Chasing the Standard Model within Heterotic Line Bundle Constructions

Luca A. Nutricati

In collaboration with Andrei Constantin, Lucas T. Y. Leung, Andre Lukas

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Motivations

Standard Models from **String Theory**

Pursued in various string constructions (Heterotic, Type I/II, M-theory, F-theory) with

- Orbifolds
- Intersecting branes
- Orientifolds
- Line bundles
- …

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(**Heterotic**, Type I/II, M-theory, F-theory) with

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To realistically connect **string theory** to **particle physics**, we must:

1. Identify string models with **correct gauge group** and **particle spectrum**
2. **Compute physical Yukawa couplings** as functions of the compactification moduli
3. **Stabilise all the moduli**

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1.9 This talk

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1. Identifying models with **correct gauge group** and **particle spectrum**

In 2013, a comprehensive scan revealed several million heterotic line bundle models with

- ✓ SM gauge group
- ✓ Three chiral families
- ✓ No exotics

[Anderson, Constantin, Gray, Lukas, Palti, '13, JHEP]

Motivations

2. **Computing physical Yukawa couplings** as functions of the moduli

The ingredients for this recipe are

Holomorphic Yukawa couplings

↓

Do not depend on the Ricci-flat CY metric

↓

Can be done analytically ✓

Matter field Kähler metric

↓

Required full knowledge of the compactification geometry

↓

In general cannot be done analytically (except for very limited cases) ✗

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2. **Computing physical Yukawa couplings** as functions of the moduli

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For illustration, the **method has been applied on a specific model** and it **required half a day** on a single twelve-core CPU.

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For illustration, the **method has been applied on a specific model** and it **required half a day** on a single twelve-core CPU.

✗ **It is not ideal for scanning over a large database of models.**

Motivations

Rather than applying machine learning across millions of models, we may ask a **key question.**

Given a **list of models** with the **correct gauge group** and **chiral spectrum**,

- ? **Is there any room in the moduli space of these models to accommodate the SM flavour parameters?**
- ? **If so, what's the fraction of models that can realise this?**

Outline

- Model building: heterotic line bundle models
- Search strategy
- Results
- Conclusions and outlook

Heterotic line bundle models

$E_8 \times E_8$ Heterotic

Heterotic line bundle models

$$\begin{array}{c} E_8 \times E_8 \text{ Heterotic} \\ \Big\downarrow \text{on} \\ \dfrac{\text{CY threefold}}{\Gamma} \end{array}$$

Freely-acting
symmetry group

Heterotic line bundle models

$E_8 \times E_8$ Heterotic

$$\Big\downarrow \text{on}$$

$$\frac{\text{CY threefold}}{\Gamma} \longrightarrow$$

Freely-acting symmetry group

with vector bundle over it

$$V = \bigoplus_{a=1}^{5} \mathcal{L}_a$$

is a sum of line bundles

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$$\mathcal{G} = S\left(U(1)^5\right)$$

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GUT-type theory

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$$\mathcal{G} \times SU(3) \times SU(2) \times U(1)$$

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Use this **global symmetry** to **impose constraints** on **the possible operators** present in the theory.

This provides a stringy **Froggatt-Nielsen** mechanism to **get the observed quark and lepton masses**

Heterotic line bundle models

Field content:

field	SM rep	name	SU(5)	$\mathcal{G}$ charge pattern
Q u e	$(\mathbf{3},\mathbf{2})_1$ $(\bar{\mathbf{3}},\mathbf{1})_{-4}$ $(\mathbf{1},\mathbf{1})_6$	LH quark RH u-quark RH electron	$\mathbf{10}$	$\mathbf{e}_a$
d L	$(\bar{\mathbf{3}},\mathbf{1})_2$ $(\mathbf{1},\mathbf{2})_{-3}$	RH d-quark LH lepton	$\bar{\mathbf{5}}$	$\mathbf{e}_a+\mathbf{e}_b$
H^d H^u	$(\mathbf{1},\mathbf{2})_{-3}$ $(\mathbf{1},\mathbf{2})_3$	down-Higgs up-Higgs	$\bar{\mathbf{5}}^{H^d}$ $\mathbf{5}^{H^u}$	$\mathbf{e}_a+\mathbf{e}_b$ $-\mathbf{e}_a-\mathbf{e}_b$
ϕ Φ_i	$(\mathbf{1},\mathbf{1})_0$ $(\mathbf{1},\mathbf{1})_0$	pert. moduli non-pert. moduli	$\mathbf{1}$ $\mathbf{1}$	$\mathbf{e}_a-\mathbf{e}_b$ $(k_1^i,\ldots,k_5^i)$

