Confinement Slingshot: Confined Monopoles in First-Order Phase Transitions

Maximilian Bachmaier

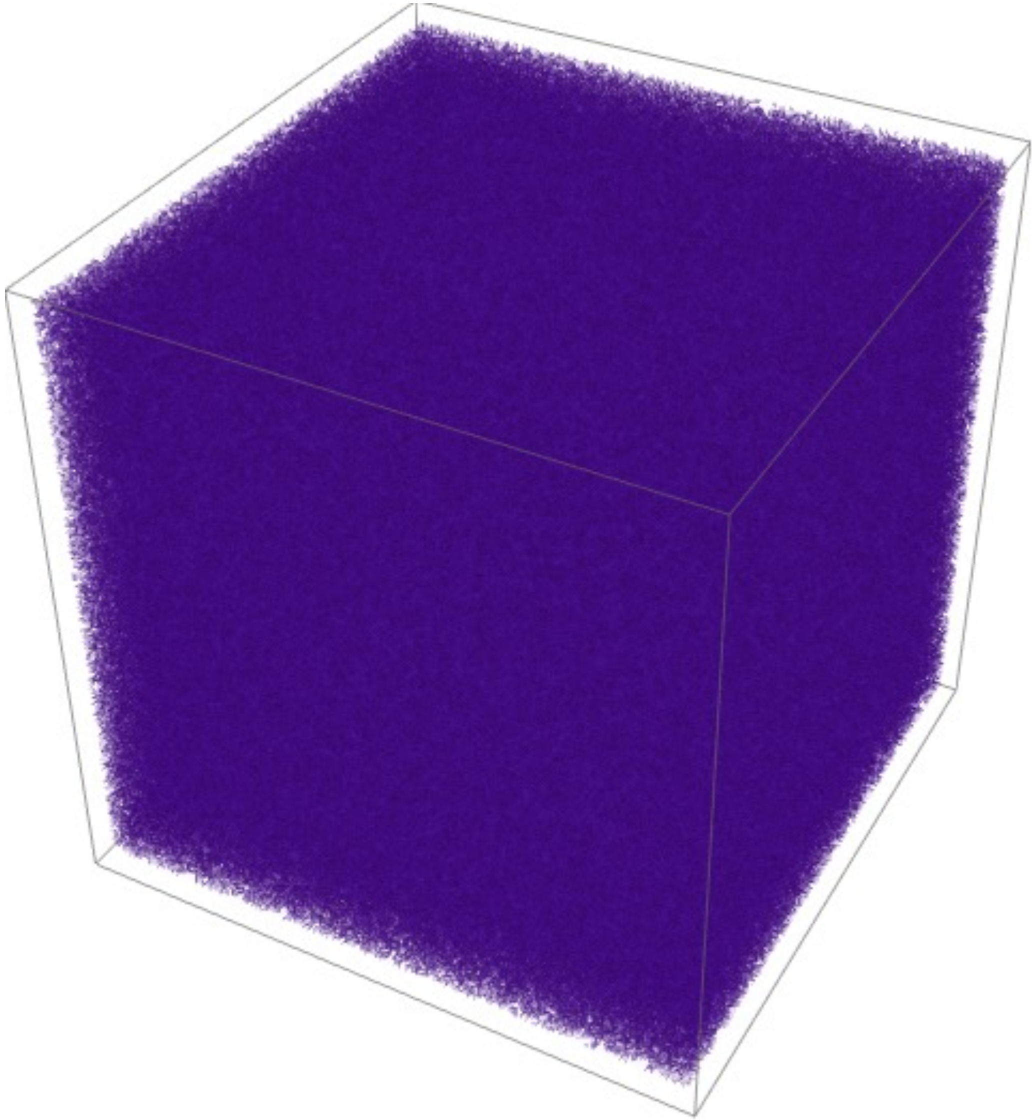
In collaboration with: Gia Dvali
Juan Sebastián Valbuena-Bermúdez
Michael Zantedeschi

PASCOS 2025 (Durham, July 2025)

The Magnetic Monopole Problem

Grand Unified Theories predict Magnetic Monopoles
(Creation through Kibble Mechanism)

Zeldovich and Khlopov 1978, Preskill 1979:
Magnetic Monopole Problem

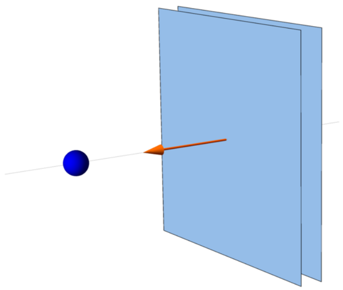


The Magnetic Monopole Problem - Solutions

Inflation
BUT: GUT phase transition can happen after inflation[1]

Symmetry Non-Restoration[2] (Dvali, Melfo and Senjanovic 1995)

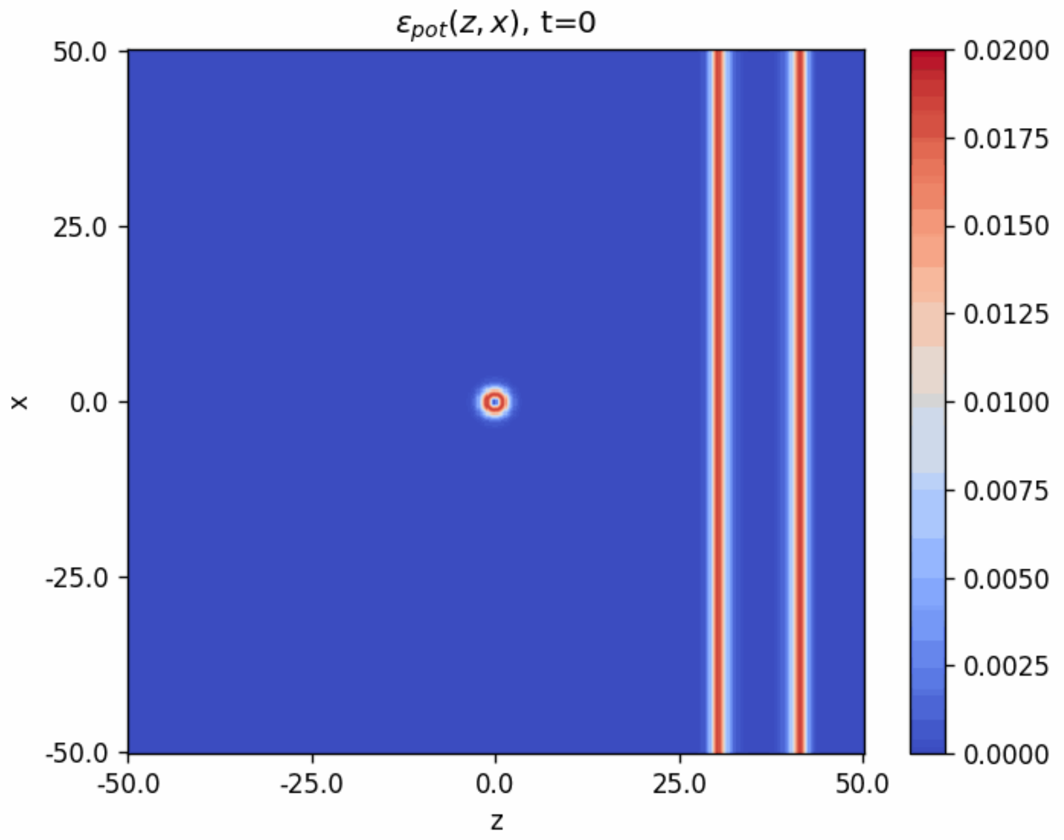
Domain Walls erase Magnetic Monopoles[2] (Dvali, Liu and Vachaspati 1997)
Numerical Studies: Brush, Pogosian, Vachaspati 2015
Dvali, Valbuena-Bermudez 2022
Bachmaier, Dvali, Valbuena-Bermudez 2023



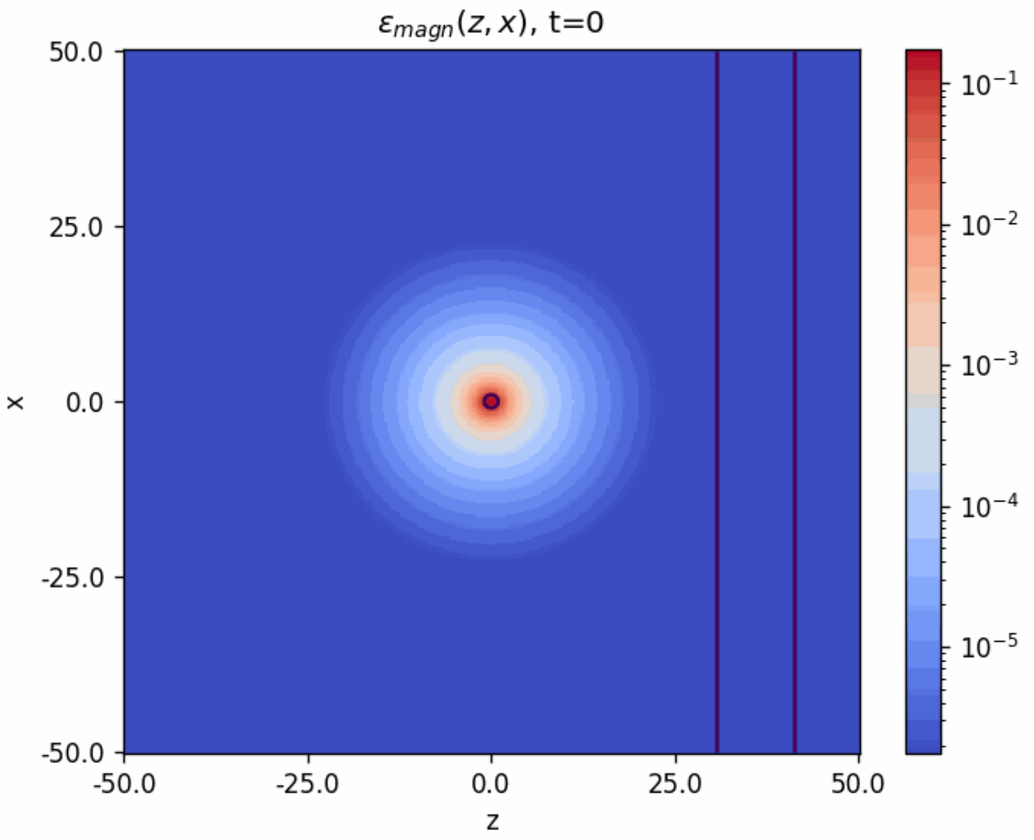
[1]see for example G. Dvali, H. Tye – Brane Inflation (1998)
[2]minimal solutions, since they don't require physics beyond minimal SU(5)

Magnetic Monopole - Domain Wall Collision

Potential Energy Density



Magnetic Energy Density



M. B., G. Dvali, J. S. Valbuena-Bermudez (2023)

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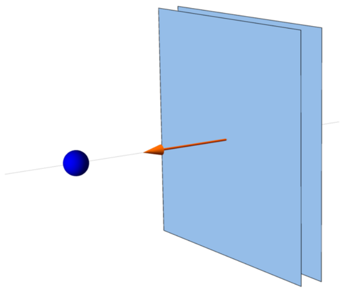

