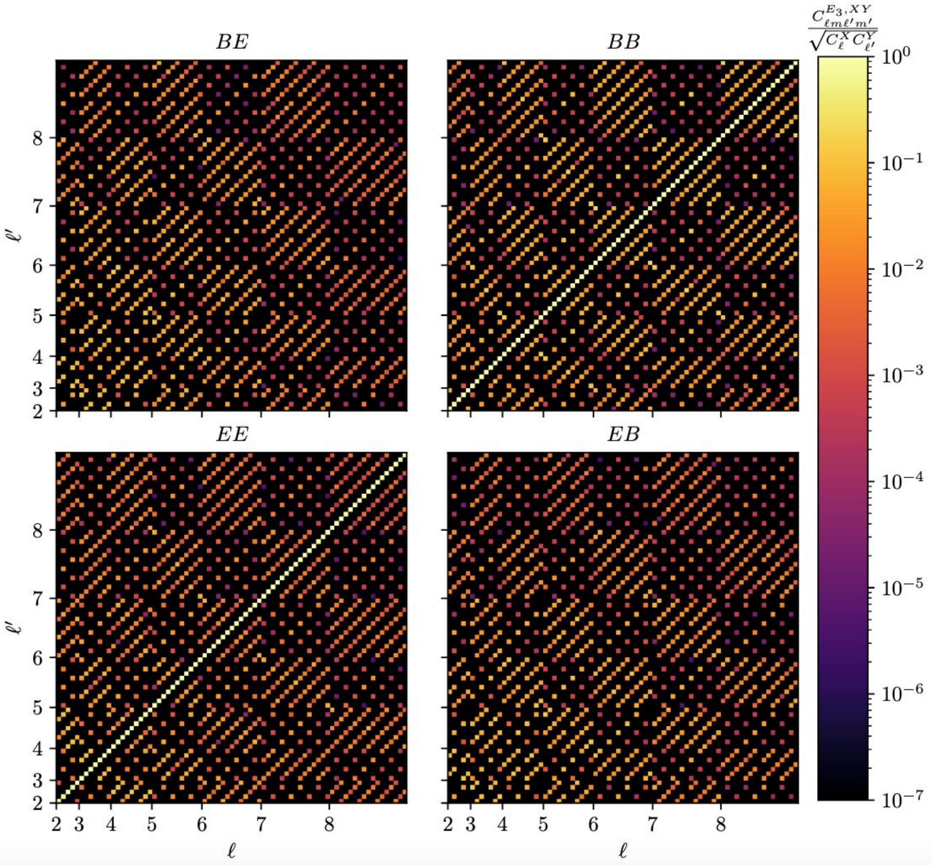
$$= \frac{\pi^2}{2V_{E_i}} \sum_{\lambda=\pm 2} \sum_{\boldsymbol{n}\in\mathcal{N}^{E_i}} \overbrace{\frac{\mathcal{P}^{\mathrm{t}}(k_{\boldsymbol{n}})}{k^3_{\boldsymbol{n}}} \Delta^X_\ell(k_{\boldsymbol{n}}) \Delta^{Y*}_{\ell'}(k_{\boldsymbol{n}})}^{\text{PS and TF}} \overbrace{\xi^{E_i;X,\hat{\boldsymbol{k}}_{\boldsymbol{n}}}_{k_{\boldsymbol{n}},\ell m,\lambda} \, \xi^{E_i;Y,\hat{\boldsymbol{k}}_{\boldsymbol{n}}*}_{k_{\boldsymbol{n}},\ell' m',\lambda}}^{\text{Topology Coefficients}}.$$

