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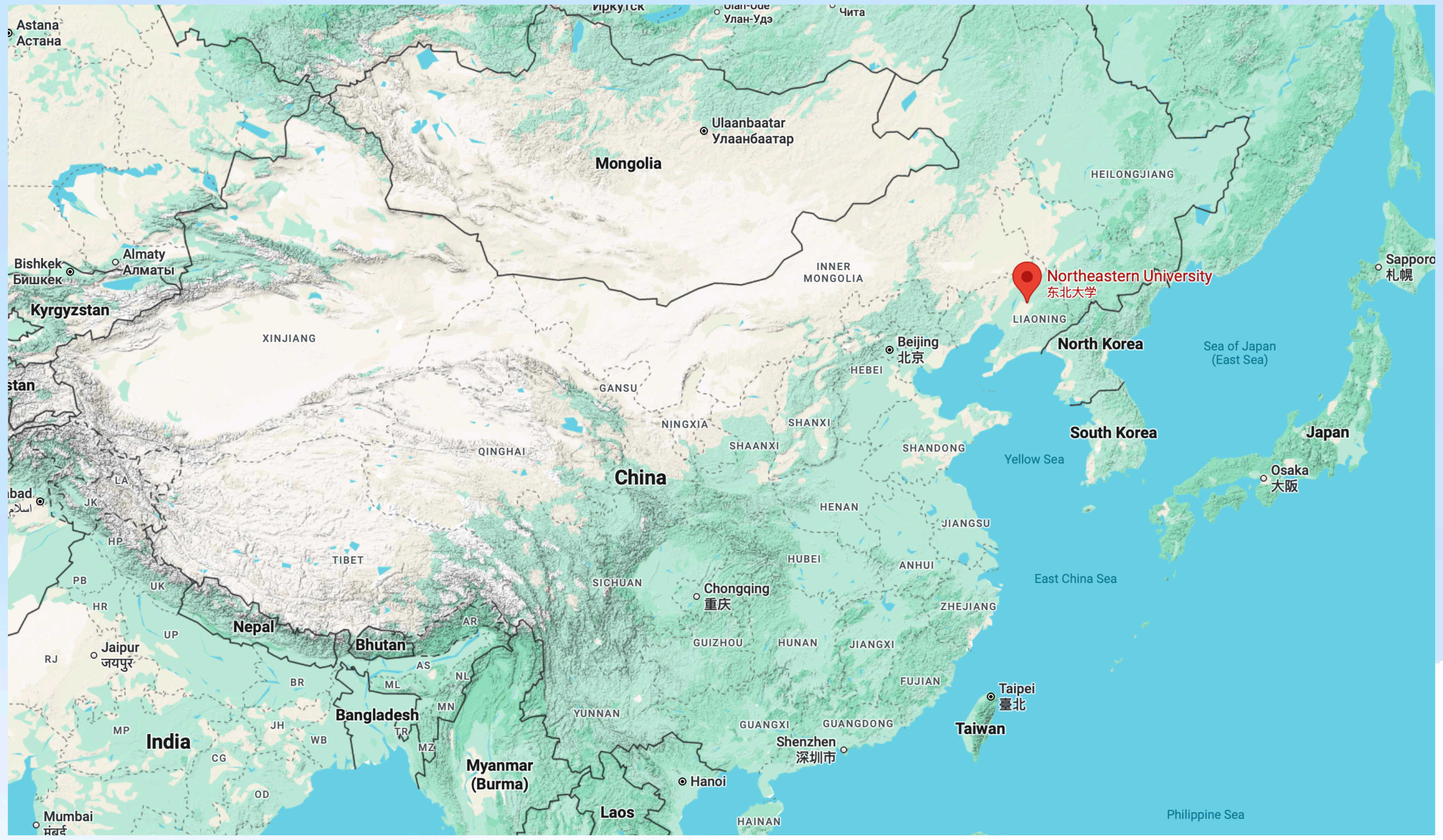
Non-Abelian Domain walls



Bowen Fu (付博文) **24 July 2025**

The 30th International Symposium on Particles, Strings and Cosmology (PASCOS 2025)

Based on **BF**, S. F. King, L. Marsili, S. Pascoli, J. Turner, Y-L Zhou, 2409.16359

Domain walls

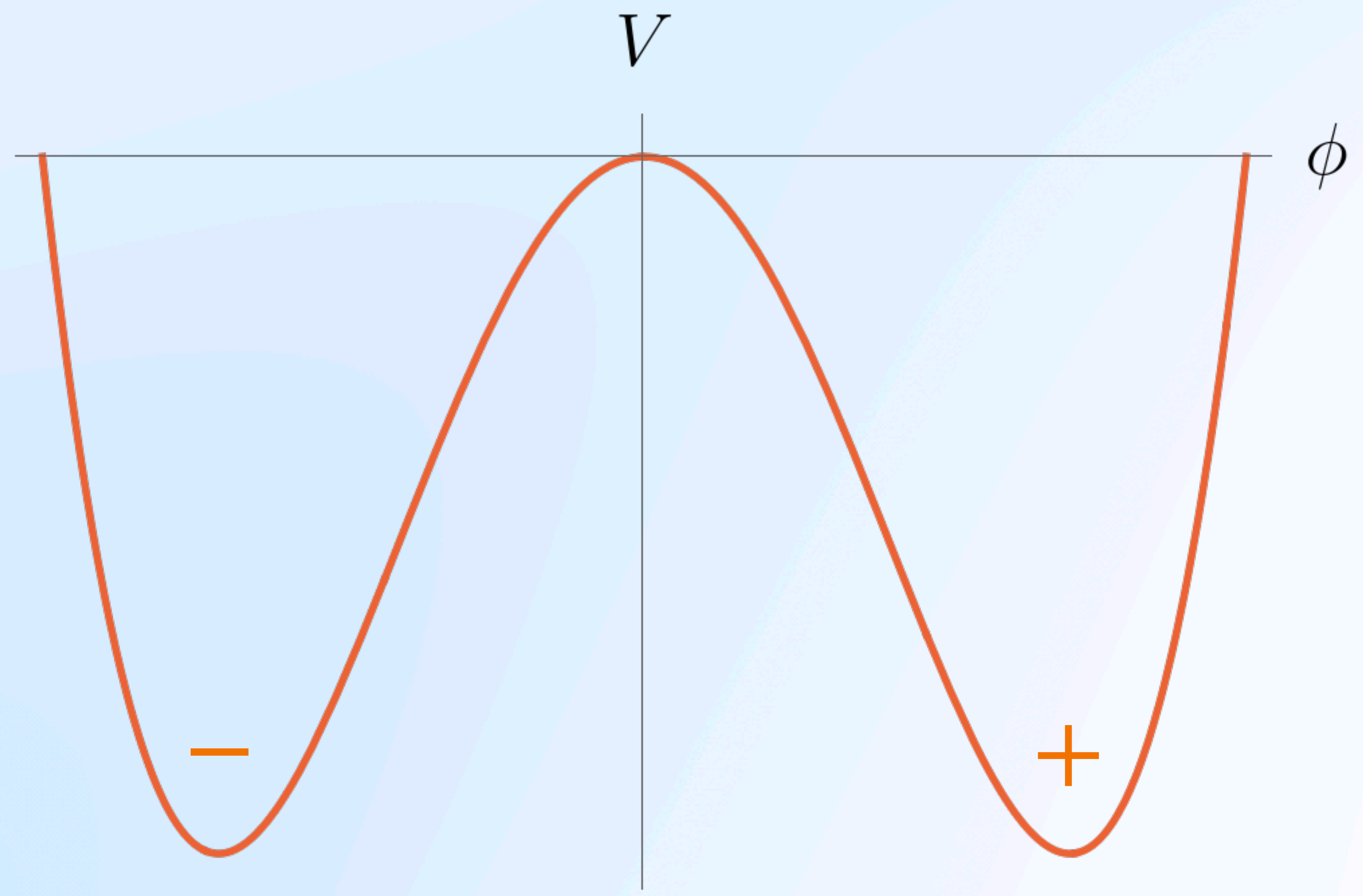
Domain wall formation

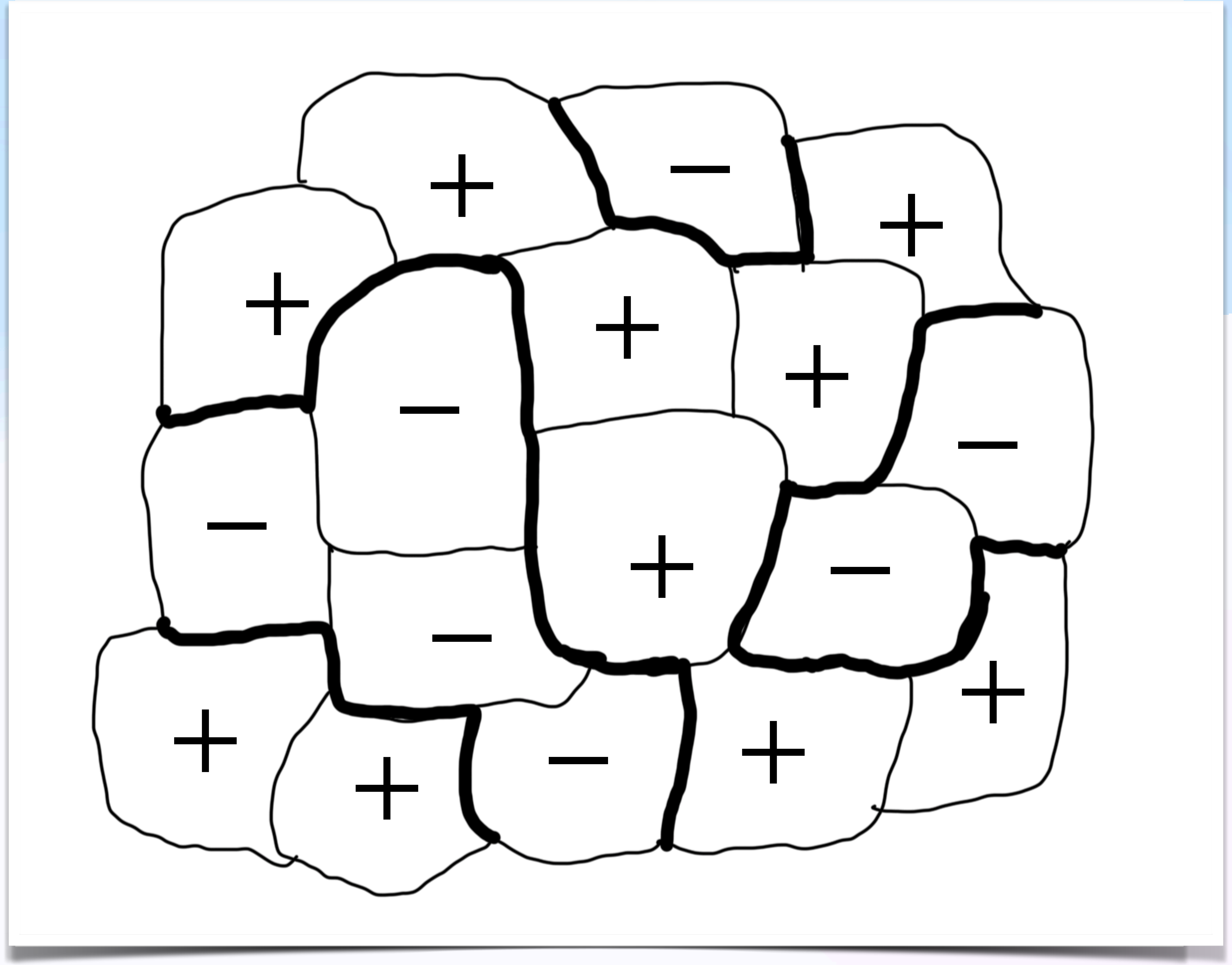
Kibble mechanism:

Z_2 - a simplest case

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\langle\phi\rangle = \pm v \qquad v = \sqrt{\mu^2/\lambda}$$