Magnetic Monopoles connected by Strings (Langacker and Pi 1980)
Numerical Studies: Dvali, Valbuena-Bermudez, Zantedeschi 2022
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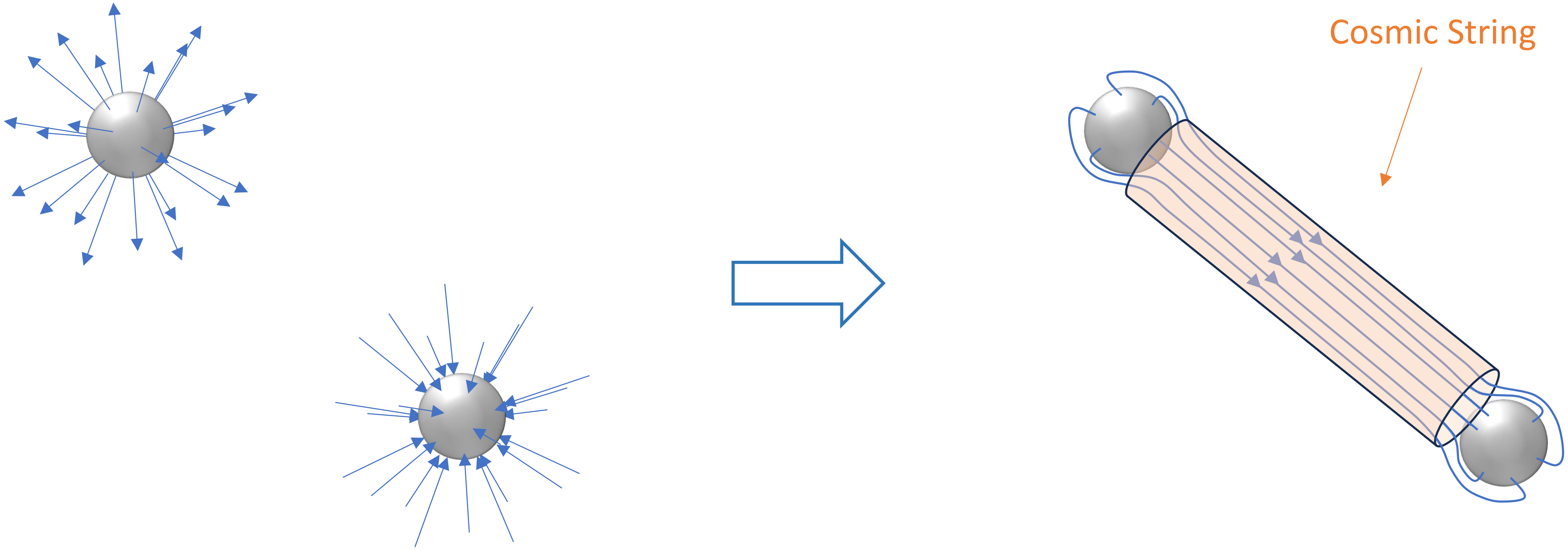
Magnetic Monopoles Connected by a String

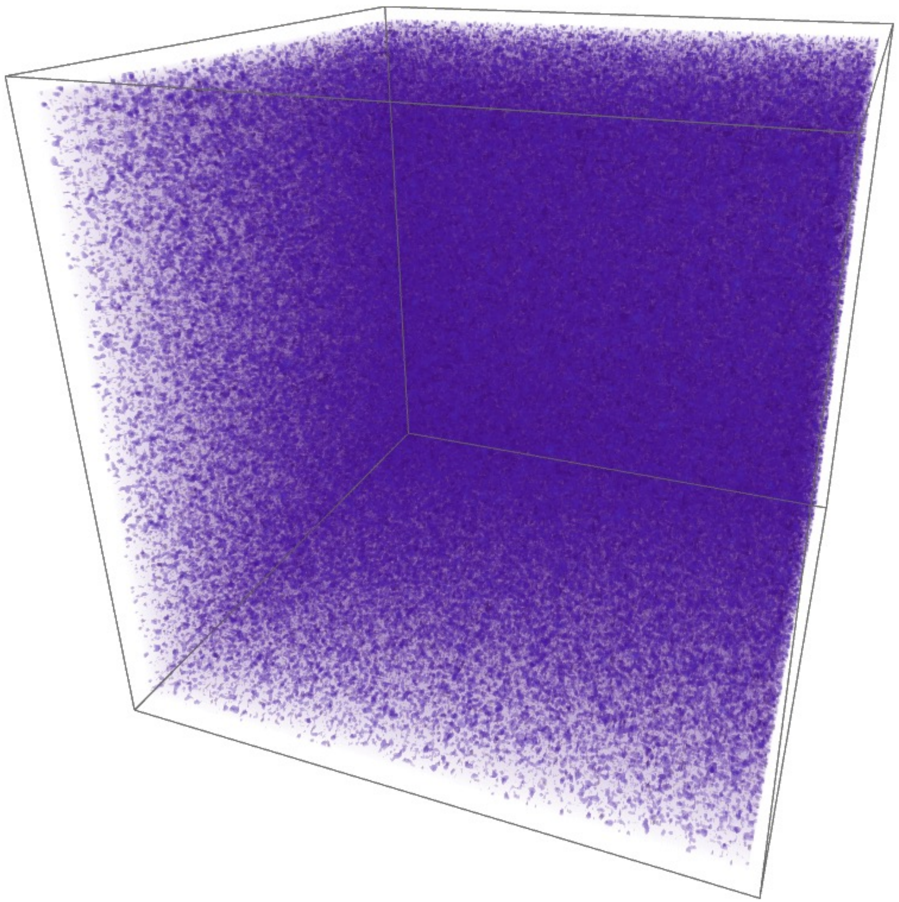
First Phase Transition: $G \to \ldots \times U(1)$

Second Phase Transition: $\ldots \times U(1) \to \ldots \times \cancel{U(1)}$



P. Langacker, S. Pi (1980)
X. Martin, A. Vilenkin (1997)
G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2022)

Second-Order Phase Transition



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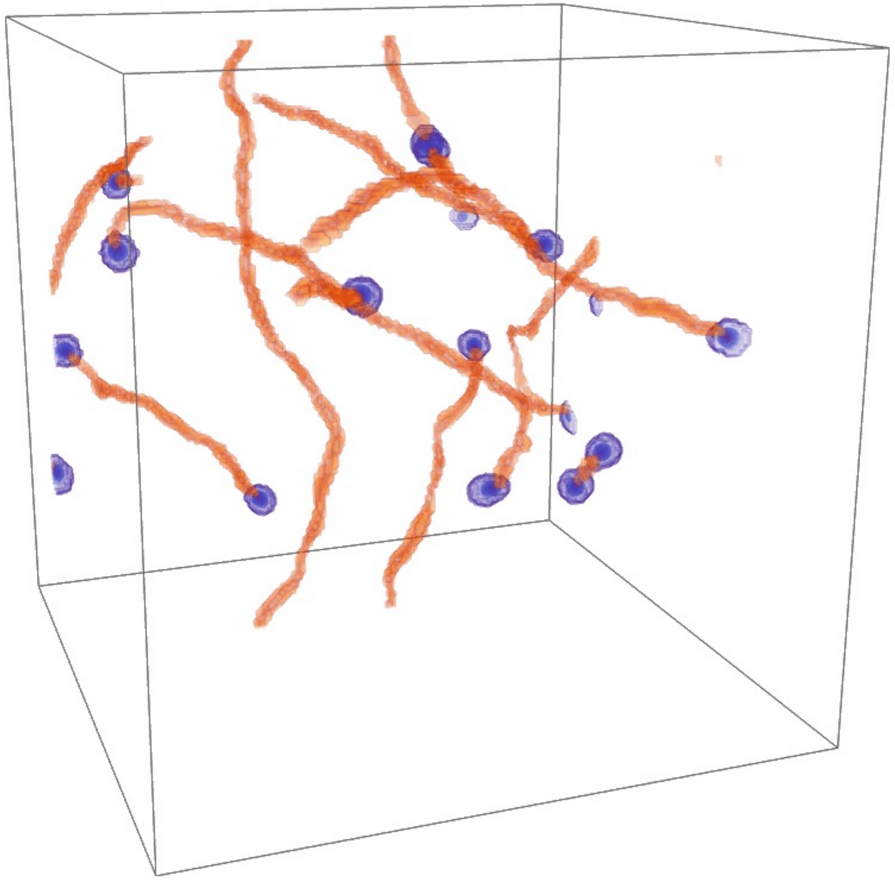
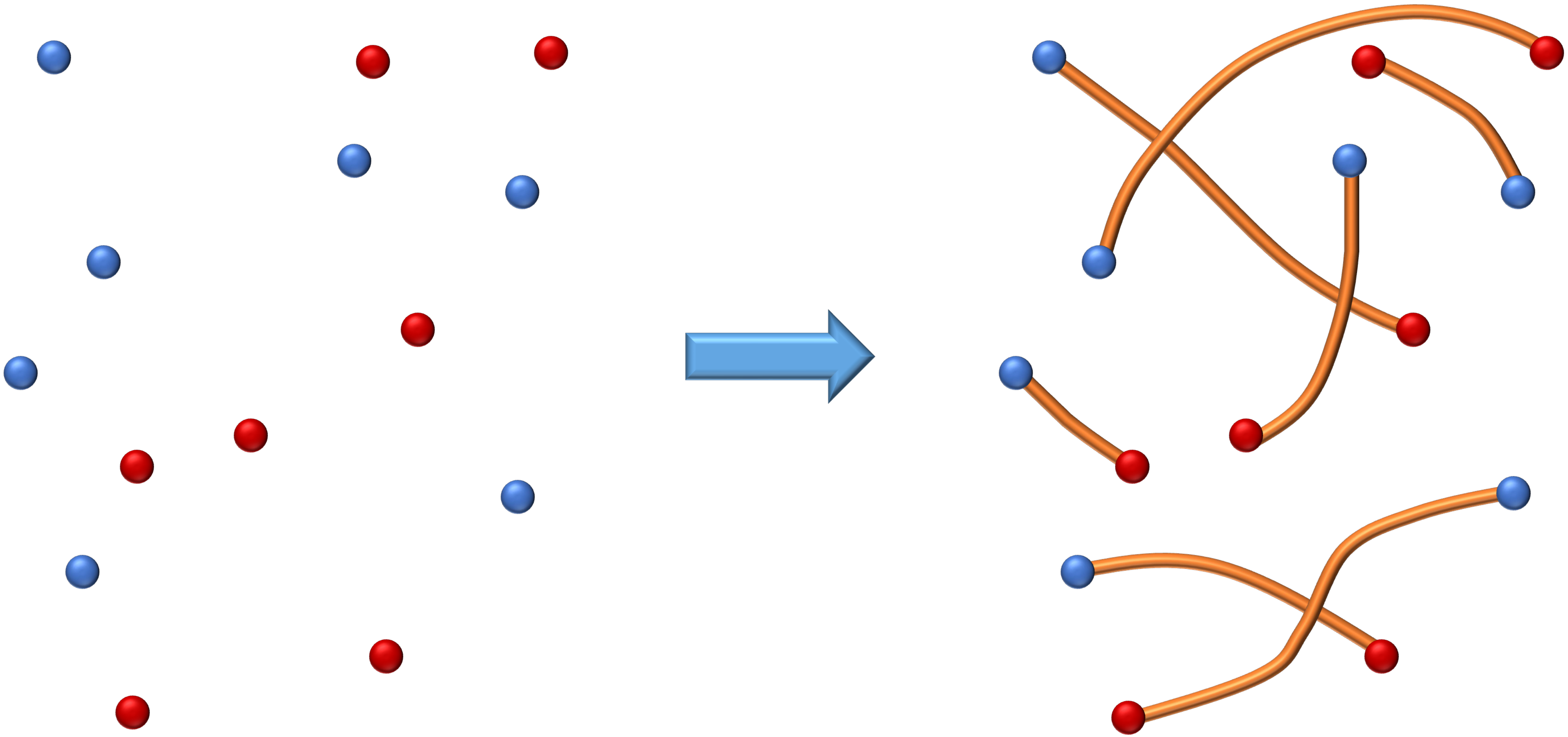
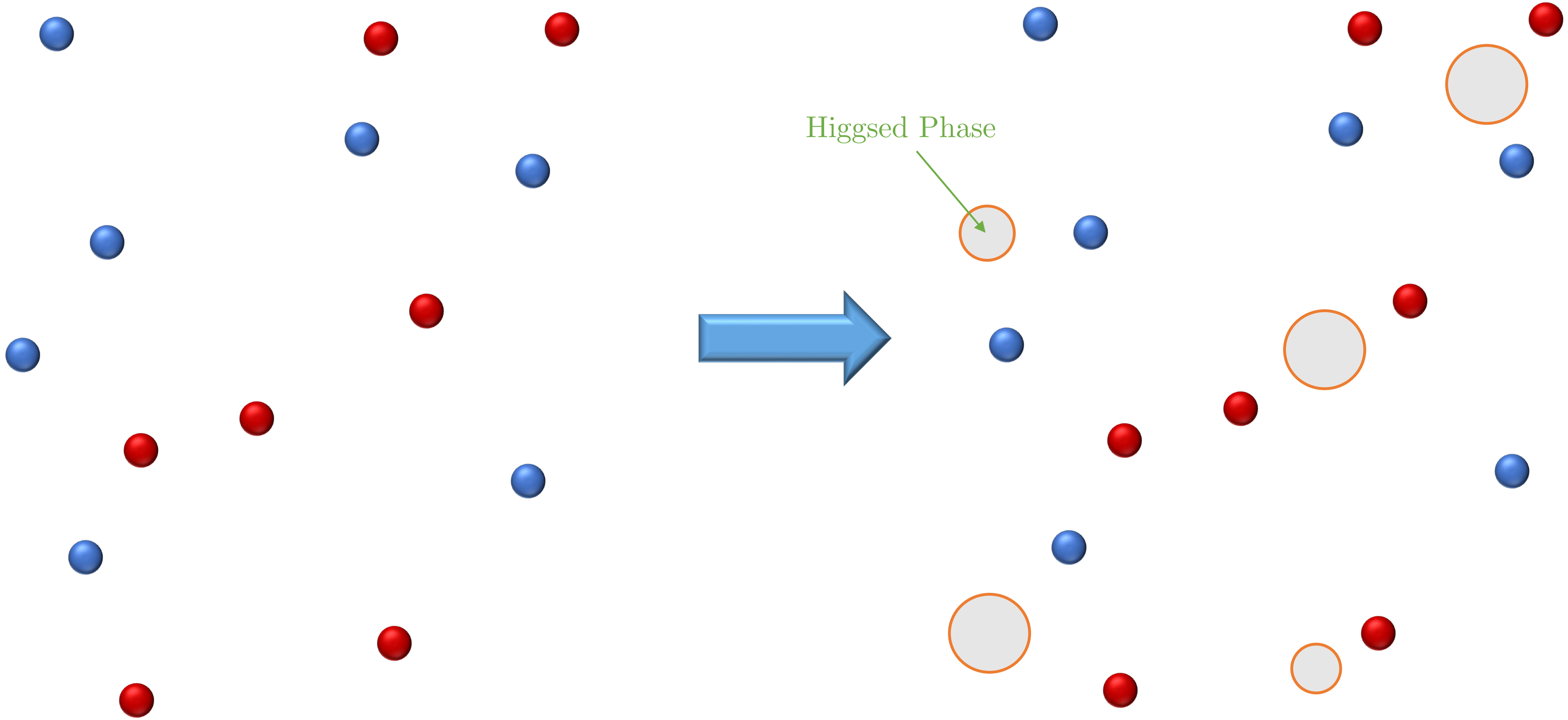


