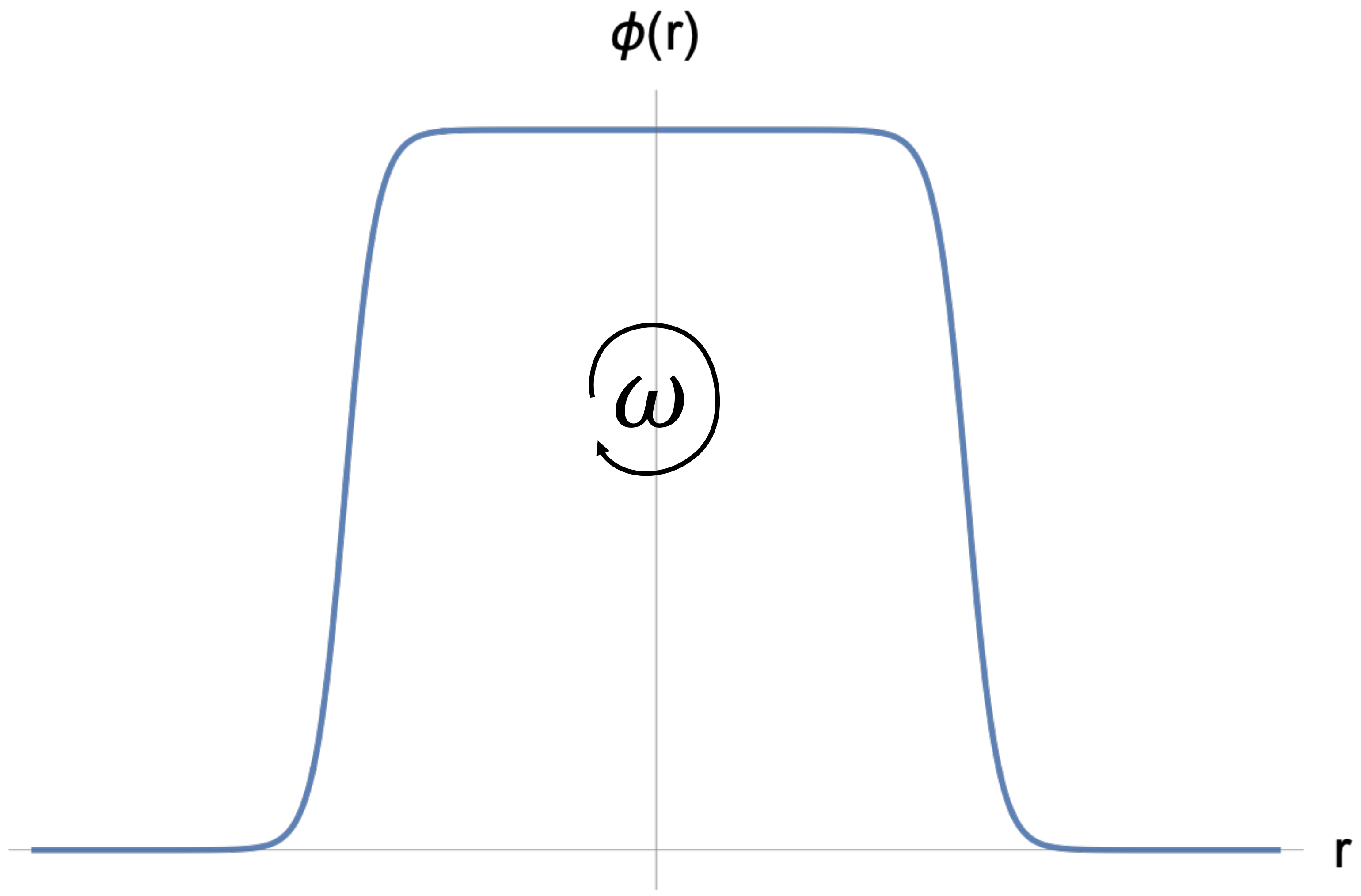
Similarities in the evaporation of saturated solitons and black holes

[1] GC, Dvali, Sakhelashvili 2025 (to appear)

Giacomo Contri *with Gia Dvali and Otari Sakhelashvili* [1]

Are black holes unique?



No[1] ☺

In this talk, we'll discuss how to properly frame the question and study a specific example

[1]Dvali, 2019, 2021

The phenomenon of Saturation

Black holes

- Area law entropy $S = \frac{A}{L_P^2}$
- Thermal evaporation rate $T \sim \frac{1}{R}$
- Classical horizon
- Information retrieval time: $t \sim SR$

[1]Dvali, 2019, 2021 [2]Dvali Sakhelashvili 2021, Dvali Valbuena Kaikov 2021, Dvali Venogupalan 2021

The phenomenon of Saturation

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- $S = L^{(d-2)} f_{poincare}^2$ ✔

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Area law entropy $S = \frac{A}{L_P^2}$	$S = L^{(d-2)} f_{poincare}^2$ ✓
Thermal evaporation rate $T \sim \frac{1}{R}$	Evaporation rate $\Gamma \sim \frac{1}{L}$ ✓
Classical horizon	Classical Information horizon ✓
Information retrieval time: $t \sim SR$	$t \sim SL$ ✓

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• Classical horizon	• Classical Information horizon ✓
• Information retrieval time: $t \sim SR$	• $t \sim SL$ ✓

Black holes are one instance of the broader universality class of saturons[2]

The unifying principle is saturation of unitarity[1]*:* $S \leq \frac{1}{\alpha}$

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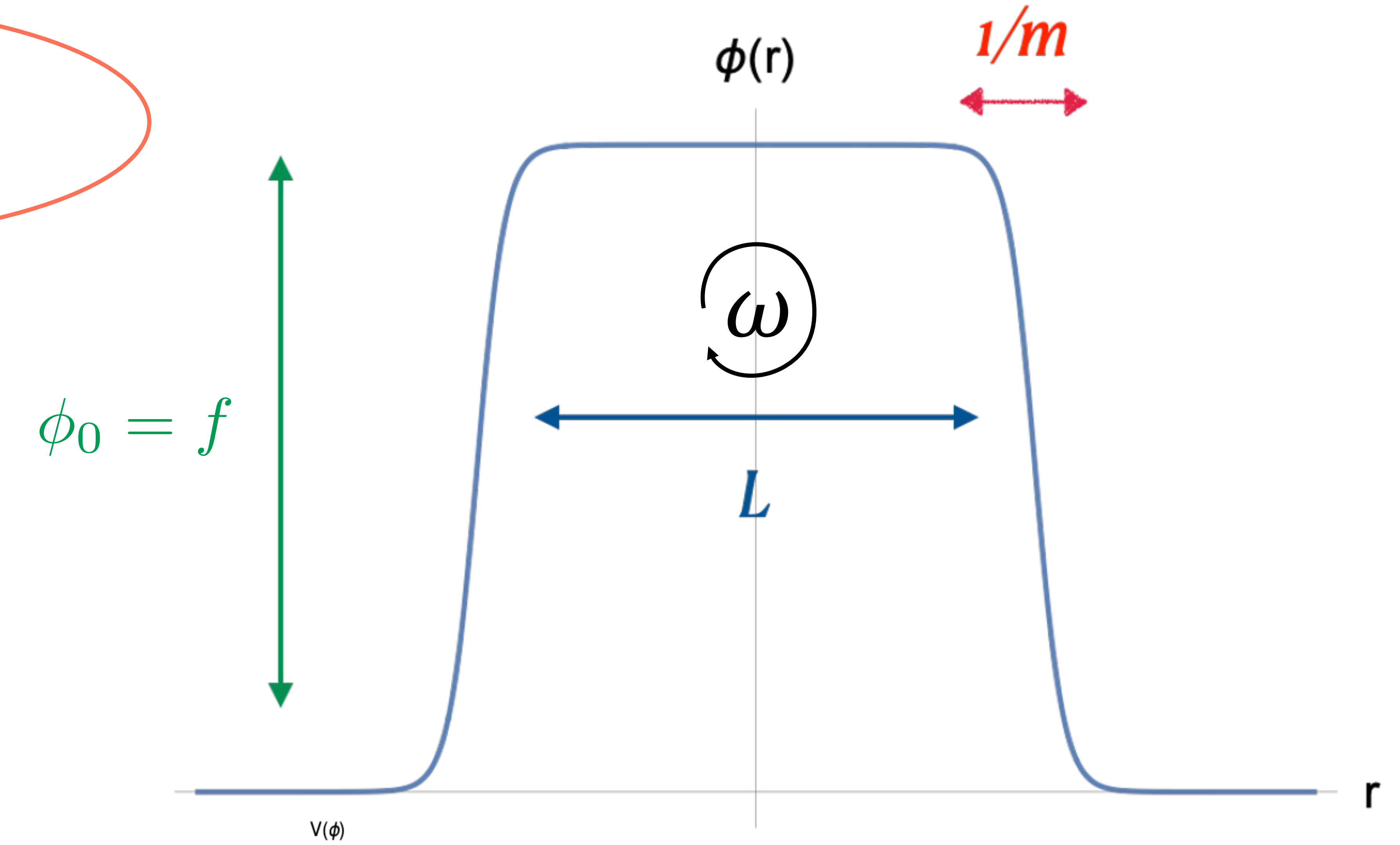
A model of a saturon

