# Superradiance of Non-topological Solitons


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P. M. Saffin, **QXX**, and S.-Y. Zhou, PRL 131 (2023) 11 11, (2212.03269)
H.-Y. Gao, P. M. Saffin, Y.-J. Wang, **QXX**, and S.-Y. Zhou, SCPMA 67 260413 (2024), (2306.01868)
G.-D. Zhang, C.-H. Li, **QXX**, and S.-Y. Zhou, Phys.Rev.D 111 (2025) 10, 103027 (2503.04657)

PASCOS 2025

# Outline

- Non-topological solitons & superradiance
- Superradiance of Non-topological solitons
- Coupling to other fields

  scalar field: Friedberg-Lee-Sirlin solitons

  gravitational field: boson stars

Rosen 1968
Friedberg, Lee, and Sirlin 1976
Coleman 1985

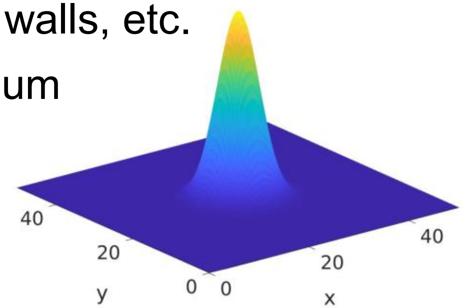
# Non-topological solitons / Q-balls

see Bowen Fu's talk next

## Localized, extended, and stationary

Topological solitons: t'Hooft-Polyakov monopoles, domain walls, etc.

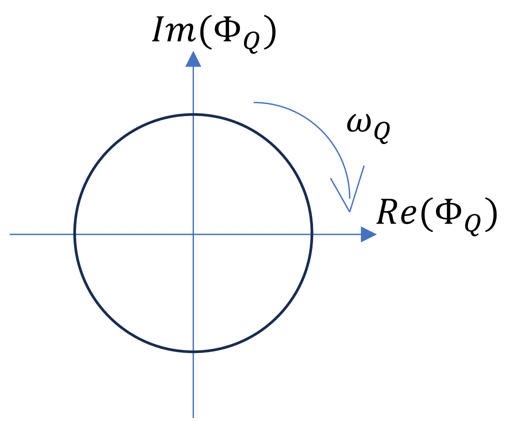
Non-topological solitons: boundaries are in the same vacuum




## Q-balls

$$\mathcal{L} = -\partial^\mu \Phi^* \partial_\mu \Phi - V(|\Phi|)\ ,$$ Noether charge Q: $\Phi \to \Phi e^{i\theta}$

$$\Phi_Q = \frac{1}{\sqrt{2}} f(r) e^{-i\omega_Q t},\ f(r\to\infty) = 0$$




Attractive interactions in the potential

Minimal of the energy functional with fixed Q

# Black hole superradiance

Dicke 1954

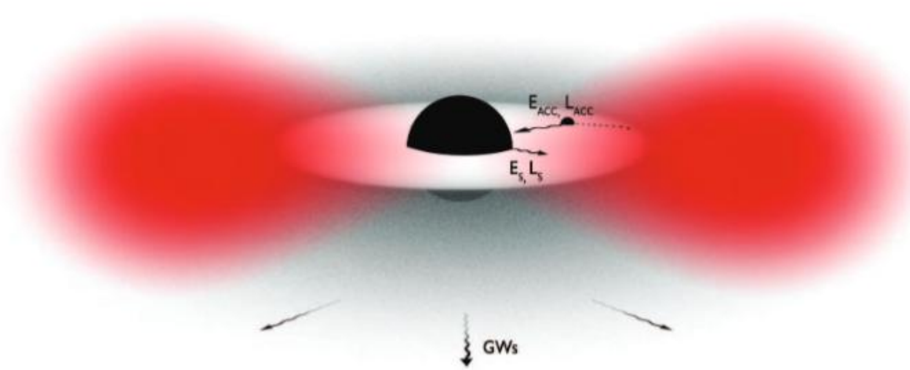
Radiation enhancement effect

Kerr BHs $\psi \sim e^{-i\omega t + im\varphi}, \quad 0 < \omega < m\Omega_H$

Superradiant instability

bosonic clouds and GWs

cosmic particle detectors




$$f_{\rm GW} \sim 5\ {\rm kHz} \left( \frac{\mu\hbar}{10^{-11}{\rm eV}} \right)$$

Observational results of supermassive BHs

Priyanka Sarmah's
talk on Monday

arXiv:1501.06570

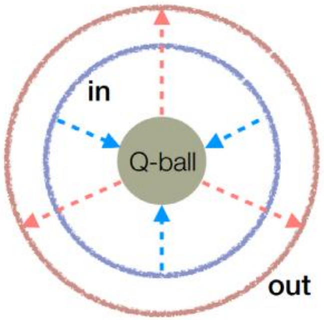
# Do solitons superradiate?

