What Keeps string Theory Stable? Moduli, Tachyons and Vacuum Energy !

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Motivation

Outline

- Moduli fields
  - Free Fermionic Formalism
  - Thirring Interactions
- Vacuum Energy
  - Partition Function
  - Cosmological Constant
- Models (Examples)
  - S-Model
  - $\tilde{S}$–Model
- Conclusions and future work

Moduli Fields

Moduli Fields In String Theory

Scalar fields that describe shape and size of extra dimensions

↓

Fix Moduli (Moduli can be projected out)

↓

Only the dilaton survives

Moduli Fields

We started with a free fermionic formalism

$$i\partial X^I_L \sim y^I \omega^I$$

and

$$i\partial X^I_R \sim \bar{y}^I \bar{\omega}^I$$

Deformations including world-sheet Thirring interactions among fermions

$$S \sim \int h_{IJ} \partial X^I \partial \bar{X}^J \sim \int h_{IJ} y^i w^i \bar{y}^i \bar{w}^i$$

Moduli Fields

Thirring Interactions

$$J^i_L(z)\bar{J}^j_R(\bar{z})$$

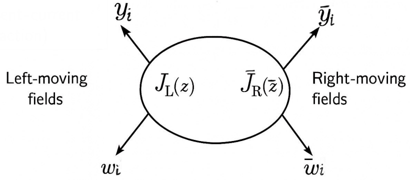
$$J^i \sim y^i w^i, \bar{J}^j \sim \bar{y}^i \bar{w}^i$$

So the Thirring interactions:

$$J^i_l \bar{J}^j_R \sim y^i w^i \bar{y}^i \bar{w}^i$$

$\Longrightarrow$

Thirring Interactions



The interaction is effectively a product of fields

$$J_L \bar{J}_R \approx y_i \, w_i \quad \bar{y}_i \, \bar{w}_i$$

Moduli Fields

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	1

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	0	0	1	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	1	0	1	0	1
γ	0	1	0	0	0	1	0	0	1	0	0	0

Vacuum energy(Potential)

Vacuum energy(Potential)

Calculating one-loop vacuum energy

$$\mathbf{Z} = Z_B \sum_{St} C \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \prod Z \begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix} \ ,$$

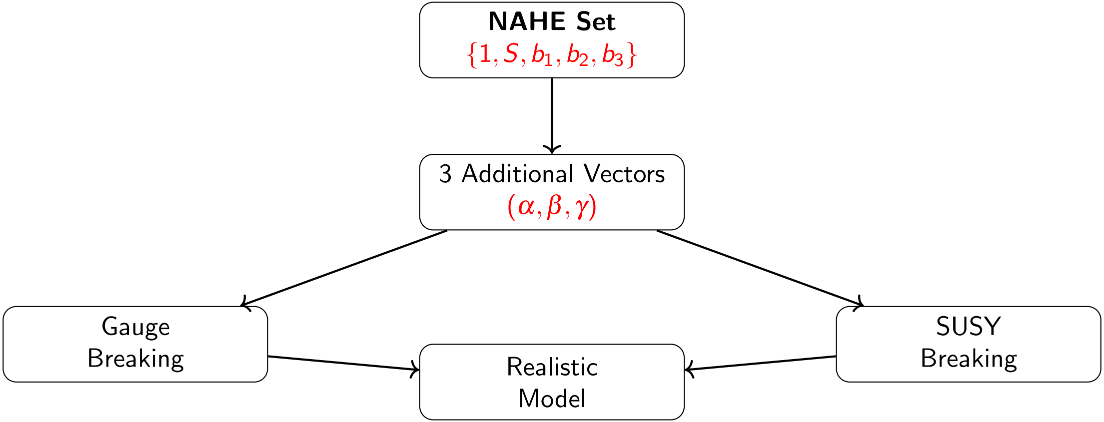
$$\begin{aligned} V_{1-loop} &= -\frac{1}{2} \frac{\mathscr{M}^4}{(2\pi)^4} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau, \bar{\tau}; T^{(i)}, U^{(i)}) \\ &= -\frac{1}{2} \frac{\mathscr{M}^4}{(2\pi)^4} \int_{\mathscr{F}} \frac{d^2\tau}{\tau_2^3} \sum a_{mn} q^m \bar{q}^n \quad ; \quad q = e^{2i\pi\tau} \\ &= \sum a_{mn} I_{mn} \ , \end{aligned}$$