What happens in a first-order phase transition?

Second-Order Phase Transition



First-Order Phase Transition



The Model

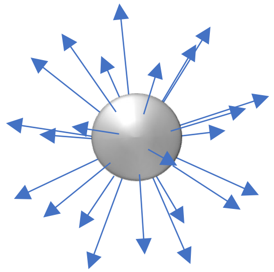
We consider an $SU(2)$ gauge theory with the following potential

$$V(\phi) = \lambda_\phi \left(\phi^a\phi^a - v_\phi^2\right)^2 + \lambda_\psi \left(\psi^\dagger\psi - v_\psi^2\right)^2 \left(\psi^\dagger\psi\right) + \beta\psi^\dagger\phi\psi$$

ϕ: $SU(2)$ adjoint
ψ: $SU(2)$ fundamental

$\langle\phi^a\phi^a\rangle = v_\phi^2 \rightarrow SU(2)$ breaks down to $U(1)$
$\rightarrow$ Magnetic Monopoles

't Hooft-Polyakov
Magnetic Monopole



Coulomb (unconfined)
Vacuum

The Model

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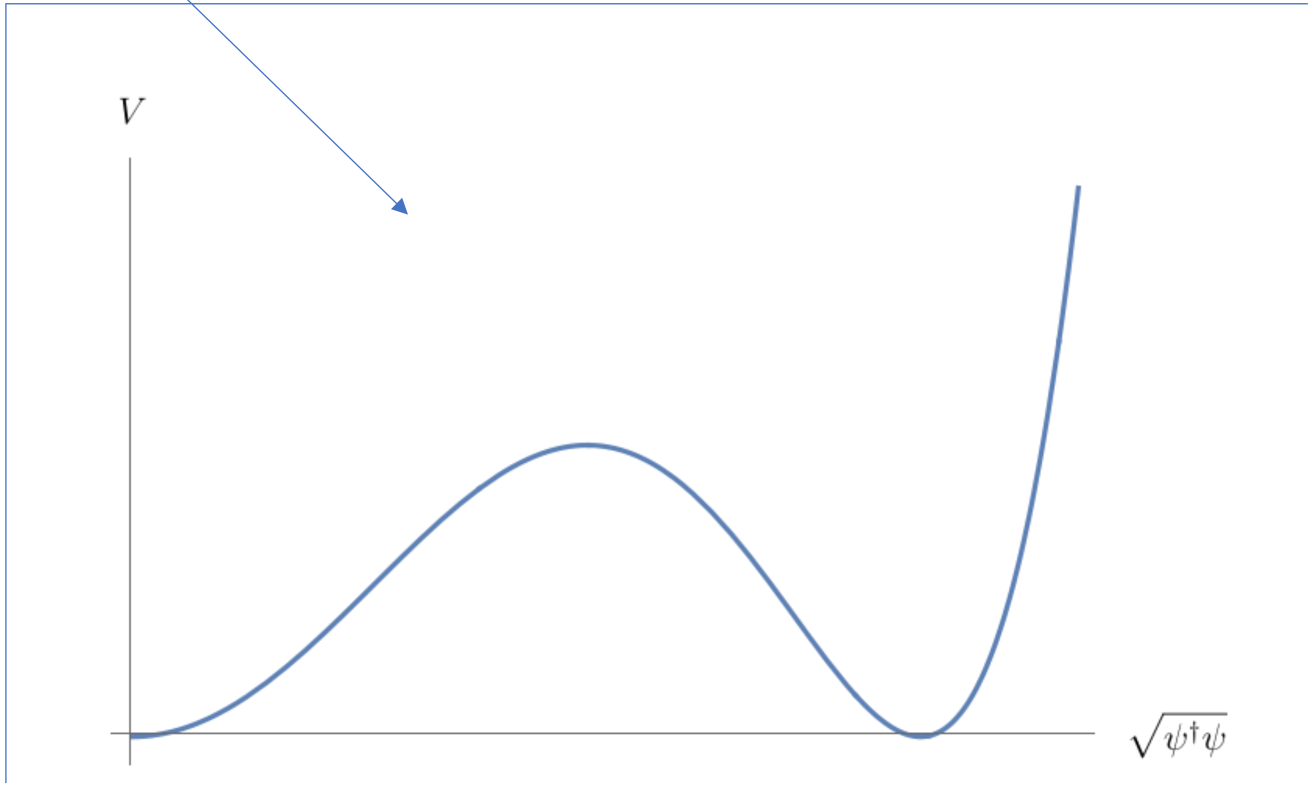

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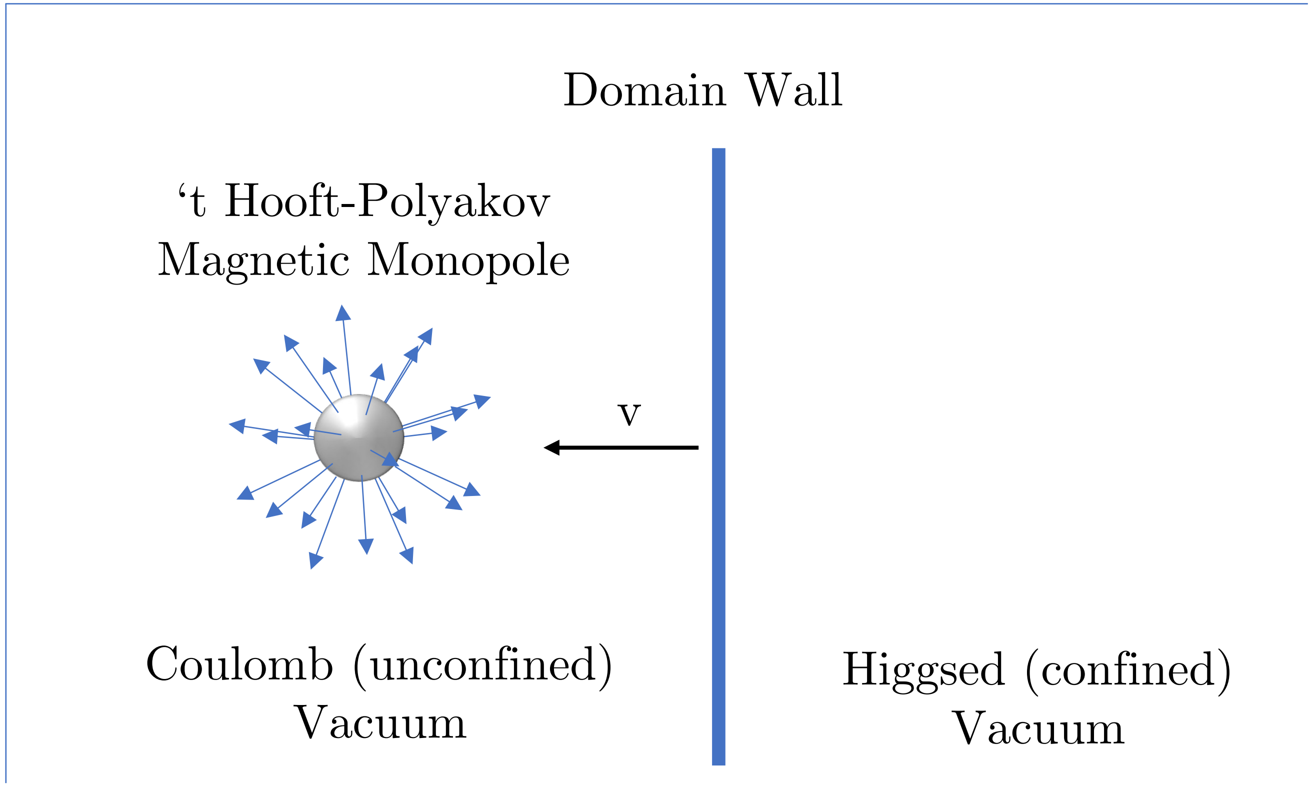
$\langle\phi^a\phi^a\rangle = v_\phi^2 \to SU(2)$ breaks down to $U(1)$
$\to$ Magnetic Monopoles