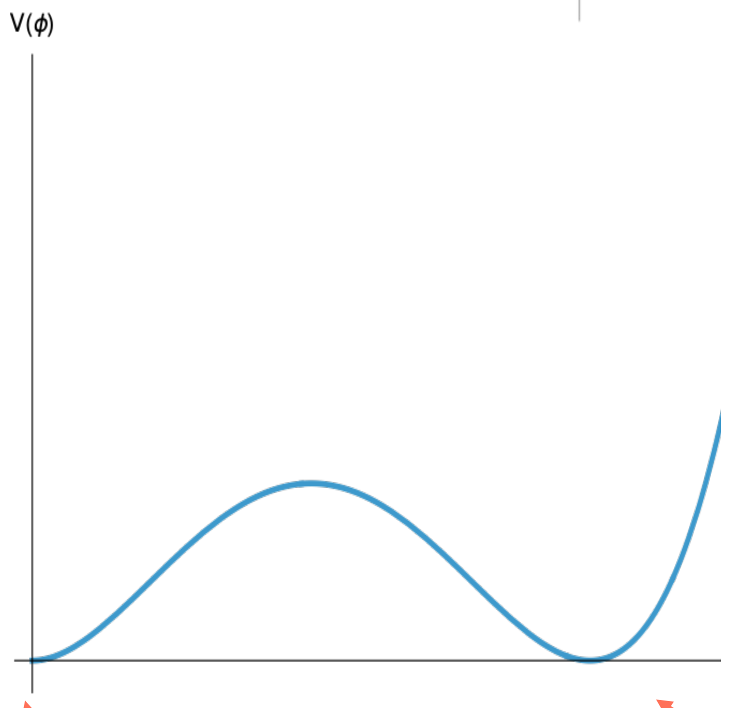
Scalar field in the adjoint of SU(N)

- The model[1]: $\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\left[(\partial_\mu \phi)\,(\partial^\mu \phi)\right] - V[\phi]$

$$V[\phi] = \frac{\alpha}{2}\,\mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\,\mathrm{tr}\left[\phi^2\right]\right)^2\right]$$

- We study the large-N limit of the 1+1 dimensional theory





SU(N) unbroken

SU(N) broken to`SU(N-1)xU(1)

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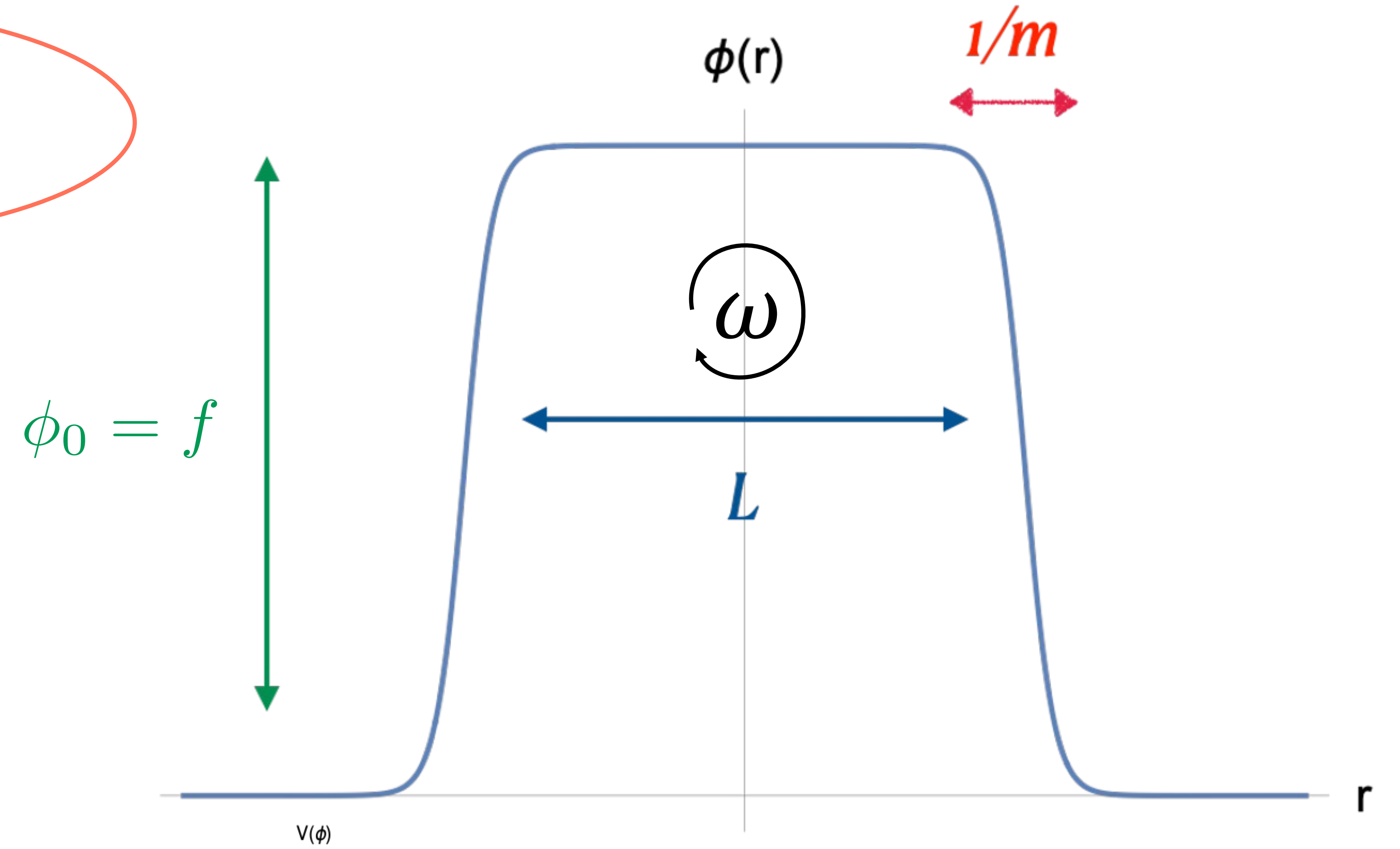
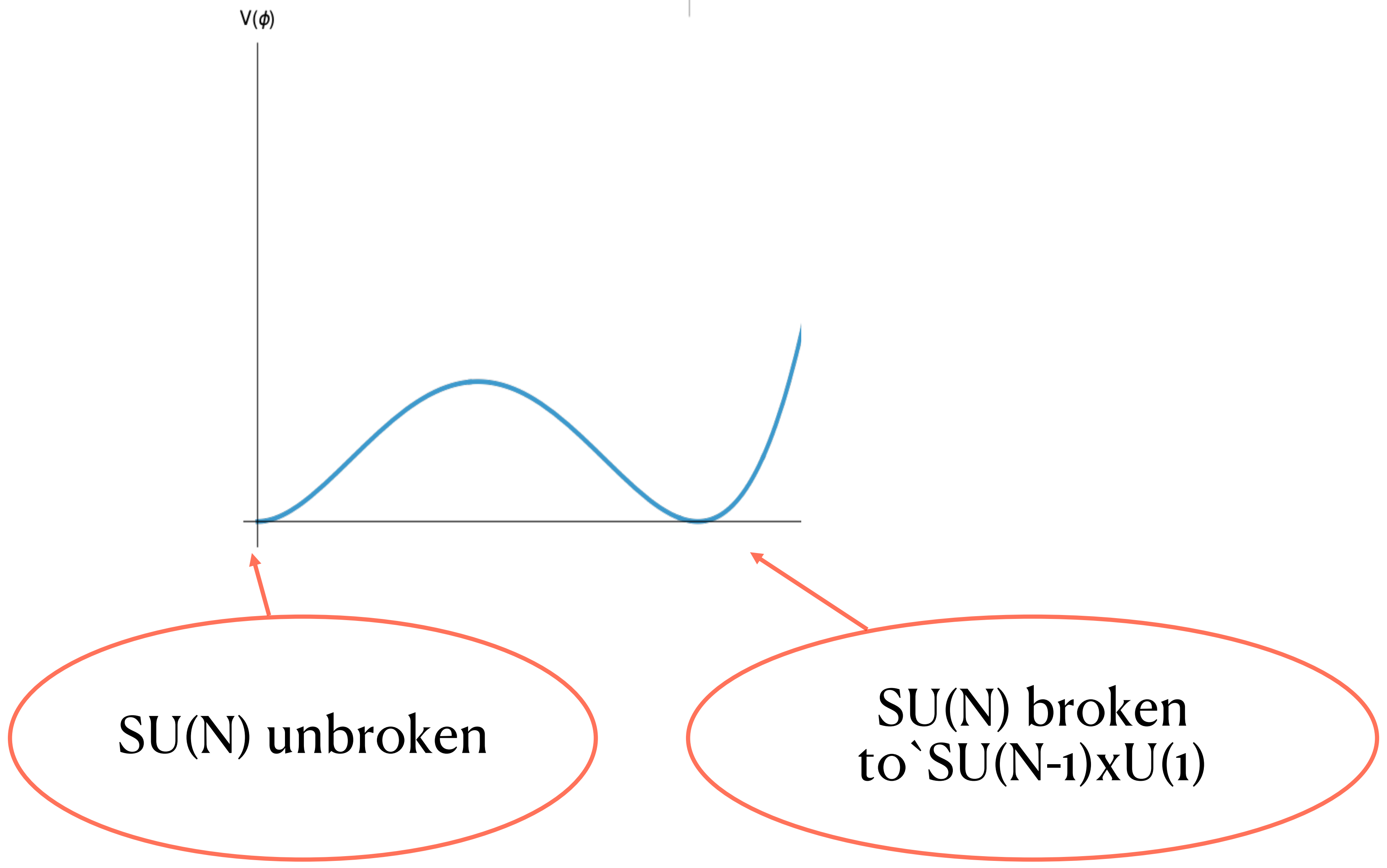
- We study the large-N limit of the 1+1 dimensional theory
- The model admits SU(N)-charged bubbles with internal rotation with frequency ω: $\phi_{bubble} \sim e^{-i\omega t T_a}\phi(r)e^{i\omega t T_a}$
- The scale of Poincaré symmetry breaking is $f^2_{Poincare} = (mL)^2 f^2$