Vacuum energy

Cosmological Constants

- **Positive** — de Sitter (dS) space
- **Zero** — Unbroken SUSY
- **Negative** — Anti-de Sitter (AdS) space

Example: (S-Model)



(S-Model)

$$1=\left\{\psi^{1,2},\chi^{1,\dots 6}y^{1,\dots 6},w^{1,.6}\mid \bar{y}^{3,.6},\bar{w}^{1,.6},\bar{\psi}^{1,\dots 5},\bar{\eta}^{1,2,3},\bar{\phi}^{1,.8}\right\}$$
$$S=\left\{\psi^{1,2},\chi^{1,\dots 6}\right\}$$
$$b_1=\left\{\psi^{1,2},\chi^{1,2},y^{3,6}\mid \bar{y}^{3,\dots 6},\bar{\psi}^{1,.5},\bar{\eta}^{1}\right\}$$
$$b_2=\left\{\psi^{1,2},\chi^{3,4},y^{1,2},\omega^{5,6}\mid \bar{y}^{1,2},\bar{\omega}^{5,6},\bar{\psi}^{1,\dots 5},\bar{\eta}^{2}\right\}$$
$$b_3=\left\{\psi^{1,2},\chi^{5,6},\omega^{1,.4}\mid \bar{\omega}^{1,.4},\bar{\psi}^{1,..5},\bar{\eta}^{3}\right\}$$
$$\alpha=\left\{y^3y^6,\omega^6\bar{\omega}^6,\omega^3\bar{\omega}^3,\bar{y}^1\bar{\omega}^5,\bar{\omega}^2\bar{\omega}^4,\bar{\psi}^{1,\dots,3},\bar{\phi}^{1,\dots,4}\right\}$$
$$\beta=\left\{y^1\omega^5,y^5\bar{y}^5,\omega^1\bar{\omega}^1,\bar{y}^3\bar{y}^6,\bar{\omega}^2\bar{\omega}^4,\bar{\psi}^{1,\dots,3},\bar{\phi}^{1,\dots,4}\right\}$$
$$\gamma=\left\{\omega^2\omega^4,y^2\bar{y}^2,y^4\bar{y}^4,\bar{y}^3\bar{y}^6,\bar{y}^1\bar{\omega}^5,\frac{1}{2}\left(\bar{\psi}^{1,\dots,5},\bar{\eta}^{1,2,3},\bar{\phi}^{1,5,67}\right),\bar{\phi}^{3,4}\right\}$$

(S-Model)

with the set of GGSO phases given by:

$$
\begin{array}{c} \\ 1 \\ S \\ \\ b_1 \\ b_2 \\ b_3 \\ \\ \alpha \\ \beta \\ \gamma \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma \end{array} \\
\left(\begin{array}{cccccccc}
1 & 1 & -1 & -1 & -1 & 1 & 1 & i \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
 & & & & & & & \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & i \\
 & & & & & & & \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1
\end{array}\right)
\end{array} .
$$

(S-Model)

by the following modification:

$$C\begin{bmatrix} S \\ \alpha \end{bmatrix} \to 1, \text{ and } C\begin{bmatrix} S \\ \beta \end{bmatrix} \to 1$$

Then recalculated the PF and by integrating it we get:

$$V = -15.5792$$

which gives us the cosmological constant:

$$\Lambda = 0.00499799 \mathscr{M}_s^4 \ .$$

($\tilde{S}$–Model)