# Perturbations on a Q-ball

Fixed background perturbations $\Phi = \Phi_Q + \phi$

$$\partial^2\phi - U(r)\phi - W(r)e^{-2i\omega_Q t}\phi^* = 0$$

coherent internal rotation




Two coupled perturbative modes

$$\phi = \eta_+(r)e^{-i\omega_+ t} + \eta_-(r)e^{-i\omega_- t},\ \omega_\pm = \omega_Q \pm \omega$$

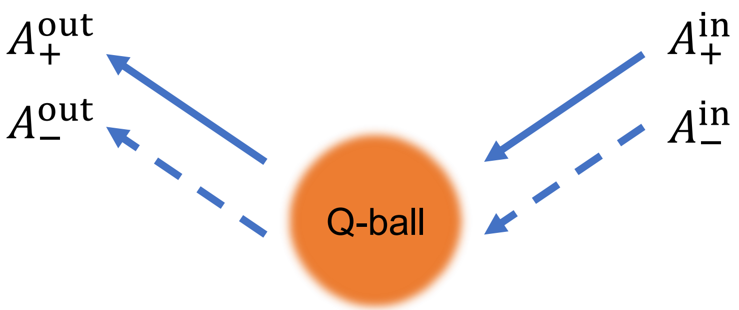
$$\eta''_\pm + \frac{2}{r}\eta'_\pm + (\omega_\pm^2 - U)\eta_\pm - W\eta^*_\mp = 0,\ W(r\to\infty) = 0$$

Energy can be transferred between modes

# Scattering and particle number conservation

$$\eta_\pm(r\to\infty) = \frac{1}{\sqrt{|k_\pm| r}}(A_\pm^{\rm in} e^{-ik_\pm r} + A_\pm^{\rm out} e^{ik_\pm r}),\; k_\pm = {\rm sign}(\omega_\pm)(\omega_\pm^2-\mu^2)^{1/2}$$

Scattering states: $|\omega| > |\omega_Q| + \mu$




Effective Lagrangian

$$L(\xi_s,\xi_s') = \sum_{s=\pm}\left[|\xi_s'|^2 - (\omega_\pm^2 - U)|\xi_s|^2\right] + W(\xi_+^*\xi_-^* + \xi_+\xi_-),\; \xi_s = r\eta_s$$

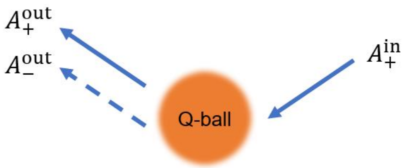
U(1) symmetry $\xi_+ \to \xi_+ e^{i\alpha}, \xi_- \to \xi_- e^{-i\alpha}$

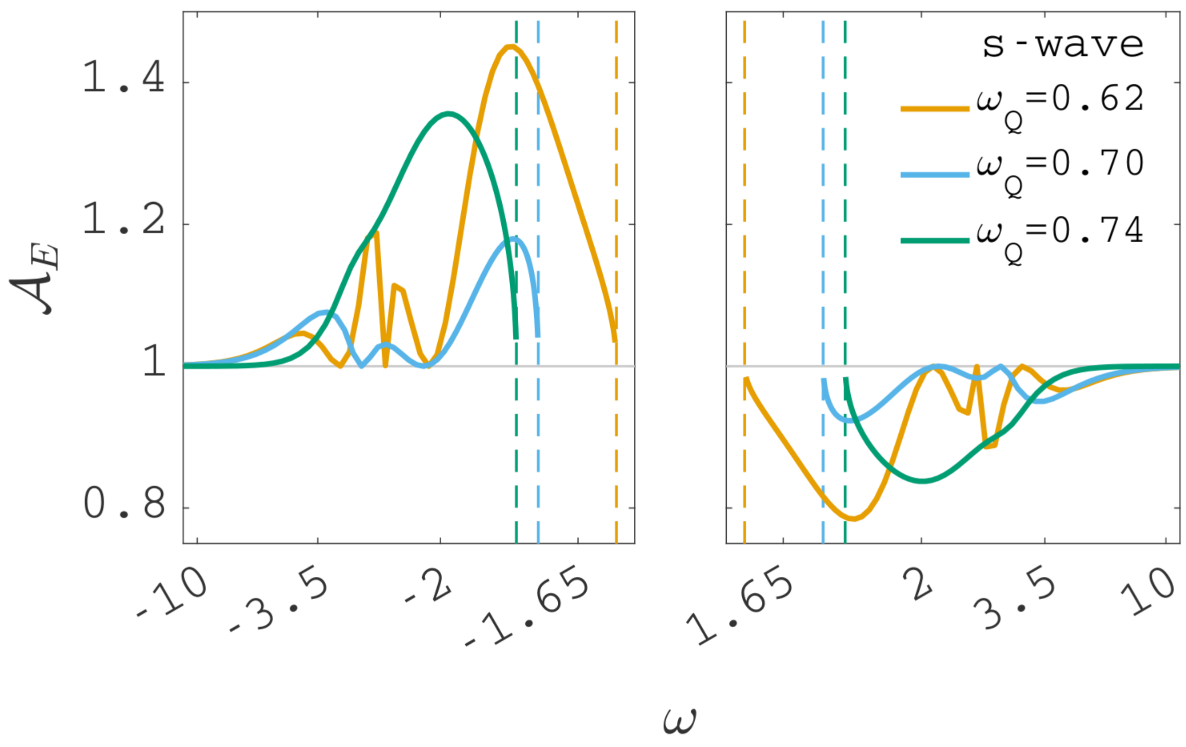
Particle number conservation $|A_+^{\rm out}|^2 + |A_-^{\rm out}|^2 = |A_+^{\rm in}|^2 + |A_-^{\rm in}|^2$