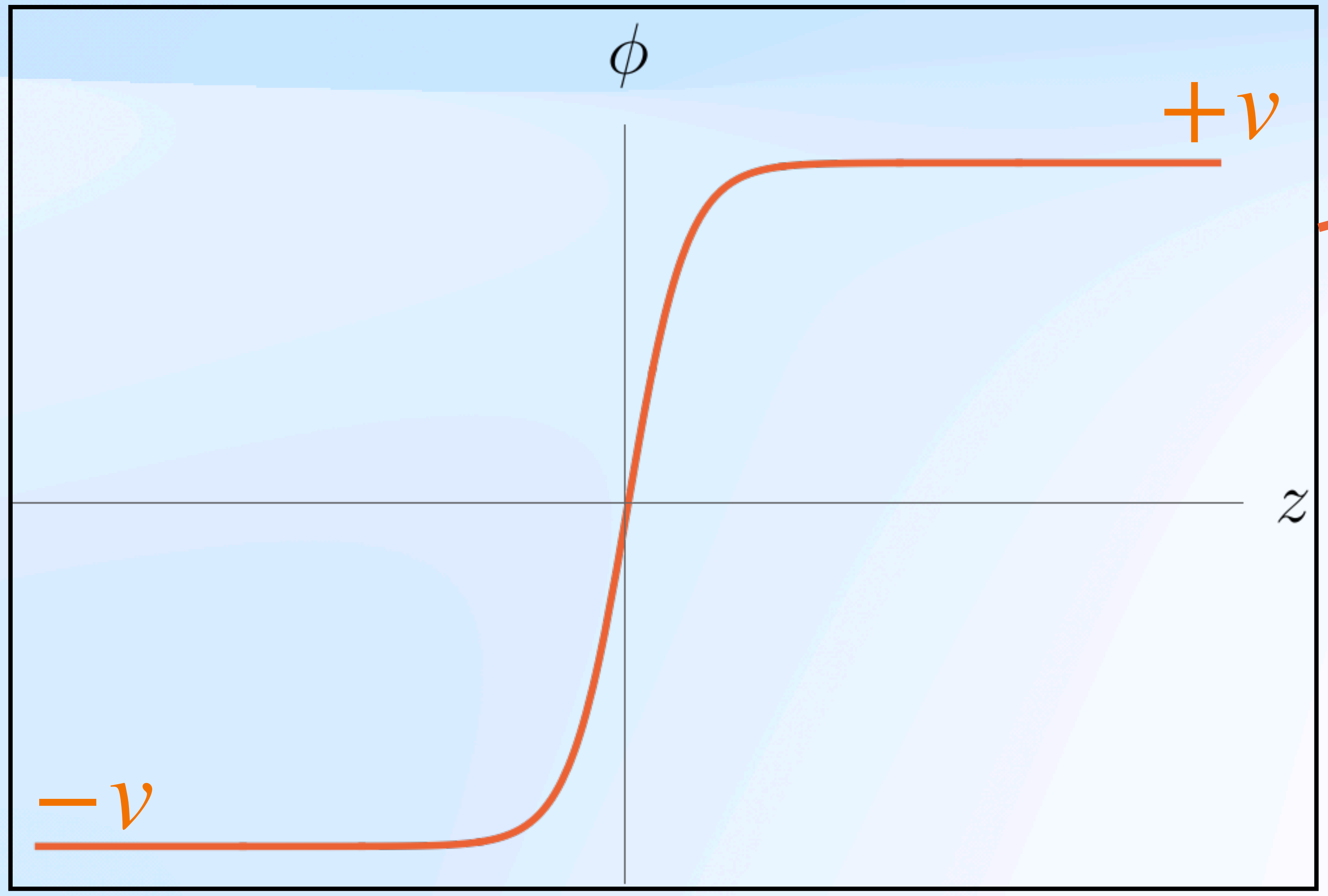


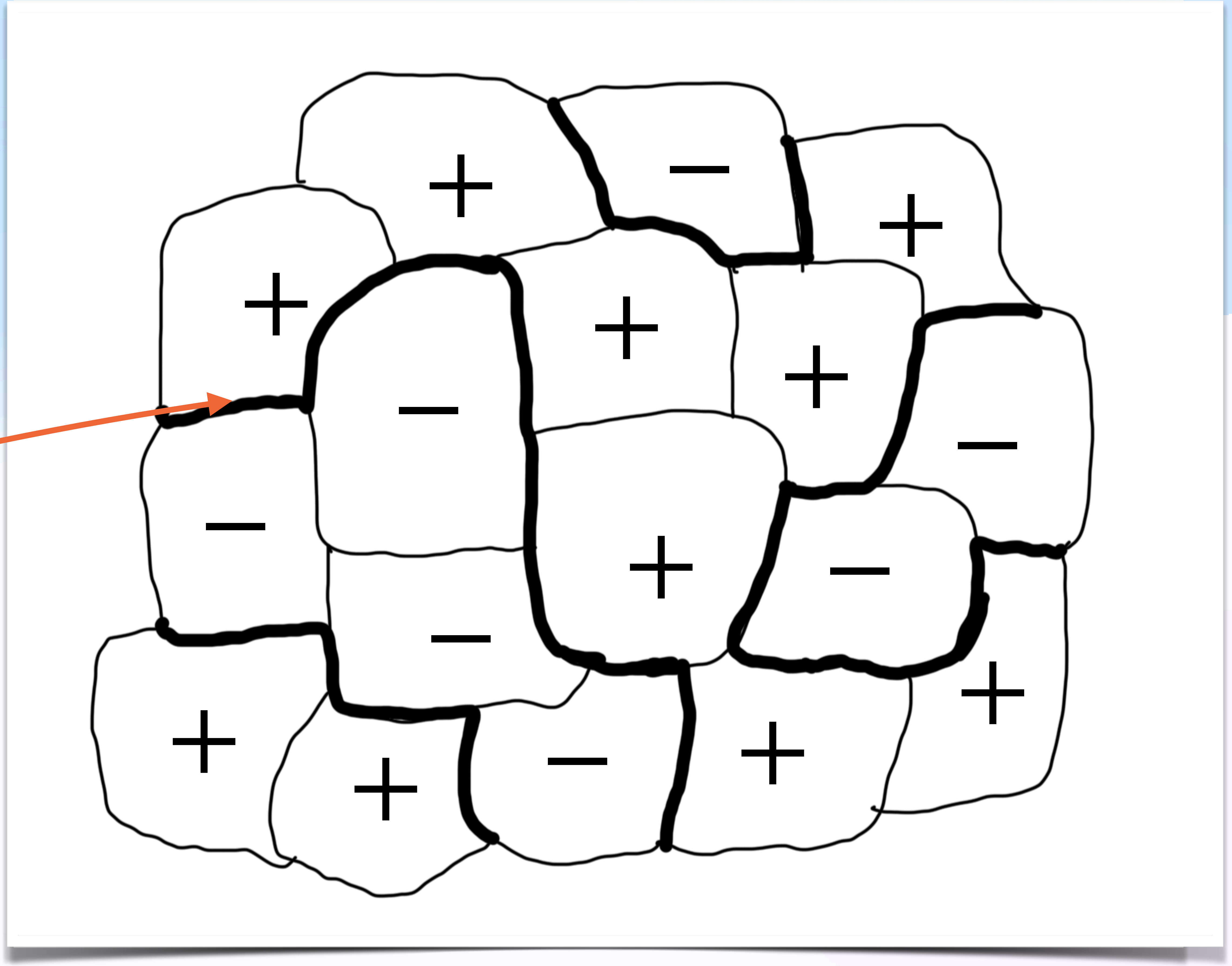


Domain wall formation

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\partial V}{\partial \phi}, \quad \phi(\pm\infty) = \pm v$$

$$\phi(z) = v \tanh \frac{z}{\Delta} \qquad \Delta = \sqrt{\frac{2}{\lambda v^2}}$$