NAHE Set

$\{1, S, b_1, b_2, b_3\}$

↓

3 Additional Vectors

$\{\alpha, \beta, \gamma\}$

↓

$$\alpha = \{y^3y^6,\ \bar{y}^3\bar{y}^6,\ w^6\bar{w}^6,\ \bar{y}^1\bar{w}^5,\ w^3\bar{w}^3,\ \bar{w}^2\bar{w}^4,\ \bar{\psi}^{1,2,3},\ \bar{\eta}^1,\ \bar{\phi}^{1,2}\}$$

$$\beta = \{y^5\bar{y}^5,\ \bar{y}^3\bar{y}^6,\ y^1w^5,\ \bar{y}^1\bar{w}^5,\ w^1\bar{w}^1,\ \bar{w}^2\bar{w}^4,\ \bar{\psi}^{1,2,3},\ \bar{\eta}^2,\ \bar{\phi}^{3,4}\}$$

$$\gamma = \{y^4\bar{y}^4,\ y^2\bar{y}^2,\ w^2w^4,\ \bar{\psi}^{1,2,3,4,5} = \tfrac{1}{2},\ \bar{\eta}^{1,2,3} = \tfrac{1}{2},\ \bar{\phi}^{5,6,7,8} = \tfrac{1}{2}\}$$

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($\tilde{S}$–Model)

$$
\begin{array}{c} \\ 1 \\ S \\ \\ b_1 \\ b_2 \\ b_3 \\ \\ \alpha \\ \beta \\ \gamma \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma \end{array} \\
\left(\begin{array}{cccccccc}
1 & 1 & -1 & -1 & -1 & -1 & -1 & i \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
 & & & & & & & \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & i \\
-1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 \\
 & & & & & & & \\
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
-1 & -1 & 1 & 1 & -1 & -1 & -1 & -i
\end{array}\right)
\end{array} .
$$

($\tilde{S}$–Model)

by the following modification:

$$S \to \tilde{S} = \{\psi^\mu, \chi^{1,2}, \chi^{3,4}, \chi^{5,6} \mid \bar{\phi}^{3,4,5,6}\} \ ,$$

$$C\begin{bmatrix}\tilde{S}\\ \gamma\end{bmatrix} \to i \ , \ C\begin{bmatrix}\beta\\ \tilde{S}\end{bmatrix} \to 1.$$

We develop the matrix in the following way:

$$C\begin{bmatrix}\tilde{S}\\ \tilde{S}\end{bmatrix} \to -1 \ , \ C\begin{bmatrix}\alpha\\ \alpha\end{bmatrix} \to -1 \ , \ C\begin{bmatrix}\alpha\\ 1\end{bmatrix} \to 1 \ , \ C\begin{bmatrix}1\\ \alpha\end{bmatrix} \to 1.$$

$$\Lambda = -0.0199\mathscr{M}_s^4 \ .$$

Conclusions and Future Work

- Break SUSY either explicitly or a' la Scherk-Schwarz.
- Asymmetric boundary conditions as valid as symmetric one.
- Moduli fixing (except the dilaton).
- The value of this one–loop potential can be positive or negative, relating to De Sitter or anti–De Sitter spaces.
- Future work; will focus on both symmetric and asymmetric BCs not just for projecting out the moduli but also projecting out charge field.

References

[1] E. Basaad, L. A. Detraux, A. R. D. Avalos, A. E. Faraggi and B. Percival. *'Vacuum energy in nonsupersymmetric quasi-realistic heterotic string vacua with fixed moduli'* In: Eur. Phys. Jour. C85 (2025) 209. doi : 10.1140/epjc/s10052-024-13733-2.

[2] A.E.Faraggi. *Moduli fixing in realistic string vacua* In: Nucl.Phys.B728 (2005) 83. doi : 10.1016/j.nuclphysb.2005.08.028.

[3] A. E. Faraggi, V. G. Matyas, and B. Percival. *Towards Classification of $N = 1$ and $N = 0$ Flipped SU(5) Asymmetric 2×2 Heterotic String Orbifolds* In: Phys.Rev.D 106 (2022), doi : 10.1103/PhysRevD.106.026011.

[4] Zihni Kaan Baykara, Houri-Christina Tarazi, and Cumrun Vafa. *New Non-Supersymmetric Tachyon-Free Strings.* 5 (2024), arXiv:2406.00185.

THANK YOU FOR YOUR ATTENTION.!!