Saffin, **QXX**, and Zhou, PRL 131 (2023) 11 11

# Energy amplification: one in-going mode

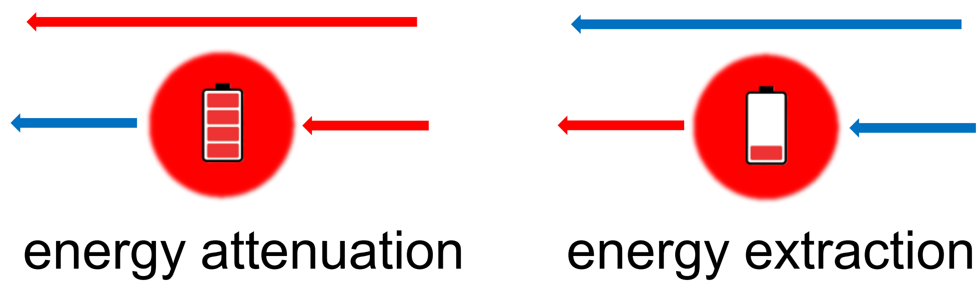
Amplification factor $\mathcal{A}_E = \left| \frac{F_E^{\text{out}}}{F_E^{\text{in}}} \right| = \frac{\omega_+ |A_+^{\text{out}}|^2 - \omega_- |A_-^{\text{out}}|^2}{\omega_+ |A_+^{\text{in}}|^2 - \omega_- |A_-^{\text{in}}|^2}$



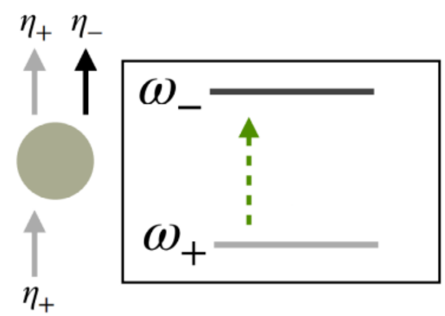
# What happened?

Elastic and non-elastic scattering




arXiv:2412.13885

For $Q > 0, \omega < 0, |\omega_+| < |\omega_-|$ , low energy states are raised to high energy states



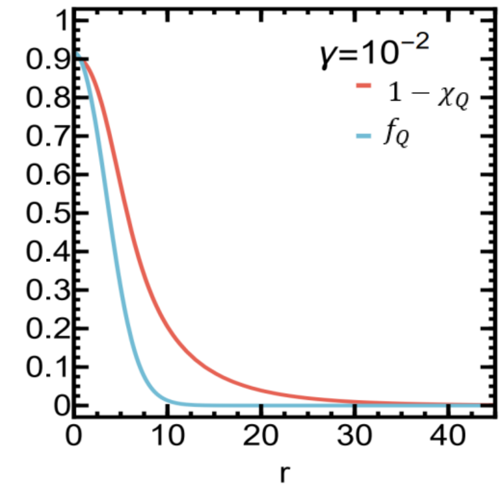

Superradiant scattering if the in-going wave has opposite charges to the Q-ball

Zhang, Li, **QXX**, and Zhou, PRD 111 (2025) 10, 103027

# Coupling to a real scalar: Friedberg-Lee-Sirlin (FLS) solitons

Friedberg, Lee, and Sirlin 1976

$$\mathcal{L} = -\partial^\mu \Phi^* \partial_\mu \Phi - \frac{1}{2}\partial^\mu \chi \partial_\mu \chi - \frac{\gamma e^2}{8}(\chi^2 - v^2)^2 - e^2 \chi^2 \Phi^* \Phi$$