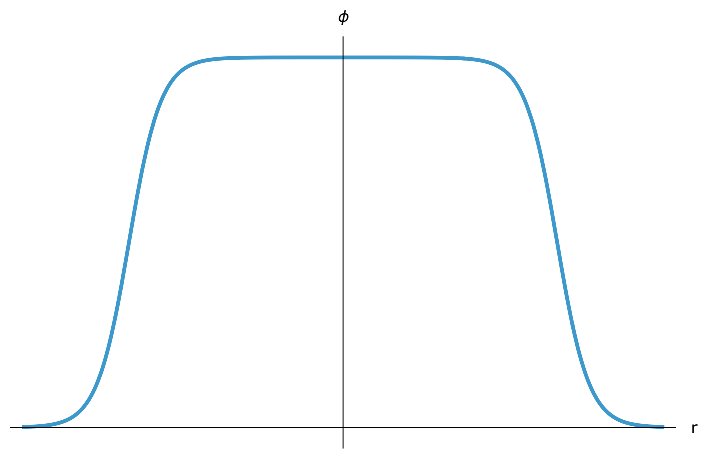


[1]Dvali, Kaikov, Valbuena 2021

Bubbles zoology

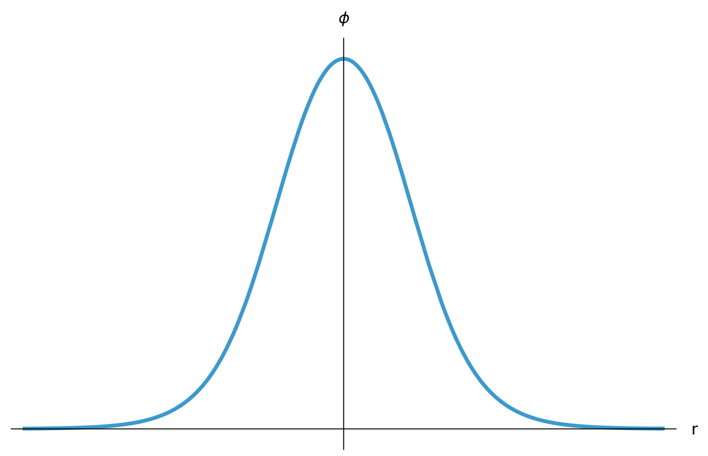
Thin-wall bubbles

$$L \gg \frac{1}{m} \qquad m \gg \omega$$



Thick-wall bubbles

$$m \sim \omega \sim \frac{1}{L}$$



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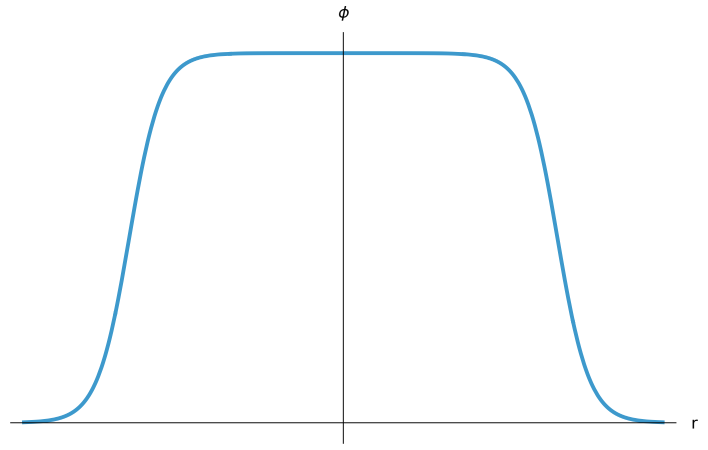

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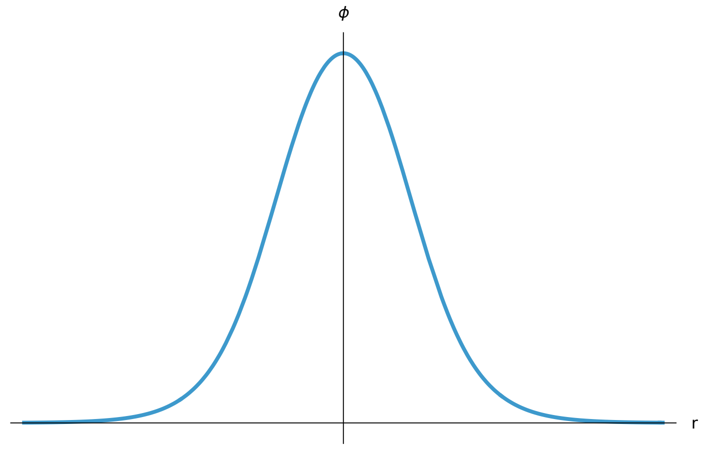

- We can resolve the bubbles as multi-particle states
- The occupation numbers are: $N_g \sim \phi_0^2 L\omega \quad N_\phi \sim \phi_0^2$

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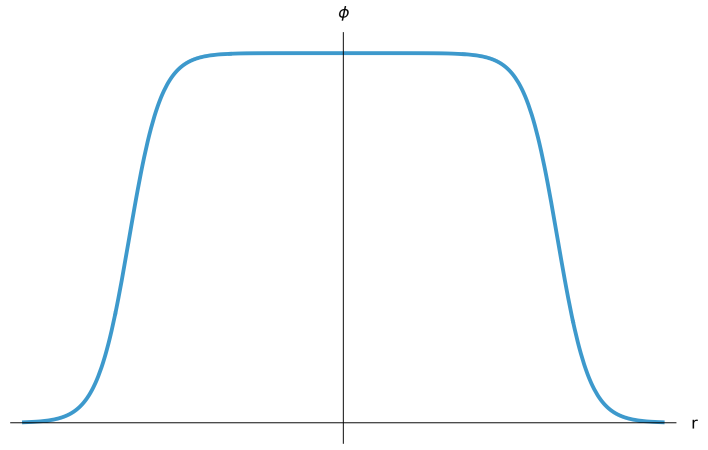

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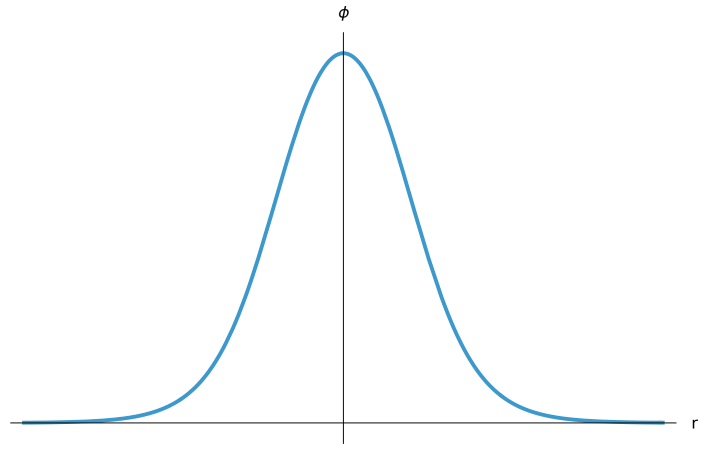

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$$\sum_a^{2N} n^a = N_g \qquad |\,state\rangle = |n_\omega^1, n_\omega^2, \dots\rangle$$

- At saturation the ‘t Hooft coupling becomes $\lambda := \frac{N}{f^2} \sim 1$

$$\boxed{S \ll f^2}$$

$$\boxed{S = f^2 = N = L^{(d-2)} f^2_{poincare}}$$

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Bubbles zoology

Saturon!