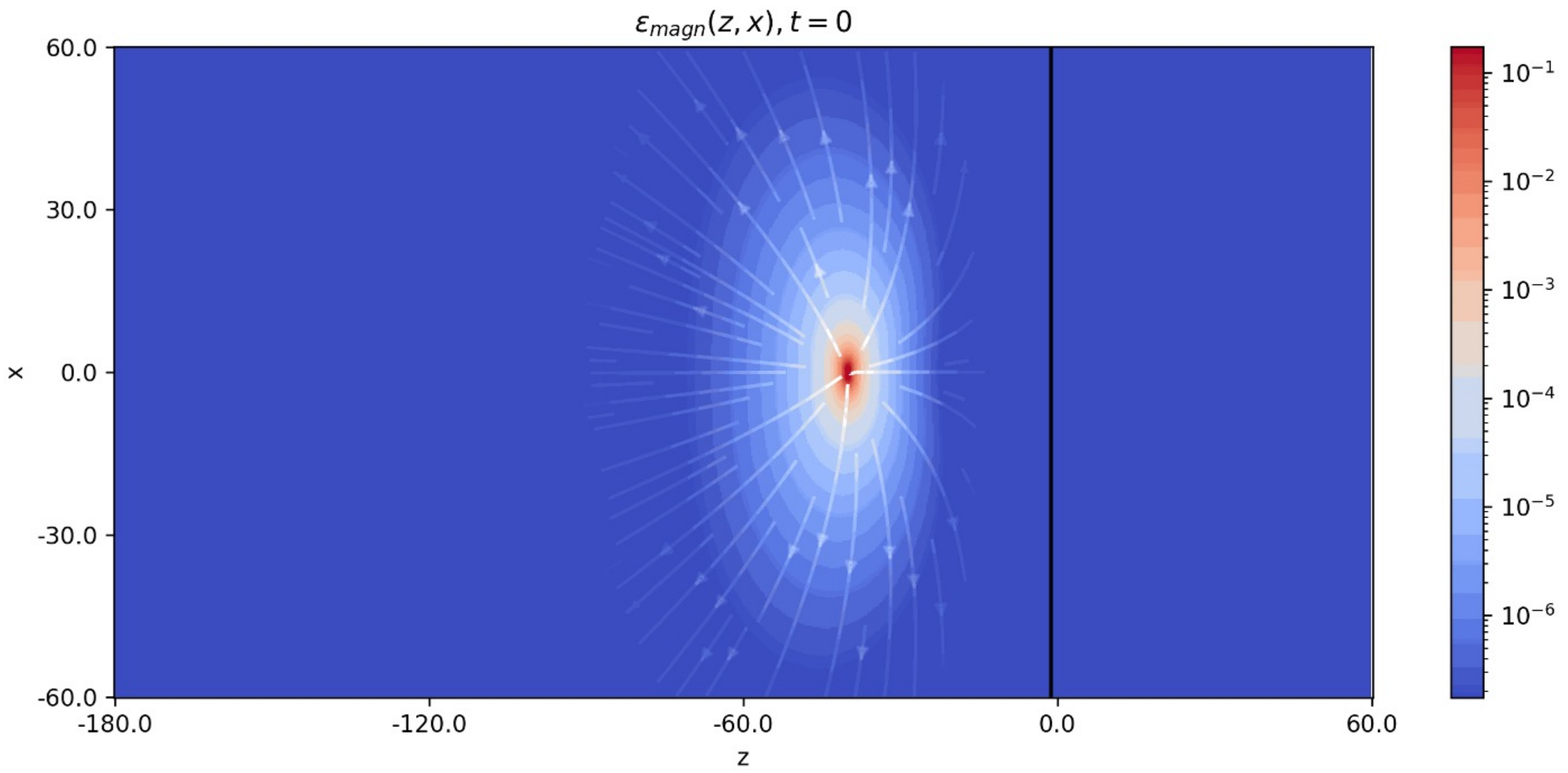
Disconnected Vacuum Manifold for ψ
$\to$ Domain Walls

$\langle\psi^\dagger\psi\rangle = v_\psi^2 \to U(1)$ breaks down to 1
$\to$ Cosmics Strings

Breaking Pattern: $SU(2) \xrightarrow{\phi} U(1) \xrightarrow{\psi} 1$



Confinement Slingshot



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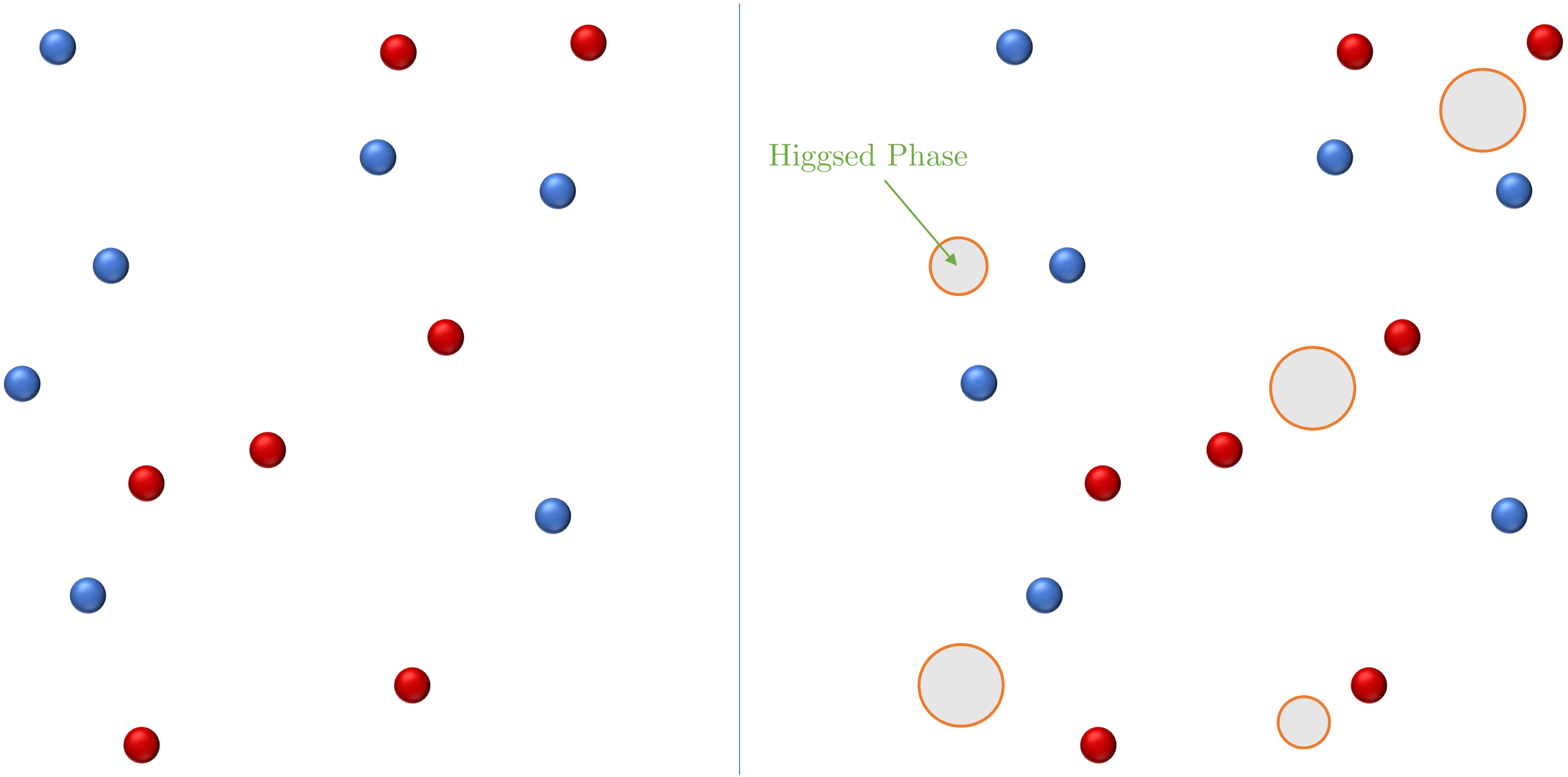
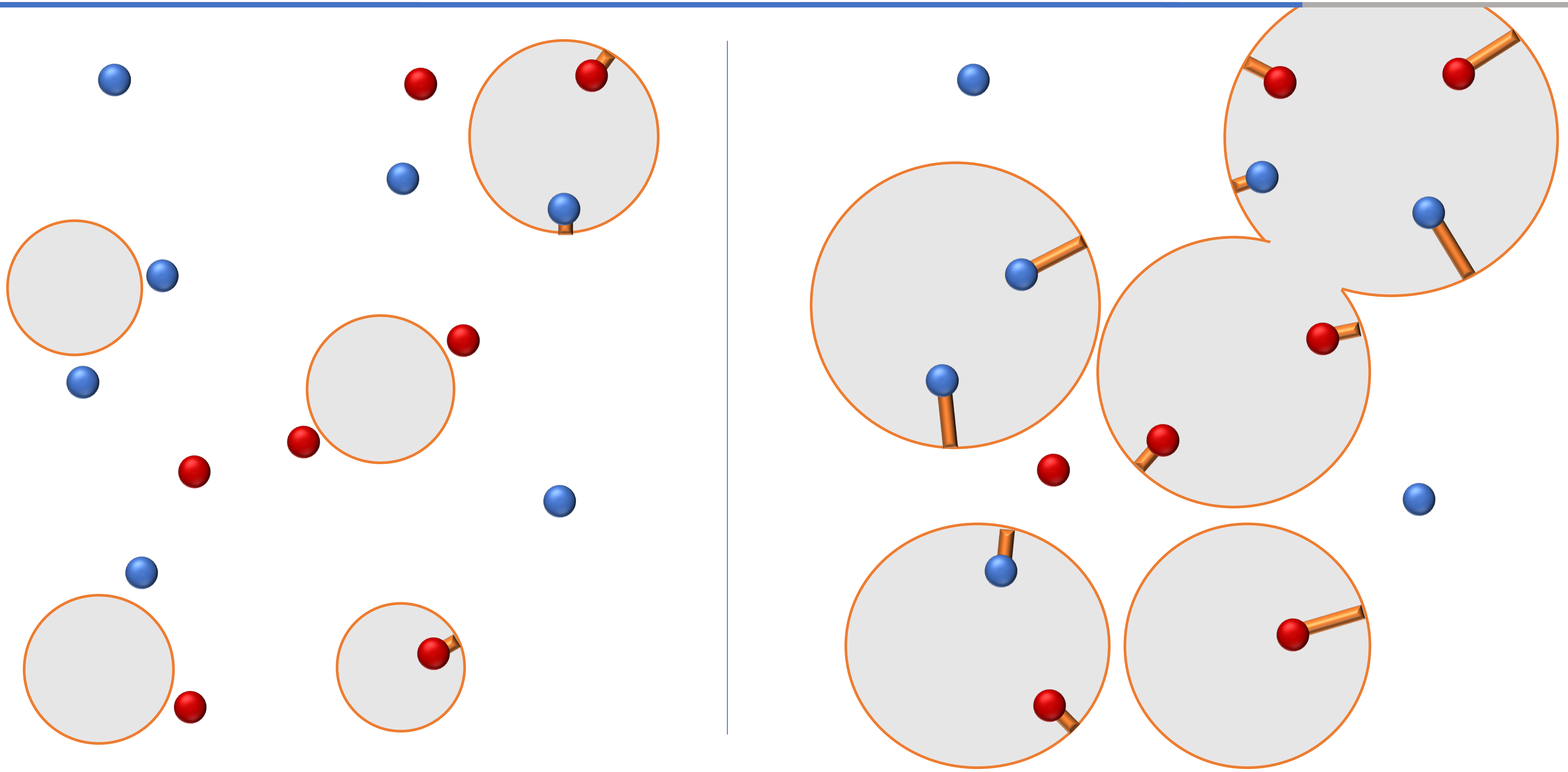
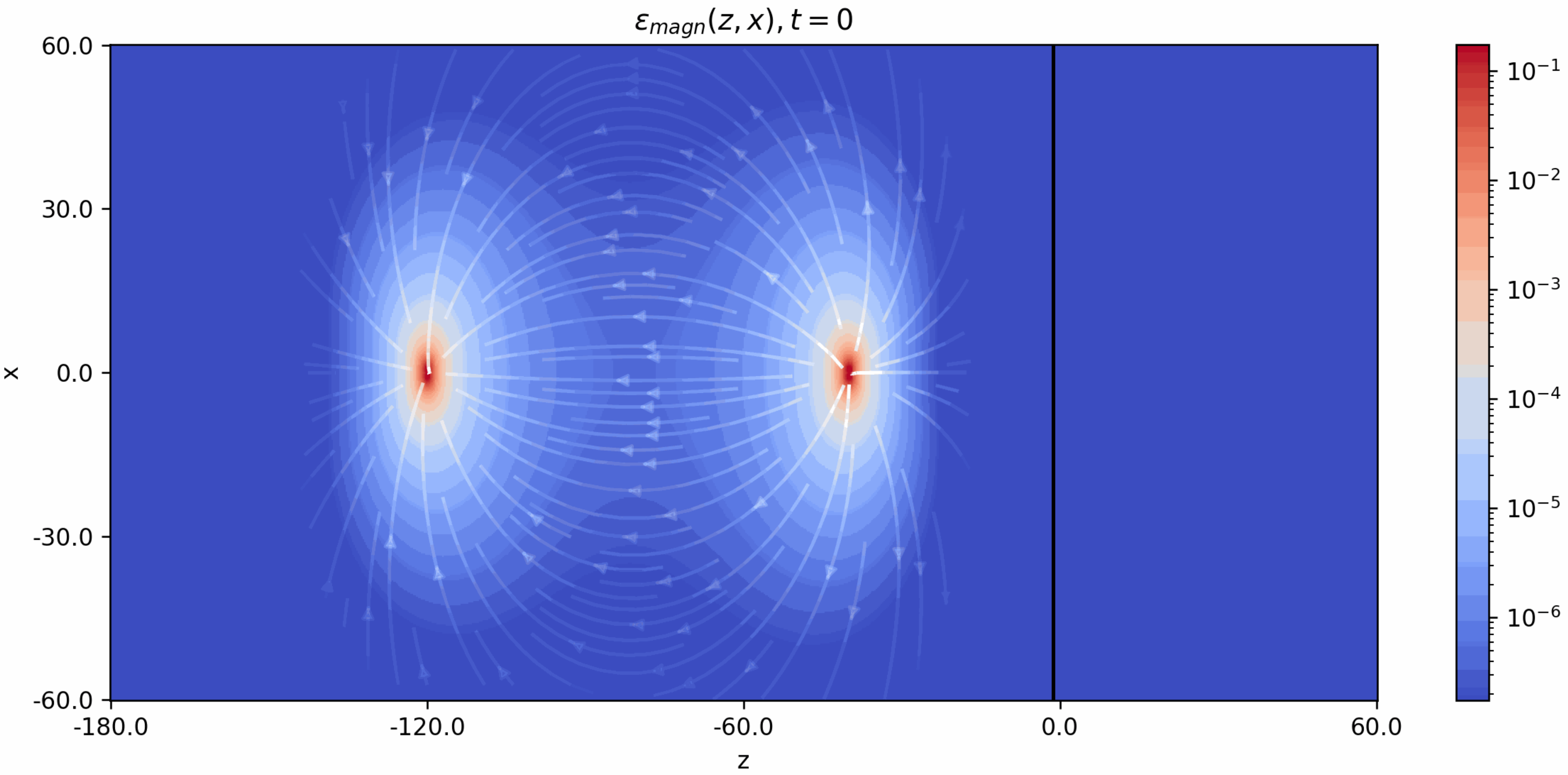
First-Order Phase Transition