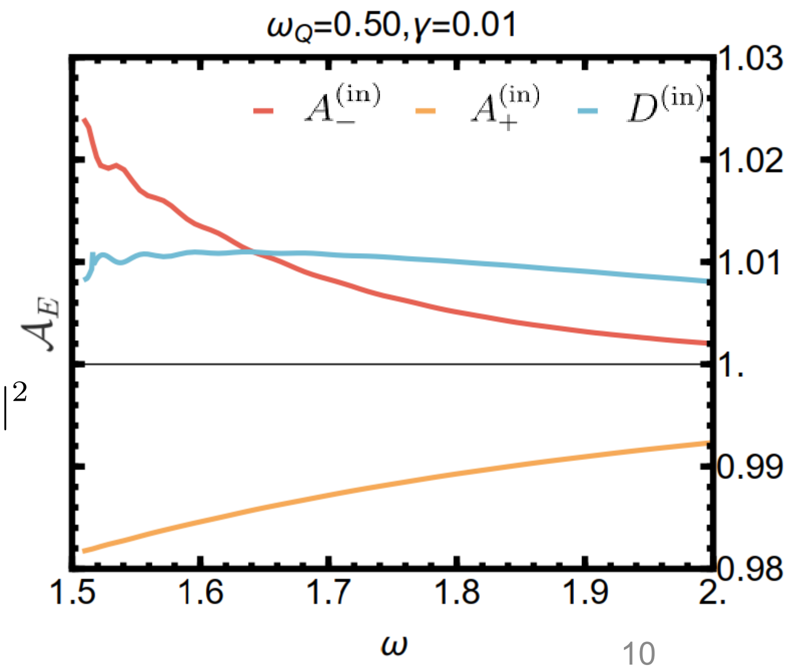

## Renormalizable model

$$\Phi_Q = f_Q(r) e^{-i\omega_Q t}, \quad \chi_Q = \chi_Q(r)$$

## Modes coupling

$$\eta(\omega_Q + \omega) \leftrightarrow \chi(\omega) \leftrightarrow \eta(\omega_Q - \omega)$$

$A_+^{(\mathrm{in})}, A_+^{(\mathrm{out})}$ → $\eta(\omega_Q + \omega)$; $D^{(\mathrm{in})}, D^{(\mathrm{out})}$ → $\chi(\omega)$; $A_-^{(\mathrm{in})}, A_-^{(\mathrm{out})}$ → $\eta(\omega_Q - \omega)$

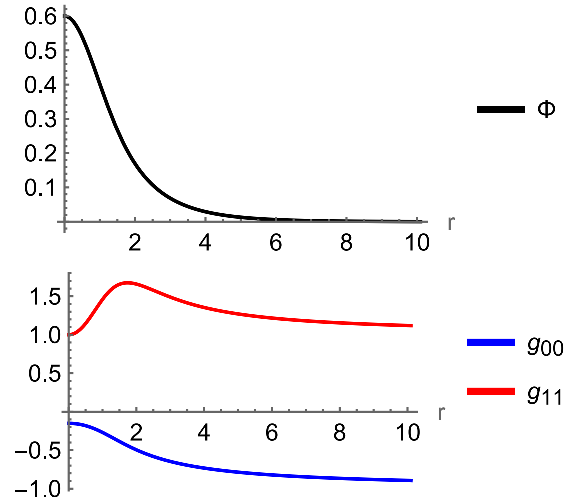



## Particle number conservation

$$|A_+^{\mathrm{out}}|^2 + |A_-^{\mathrm{out}}|^2 + 2|D^{\mathrm{out}}|^2 = |A_+^{\mathrm{in}}|^2 + |A_-^{\mathrm{in}}|^2 + 2|D^{\mathrm{in}}|^2$$

Gao, Saffin, Wang, **QXX**, and Zhou, SCPMA 67 260413 (2024)

# Coupling to the gravitational field: Boson stars




D. J. Kaup 1968

$$\mathcal{L} = \sqrt{-g}\left(\frac{R}{16\pi G} - \nabla^{\mu}\Phi^{*}\nabla_{\mu}\Phi - V(|\Phi|)\right)$$