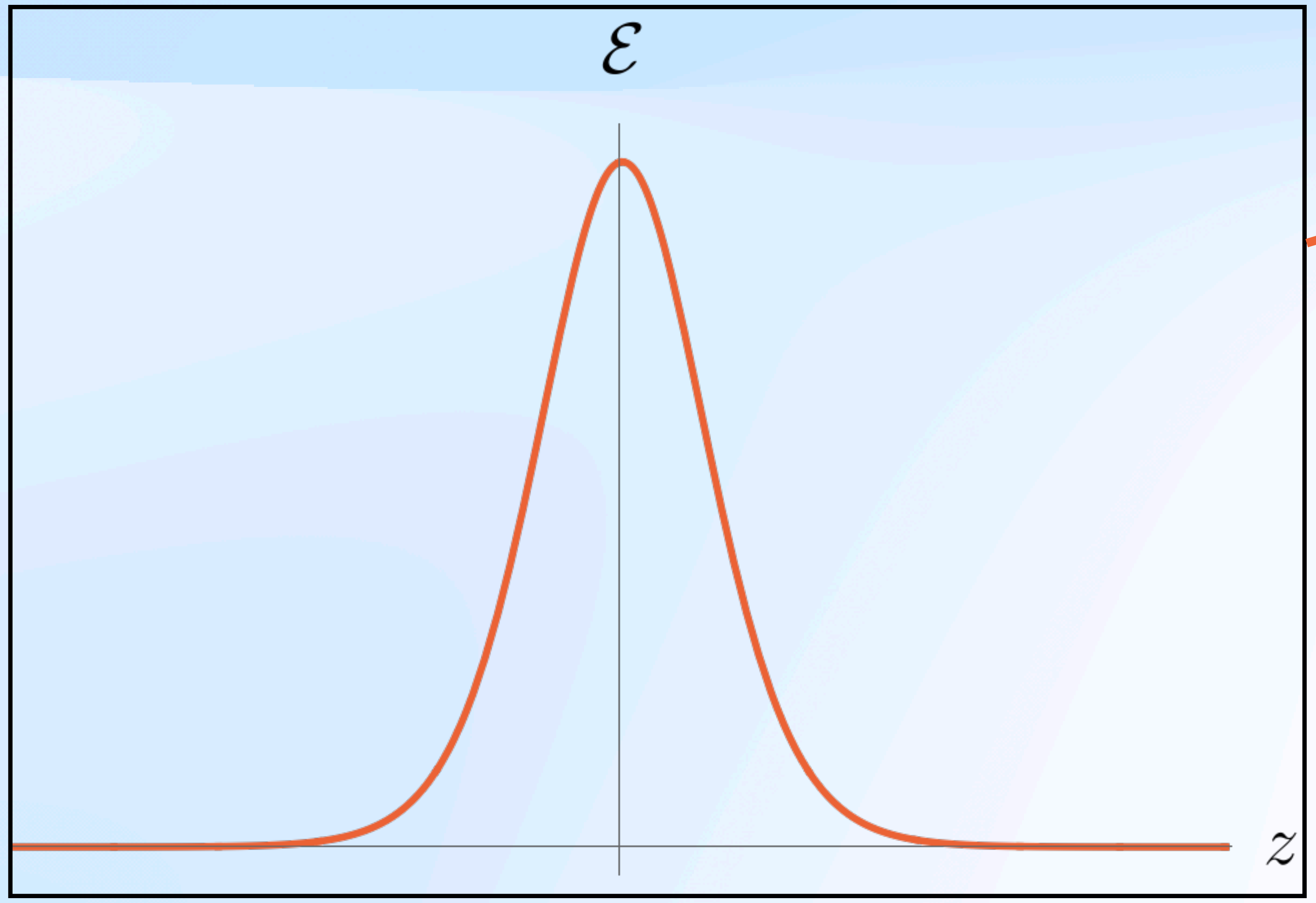


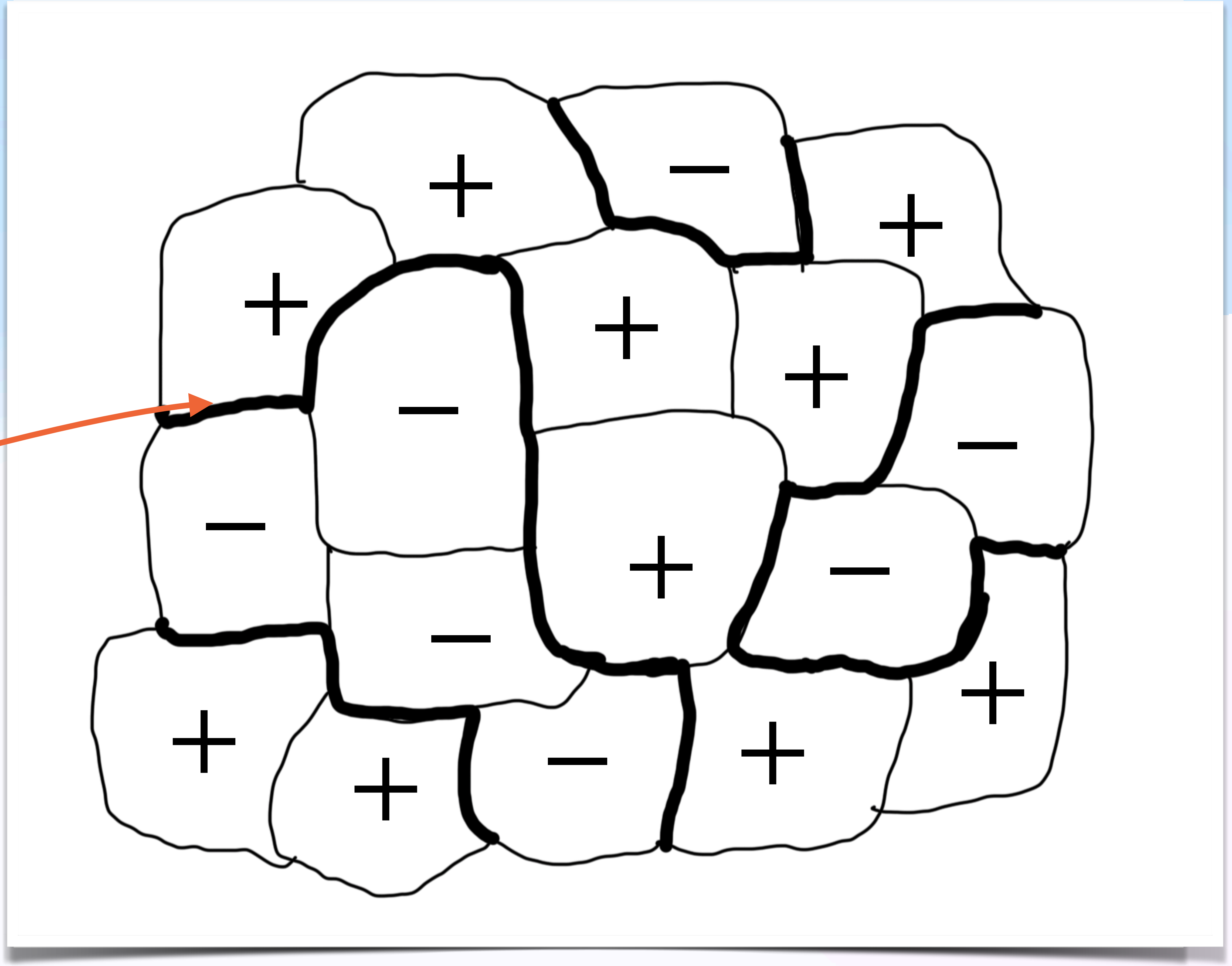


Domain wall formation

$$\mathcal{E} = \frac{1}{2}\phi'^2 + \Delta V$$

$$\sigma = \int \mathcal{E}\, dz = \frac{2\sqrt{2}}{3}\sqrt{\lambda}v^3$$





Discrete Symmetries

- Abelian: Z_n
- Non-Abelian： $A_n, S_n, \Delta(27)$……

Roles: flavour symmetries, dark matter, …

S_4 domain walls

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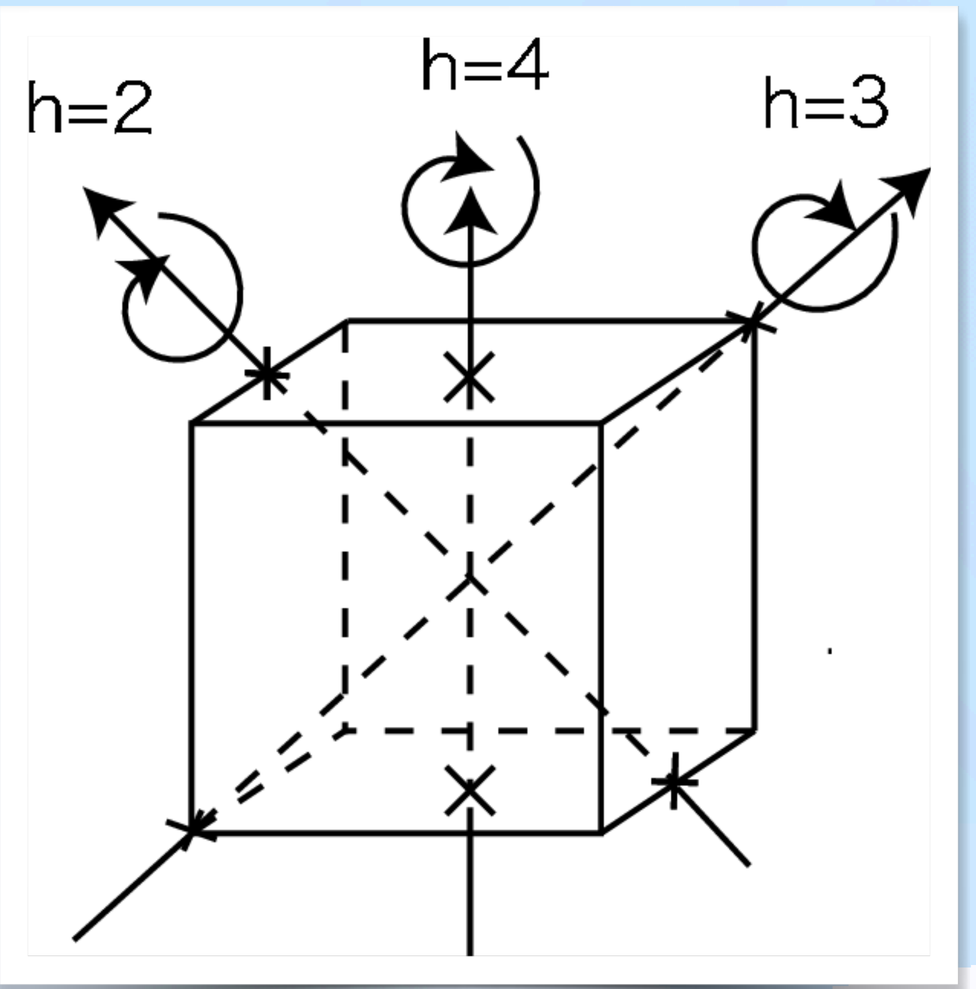
S_4 scalar theory



Ishimori, etc 1003.3552

- The octahedral/cube group S_4:

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- The most general renormalisable flavon potential:

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

Also for $A_4 \times Z_2$

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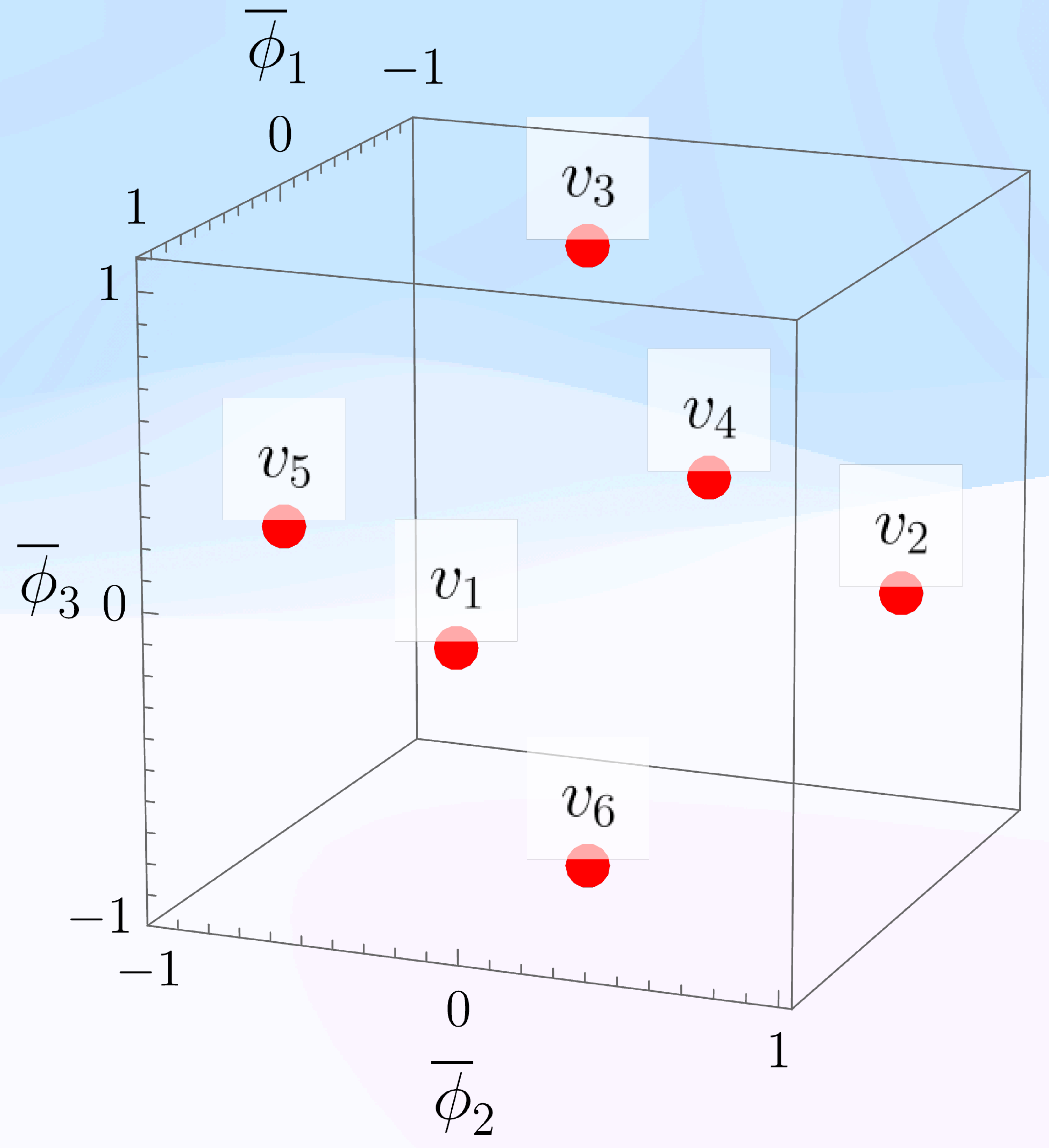
S_4 vacuum structure

$g_2 > 0$

$$\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix} \right\} v$$

$$v = \frac{\mu}{\sqrt{g_1}} \qquad \overline{\phi}_i = \frac{\phi_i}{v}$$



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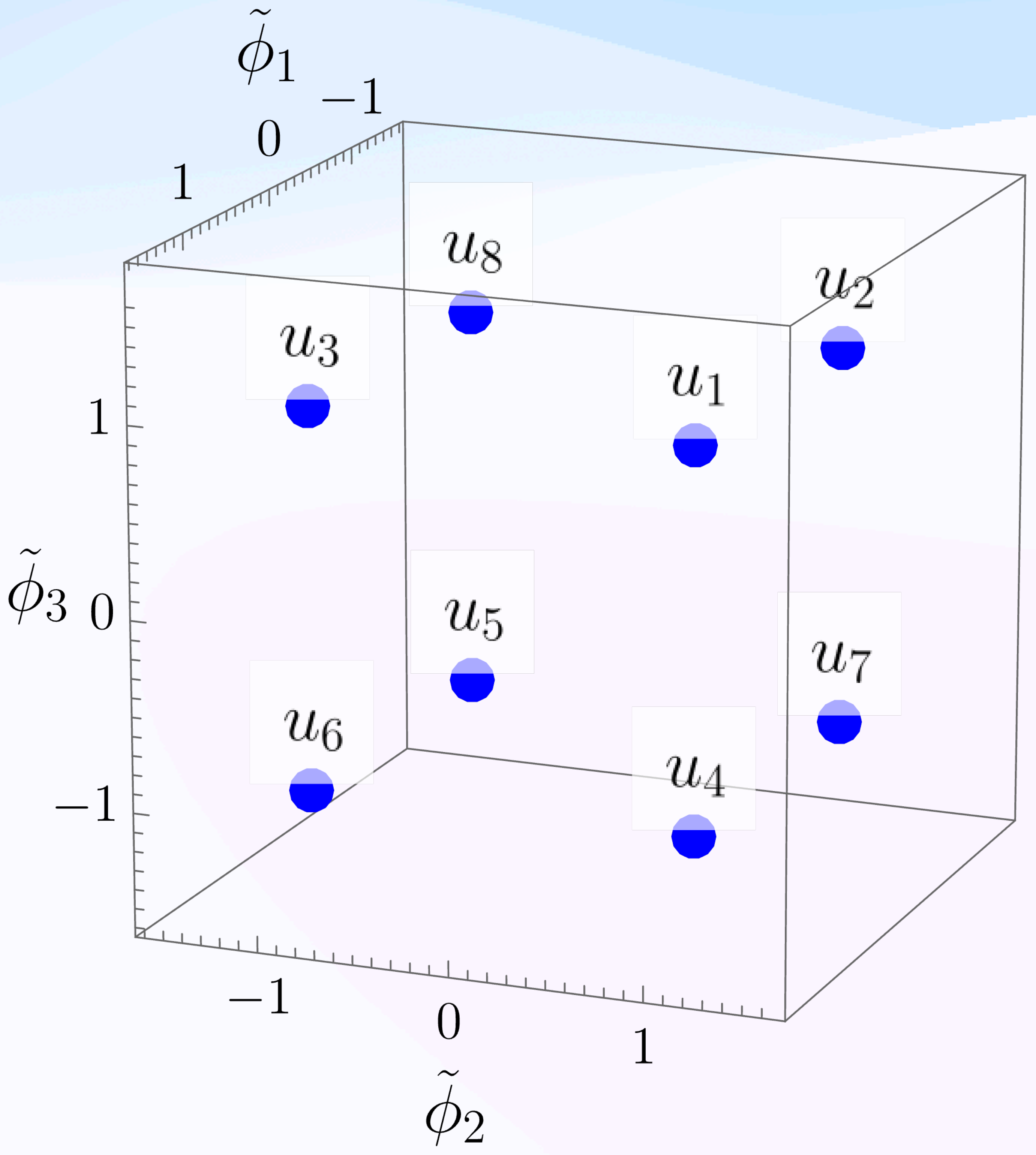
S_4 vacuum structure

$$-\frac{3}{2}g_1 < g_2 < 0$$

$$\left\{\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\end{pmatrix},\begin{pmatrix}-1\\1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}u$$

$$u = \frac{\mu}{\sqrt{3g_1 + 2g_2}} \qquad \tilde{\phi}_i = \frac{\phi_i}{u}$$



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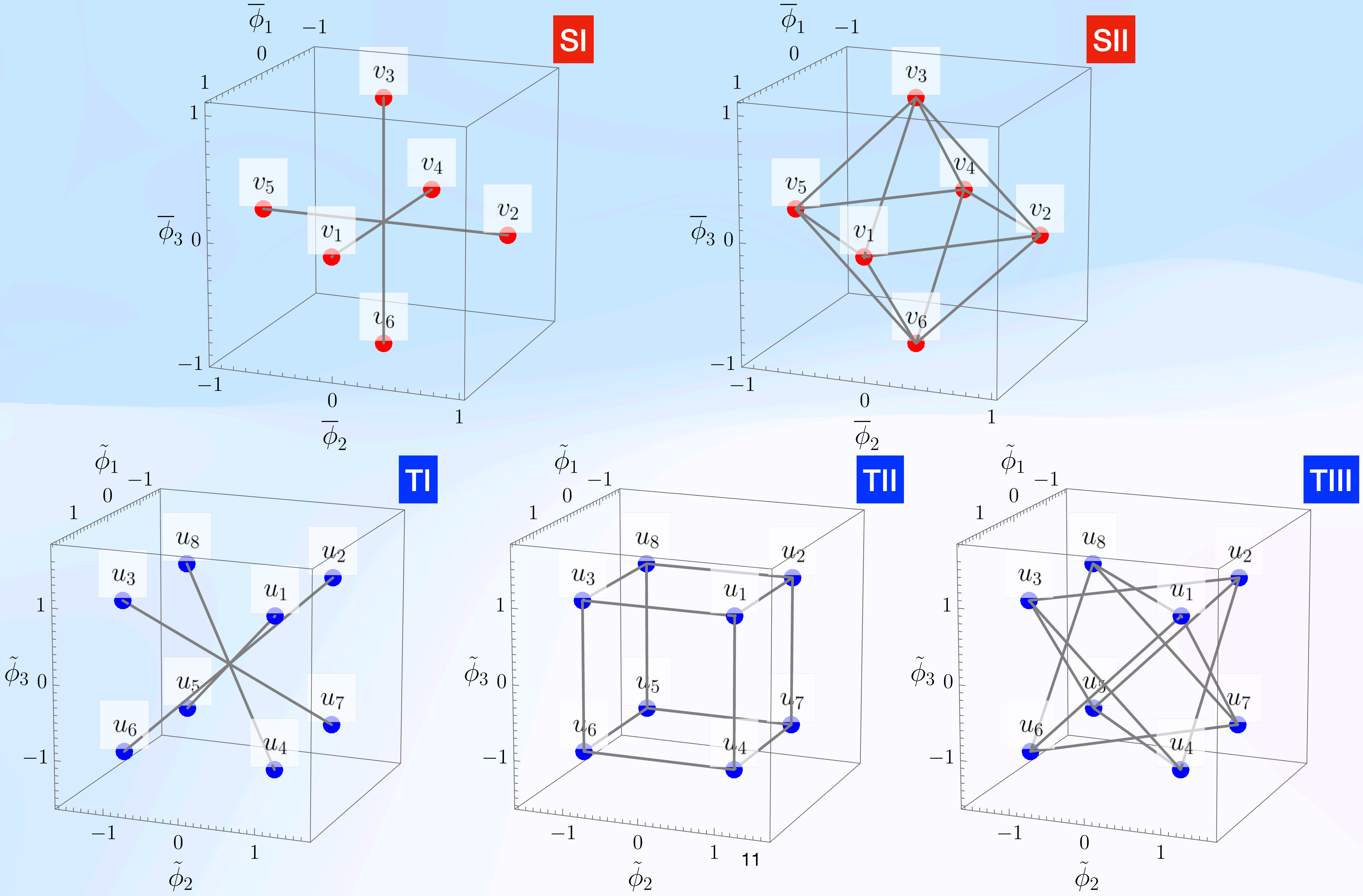
S_4 domain walls