First-Order Phase Transition



Confinement Slingshot – Two Monopoles



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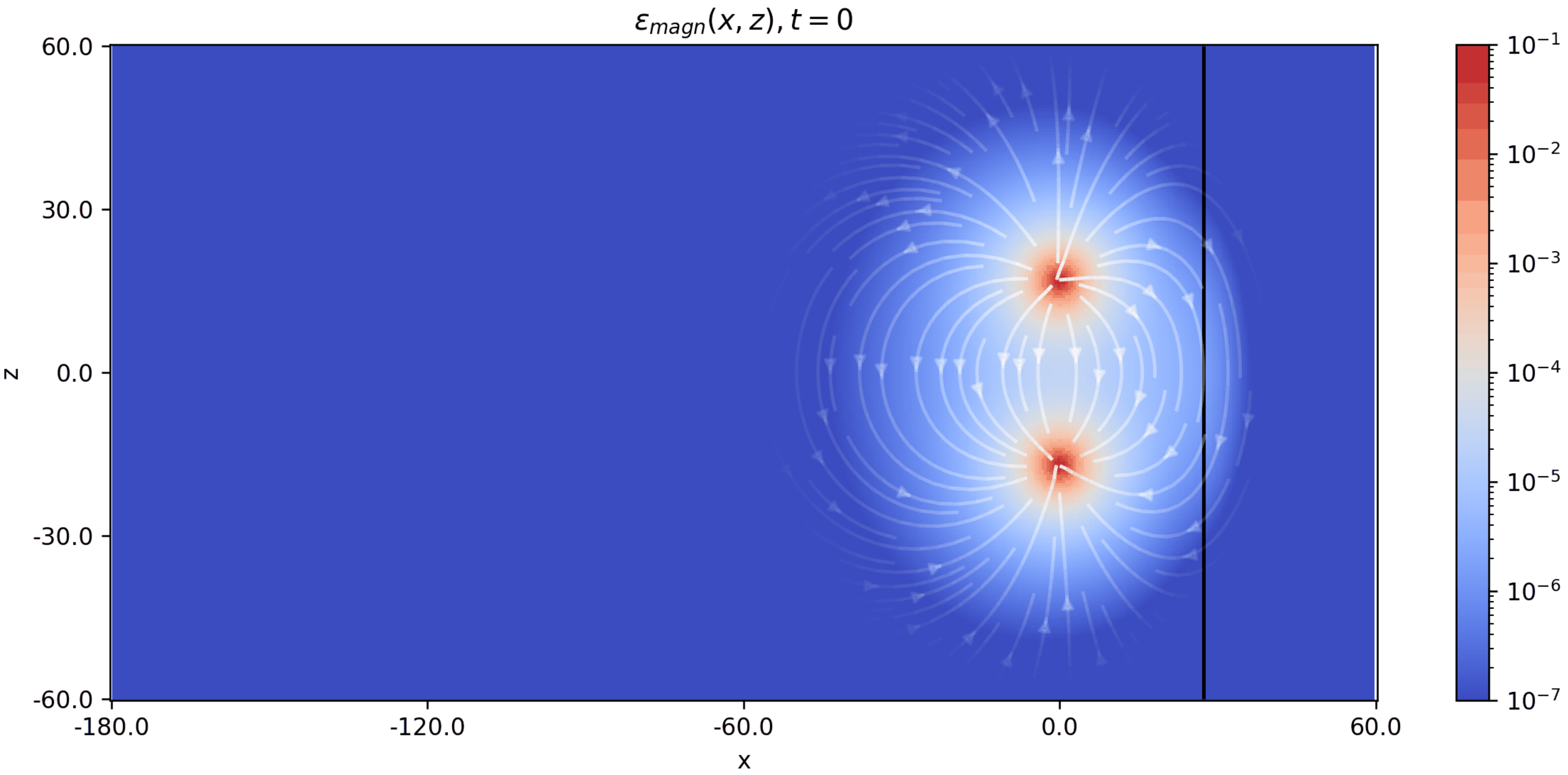
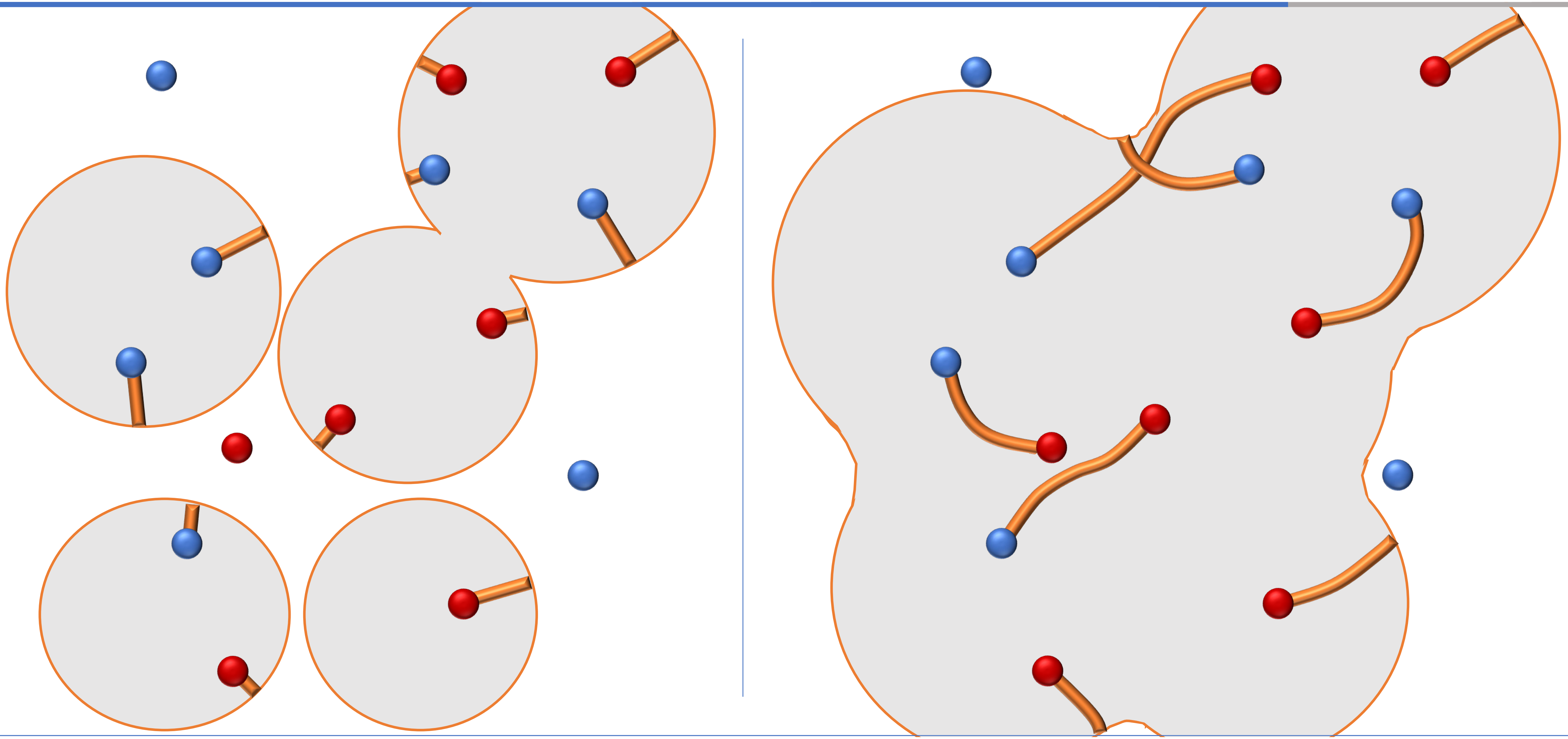
Multiple Confinement Slingshots