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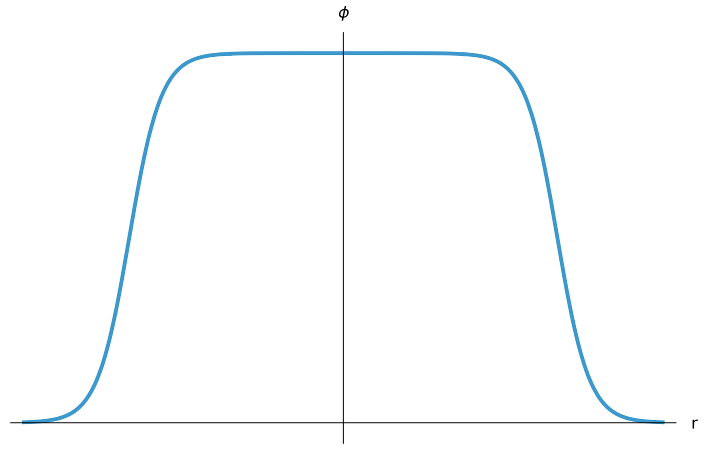

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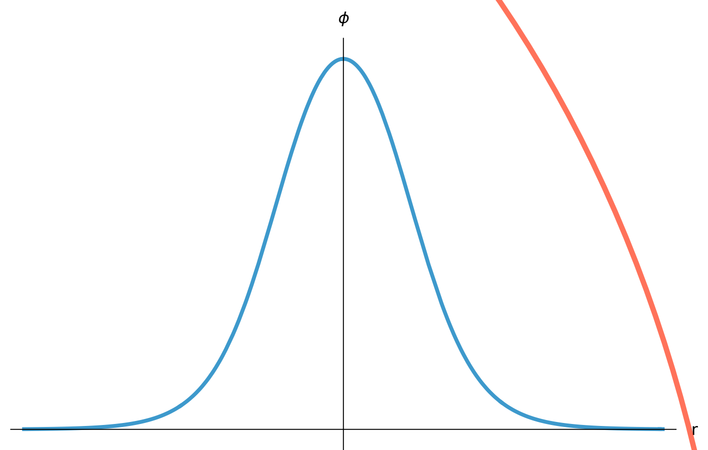

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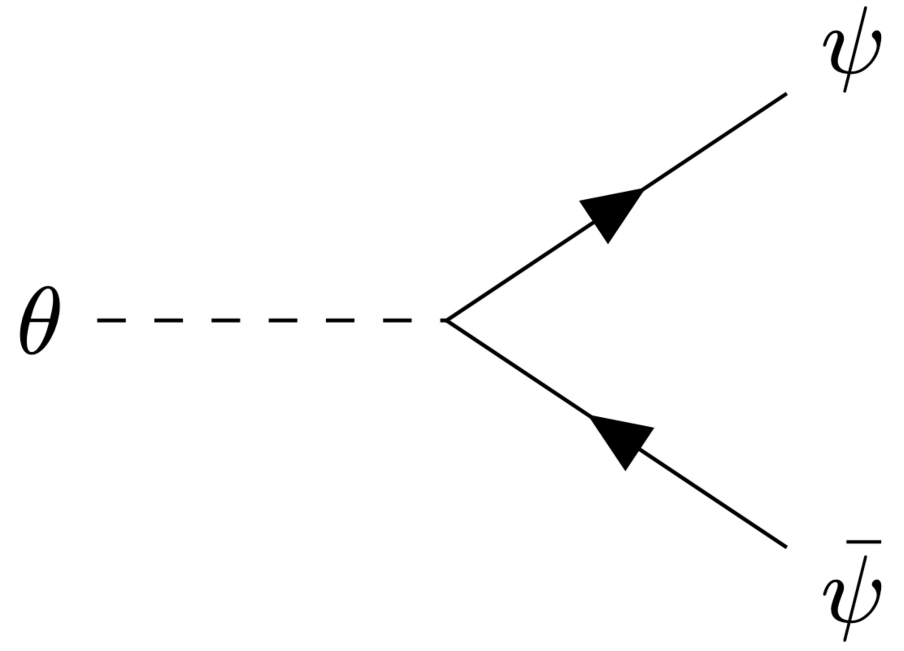
Saturon evaporation

Can we reproduce the $\Gamma = \frac{1}{R}$ evaporation rate[1]*?*

- To allow the bubble to decay, we can couple it to a massless fermionic field

$$\mathcal{L} = \mathcal{L}_\phi + i\bar{\psi}\gamma_\mu\partial^\mu\psi - g\bar{\psi}^\alpha\phi_{\alpha\beta}\psi^\beta$$

- This opens up a decay channel through pair creation:



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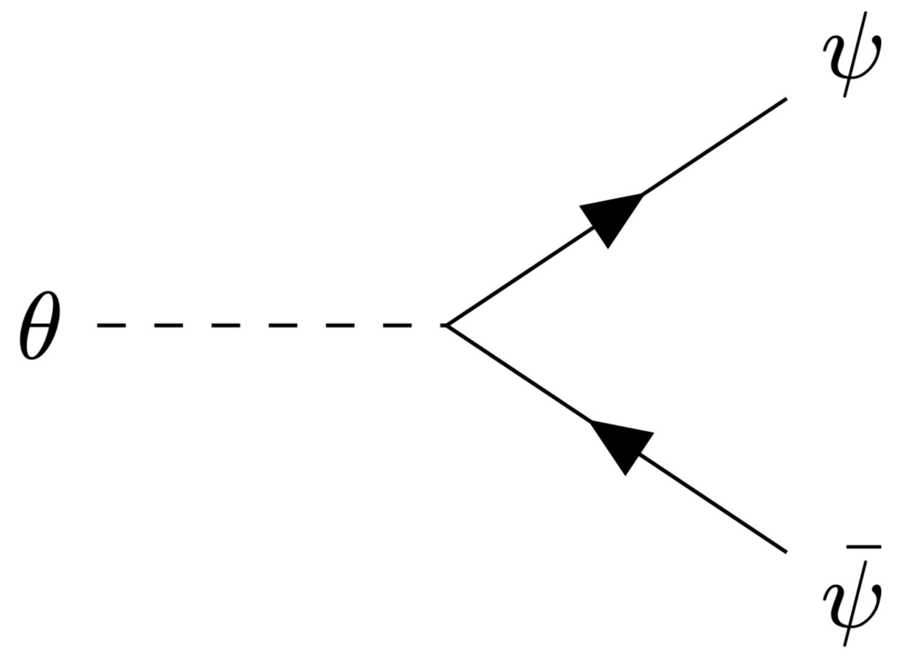

- We studied the evaporation both semiclassically and quantum mechanically, for thin and thick-wall saturated bubbles

[1]GC, Dvali, Sakhelashvili 2025

Saturon evaporation: the semi-classical picture

- Semiclassical approach: quantization of the fermionic field in the bubble background
- The fermions get a space and time dependent mass term from the interaction: $g\bar{\psi}\phi_{bubble}(r,t)\psi$

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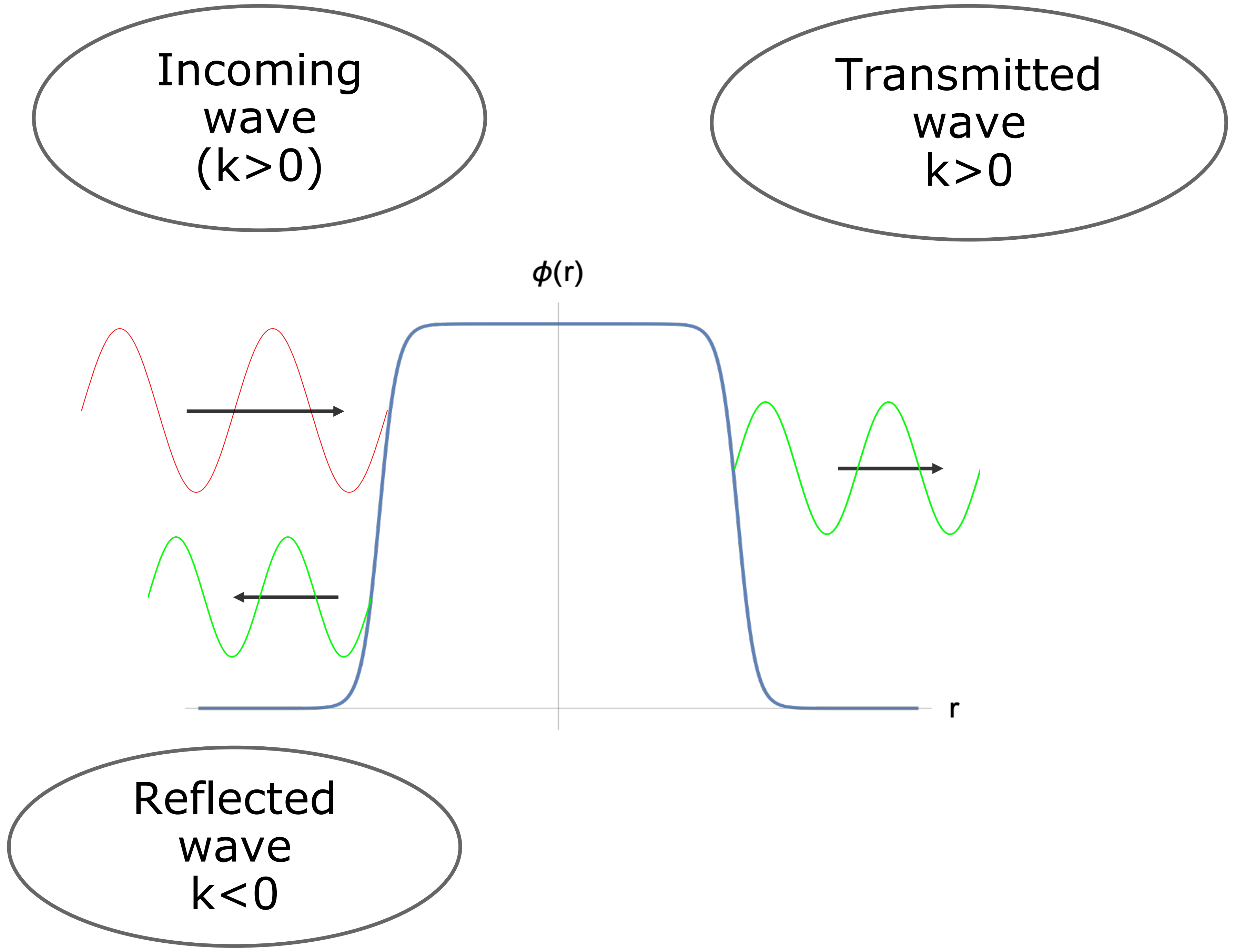
$$a_k^{(incoming)} = \alpha a_k^{(outgoing)} + \beta a_k^{\dagger(incoming)}$$