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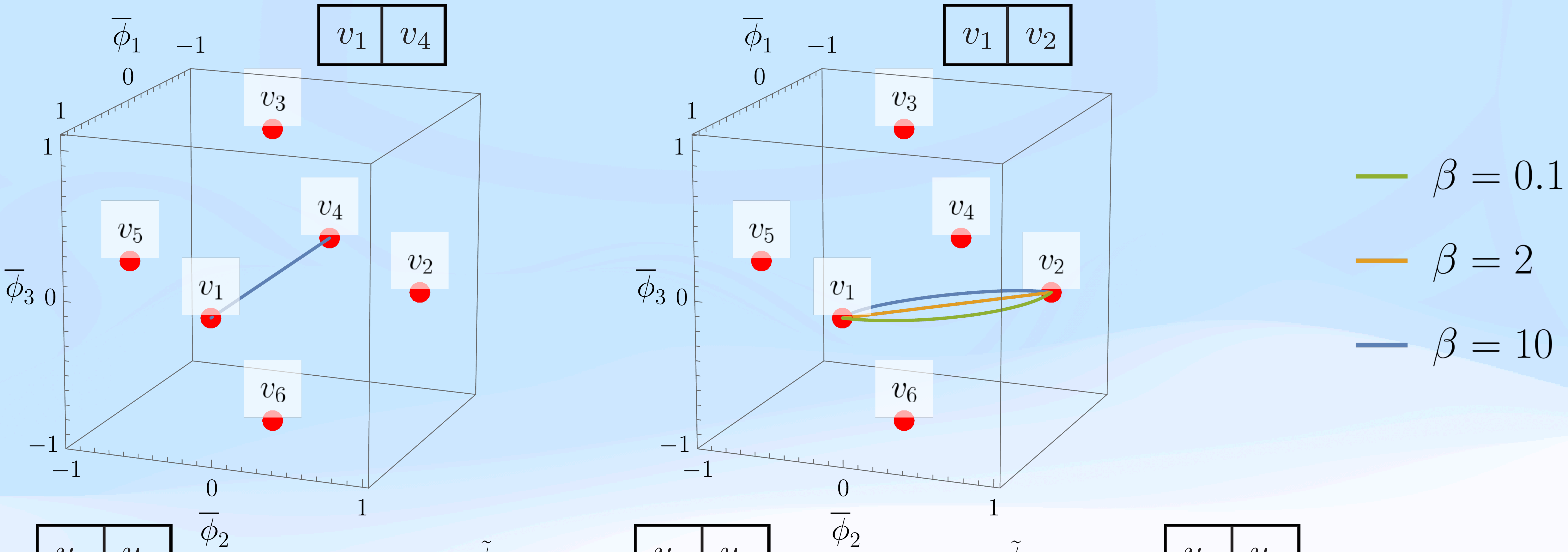
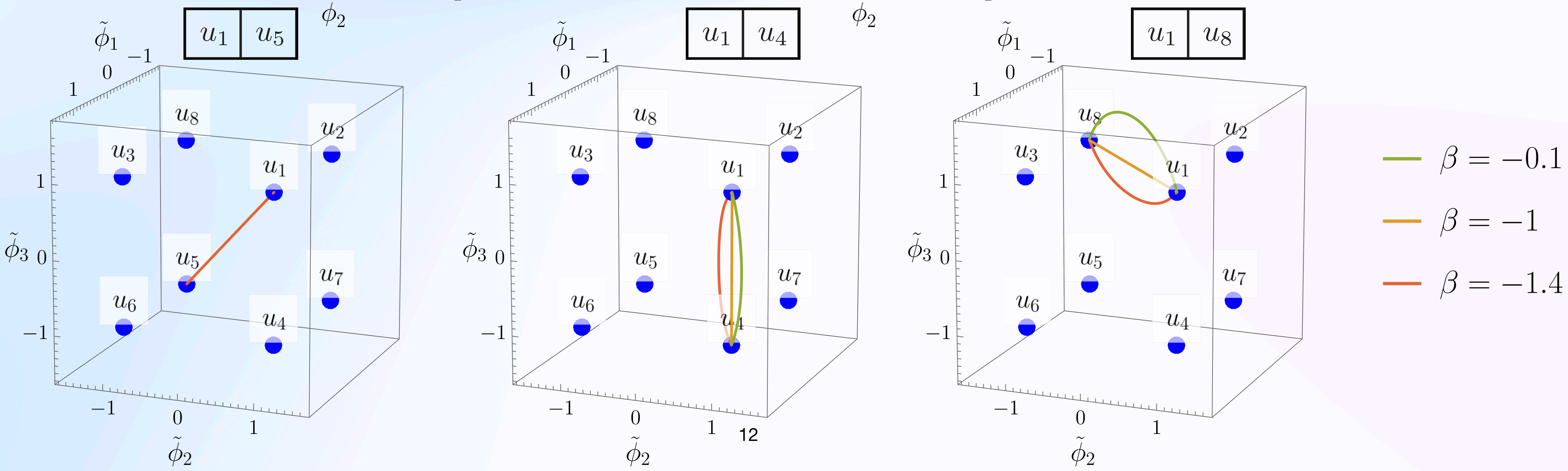
S_4 domain walls





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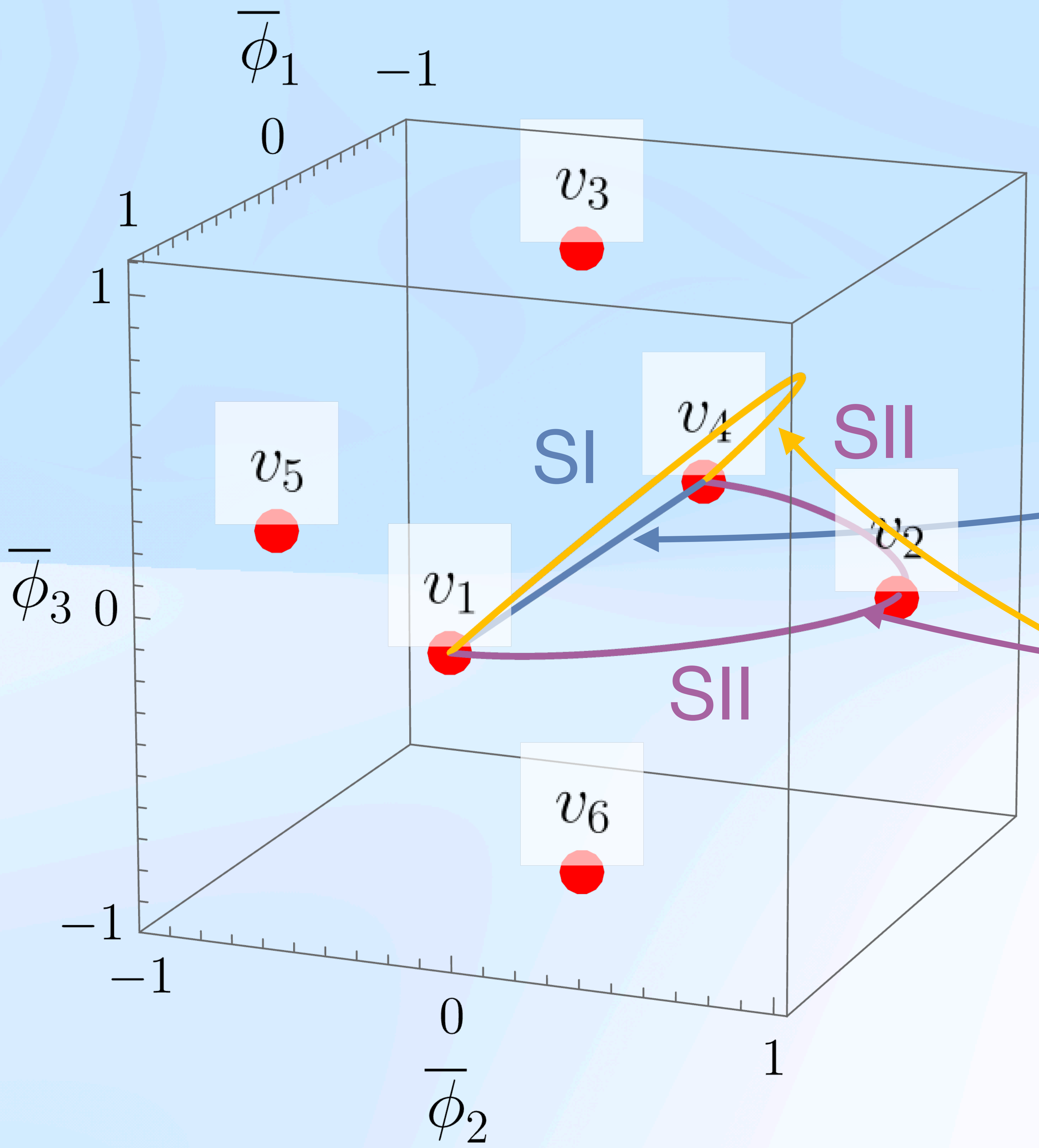
Stability of DWs



Straight line SI solution
Independent of $\beta = g_2/g_1$

Two SIII solutions with pitstop at v_2

Intermediate solution (still satisfies EoM)