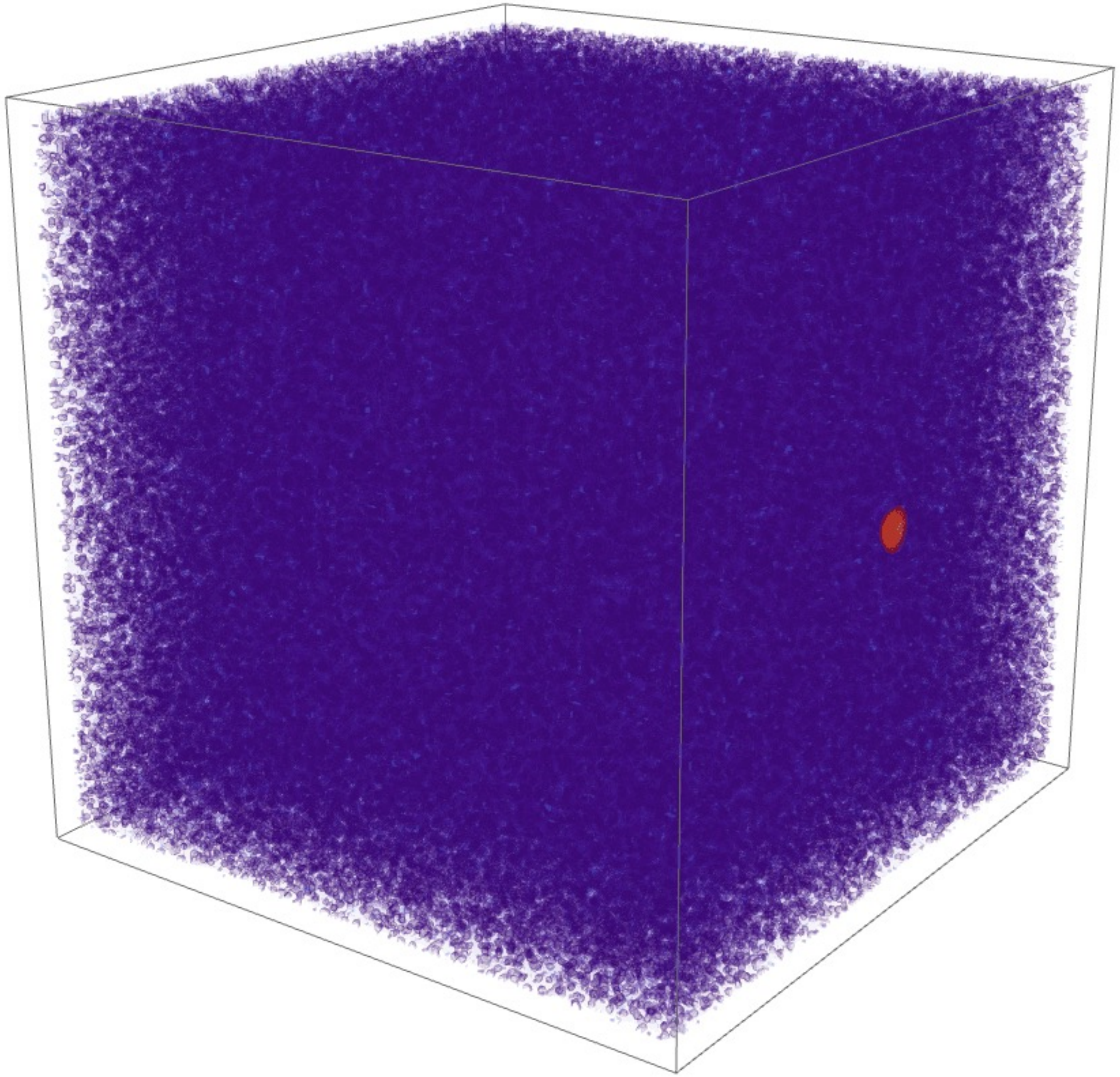


M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (to be published)

First-Order Phase Transition



First-Order Phase Transition



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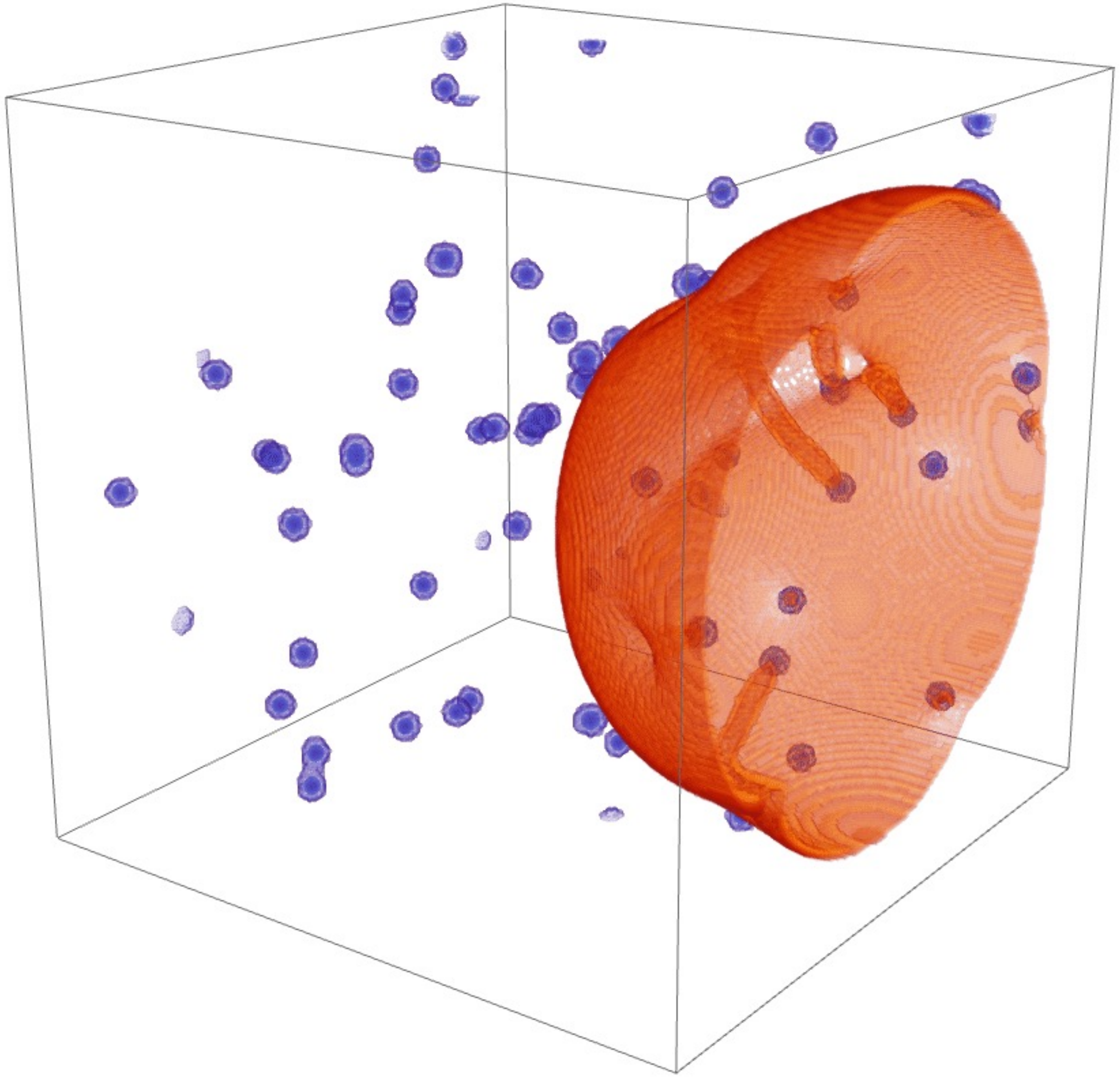
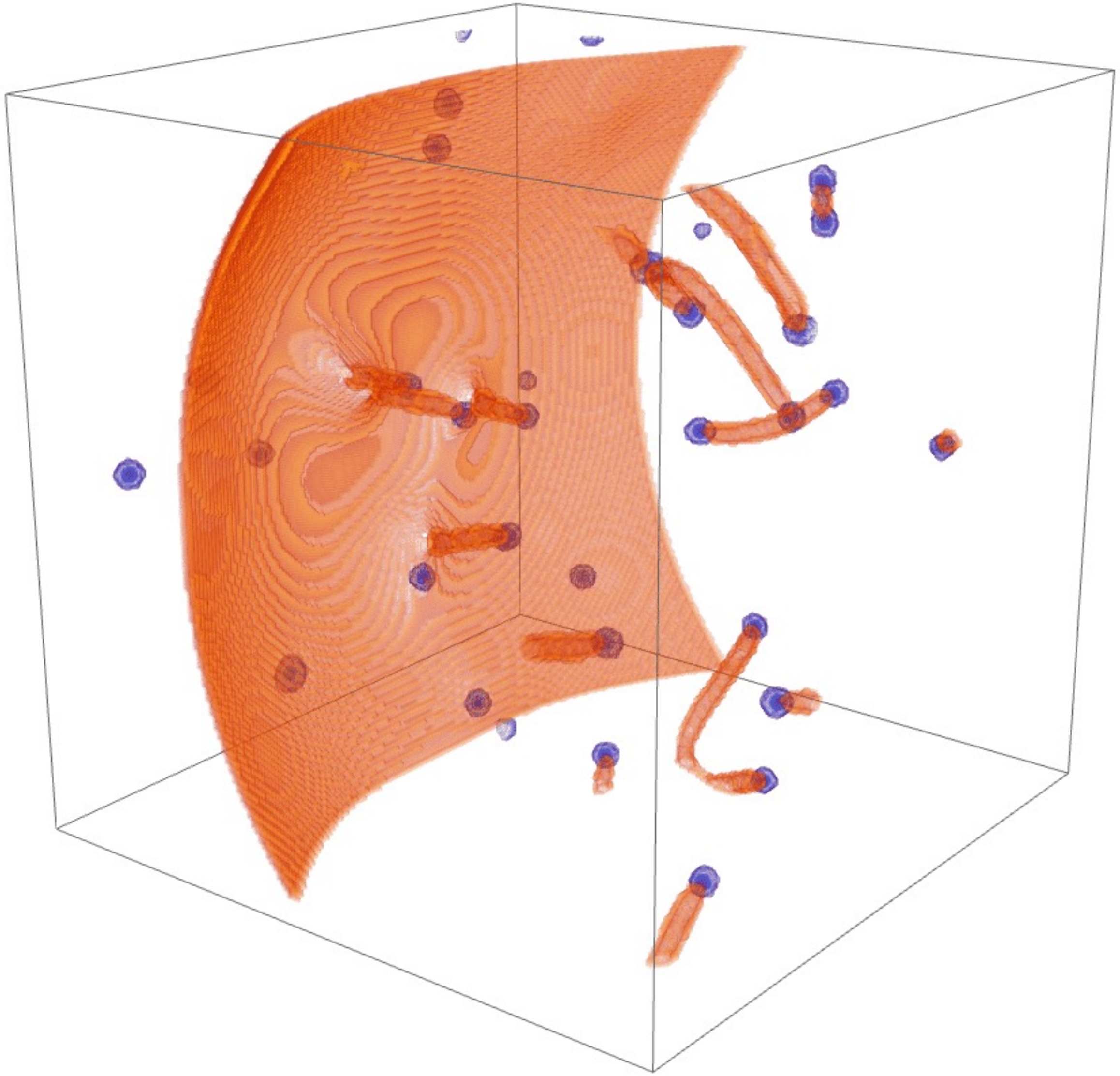


