Modified Microcausality from Perturbation Theory

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Giordano Cintia

based on arXiv:2504.16992 with F. Piazza and S. Ramos

Microcausality is the statement that quantum fields commute if they are space-like separated

1 $$[\phi(x), \phi(0)] = 0 \quad \text{if} \quad x^2 = t^2 - \vec{x}^2 < 0$$ *(Minkowski for now...)*

This is an operatorial statement: depends on the theory, and not on the state

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Microcausality implies that the *retarded Green function vanishes outside the light-cone*

2. $$G_R = \theta(t)\langle[\phi(x), \phi(0)]\rangle$$

Vanishing commutator outside the light-cone → Causal evolution of the theory

Microcausality is a requirement for a healthy theory. Hard to prove for a general QFT…

1) Free massless scalar field → The commutator is supported only on the light-cone

$$[\phi(x), \phi(0)] = \frac{1}{2\pi i}\delta(\vec{x}^2 - t^2)$$

2) Massive scalar field → One can show it vanishes for space-like separations

3) Lorentz invariant states
(Expectation value) → Interacting commutators decomposed as massive ones using Källén-Lehmann

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4) Lorentz invariant theories… (Operatorial) → Boosts preserve commutators + Equal-time commutation relations

$$[\Phi(\vec{x}, t_x), \Phi(\vec{y}, t_y)] = U^\dagger(\Lambda)[\Phi(\vec{z}_1, t_z), \Phi(\vec{z}_2, t_z)]U(\Lambda) = 0$$

if $(x-y)^2 < 0$

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Not always true… problem with EFTs: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda}(\partial_\mu\phi\partial^\mu\phi)^2$

[Adams, Arkhani-Hamed, Dubovsky, Nicolis, Rattazzi; arXiv:0602178]

What happens to Microcausality without Lorentz?

The theory of scalar field on a classical curved background $g_{\mu\nu}$ (Not necessarily gravity…)

$$g^{\mu\nu}(x)\partial_\mu\partial_\nu\phi = F(x,\phi,\partial\phi)$$

1) At best linear in the field second derivative

2) Covariant form for the LHS in terms of an effective metric

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The causal dependence of solutions on initial conditions determined by the light-cones of the metric $g_{\mu\nu}$

1

$$[\phi(x),\phi(0)] = 0 \quad \text{if} \quad x^2 = g_{\mu\nu}x^\mu x^\nu < 0$$

Valid for any scalar field theory theory with an operatorial EOM of the above form

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For non-renormalizable theories, the metric is a function of the field itself

2

$$g^{\mu\nu}(x,\phi,\partial\phi) \neq g^{\mu\nu}_{\rm cl}(x) \equiv g^{\mu\nu}(x,0,0)$$

As in $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda}(\partial_\mu\phi\partial^\mu\phi)^2$!

The effective metric is different than the one of the background spacetime!

[S. Dubovski, A. Nicolis, E. Trincherini, G.Villadoro; arXiv:0709.1483]

In the **non-perturbative** approach, the light-cone is defined by $g_{\mu\nu}$

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi(x) = 0$$

$$\downarrow$$

$$[\phi(x), \phi(0)] = 0 \quad \text{if} \quad x^2 = g_{\mu\nu}x^\mu x^\nu < 0$$

In a **perturbative** approach, we can construct the commutator by expanding around Minkowski

$$\Box\phi = (\eta^{\mu\nu} - g^{\mu\nu})\partial_\mu\partial_\mu\phi \sim h^{\mu\nu}\partial_\mu\partial_\nu\phi$$

Green's function — source

$$\downarrow$$

But the Green function is the commutator of the free theory and has support only on the Minkowski light-cone…

$$G_R(x) = \theta(t)[\phi_0(x), \phi_0(0)] \sim \delta(x^2 - t^2)$$

Question and motivations

1) How can the sum of terms that have support only on a given light-cone generate a different light-cone?

2) It is important to identify a change in the causal structure already at the perturbative level.
Ultimate goal is *Dynamical Gravity*: What is the effect of quantum gravity on the light-cone and microcausality?