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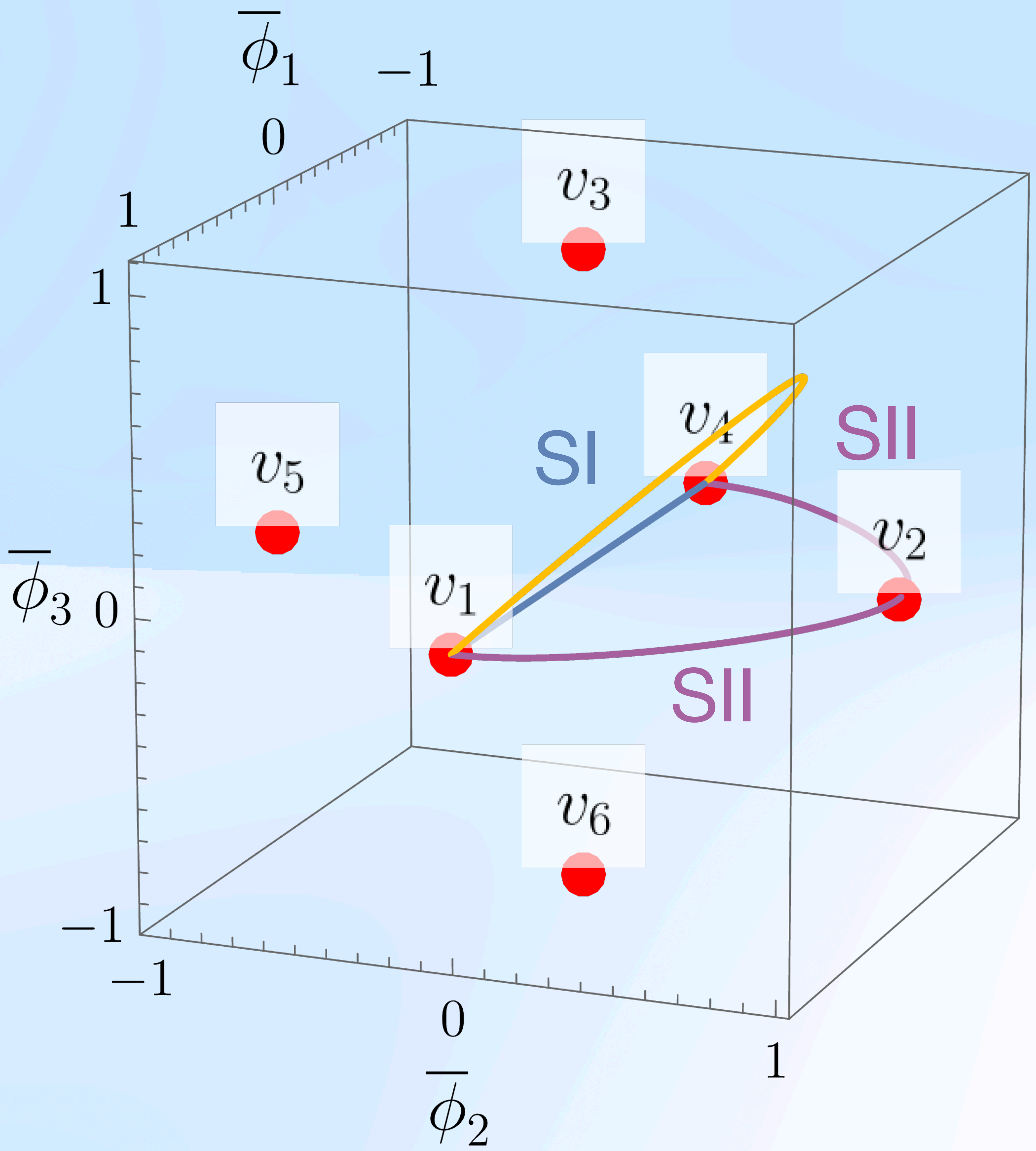
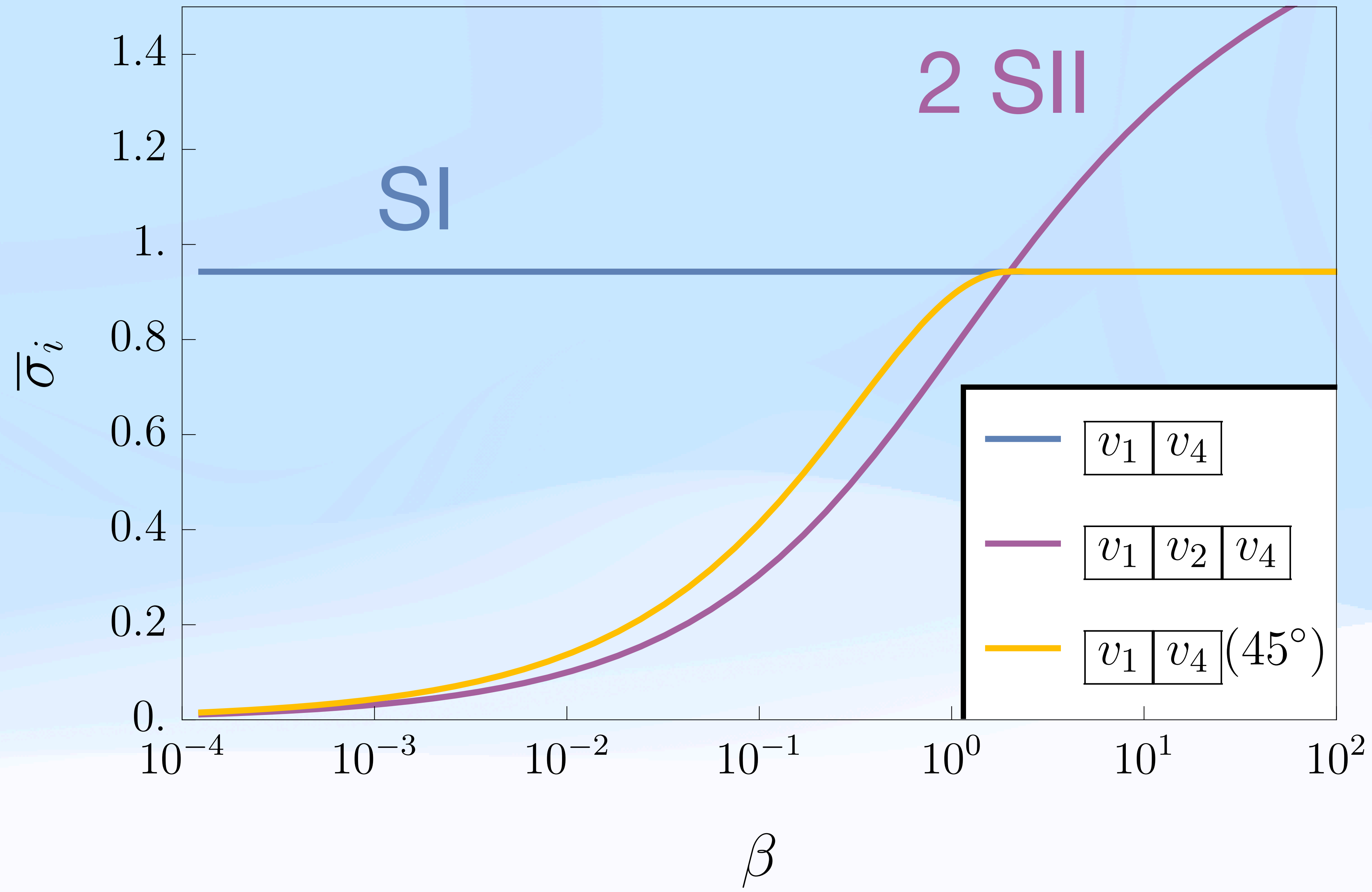
Stability of DWs





$$\beta \ll 2 : \sigma\,(\mathrm{SI}) > 2\sigma\,(\mathrm{SII})$$

SI DW **unstable** & would decay to SII

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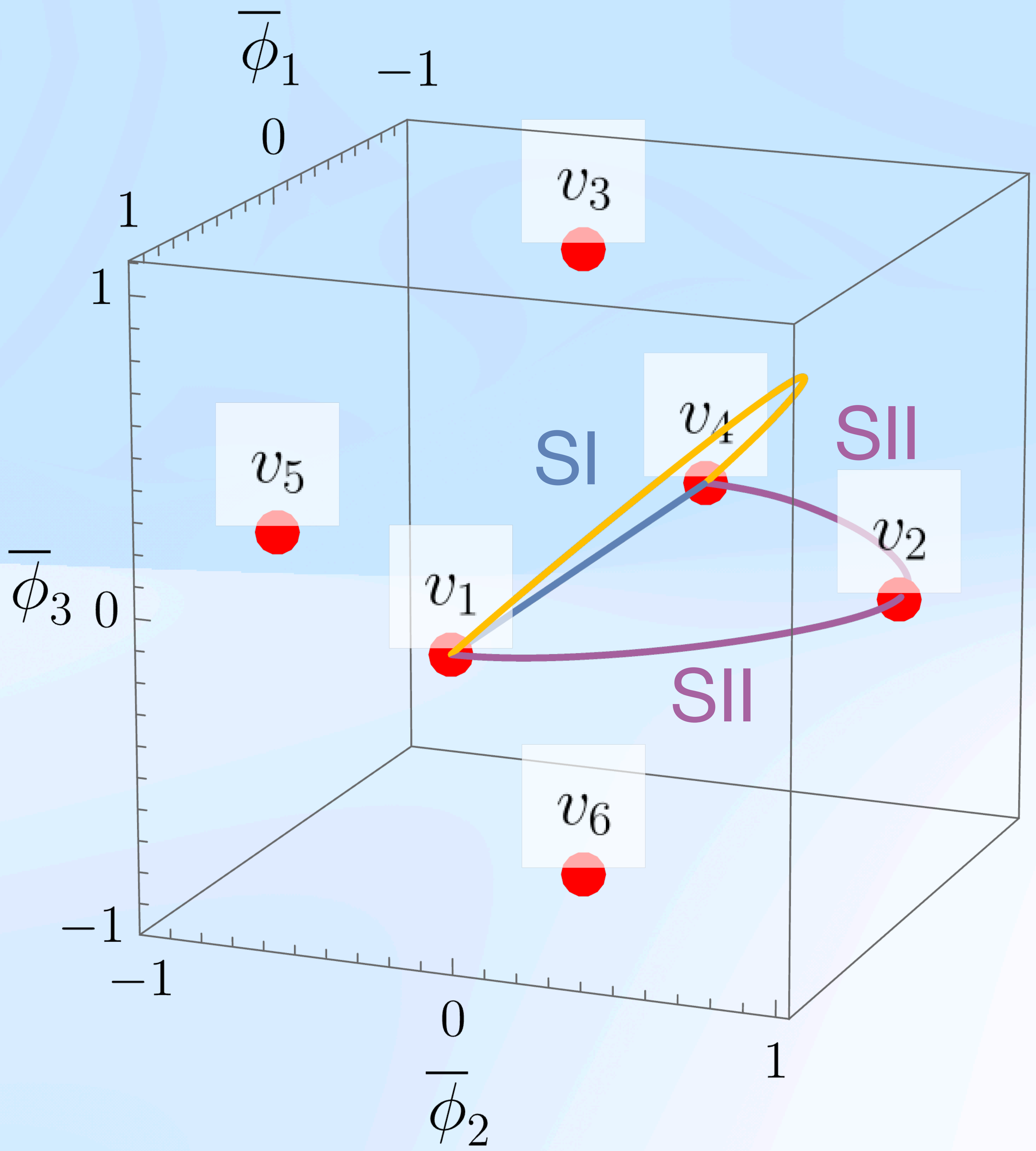
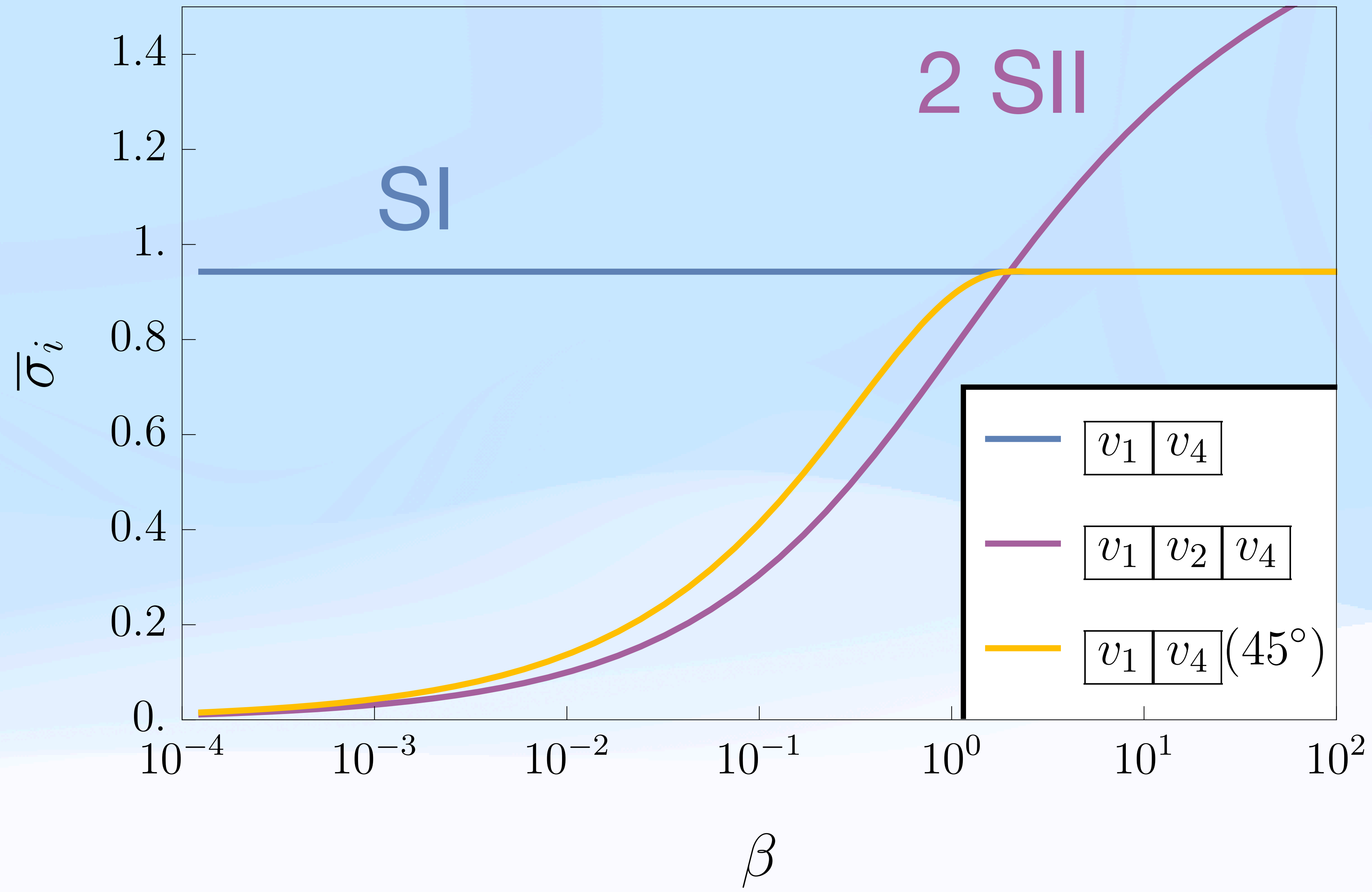
Stability of DWs