First-Order Phase Transition

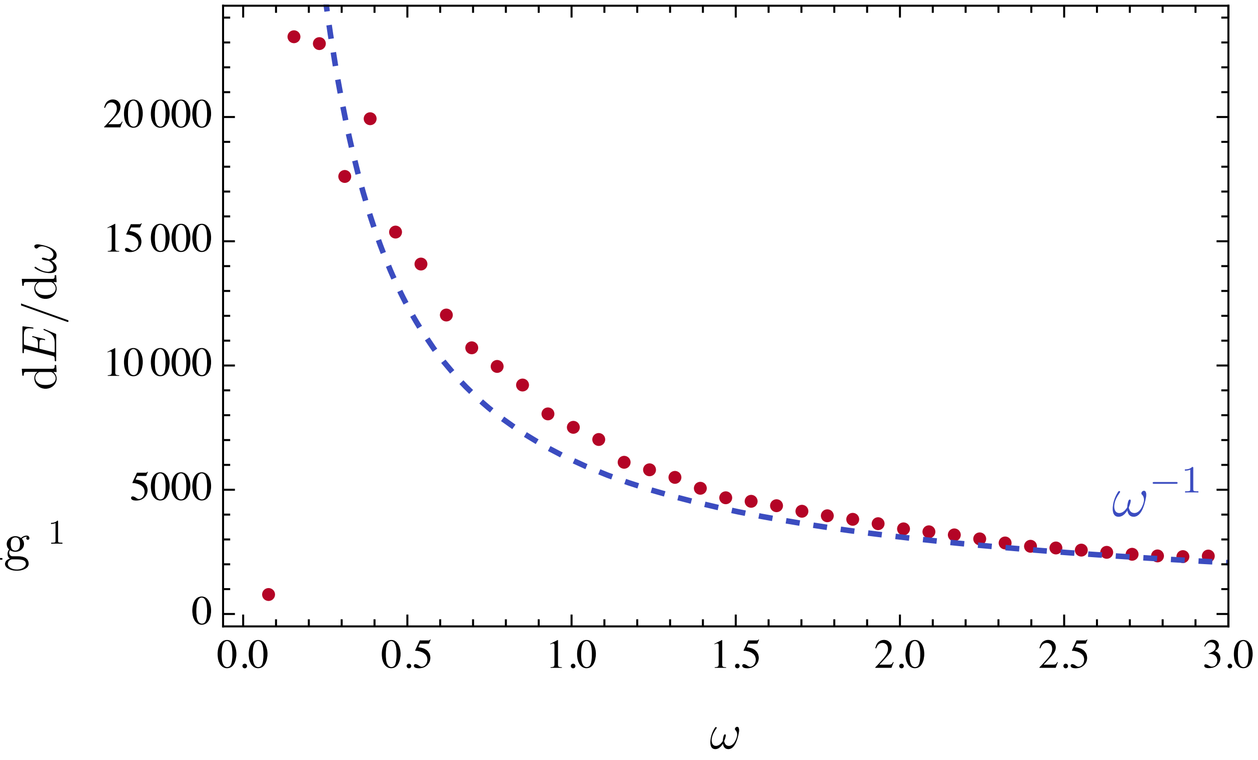


Confinement Slingshot – Gravitational Waves

The slingshot effect leads to the emission of gravitational waves

The energy spectrum decays with ω^{-1}

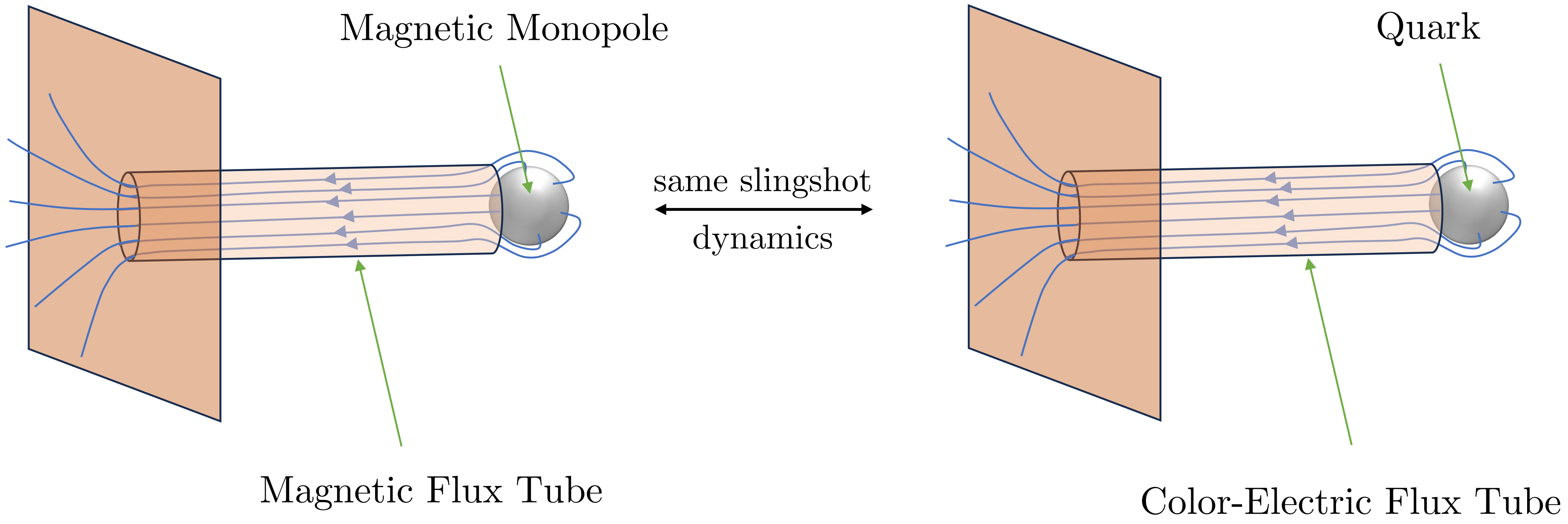
→ Similar to two monopoles connected by a string [1]



[1] X. Martin, A. Vilenkin (1997) / G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (2022)

Confinement Slingshot – Quark Confinement

The similar slingshot effect is expected in a "dual" picture when a heavy quark crosses into a confined vacuum of QCD.



(In the case of light quarks, quark-antiquark pairs can emerge and break the string.)

Summary

We analyzed the slingshot effect that can appear in first-order phase transitions involving magnetic monopoles/quarks

The slingshot can also happen for vortices/strings in 2+1/3+1 dimensions

The slingshot effect leads to the emission of gravitational waves
$\rightarrow$ observable? bounds on monopoles, GUT's, etc.?

Summary

We analyzed the slingshot effect that can appear in first-order phase transitions involving magnetic monopoles/quarks

The slingshot can also happen for vortices/strings in 2+1/3+1 dimensions

The slingshot effect leads to the emission of gravitational waves
$\rightarrow$ observable? bounds on monopoles, GUT's, etc.?

Thank you!

More Videos on Youtube:

Maximilian Bachmaier

Backup

Numerical Simulation

Python Package Numba:

- Translates Python and NumPy code into fast machine code
- Easy implementation of parallelization

Numerical Simulation – Axial Symmetry

Axial Symmetry:

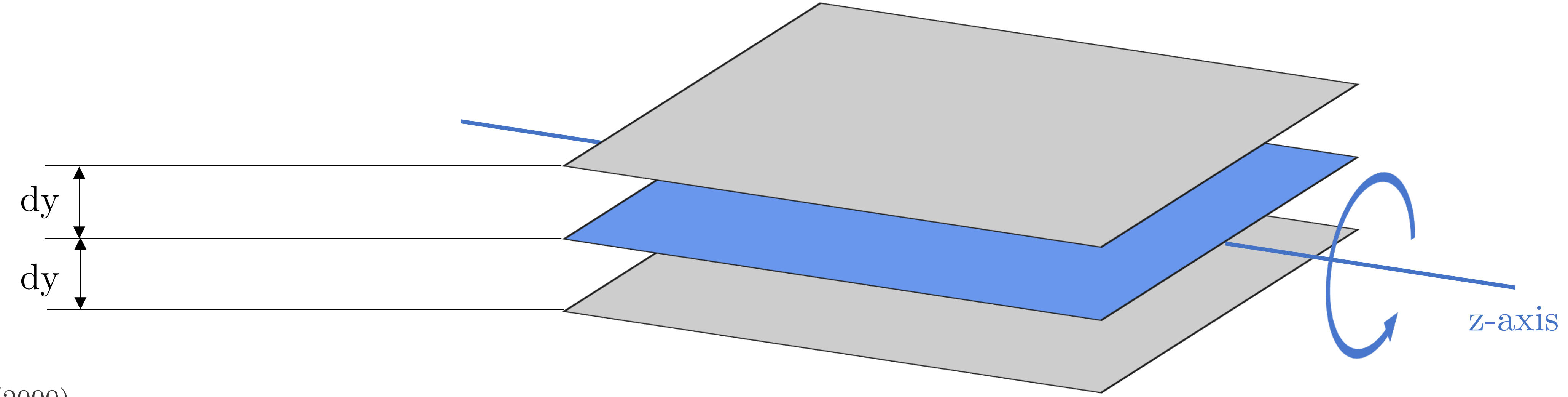
$$\phi^1 = xf_1 + yf_2 \qquad \phi^2 = yf_1 - xf_2 \qquad \phi^3 = f_3$$

$$W_x^1 = xyf_4 + y^2 f_5 + f_6 \qquad W_x^2 = -x^2 f_4 - xyf_5 + f_7 \qquad W_x^3 = xf_8 + yf_9$$

$$W_y^1 = y^2 f_4 - xyf_5 - f_7 \qquad W_y^2 = -xyf_4 + x^2 f_5 + f_6 \qquad W_y^3 = yf_8 - xf_9$$

$$W_z^1 = xf_{10} + yf_{11} \qquad W_z^2 = -xf_{11} + yf_{10} \qquad W_z^3 = 0$$

$$W_t^1 = xf_{12} + yf_{13} \qquad W_t^2 = -xf_{13} + yf_{12} \qquad W_t^3 = 0$$



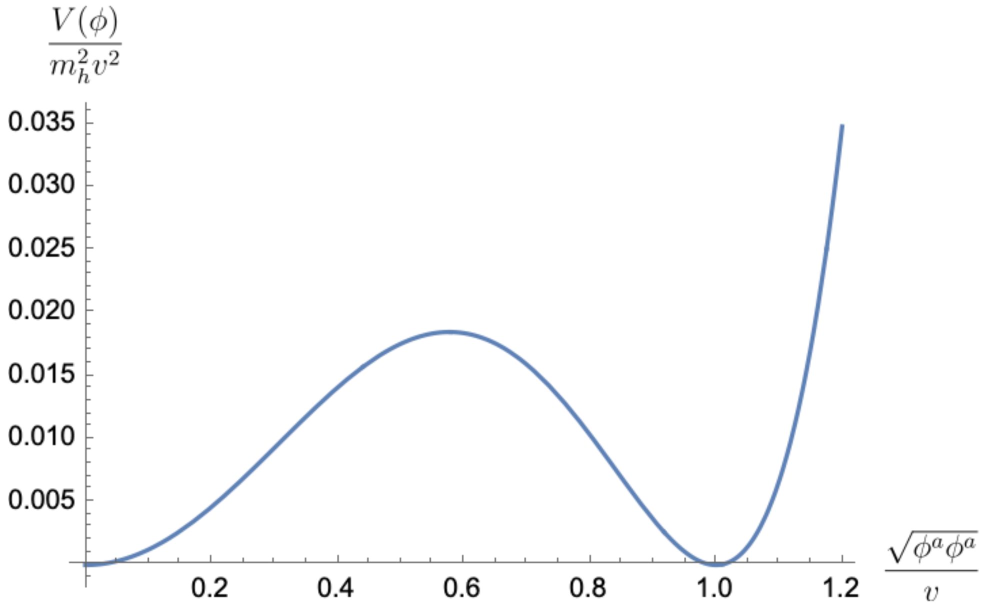
L. Pogosian, T. Vachaspati (2000)