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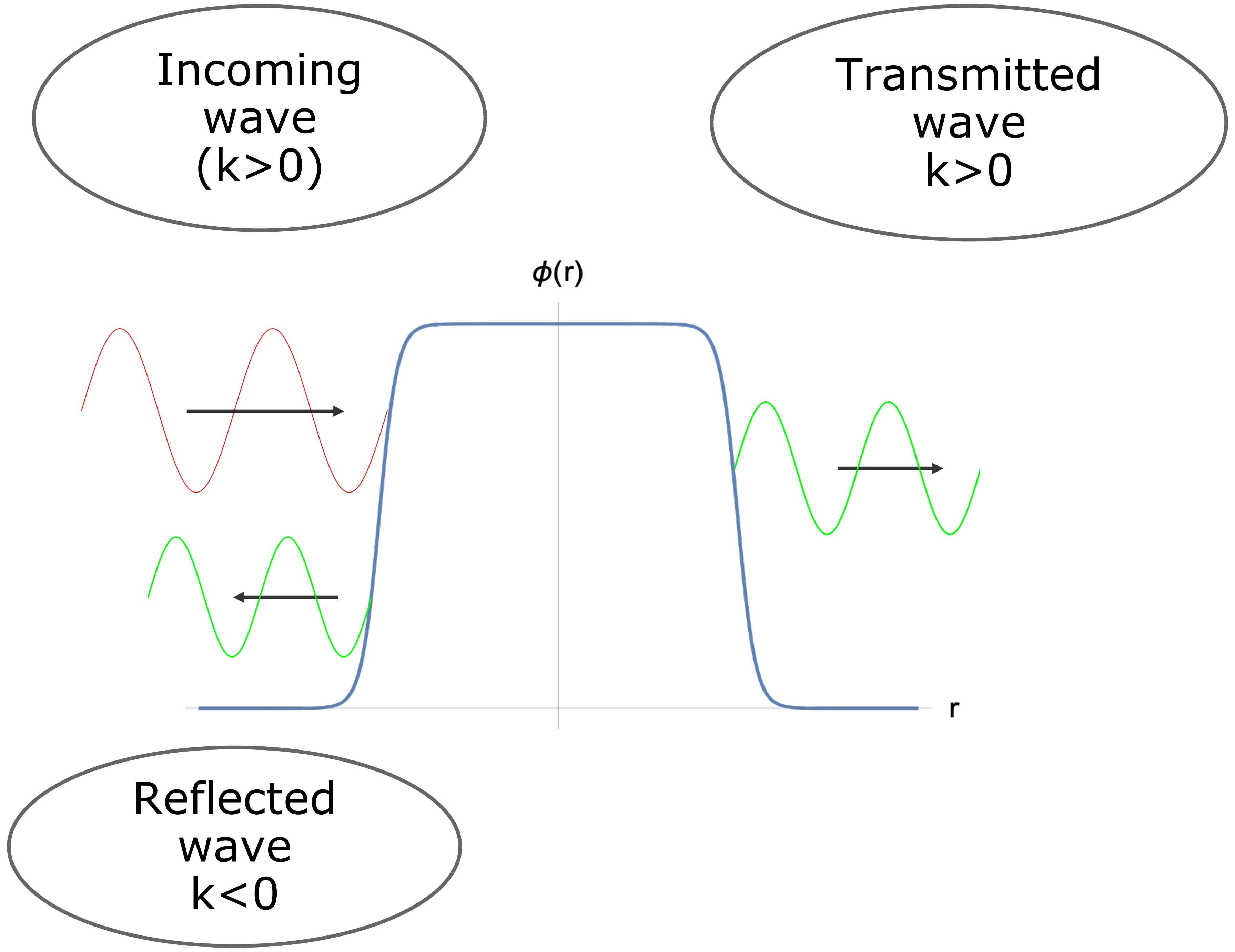

Thin-wall

Weak fermion coupling: $\Gamma = \dfrac{g^2\phi_0^2\left(2L\omega \mathrm{SinIntegral}(2L\omega) + \cos(2L\omega) - 1\right)}{4\omega}$

Fermion Coupling at saturation $\beta := \dfrac{g^2 N}{\alpha f^2} \sim 1$*:* $\Gamma = \omega \tanh^2\left(Lg\phi_0\right)$

Saturated Thick-wall

$$\Gamma \sim \frac{g^2\phi_0^2}{\omega} \sim \frac{1}{L}$$

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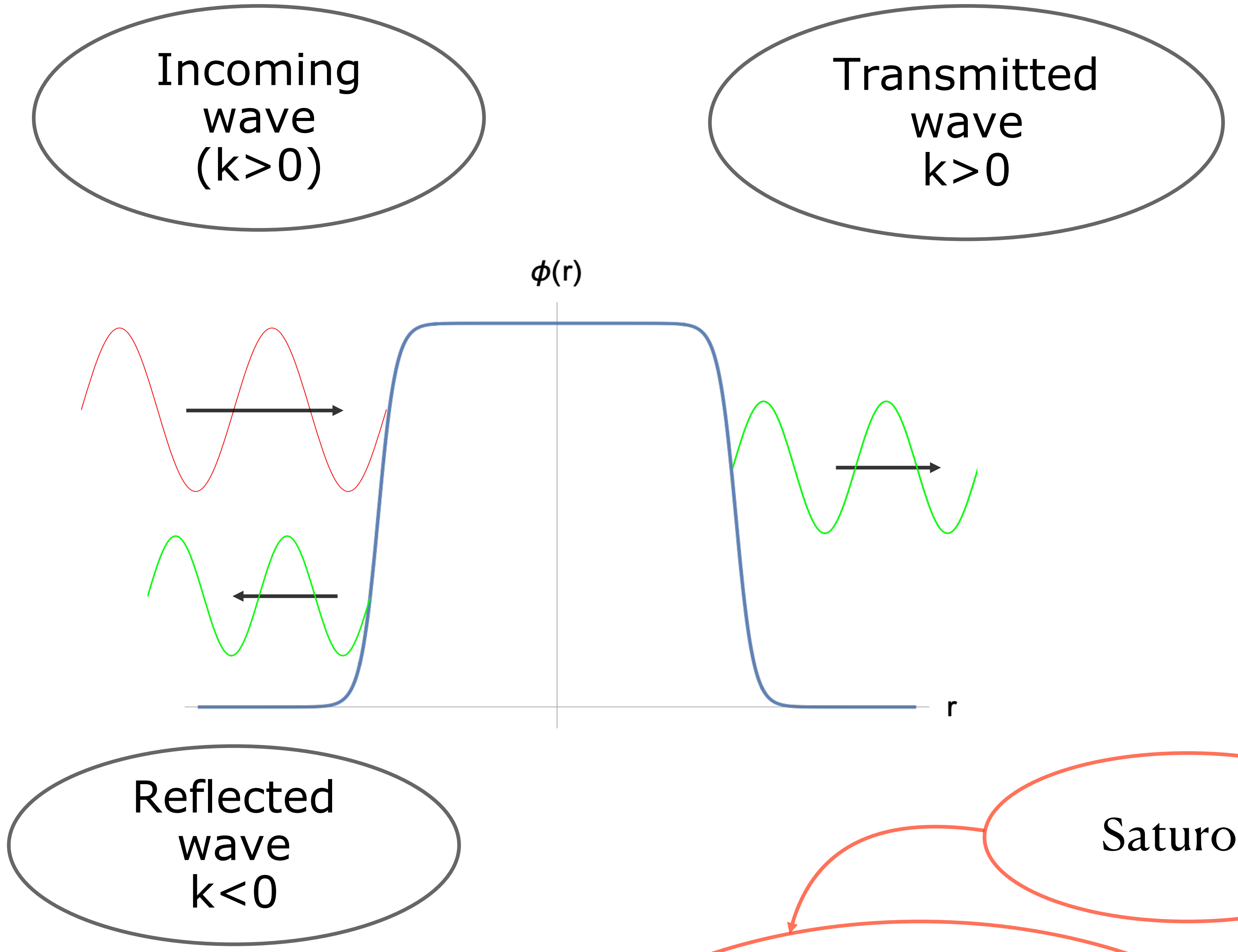