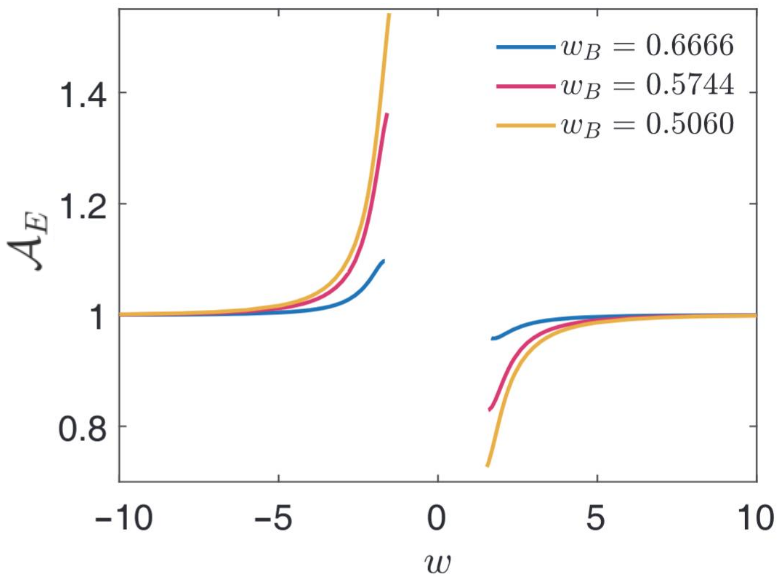
## Gravitational regular solution

$$\Phi_B = \frac{1}{\sqrt{2}} f(r) e^{-i\omega_B t},$$

## Modes coupling

$$\eta(\omega_Q + \omega) \leftrightarrow \eta(\omega_Q - \omega)$$

$$A_+^{(\mathrm{in})}, A_+^{(\mathrm{out})} \qquad A_-^{(\mathrm{in})}, A_-^{(\mathrm{out})}$$




## Particle number conservation

$$|A_+^{\mathrm{out}}|^2 + |A_-^{\mathrm{out}}|^2 = |A_+^{\mathrm{in}}|^2 + |A_-^{\mathrm{in}}|^2$$

# Summary and outlook

Internal field space rotation can induce superradiance, except for real space rotation.

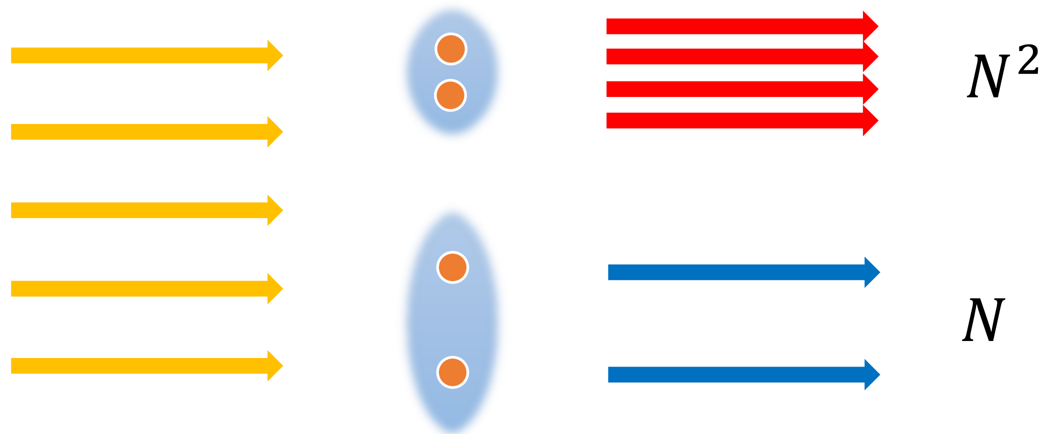
Generalization to various types of non-topological solitons: FLS solitons, boson stars, and more.

Future research: instability mechanism, potential application in particle cosmology (compact objects, new particles, etc.)