Volume

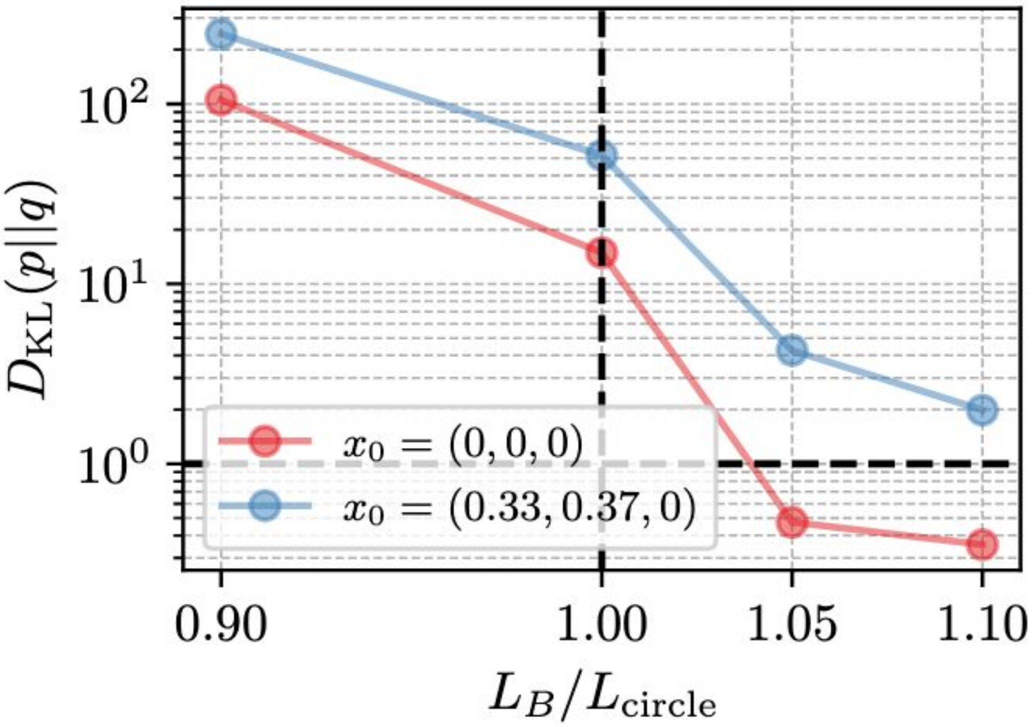
Helicities

Analogue of $\int d^3k$ for compact topologies

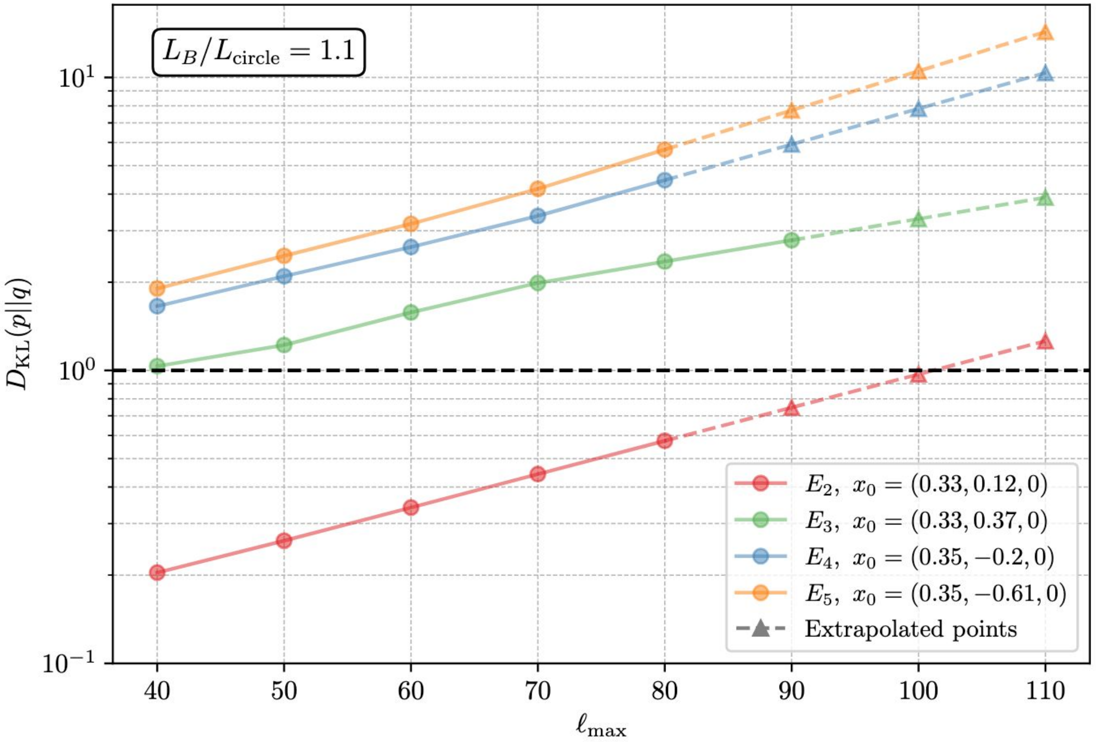
Cubic E_3, $L = 0.8L_{\rm LSS}$



E3



(c) KL divergence vs L_B



Unlike in the temperature-only case, including higher multipoles keeps introducing new information!

However, it is computationally heavy to deal with matrices with such high ell

Future (& current) directions

- ➔ Introduce noise, mask and combine with scalar perturbations. How much “distinguishability” is lost when doing so?

- ➔ How would one disentangle the non-zero EB signal coming from birrefringence?

- ➔ Do the extra non-diagonal correlations mean we could potentially detect a lower value of r ?