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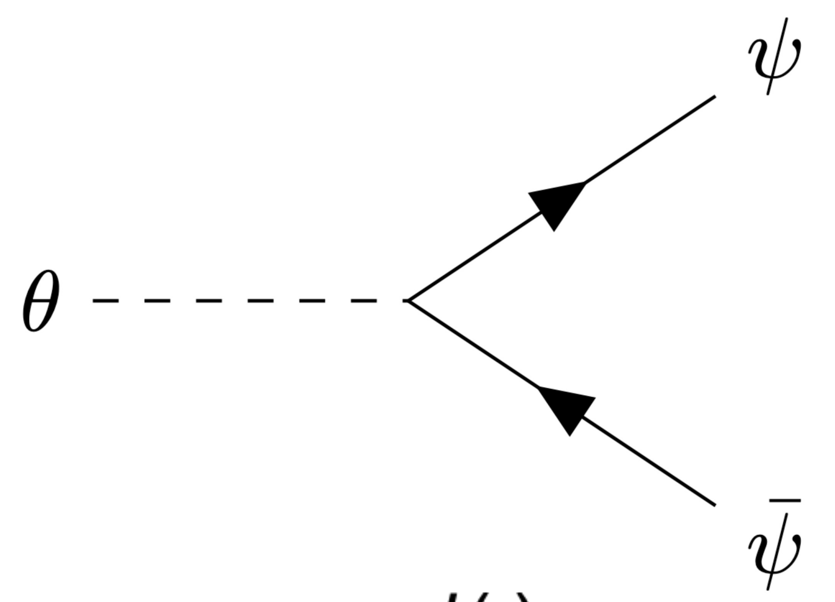
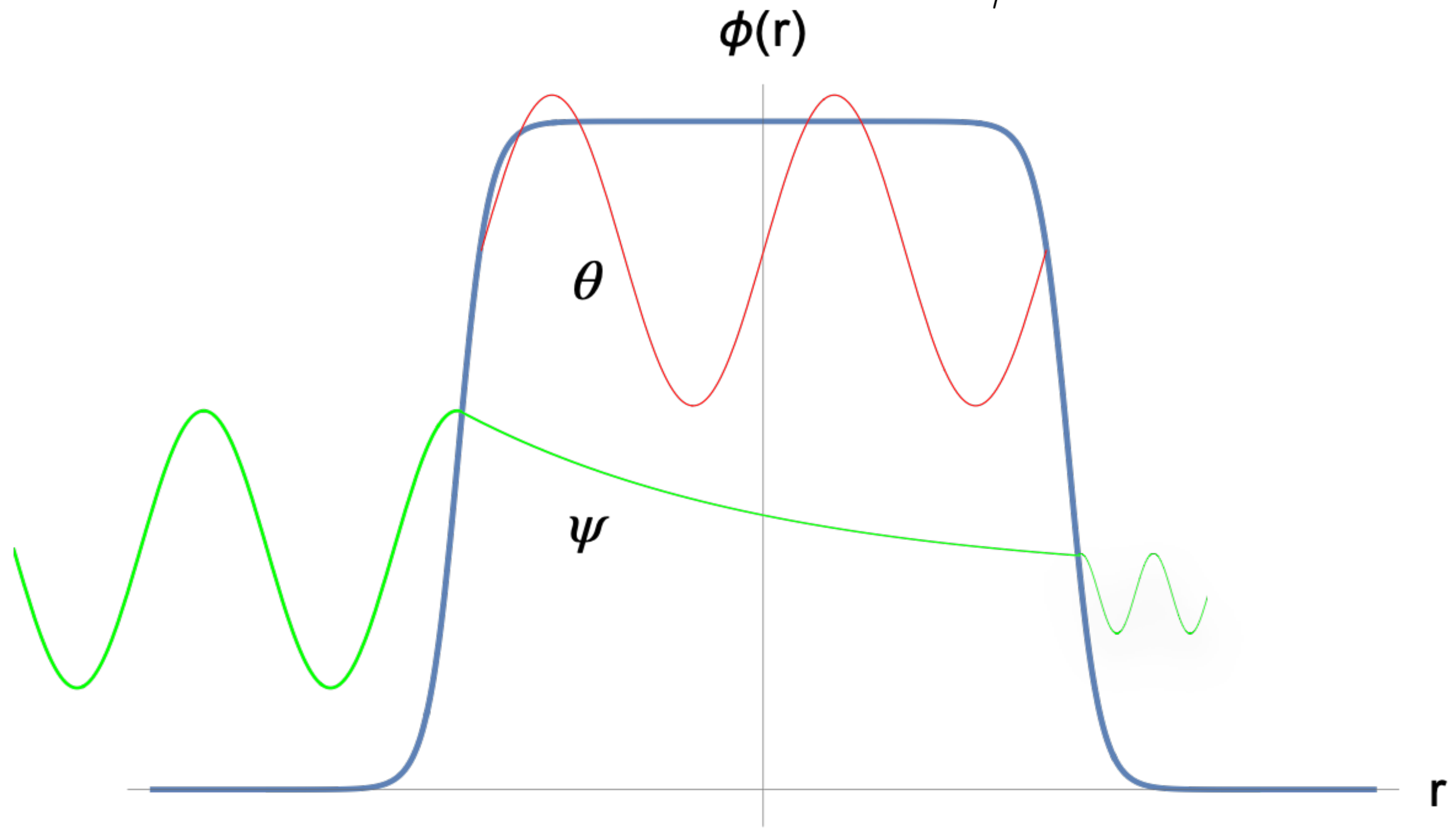
Saturon evaporation: the quantum picture

- We can understand the bubble evaporation as a quantum process
- Recall: the bubble is made of $N_g \sim \phi_0^2 L\omega$ Goldstones of energy ω
- The main decay process is the decay of a Goldstone particle θ in a particle-antiparticle pair.
- The Goldstone particle is localized inside the bubble.
- The fermions get a mass inside the bubble: for $m_\psi > \omega$ the rate is suppressed by the finite penetration length.





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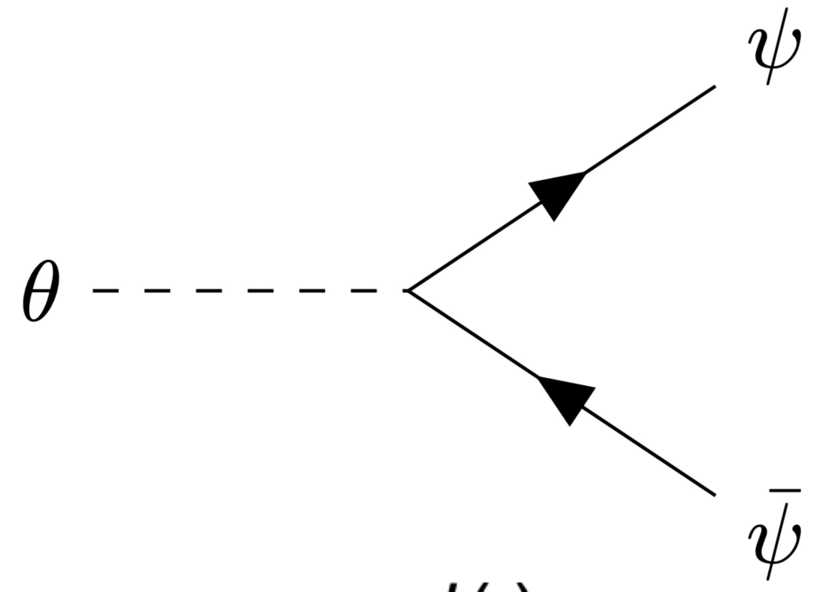
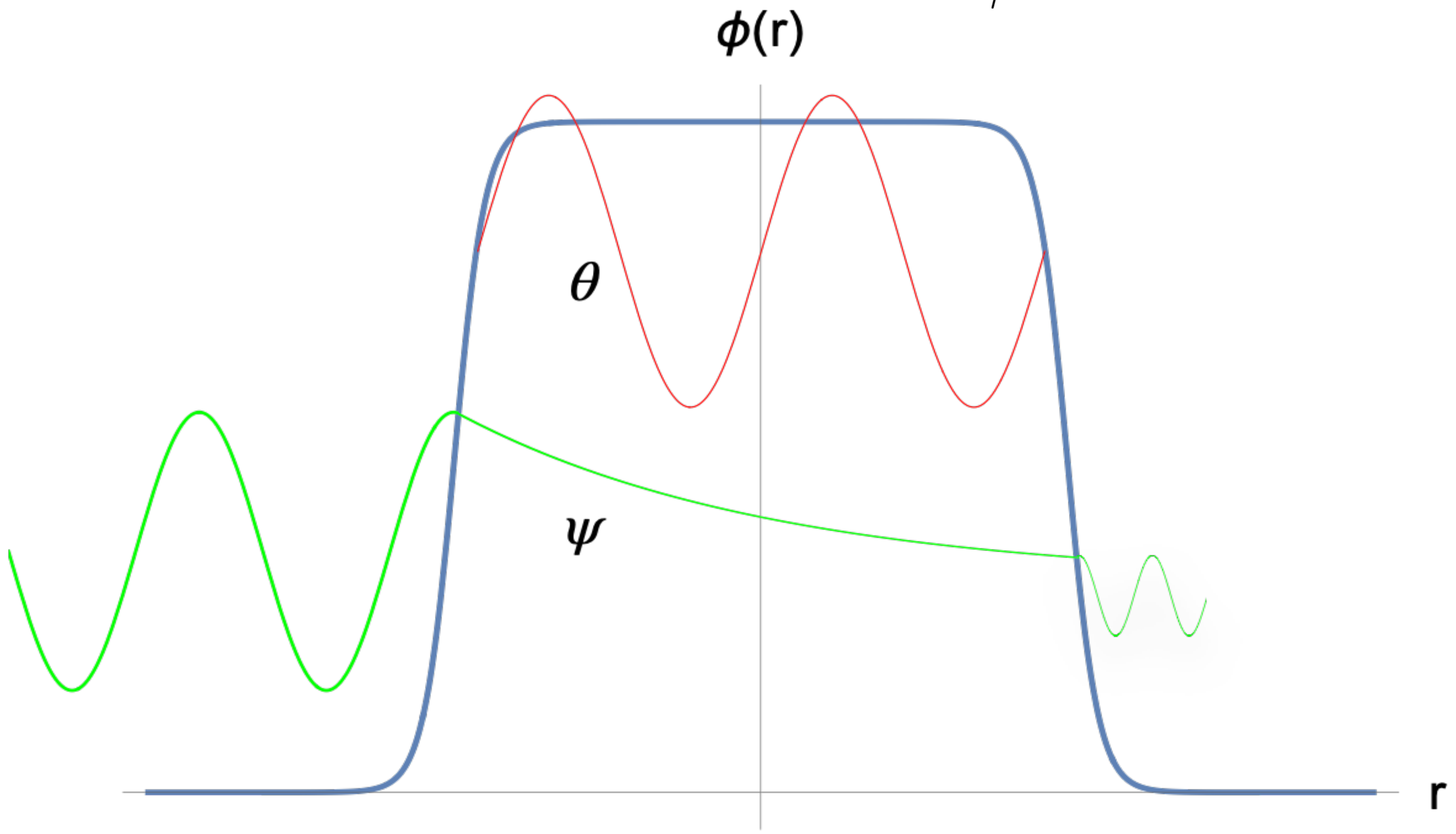