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

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In perturbation theory, we reconstruct the interacting light-cone as a Taylor expansion around the free one

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We use the interaction picture formalism to expand the interacting **OPERATORIAL** commutator around the free one

Simplest check: contact interactions

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!}\phi^4 \quad \longrightarrow \quad \underbrace{[\phi(x),\phi(0)]}_{\text{Interacting}} = \underbrace{[\phi_0(x),\phi_0(0)]}_{\text{Free}} + \frac{i\lambda}{2}\int_0^t d^4z\, [\phi_0(z),\phi_0(0)] \times [\phi_0(z),\phi_0(x)]\phi_0(z)^2$$

Field operator

$$[\phi(x),\phi(0)] = \frac{1}{2\pi i}\delta(\vec{x}^2 - t^2)$$

We know the commutator for a massless free scalar…

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

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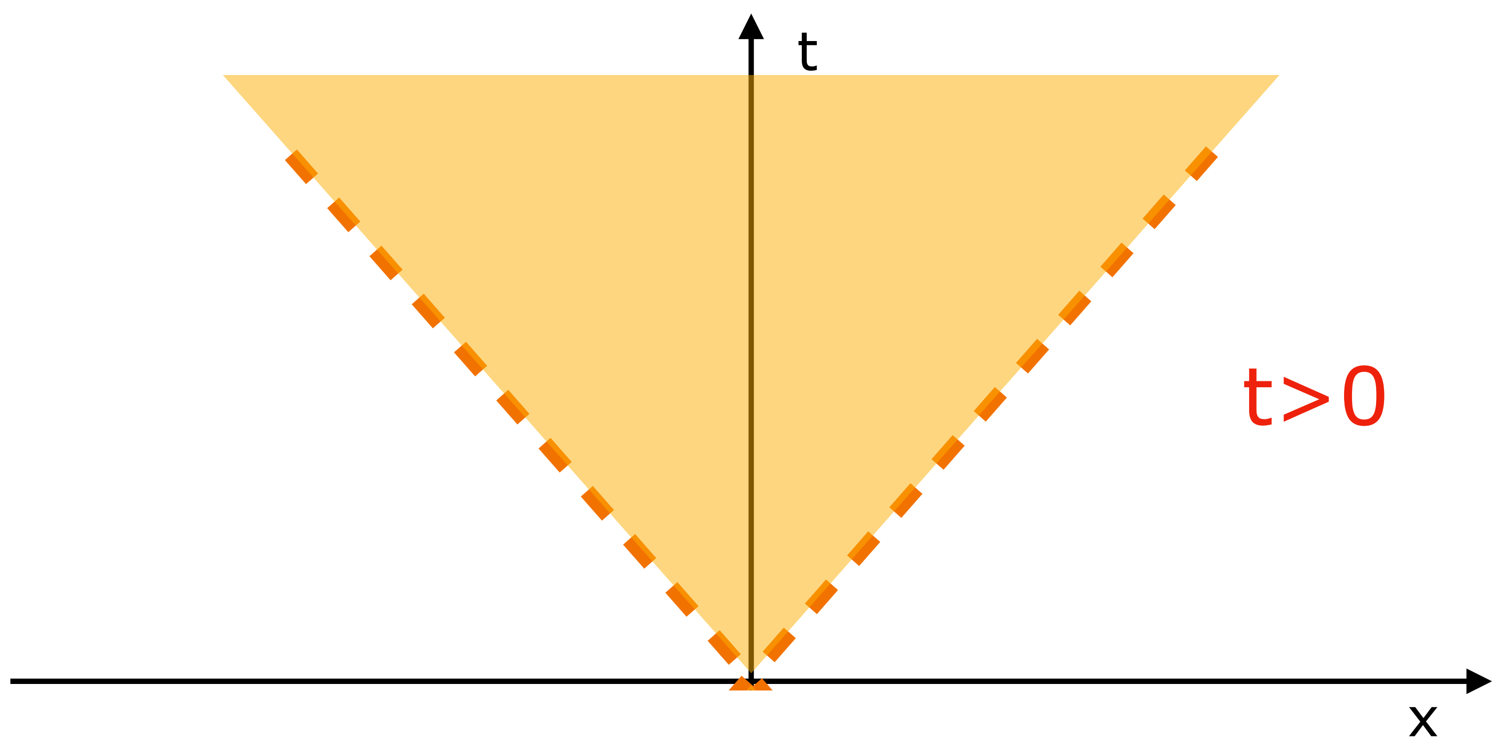
Field operator

Deltas functions have support only in the region $|\vec{x}| \leq t$

1) We have shown microcausality for the operatorial commutator!
   State independent! And holds at λ^2 also…

2) In contrast to the free theory, the commutator is supported also inside the light-cone! The value inside is **State dependent**

3) Position space analysis



[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

Change of the causal structure: derivative interactions

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda}(\partial_\mu\phi\partial^\mu\phi)^2$$

Formally, this theory is Lorentz invariant
Important point: derivative interactions!