$\mathbf{e}_1=(1,0,0,0,0)$

$\vdots$

$\mathbf{e}_5=(0,0,0,0,1)$

$(k_1^i,...,k_5^i)$

Yukawa couplings:

$$Y^u_{ij}(\{\phi\},\{\Phi\},\{\mathcal{O}(1)\text{ coeffs.}\})\ H^u_{-\mathbf{e}_a-\mathbf{b}_b}\ \mathbf{10}^i_{\mathbf{e}_c}\ \mathbf{10}^j_{\mathbf{e}_d}$$

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(No sum on i and j)

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ϕ	$(\mathbf{1},\mathbf{1})_0$	pert. moduli	$\mathbf{1}$	$\mathbf{e}_a-\mathbf{e}_b$
Φ_i	$(\mathbf{1},\mathbf{1})_0$	non-pert. moduli	$\mathbf{1}$	$(k_1^i,\ldots,k_5^i)$

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Must be $\mathcal{G}$ – invariant

$$\mathcal{G}\text{-charge }(Y^u_{ij}(\{\phi\},\{\Phi\}))=\mathbf{e}_a+\mathbf{e}_b-\mathbf{e}_c-\mathbf{e}_d$$

Strategy

- **Fill both Yukawa matrices** with allowed insertions

$$Y^u_{ij}(\{\phi\},\{\Phi\},\{\mathcal{O}(1)\ \text{coeffs.}\})\ H^u_{-\mathbf{e}_a-\mathbf{b}_b}\ \mathbf{10}^i_{\mathbf{e}_c}\ \mathbf{10}^j_{\mathbf{e}_d}$$

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As an illustrative example, the up-type Yukawa matrix can take the form

$$Y^u_{ij} = \begin{pmatrix} \phi_{2,5}\phi_{5,2}+\cdots & \phi_{2,5}\phi_{5,2}^3+\cdots & \phi_{2,5}^2\phi_{5,2}+\cdots \\ \phi_{2,5}\phi_{5,2}^3+\cdots & \phi_{5,2}+\cdots & \phi_{2,5}+\cdots \\ \phi_{2,5}^2\phi_{5,2}+\cdots & \phi_{2,5}+\cdots & 0 \end{pmatrix}$$

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Each entry is multiplied by a o(1) coefficient
that needs to be determined

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- **Find numerical value for VEVs and o(1) coeffs** such that the **observed quark and lepton masses can be reproduced along with the CKM matrix**

Assign VEVs $\left\{\frac{\langle\phi_{2,5}\rangle}{M_{\text{comp}}}=0.3,\ \frac{\langle\phi_{2,5}\rangle}{M_{\text{comp}}}=0.1,\ \ldots\right\}$

to induce effective Yukawa couplings such that

$$\langle H^u\rangle Y^u = (U^u)^\dagger\ \text{diag}(m_u,m_c,m_t)\ (V^u)^\dagger$$
$$\langle H^d\rangle Y^d = (U^d)^\dagger\ \text{diag}(m_d,m_s,m_b)\ (V^d)^\dagger$$
$$\langle H^d\rangle Y^e = (U^e)^\dagger\ \text{diag}(m_e,m_\mu,m_\tau)\ (V^e)^\dagger$$
$$(U^u)^\dagger U^d = \text{CKM}$$

Strategy

- **Control the μ-term at the same time**

  Sticking with the previous example, this term can take the form

$$\mu \;=\; \mu(\{\phi\},\{\Phi\})H^d H^u \;=\; (\phi_{2,5}\phi_{5,2} \;+\; \ldots)H^d H^u$$

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BUT this may kill Yukawa matrices. Indeed, in our example

$$Y^u_{ij} = \begin{pmatrix} \cancel{\phi_{2,5}}\phi_{5,2}+\cdots & \cancel{\phi_{2,5}}\phi^3_{5,2}+\cdots & \phi^2_{2,5}\cancel{\phi_{5,2}}+\cdots \\ \cancel{\phi_{2,5}}\phi^3_{5,2}+\cdots & \phi_{5,2}+\cdots & \cancel{\phi_{2,5}}+\cdots \\ \cancel{\phi^2_{2,5}}\phi_{5,2}+\cdots & \cancel{\phi_{2,5}}+\cdots & 0 \end{pmatrix}$$