*Thank You !*

*Backup*

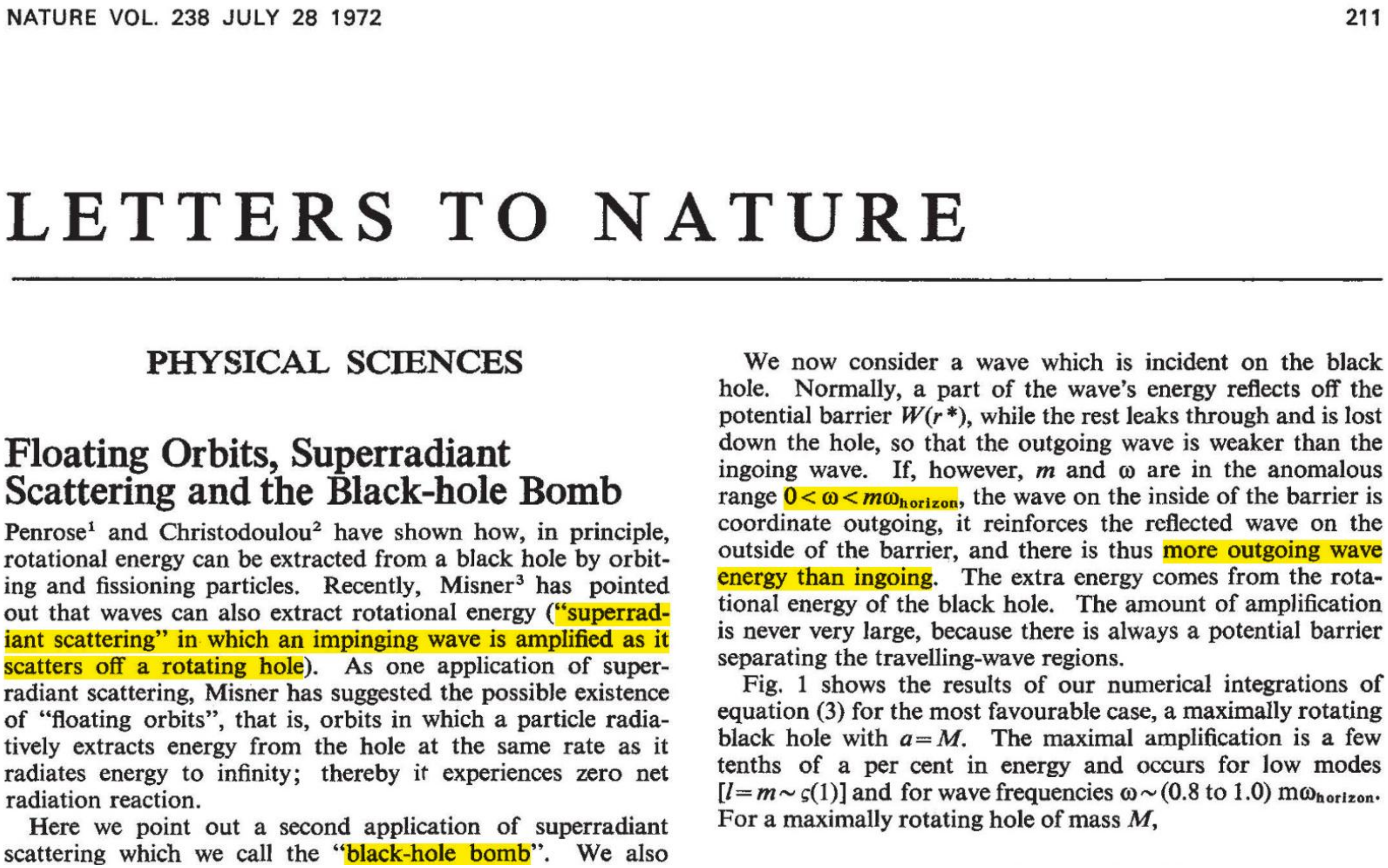
# Superradiance in quantum optics




*For want of a better term, a gas which is radiating strongly because of coherence will be called "super-radiant".*

*—Robert H. Dicke, Phys. Rev. 93 99 (1954).*

# Superradiance: history



NATURE VOL. 238 JULY 28 1972 211

# LETTERS TO NATURE

## PHYSICAL SCIENCES

### Floating Orbits, Superradiant Scattering and the Black-hole Bomb

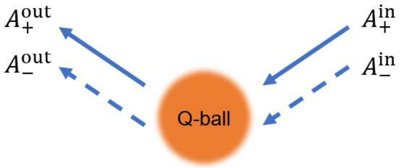
Penrose[1] and Christodoulou[2] have shown how, in principle, rotational energy can be extracted from a black hole by orbiting and fissioning particles. Recently, Misner[3] has pointed out that waves can also extract rotational energy ("superradiant scattering" in which an impinging wave is amplified as it scatters off a rotating hole). As one application of superradiant scattering, Misner has suggested the possible existence of "floating orbits", that is, orbits in which a particle radiatively extracts energy from the hole at the same rate as it radiates energy to infinity; thereby it experiences zero net radiation reaction.