$$\beta \gg 2 : \sigma(\mathrm{SI}) < 2\sigma(\mathrm{SII})$$

SII DW **unstable** & would decay to SI

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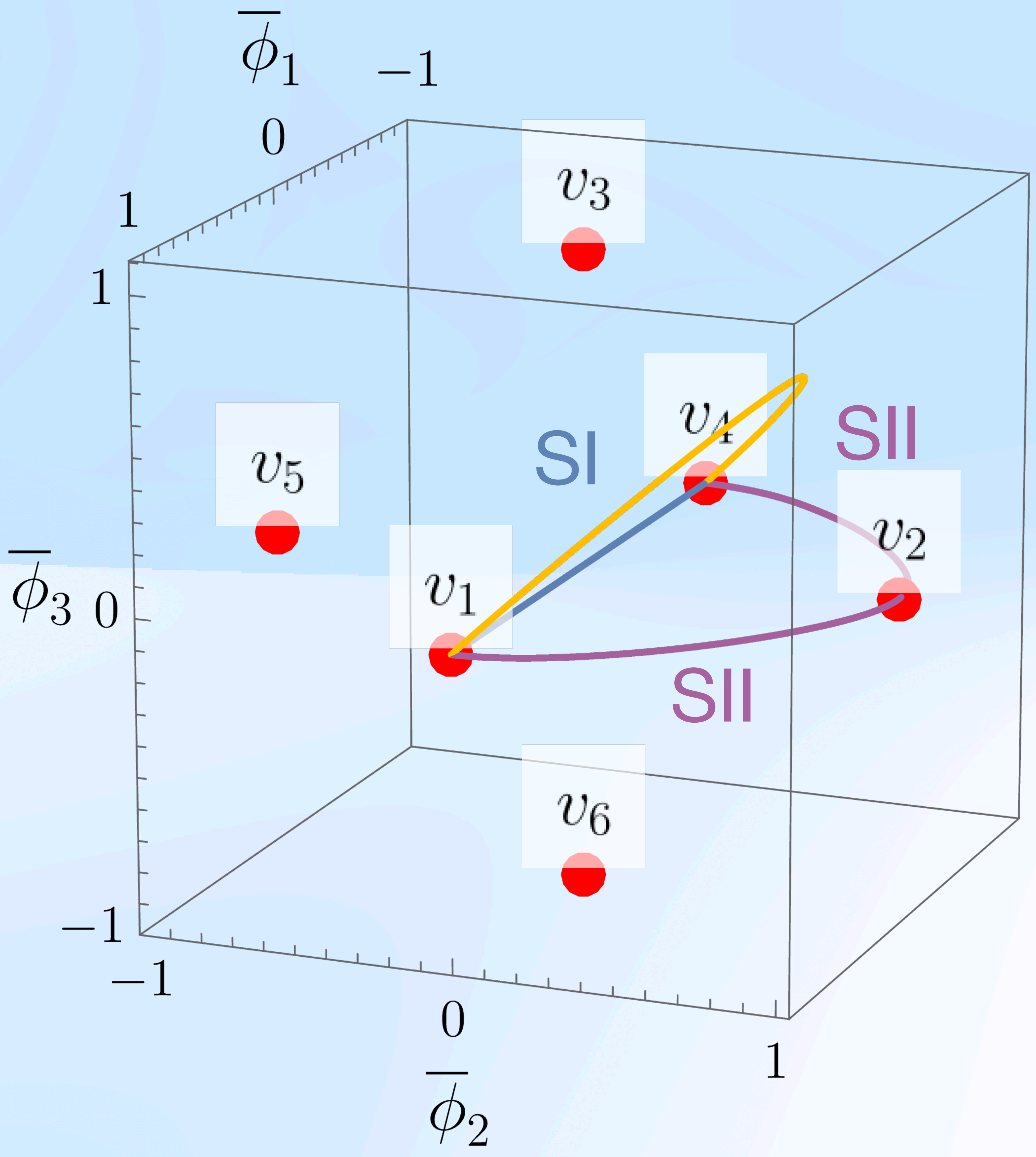
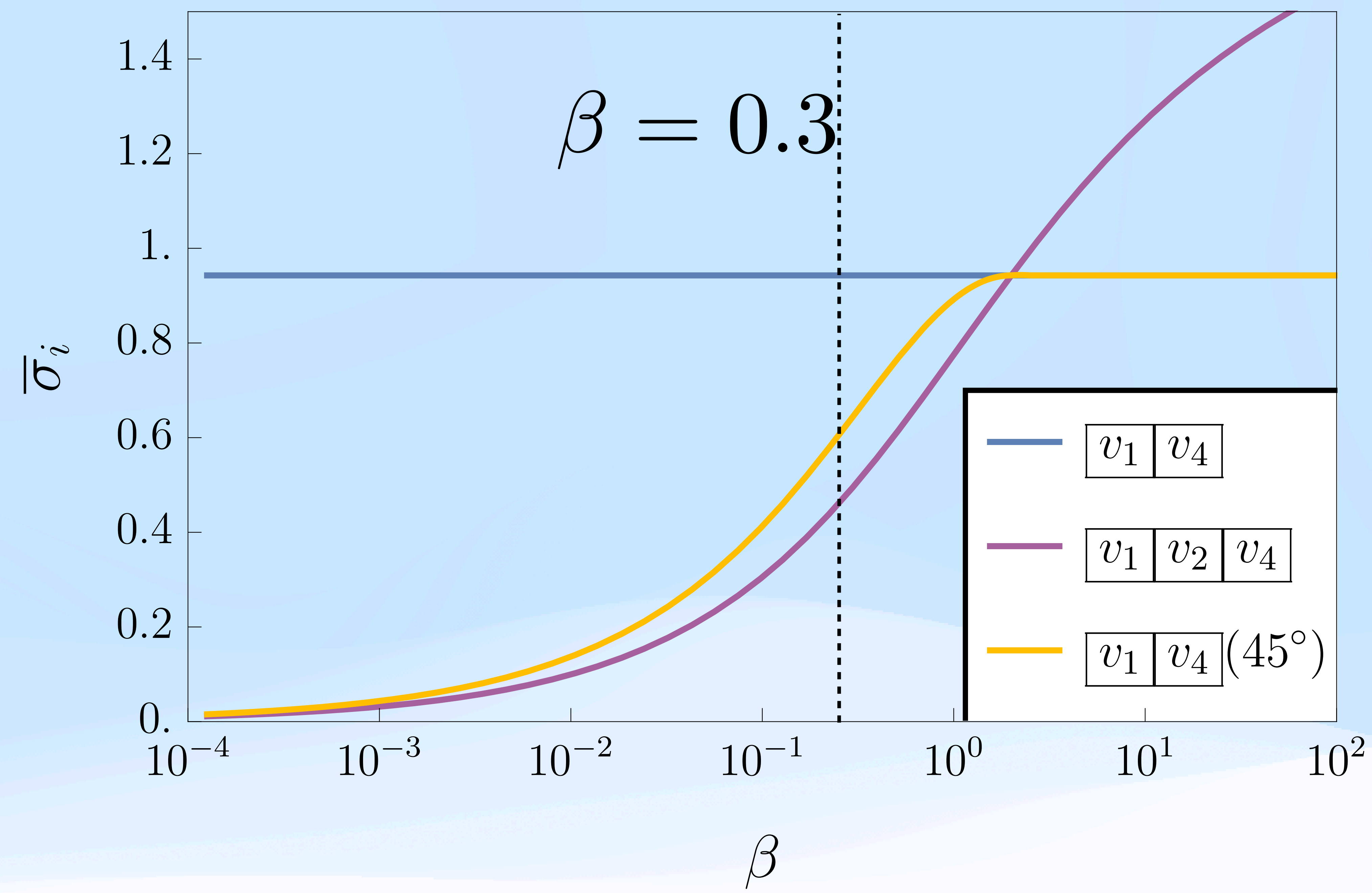
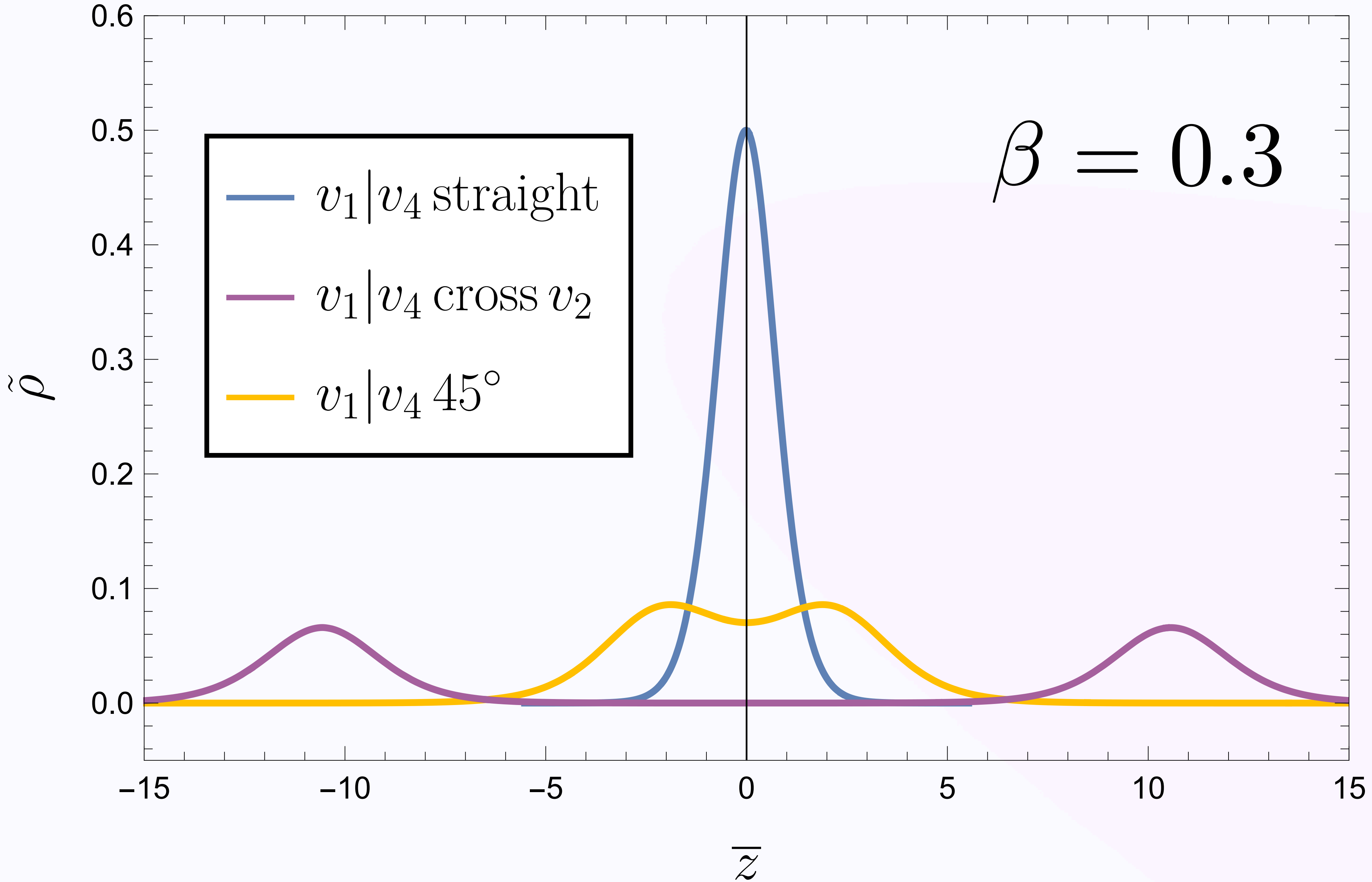
Stability of DWs