↓

The free commutator is corrected by a convolution of **Field operators** and **derivative of Delta functions**!

$$[\phi(x), \phi(0)] = \frac{1}{2\pi i}\delta(x^2 - t^2) - \frac{i}{4\pi^2\Lambda}\int_0^t \mathcal{A}(\phi)\partial_0\delta(z^2 - x^2)\partial_0(z^2) + \mathcal{B}_{ij}(\phi)\partial_i\delta(z^2 - x^2)\partial_j\delta(z^2) + \ldots$$

Field operators

Derivative of Deltas

1) We evaluate expectation values of the commutator (so of $\mathcal{A}(\phi)$ and $\mathcal{B}_{ij}(\phi)$) and show that these derivative corrections persist only in Lorentz breaking states.

2) Then, we show that derivative interactions are associated to a change in the light-cone structure of the free theory

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

Change of the causal structure: derivative interactions

We consider expectation values of

$$[\phi(x), \phi(0)] = \frac{1}{2\pi i}\delta(x^2 - t^2) - \frac{i}{4\pi^2\Lambda}\int_0^t \mathcal{A}(\phi)\partial_0\delta(z^2 - x^2)\partial_0(z^2) + \mathcal{B}_{ij}(\phi)\partial_i\delta(z^2 - x^2)\partial_j\delta(z^2) + \dots$$

Lorentz invariant vacuum

+ $|0\rangle$ ⟶

All derivatives of delta-functions cancel out!

$$\langle 0|[\phi(x), \phi(0)]|0\rangle = \frac{1}{2\pi i}\delta(\vec{x}^2 - t^2)$$

Lorentz breaking coherent state

+ $|\mu\rangle$ ⟶

Key property: $\langle\mu|\hat{\phi}_0|\mu\rangle \sim \mu t$

Derivatives of delta-functions don't cancel!

$$\langle\mu|[\phi(x), \phi(0)]|\mu\rangle = \frac{1}{2\pi i}\left\{1 + \frac{\mu^2}{\Lambda}\vec{x}\,\partial_{\vec{x}}\right\}\delta(\vec{x}^2 - t^2)$$

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

Change of the causal structure: derivative interactions

We see it by the semiclassical analysis of the fluctuations

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda}(\partial_\mu\phi\partial^\mu\phi)^2 \quad + \quad \phi(z) = \mu t + \varphi$$

Quantize the fluctuation and find the commutator…

$$[\varphi(x), \varphi(0)] = \left(1 + 3\frac{\mu^2}{\Lambda}\right)^{-1/2}\left(1 + \frac{\mu^2}{\Lambda}\right)^{-1/2}\frac{1}{2i\pi}\delta\left(\vec{x}^2 - c_s^2 t^2\right)$$

with

$$c_s^2 = \frac{1 + \frac{\mu^2}{\Lambda}}{1 + 3\frac{\mu^2}{\Lambda}} \sim 1 - 2\frac{\mu^2}{\Lambda}$$

If $\Lambda<0$, $c_s>1$. True for any μ, so within the validity of the EFT!