Here we point out a second application of superradiant scattering which we call the "black-hole bomb". We also

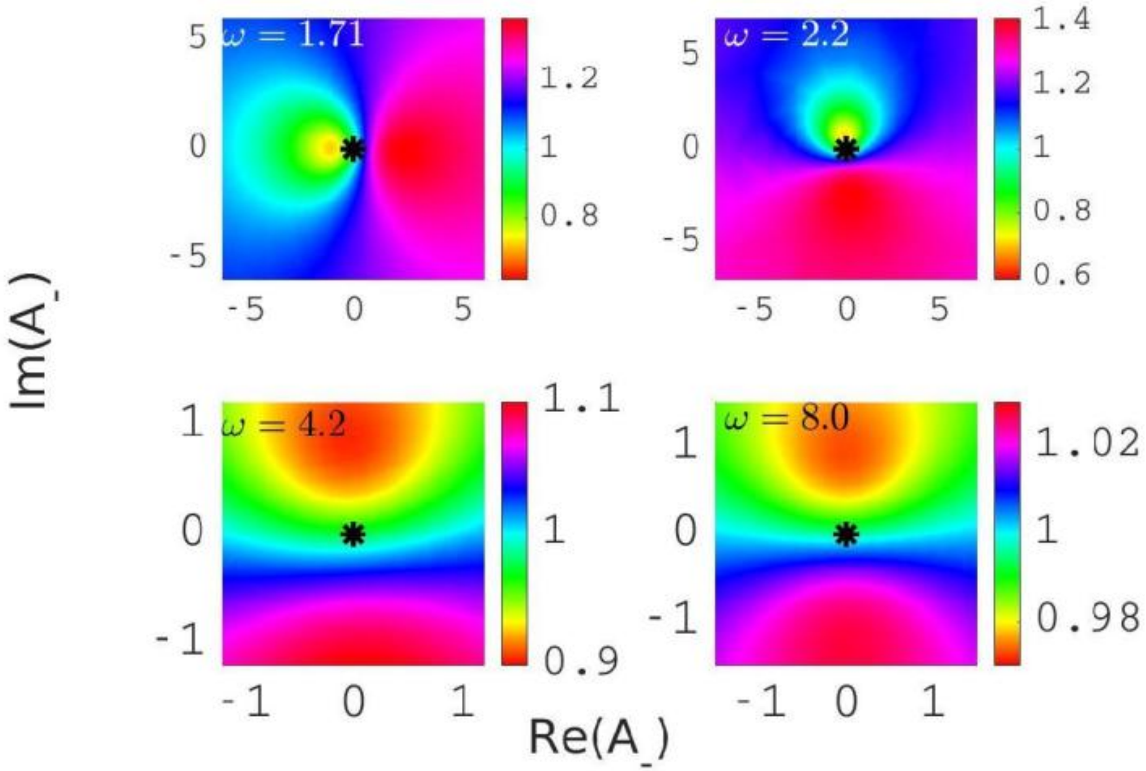
We now consider a wave which is incident on the black hole. Normally, a part of the wave's energy reflects off the potential barrier $W(r^*)$, while the rest leaks through and is lost down the hole, so that the outgoing wave is weaker than the ingoing wave. If, however, $m$ and $\omega$ are in the anomalous range $0<\omega<m\omega_{\text{horizon}}$, the wave on the inside of the barrier is coordinate outgoing, it reinforces the reflected wave on the outside of the barrier, and there is thus more outgoing wave energy than ingoing. The extra energy comes from the rotational energy of the black hole. The amount of amplification is never very large, because there is always a potential barrier separating the travelling-wave regions.

Fig. 1 shows the results of our numerical integrations of equation (3) for the most favourable case, a maximally rotating black hole with $a=M$. The maximal amplification is a few tenths of a per cent in energy and occurs for low modes $[l=m\sim\varsigma(1)]$ and for wave frequencies $\omega\sim(0.8 \text{ to } 1.0)\ m\omega_{\text{horizon}}$. For a maximally rotating hole of mass $M$,