$$\Gamma_{single} \sim \begin{cases} \frac{g^2}{\omega}\frac{1}{m^2}\frac{\omega}{L} & m_\psi \gg 1/L \\ \frac{g^2}{\omega}(\omega L) & m_\psi \ll 1/L \end{cases} \qquad \Gamma_{tot}^{(sat)} = \Gamma_{single} N_g \sim \frac{1}{L}$$

Full matching with the semiclassical analysis

Conclusions

- We studied a 1+1 dimensional example of a Saturon
- All the investigated properties match with the general features of saturation
- In particular, saturated bubbles evaporate with a $\Gamma \sim \frac{1}{L}$ rate.
- Full correspondence with black-holes

Thank you

Backups

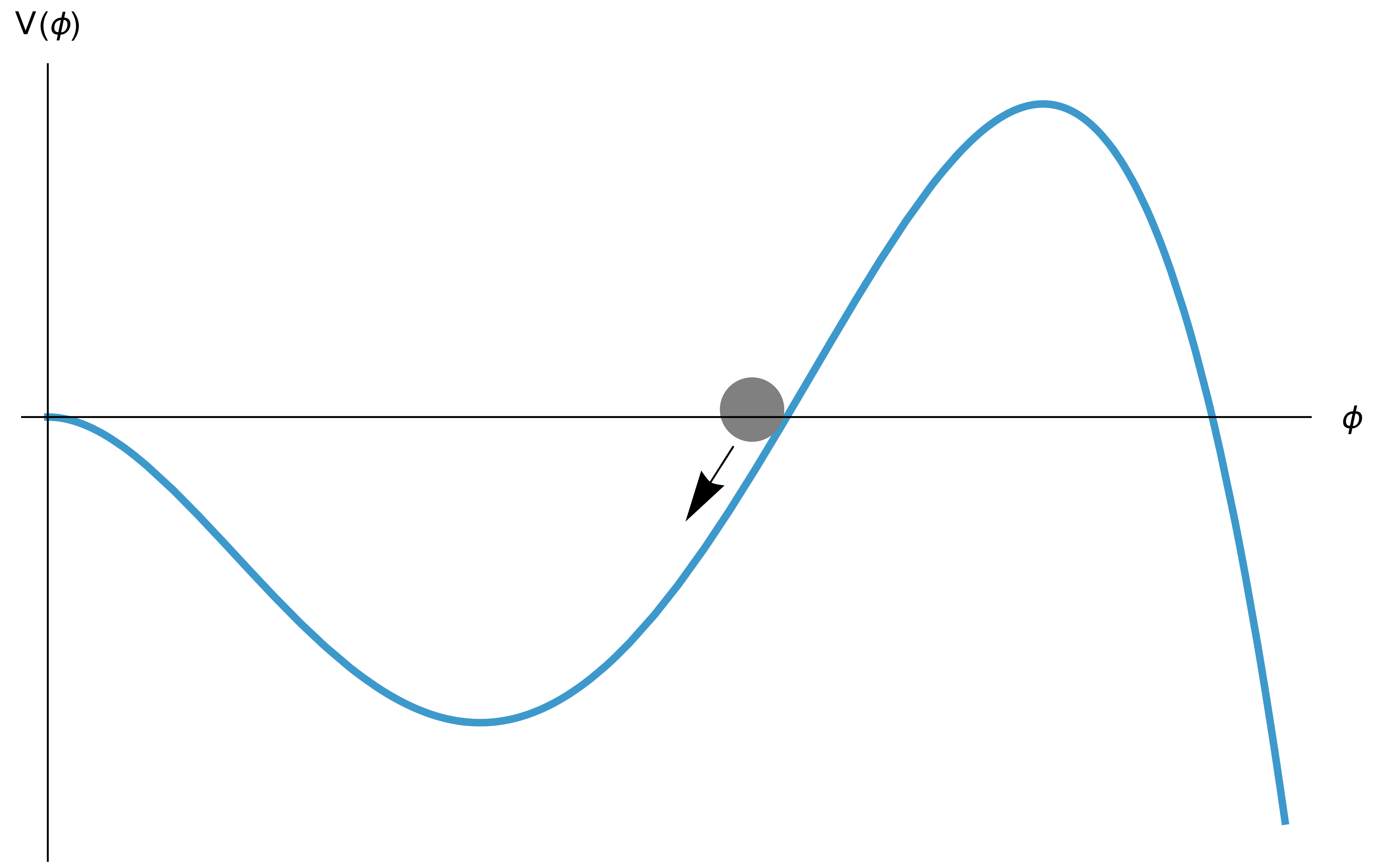
Bubble profile details

- The bubble solution in detail is given by:

$$\Phi_\alpha^\beta = \left(U^\dagger V U\right)_\alpha^\beta$$
$$U = \exp\left[-i\theta^a T^a\right]$$

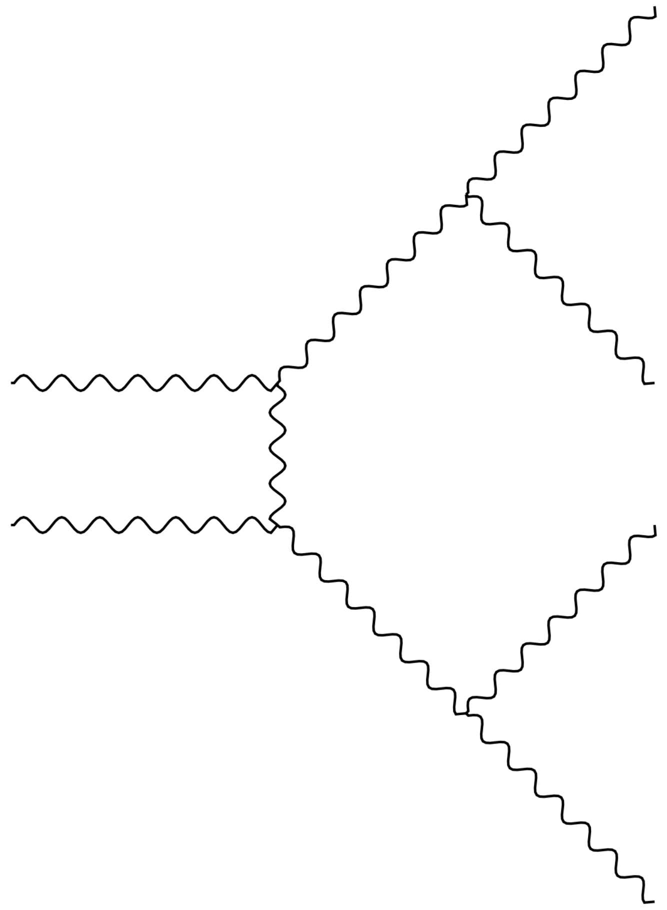
$$V = \frac{\phi(x)}{\sqrt{N(N-1)}}\,\mathrm{diag}((N-1), -1, \ldots, -1).$$

- Where ϕ is obtained as a bounded solution of the effective potential $\mathcal{V}(\phi) = \frac{1}{2}\phi^2\left(\omega^2 - \alpha(\phi - f)^2\right)$



Saturons and Unitarity

- The connection between Saturons and unitarity can be understood in terms of $2 \to N$ processes.
- $2 \to N$ processes are usually (non-perturbatively) exponentially suppressed $\sigma_{2\to N} \sim e^{-N}$
- Reason: truncation of the perturbative series at the self-susteinability point: $N = \frac{1}{\alpha}$
- The saturon‘s entropy exactly compensates for this suppression, giving rise to $O(1)$ probabiity of production: $\sigma_{tot} \sim e^S e^{-N} \sim e^{1/\alpha} e^{-1/\alpha} \sim 1$