Small μ^2/Λ expansion…

$$[\varphi(x), \varphi(0)] = \frac{1}{2\pi i}\left\{1 + \frac{\mu^2}{\Lambda}\vec{x}\partial_{\vec{x}}\right\}\delta(\vec{x}^2 - t^2) + \ldots$$

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

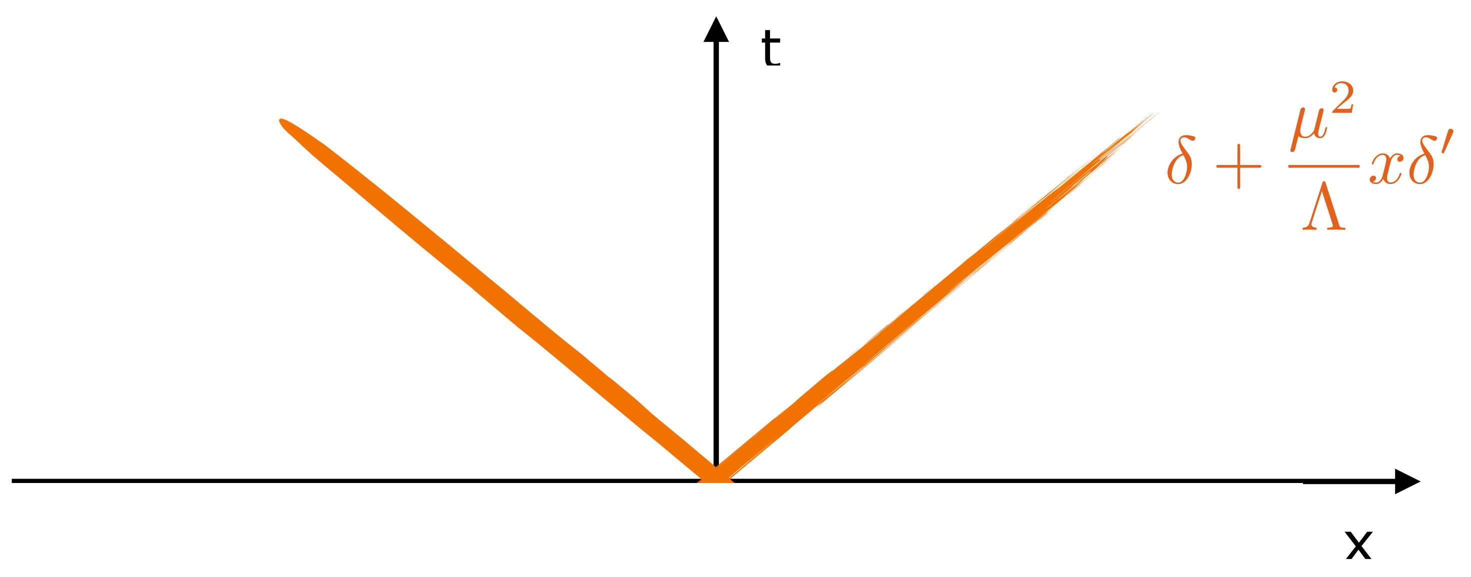
Change of the causal structure: derivative interactions

In interaction picture, we reconstruct the light-cone of the interacting theory as a Taylor expansion around the free-light-cone.

Using interaction picture: $|\mu\rangle$: $\langle\mu|\phi(z)|\mu\rangle = \mu t + \dots$