# Energy amplification: two in-going modes

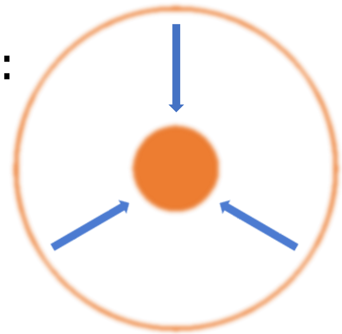


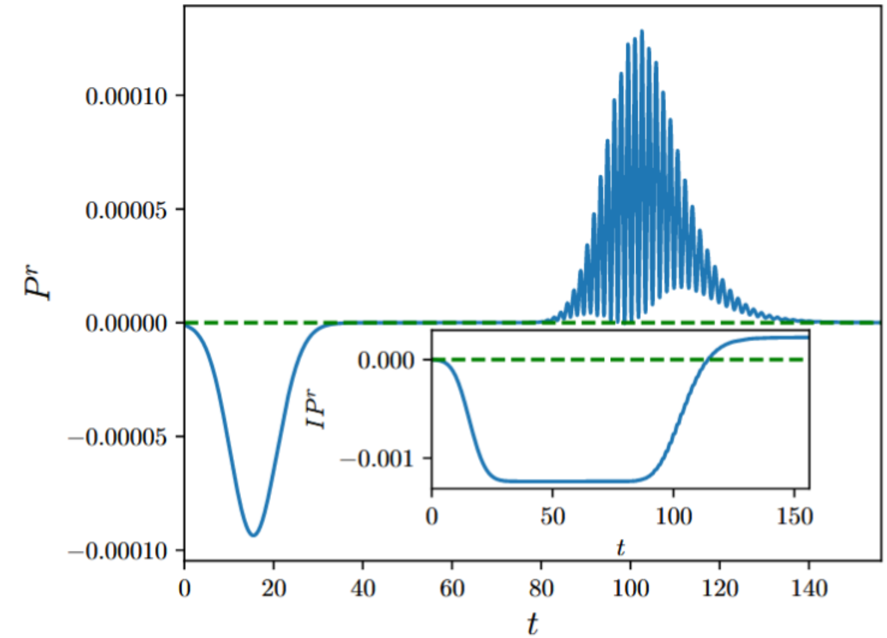

$A_+^{\rm in} = 1$, varying $A_-^{\rm in}$

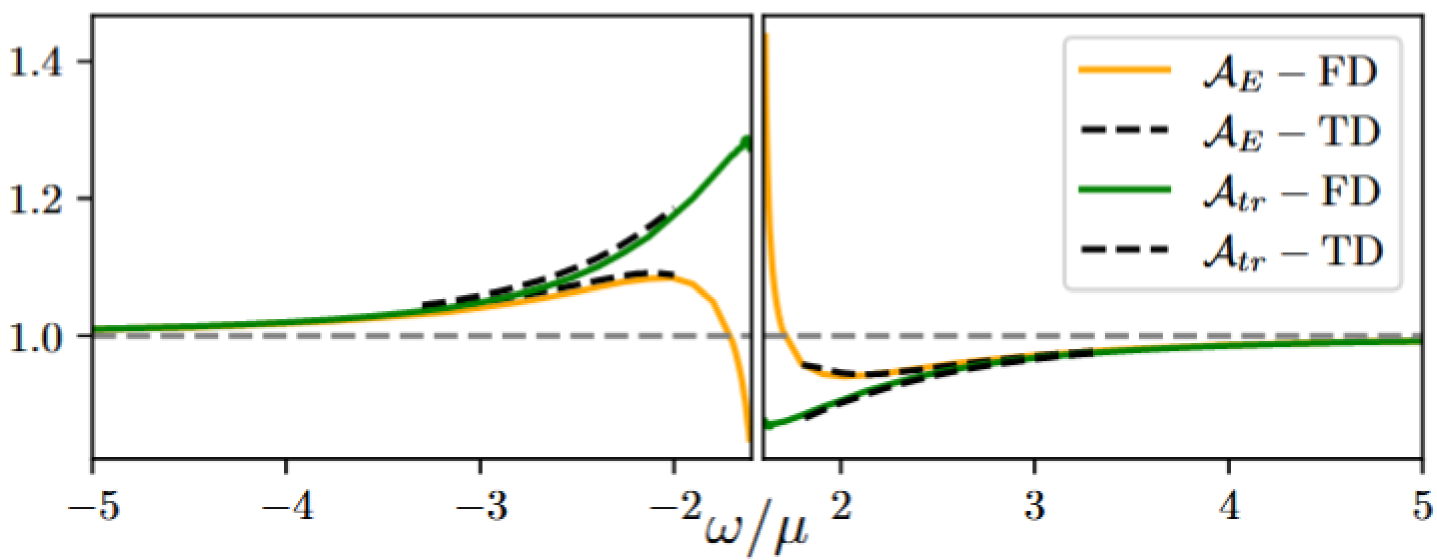


# Real-time results

$t = 0$:






arXiv:2412.01894