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One option to suppress the leading insertions is setting $\langle\phi_{2,5}\rangle \to \dfrac{M_{\mathrm{EW}}}{M_{\mathrm{comp}}}$

It seems tricky to accomplish without altering Yukawa matrices

Results

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- We have found **two models that can potentially realise realistic quark and lepton masses, CKM matrix and light Higgs.**

- **Both are constructed on** $X_{6770}/\mathbb{Z}_2$ but with different line bundles on it.

Results: first model on $X_{6770}/\mathbb{Z}_2$

- **Down spectrum**

Matter	$(Q,u,e)_{\mathbf{e}_1},\ 2\,(Q,u,e)_{\mathbf{e}_2},$ $(d,L)_{\mathbf{e}_1+\mathbf{e}_2},\ (d,L)_{\mathbf{e}_1+\mathbf{e}_5},\ (d,L)_{\mathbf{e}_2+\mathbf{e}_5}$
Higgs	$H^u_{-\mathbf{e}_1-\mathbf{e}_2},\ H^d_{\mathbf{e}_1+\mathbf{e}_2}$
Moduli	$\phi_{1,2} := \phi_{\mathbf{e}_1-\mathbf{e}_2},\quad 4\,\phi_{2,1} := 4\,\phi_{\mathbf{e}_2-\mathbf{e}_1},$ $4\,\phi_{1,3} := 4\,\phi_{\mathbf{e}_1-\mathbf{e}_3},\ 3\,\phi_{5,1} := 3\,\phi_{\mathbf{e}_5-\mathbf{e}_1},$ $2\,\phi_{2,3} := 2\,\phi_{\mathbf{e}_2-\mathbf{e}_3},\ 7\,\phi_{2,5} := 7\,\phi_{\mathbf{e}_2-\mathbf{e}_5}$ $\Phi_1 := \Phi_{(1,0,0,0,-1)},\ \Phi_2 := \Phi_{(-2,1,1,0,0)},$ $\Phi_3 := \Phi_{(0,1,0,-1,0)},\ \Phi_4 := \Phi_{(0,-2,0,1,1)},$ $\Phi_5 := \Phi_{(0,0,-1,1,0)}$

- **Yukawas**

$$Y^u = \begin{pmatrix} 0 & 1 & 1 \\ 1 & \phi_{1,2} & \phi_{1,2} \\ 1 & \phi_{1,2} & \phi_{1,2} \end{pmatrix}$$

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- **VEV assignments**

$$\langle\phi_{1,2}\rangle=\langle\phi_{2,5}\rangle=0.7\,,\quad \langle\Phi_1\rangle=\langle\Phi_2\rangle=\langle\Phi_4\rangle=\langle\Phi_5\rangle=0.07\,.$$

$$\langle\Phi_3\rangle, \langle\phi_{2,1}\rangle, \langle\phi_{1,3}\rangle, \langle\phi_{5,1}\rangle, \langle\phi_{2,3}\rangle = \mathcal{O}(M_{\mathrm{EW}}/M_{\mathrm{GUT}})\,,$$

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- **Quark and lepton masses reproduced within experimental error (deviations less than 1%)**

Results: first model on $X_{6770}/\mathbb{Z}_2$

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- **VEV assignments**

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$$\langle\Phi_3\rangle,\langle\phi_{2,1}\rangle,\langle\phi_{1,3}\rangle,\langle\phi_{5,1}\rangle,\langle\phi_{2,3}\rangle=\mathcal{O}(M_{\mathrm{EW}}/M_{\mathrm{GUT}})\,,$$

- **Yukawas**

$$Y^u = \begin{pmatrix} 0 & 1 & 1 \\ 1 & \phi_{1,2} & \phi_{1,2} \\ 1 & \phi_{1,2} & \phi_{1,2} \end{pmatrix}$$

$$Y^d = \begin{pmatrix} \Phi_2\Phi_4 + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^2 + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^3 + \ldots \\ \Phi_2\Phi_4\phi_{1,2} + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^3 + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^4 + \ldots \\ \Phi_2\Phi_4\phi_{1,2} + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^3 + \ldots & \Phi_2^2\Phi_5\phi_{1,2}^4 + \ldots \end{pmatrix}$$