First order in H-int

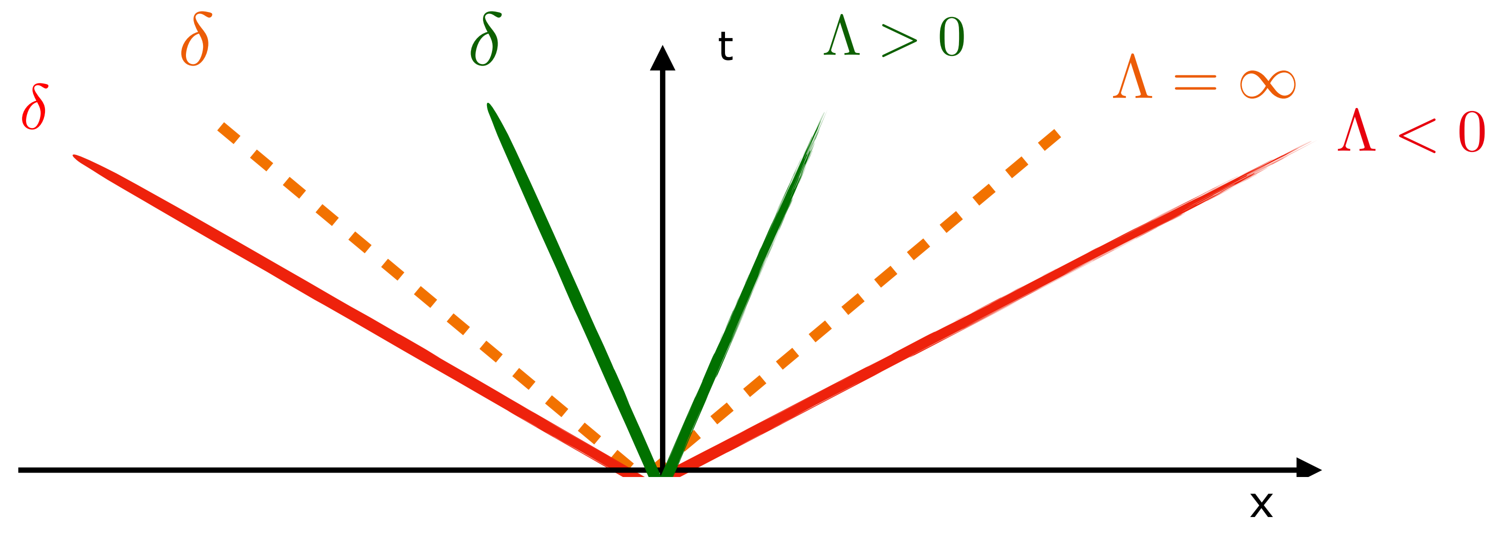
$$\langle\mu|[\phi(x),\phi(0)]|\mu\rangle = \frac{1}{2\pi i}\left\{1+\frac{\mu^2}{\Lambda}\vec{x}\,\partial_{\vec{x}}\right\}\delta(\vec{x}^2-t^2)$$



Semiclassical theory of fluctuations: $\phi(z) = \mu t + \varphi$

Expansion of of the Dirac Delta on the sound-cone

$$[\varphi(x),\varphi(0)] = \frac{1}{2\pi i}\left\{1+\frac{\mu^2}{\Lambda}\vec{x}\partial_{\vec{x}}\right\}\delta(\vec{x}^2-t^2)$$



[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

Why derivatives of delta represent a change in the light-cone?

Once you the derivative of Dirac delta, the change in the causal structure is automatic by applying **linear response theory** in the presence of an extended, but localized source…

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{\Lambda}(\partial_\mu\phi\partial^\mu\phi)^2 + J(x)\phi$$

The response of the field to an external perturbation is

$$\langle\mu|\phi(0)|\mu\rangle_J = \langle\mu|\phi(0)|\mu\rangle_{J=0} + i\int_{-\infty}^{0} d^4x\, J(x)\langle\mu|[\phi(x),\phi(0)]|\mu\rangle$$

Full-interacting commutator in the absence of the source

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

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Full-interacting commutator in the absence of the source

Choose a **gaussian source**: the signal induced by the source propagates on the effective light-cone

$$|x_0| = -t_0\left(1-\frac{\mu^2}{\Lambda}\right) \longrightarrow |x_0| \sim c_s(-t_0)$$

No need to know the full interacting light-cone to see that the causal structure is changed!

Perturbation theory is enough!

[G.C., F. Piazza, S. Ramos; arXiv:2504.16992]

Conclusions

1) The commutator encodes the causal structure of the theory.

2) We need perturbative techniques to check where it is supported ***(interaction picture)***, in particular for non-Lorentz invariant theories or EFTs. Expanding around Minkowski provide a promising technique to check this support

The distortion of the light-cone emerges perturbatively in the commutator as a series of derivatives of delta functions, representing a Taylor expansion of the interacting light-cone around the free one.

$$\langle\mu|[\phi(x),\phi(0)]|\mu\rangle = \frac{1}{2\pi i}\left\{1+\frac{\mu^2}{\Lambda}\vec{x}\,\partial_{\vec{x}}\right\}\delta(\vec{x}^2-t^2) \quad \longleftrightarrow \quad [\varphi(x),\varphi(0)] = \left(1+3\frac{\mu^2}{\Lambda}\right)^{-1/2}\left(1+\frac{\mu^2}{\Lambda}\right)^{-1/2}\frac{1}{2i\pi}\delta\left(\vec{x}^2-c_s^2t^2\right)$$

Interaction picture and coherent states — Semiclassical analysis of fluctuations

3) Derivative of delta functions can be connected to a change in the causal structure using linear response theory. It does not rely on any resummation