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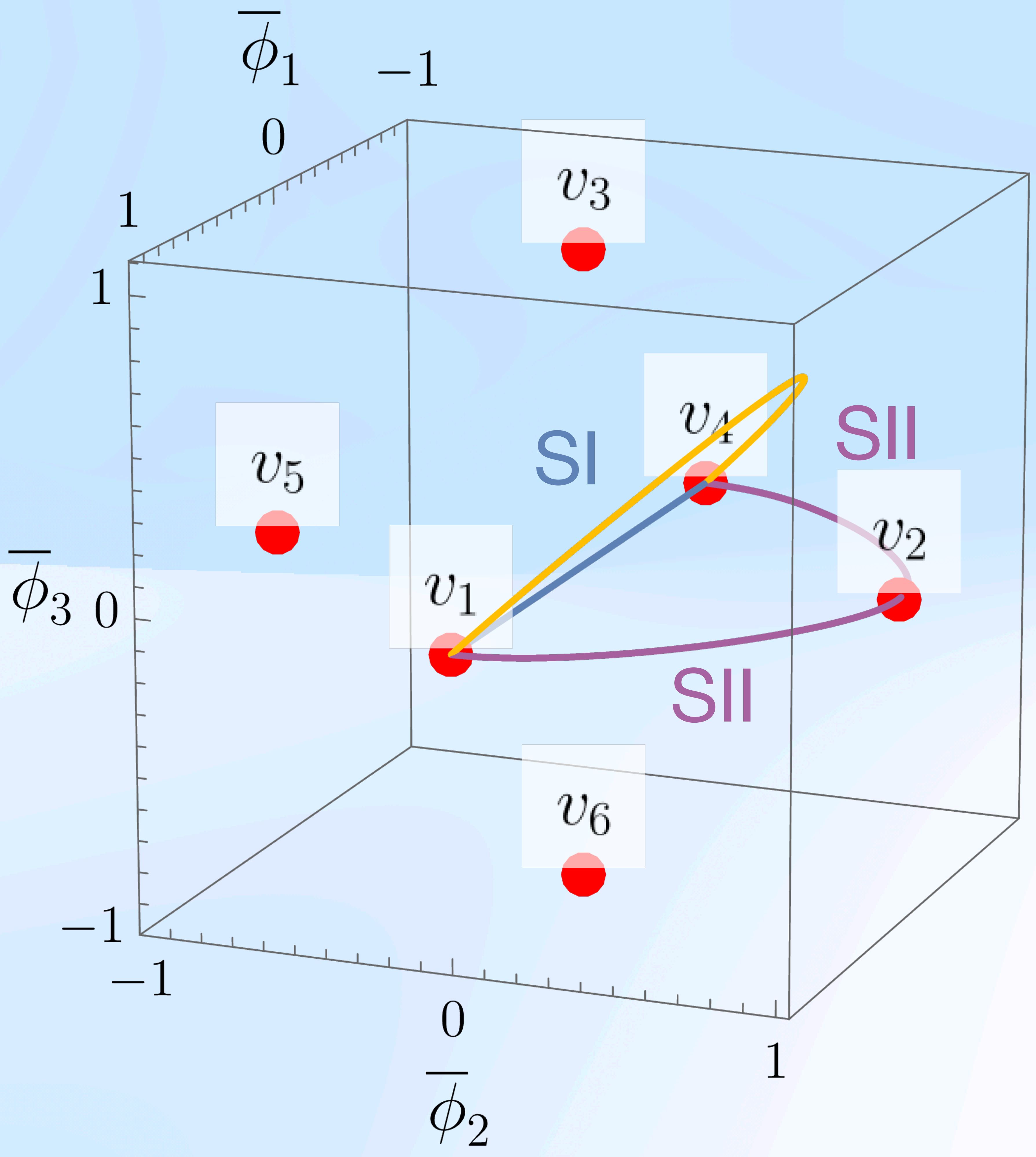
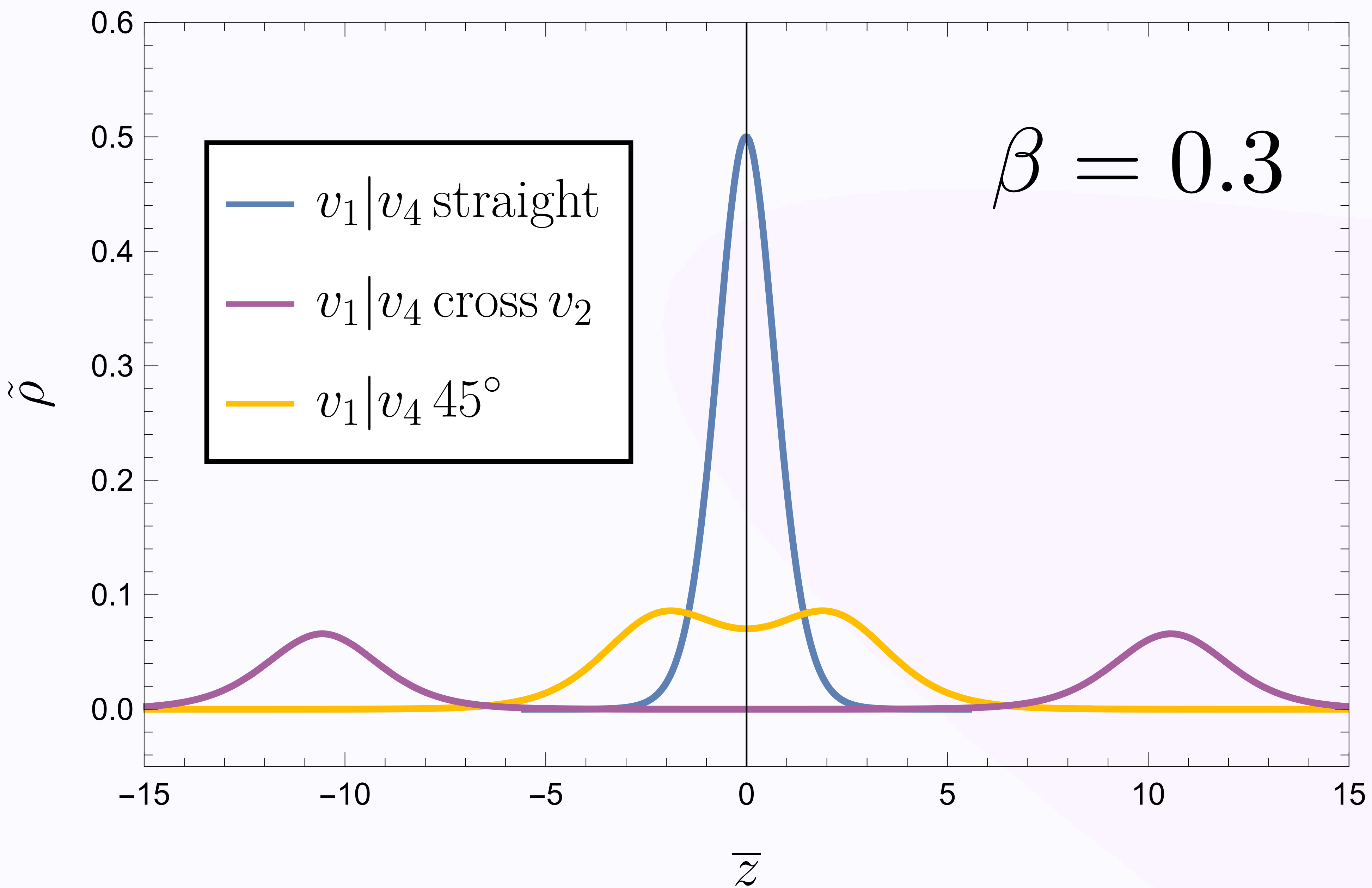
Stability of DWs

For $\beta = 0.3$, the SI-type DW will decay to two SII type DWs

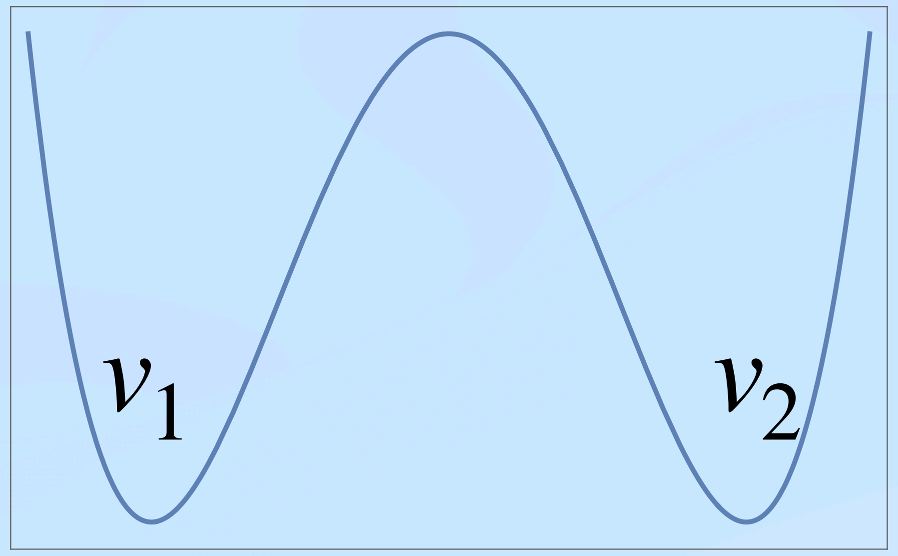




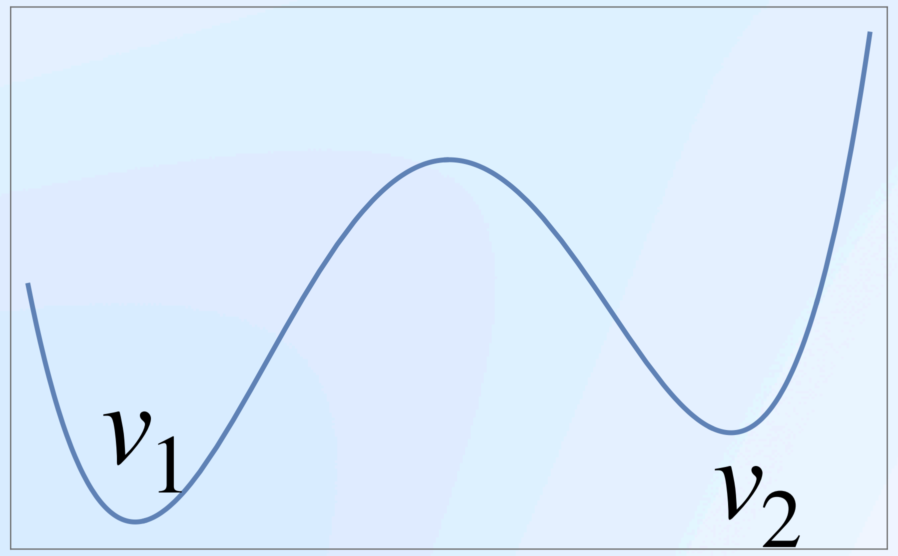
Gravitational waves

Gravitational wave from DWs

- Exact discrete symmetry $\Longrightarrow$ stable DWs



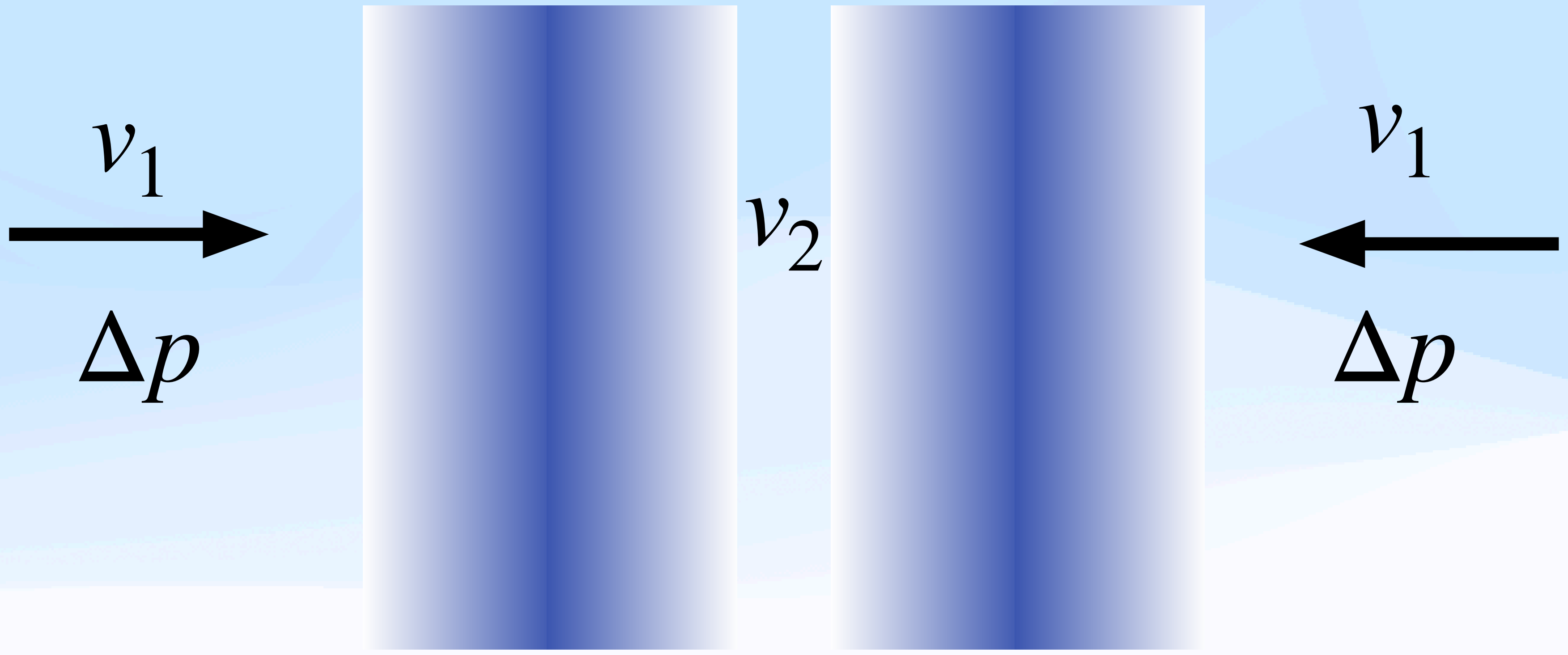
- Bias $\Longrightarrow$ unstable DWs



$$V_{\rm bias} \equiv V(v_2) - V(v_1)$$

Saikawa 1703.02576

Pressure difference $\Delta p \propto V_{\rm bias}$



$$\Omega_{\rm GW}^{\rm peak}\left(\sigma, V_{\rm bias}\right)$$

$$\begin{cases} f > f_{\rm peak}, \Omega_{\rm GW} \propto f^{-1} \\ f < f_{\rm peak}, \Omega_{\rm GW} \propto f^{3} \end{cases}$$