Counting the entropy

- The bubble is resolved as a multiparticle state:

$$|sol\rangle = \mathrm{e}^{\int d\boldsymbol{k} \sum_{j=1}^{N} \sqrt{n_j(\boldsymbol{k})}\left(\hat{a}_j(\boldsymbol{k})^\dagger\right)}|0\rangle$$

- We can count the entropy in two ways:

1) As discussed above, through the combinatorics of Godstone's constituents:

$$N_g \sim \phi_0^2 L\omega \qquad \sum_a^{2N} n^a = N_g \qquad |\, state\rangle = |n_\omega^1, n_\omega^2, \ldots\rangle$$

The combinatorics gives: $S \sim 2N \ln\left[\left(1+\frac{2N}{N_g}\right)^{\frac{N_g}{2N}}\left(1+\frac{N_g}{2N}\right)\right]$

2)We can see the bubble as composed as N_g asymptotic quanta, transforming into the adjoint representation:

- The multiplicity is the dimension of the totally simmetric representation of N_g copies of SU(N).
- Both method give the same answer, given in the above slides

Details on the radiation spectrum

- Semiclassically, we can obtain the full spectrum.

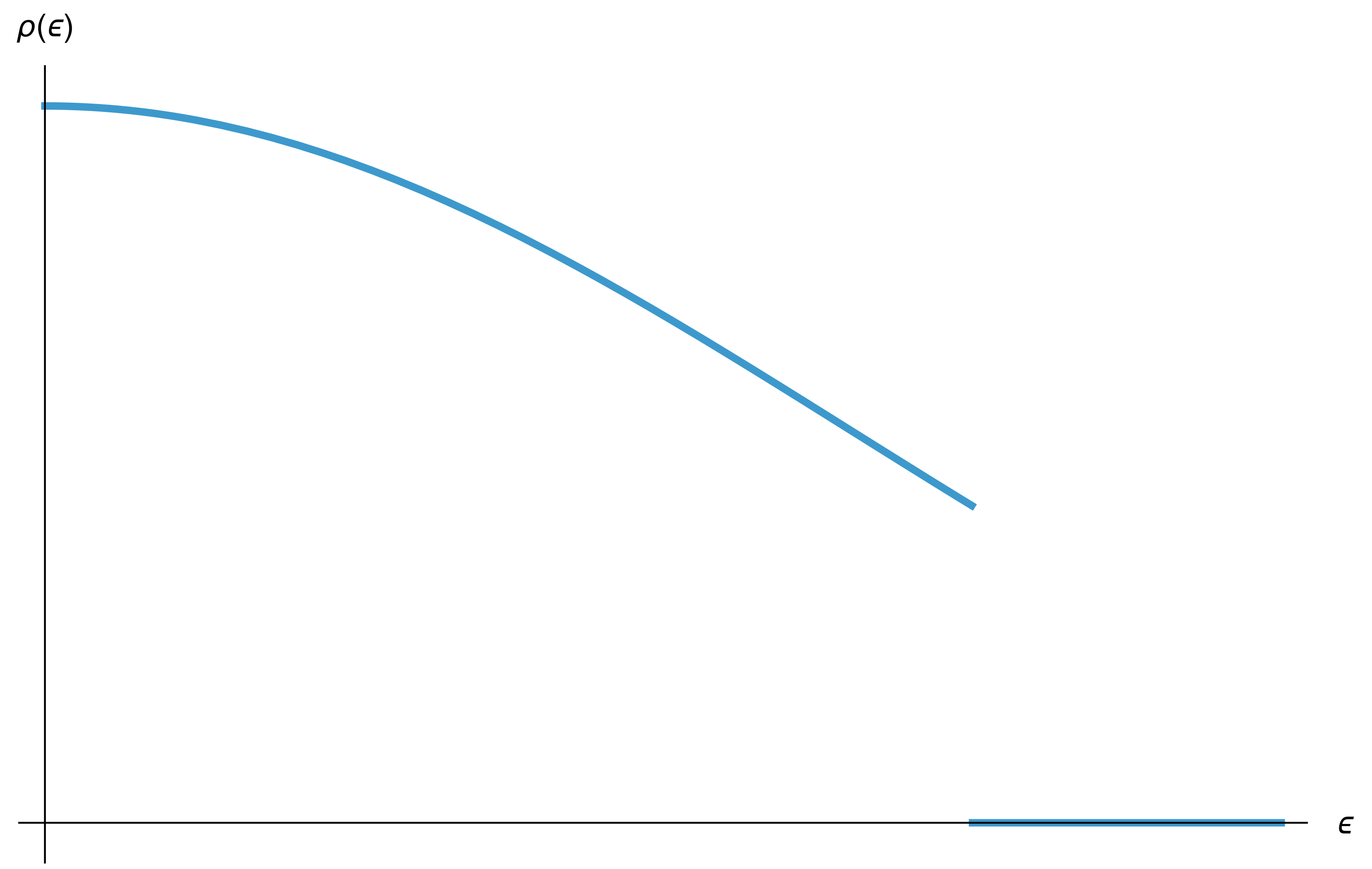
- As a series expansion in $g\phi_0/\omega$:

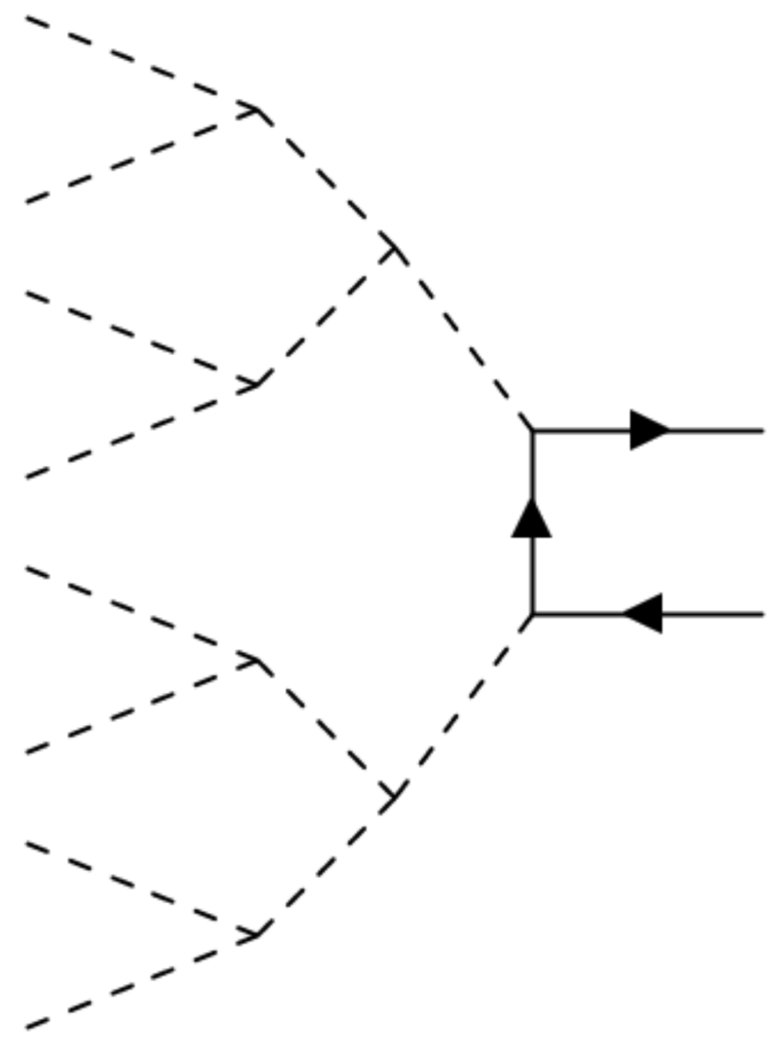
$$\rho_\epsilon = \frac{g^2\phi_0^2\sin^2(2L\epsilon)}{4\epsilon^2}$$

- Energy peaked at ω

- Democratic in particle-anti particle, flavour is conserved

- Cutoff at energy ω from charge conservation: $N \to 2$ is not only exponentially suppressed, but forbidden





Information Horizon and classical limit

- There are different views that one can take in analyzing the saturon
- Semi-classical limit: $f^2 \sim N \to \infty \quad \frac{\alpha}{m^2} \sim \frac{g^2}{m^2} \sim \frac{1}{N} \to 0 \quad m^2 \sim \alpha N \ = \text{finite}$
- The (individual) Goldstone coupling $\alpha_G \sim \frac{m^2}{E^2 f^2} \to 0$ vanishes
- One cannot resolve any non-classical difference between patterns $\sum_a^{2N} n^a = N_g \qquad |\, state\rangle = \left|n_\omega^1, n_\omega^2, \ldots\right\rangle$
- Large but finite N: quantum effects like the resolution of non-macroscopically different patter emerge as $\frac{1}{N}$ effects
- The retrival rate is $\Gamma_{\text{Gold}} \ \sim \alpha_G^2 m N_G \sim \frac{1}{Rf^2} \sim \frac{1}{SR}$