- **Quark and lepton masses reproduced within experimental error (deviations less than 1%)**
- **CKM matrix within a 10% average deviation**

Results: first model on $X_{6770}/\mathbb{Z}_2$

- **Down spectrum**

Matter	$(Q,u,e)_{\mathbf{e}_1},\ 2\,(Q,u,e)_{\mathbf{e}_2},$ $(d,L)_{\mathbf{e}_1+\mathbf{e}_2},\ (d,L)_{\mathbf{e}_1+\mathbf{e}_5},\ (d,L)_{\mathbf{e}_2+\mathbf{e}_5}$
Higgs	$H^u_{-\mathbf{e}_1-\mathbf{e}_2},\ H^d_{\mathbf{e}_1+\mathbf{e}_2}$
Moduli	$\phi_{1,2} := \phi_{\mathbf{e}_1-\mathbf{e}_2},\ 4\,\phi_{2,1} := 4\,\phi_{\mathbf{e}_2-\mathbf{e}_1},$ $4\,\phi_{1,3} := 4\,\phi_{\mathbf{e}_1-\mathbf{e}_3},\ 3\,\phi_{5,1} := 3\,\phi_{\mathbf{e}_5-\mathbf{e}_1},$ $2\,\phi_{2,3} := 2\,\phi_{\mathbf{e}_2-\mathbf{e}_3},\ 7\,\phi_{2,5} := 7\,\phi_{\mathbf{e}_2-\mathbf{e}_5}$ $\Phi_1 := \Phi_{(1,0,0,0,-1)},\ \Phi_2 := \Phi_{(-2,1,1,0,0)},$ $\Phi_3 := \Phi_{(0,1,0,-1,0)},\ \Phi_4 := \Phi_{(0,-2,0,1,1)},$ $\Phi_5 := \Phi_{(0,0,-1,1,0)}$

- **VEV assignments**

$$\langle\phi_{1,2}\rangle=\langle\phi_{2,5}\rangle=0.7\,,\quad \langle\Phi_1\rangle=\langle\Phi_2\rangle=\langle\Phi_4\rangle=\langle\Phi_5\rangle=0.07\,.$$

$$\langle\Phi_3\rangle,\langle\phi_{2,1}\rangle,\langle\phi_{1,3}\rangle,\langle\phi_{5,1}\rangle,\langle\phi_{2,3}\rangle=\mathcal{O}(M_{\rm EW}/M_{\rm GUT})\,,$$

- **Yukawas**

$$Y^u=\begin{pmatrix}0 & 1 & 1\\ 1 & \phi_{1,2} & \phi_{1,2}\\ 1 & \phi_{1,2} & \phi_{1,2}\end{pmatrix}$$

$$Y^d=\begin{pmatrix}\Phi_2\Phi_4+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^2+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^3+\ldots\\ \Phi_2\Phi_4\phi_{1,2}+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^3+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^4+\ldots\\ \Phi_2\Phi_4\phi_{1,2}+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^3+\ldots & \Phi_2^2\Phi_5\phi_{1,2}^4+\ldots\end{pmatrix}$$

- **Quark and lepton masses reproduced within experimental error (deviations less than 1%)**
- **CKM matrix within a 10% average deviation**
- **μ-term at the electroweak scale**

$$\mu=\phi_{2,1}\phi_{1,2}+\phi_{1,2}\phi_{2,5}\phi_{5,1}+\ldots\sim\mathcal{O}(M_{\rm EW}/M_{\rm GUT})$$

Summary

- **Extracted ~200 inequivalent single Higgs heterotic line bundle models from the "202 list" produced in** [Anderson, Gray, Lukas, Palti, '11, PRD]
- **Scanned their moduli space** looking for regions that **can accommodate realistic flavour parameters** while **avoiding the μ problem.**
- Result —-> only ~**1% of the models can satisfy these constraints.**

Next steps

- **Scan over larger lists** asking for **realistic neutrino physics** and other phenomenological features.
- Provide a **mechanism to stabilise the moduli** and **use ML** to actually **compute all the involved parameters** at the stabilised points.

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Thanks for your attention