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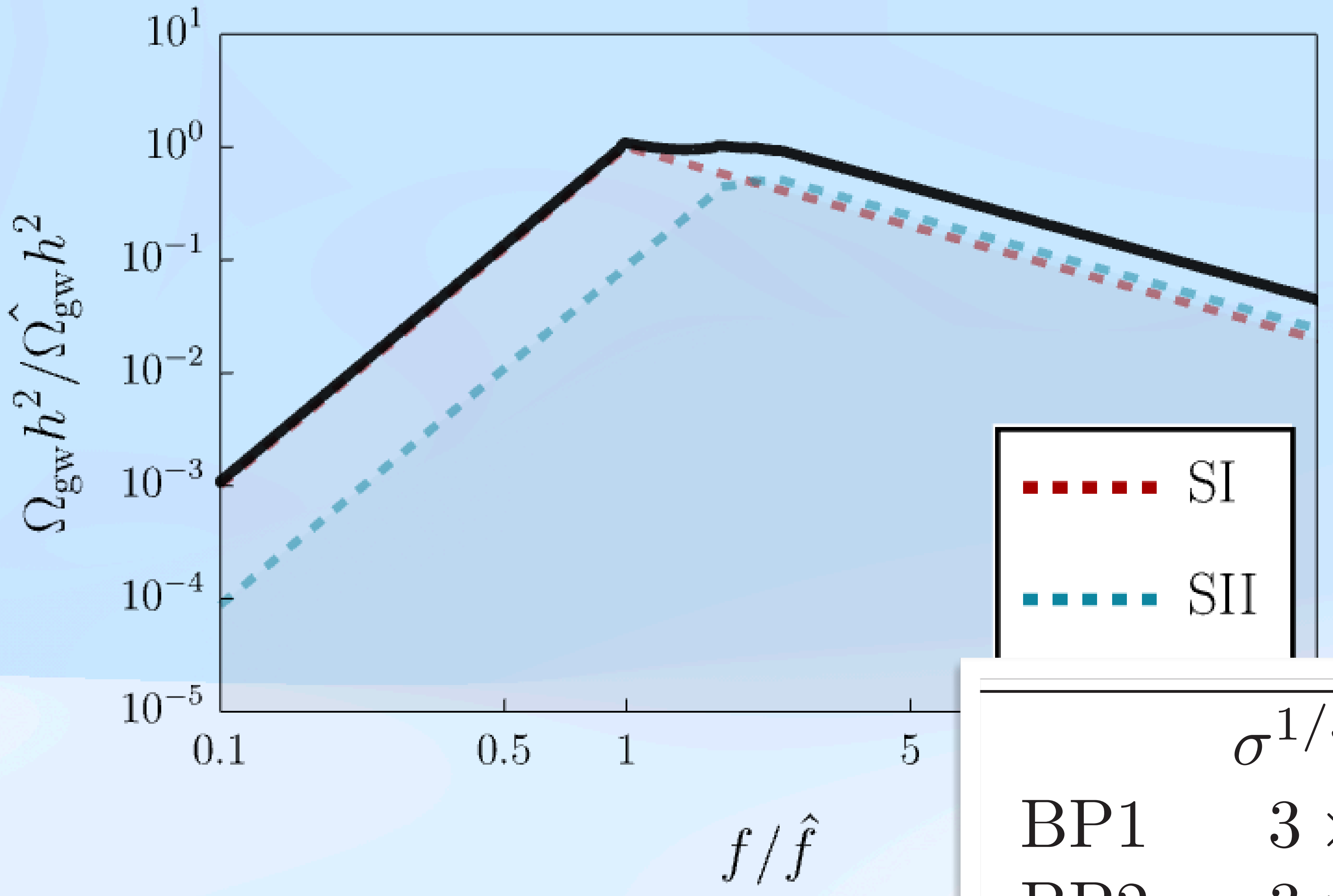
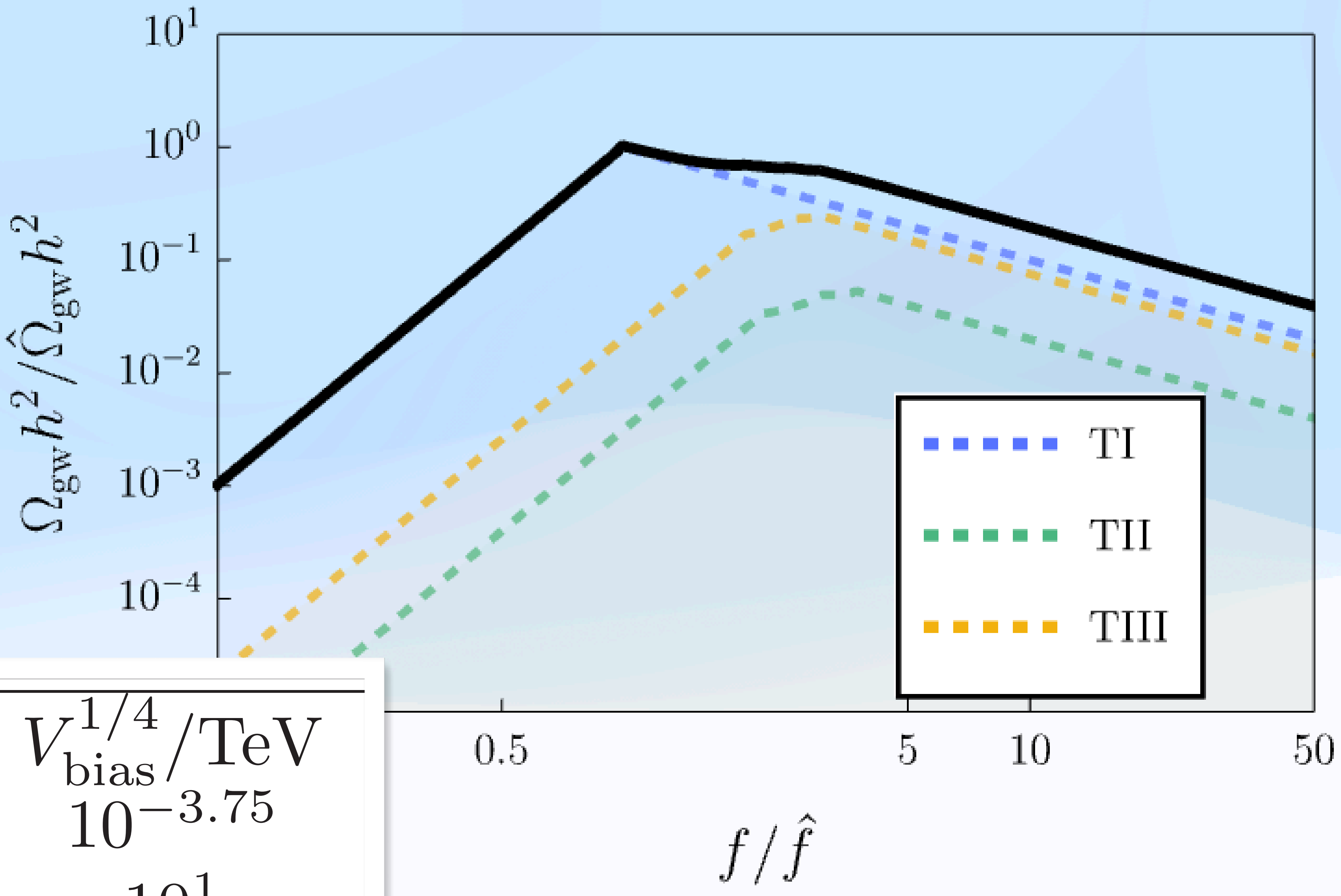
Gravitational wave

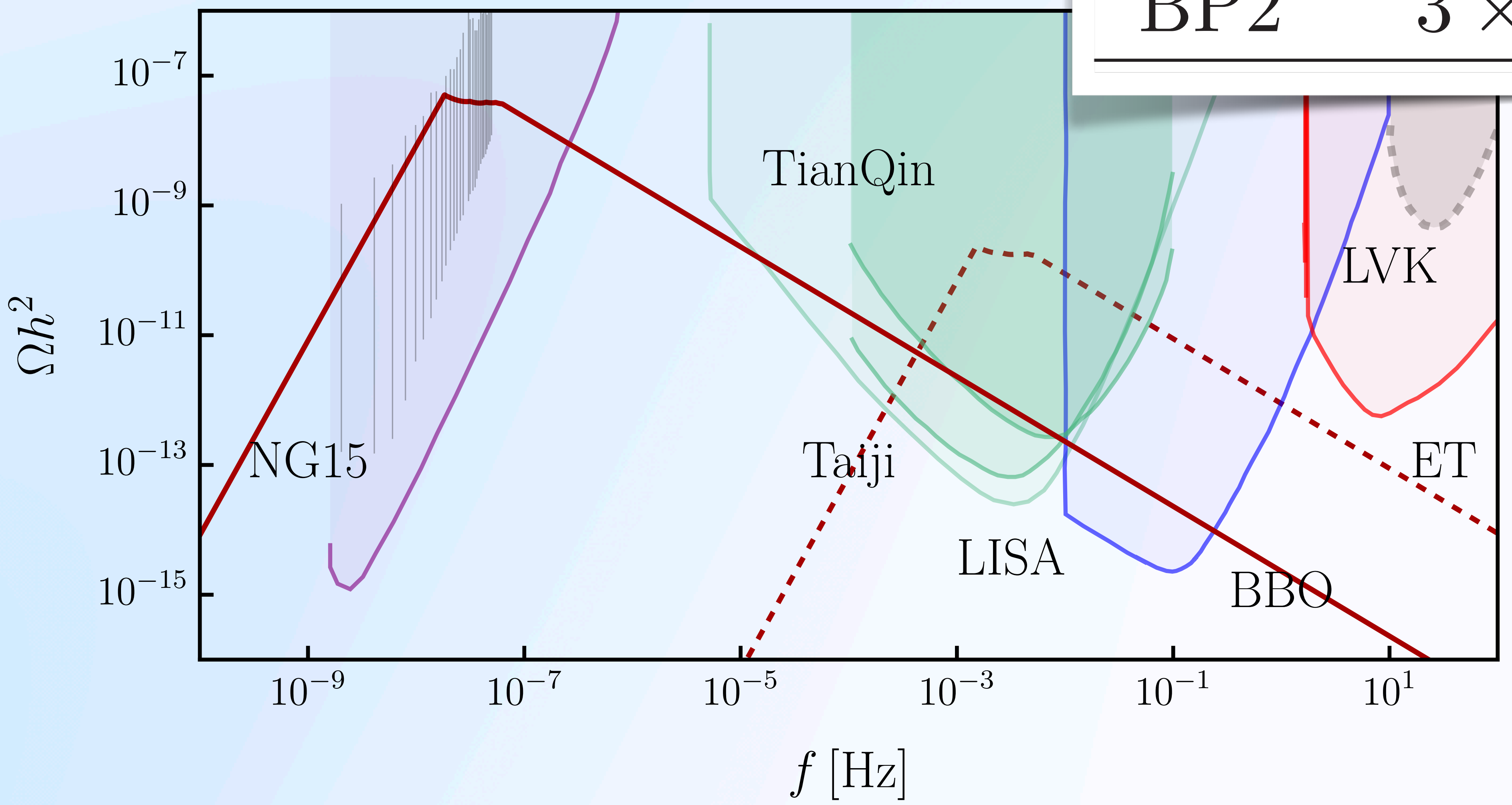
ϵ^v_{12}	ϵ^v_{13}	ϵ^v_{14}	ϵ^v_{15}	ϵ^v_{16}
$2\hat{\epsilon}$	$3\hat{\epsilon}$	$\hat{\epsilon}$	$4\hat{\epsilon}$	$5\hat{\epsilon}$

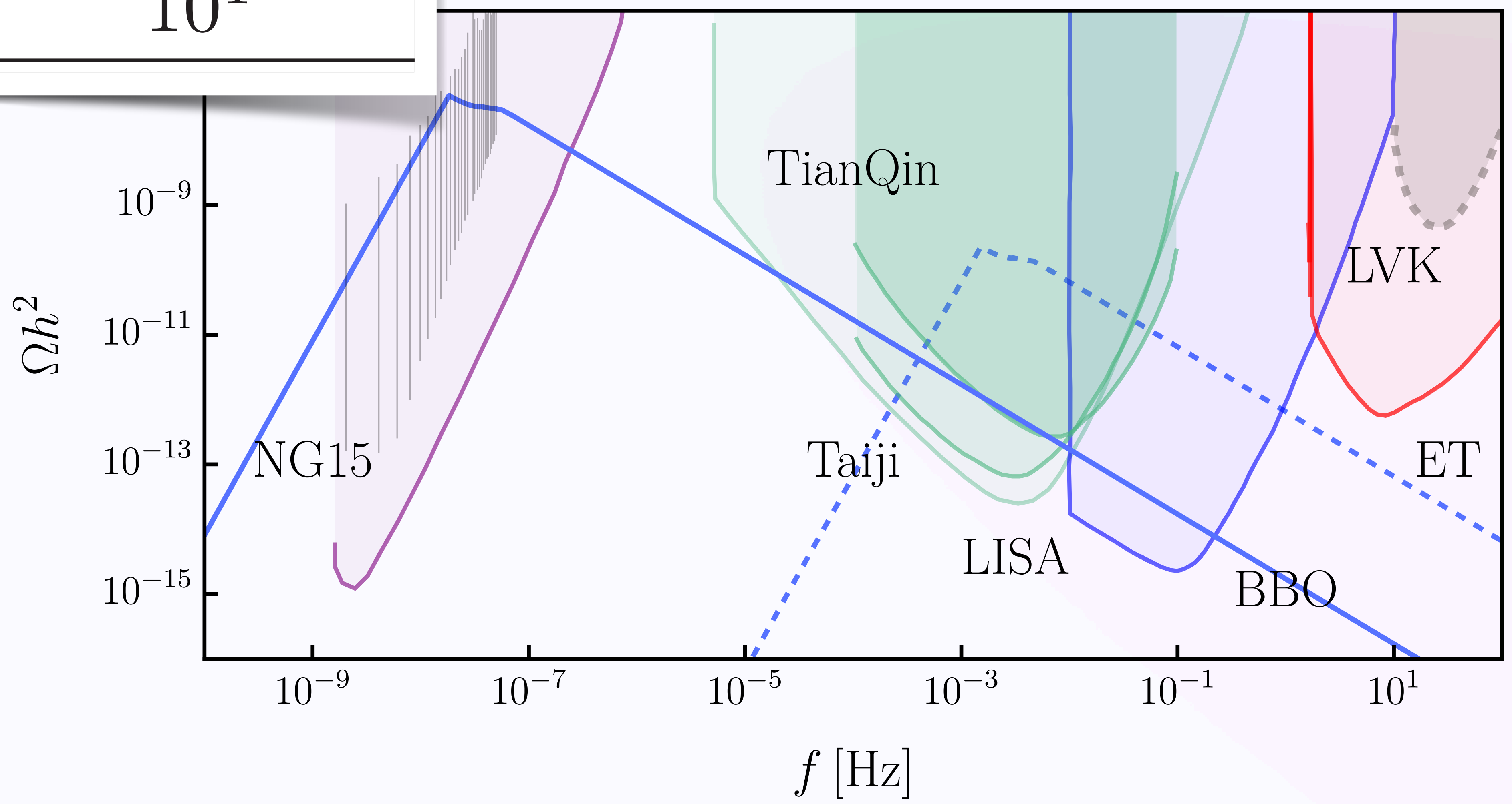
ϵ^u_{12}	ϵ^u_{13}	ϵ^u_{14}	ϵ^u_{15}	ϵ^u_{16}	ϵ^u_{17}	ϵ^u_{18}
$2\hat{\epsilon}$	$4\hat{\epsilon}$	$6\hat{\epsilon}$	$\hat{\epsilon}$	$3\hat{\epsilon}$	$5\hat{\epsilon}$	$7\hat{\epsilon}$





	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1





Summary and Outlook

- Non-abelian DWs have more interesting and non-trivial structure and phenomena
- In certain range of parameter space, unstable DWs can show up
- If the DWs are stable, they can give rise to a unique multi-peak GW signal
- The signature of GW raised by unstable domain walls is still unexplored