TAKE-HOME MESSAGE

➔ The topology of the Universe is not yet known. Interesting question on its own philosophically

➔ There are several observational effects we can look for.

➔ If the topology scale is large, this is a very hard problem

(See Andrius' talk)

➔ CMB Polarization correlations might provide a new window to uncover the shape of the Universe!

Based on:

1 **Cosmic topology. Part IIa. Eigenmodes, correlation matrices, and detectability of orientable Euclidean manifolds**

2306.17112

2 **Cosmic topology. Part IIIa. Microwave background parity violation without parity-violating microphysics**

2407.09400

3 **Cosmic topology. Part IIIb. Eigenmodes and correlation matrices of spin-2 perturbations in orientable Euclidean manifolds**

2503.08671

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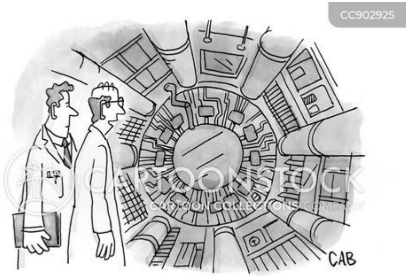
Choose One:

THANK YOU!

INSPIRATIONAL QUOTE BY FAMOUS PHYSICIST

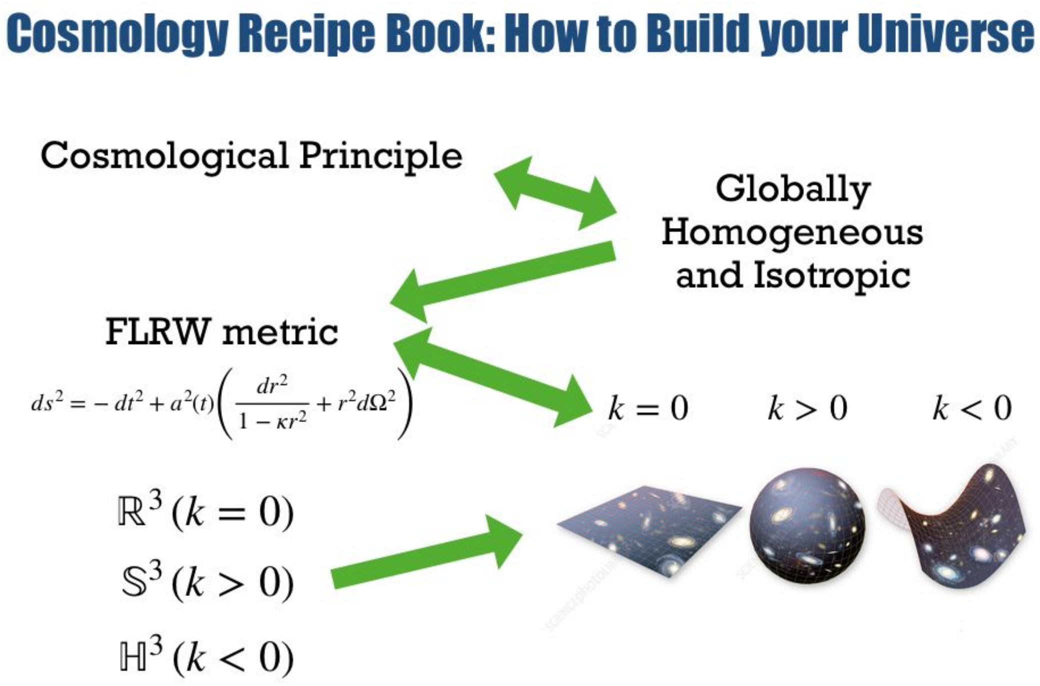
FUNNY CARTOON



"Once you have a collider, every problem starts to look like a particle."

THANK YOU!

Back-Up Slides



Back-Up Slides

$$D_{\mathrm{KL}}(p||q) = \int \mathrm{d}\{a_{\ell m}\}\; p(\{a_{\ell m}\}) \ln\left[\frac{p(\{a_{\ell m}\})}{q(\{a_{\ell m}\})}\right] \;. \tag{4.1}$$

For the CMB the $a_{\ell m}^{X}$ coefficients follow zero-mean Gaussian distributions allowing the KL divergence to be simplified to

$$D_{\mathrm{KL}}(p||q) = \frac{1}{2}\sum_{j}\left(\ln|\lambda_j| + \lambda_j^{-1} - 1\right) \;, \tag{4.2}$$

where the $\{\lambda_i\}$ are the eigenvalues of the matrix

$$C^{XY,\,p}_{\ell m \ell' m'}\,(C^{XY,\,q}_{\ell m \ell' m'})^{-1} \;.$$