The Model

We consider an $SU(2)$ gauge theory with sextic potential

$$V(\phi) = \frac{\lambda}{8}\left(\phi^a\phi^a - v^2\right)^2\left(\phi^a\phi^a\right)$$

adjoint



G. Dvali, H. B. Nielsen, N. Tetradis (2008)

$$\langle\phi^a\phi^a\rangle = \begin{cases} 0 \rightarrow SU(2) \text{ invariant vacuum} \\ v^2 \rightarrow U(1) \text{ invariant vacuum} \end{cases}$$

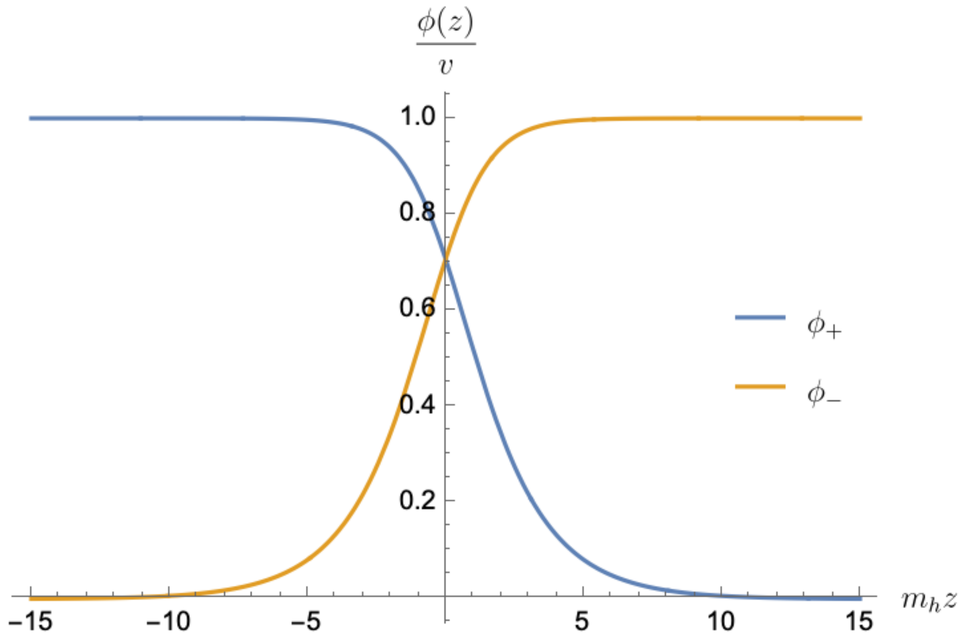
Disconnected Vacuum Manifold
$\rightarrow$ Domain Walls

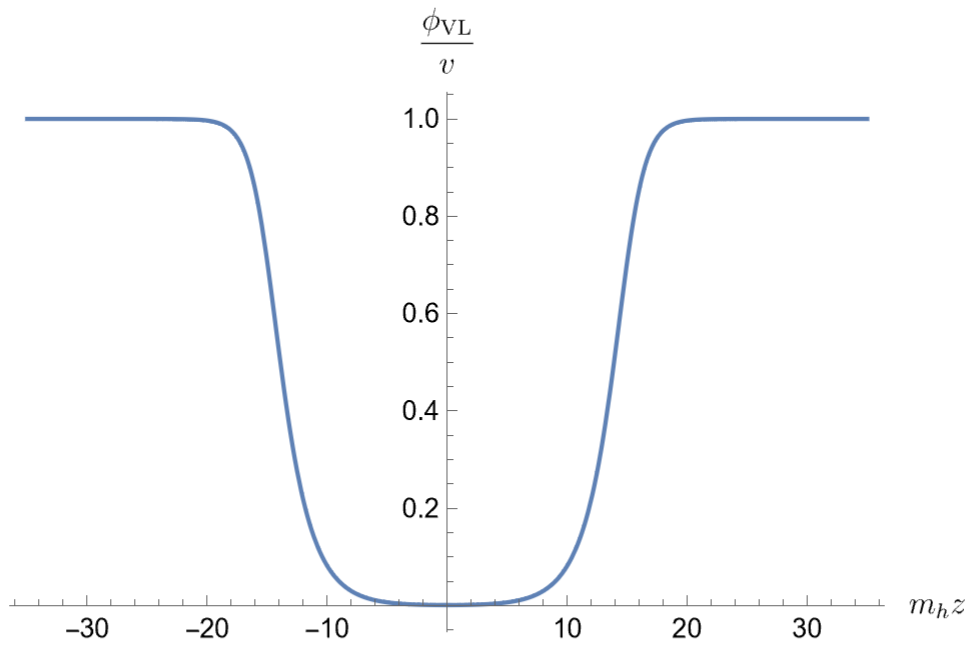
$\langle\phi^a\phi^a\rangle = v^2 \rightarrow SU(2)$ breaks down to $U(1)$
$\rightarrow$ Magnetic Monopoles

Domain Wall Solution

Two domain wall solutions are

$$\phi_{\pm} = \frac{v}{\sqrt{1 + e^{\pm m_h z}}}$$

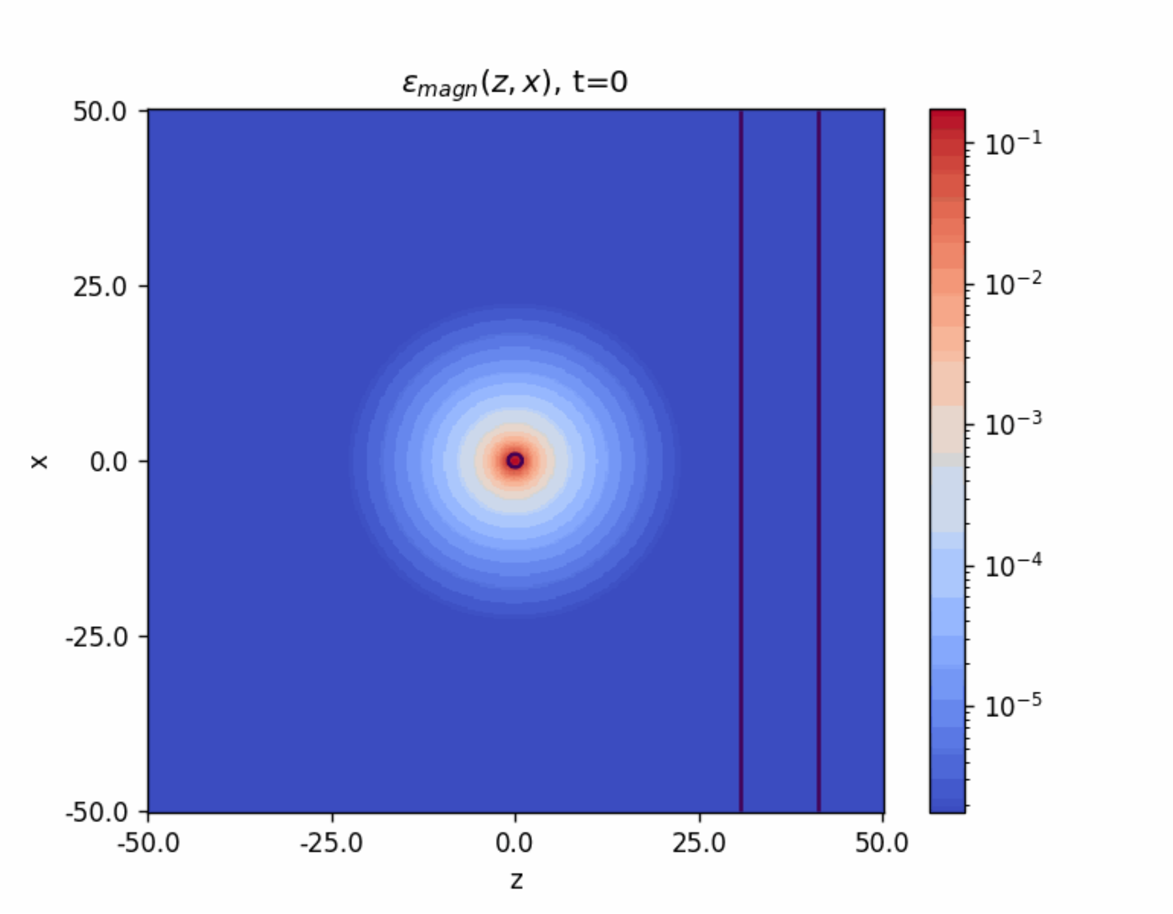




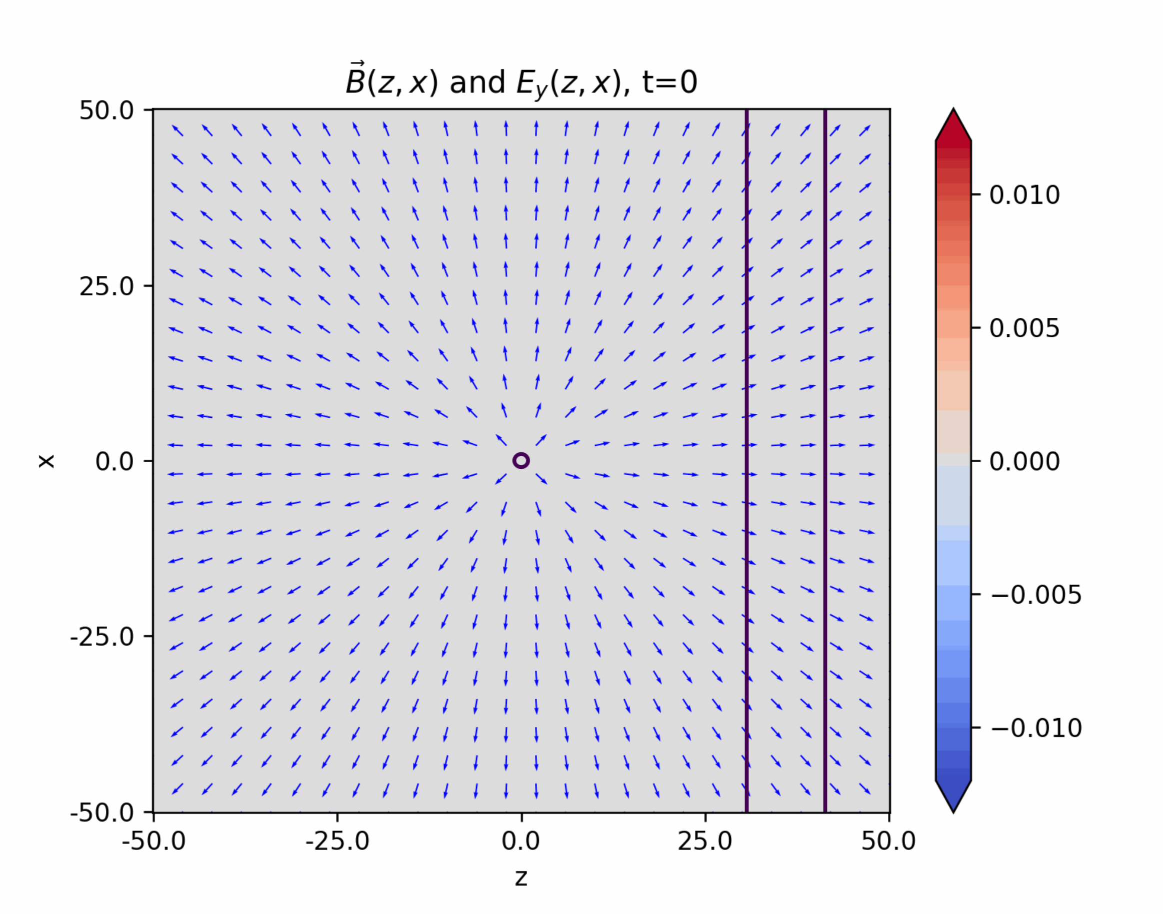
SU(2) Invariant Vacuum Layer

Erasure of the Monopole

Magnetic Energy Density



Magnetic and Electric Field



Erasure of the Monopole

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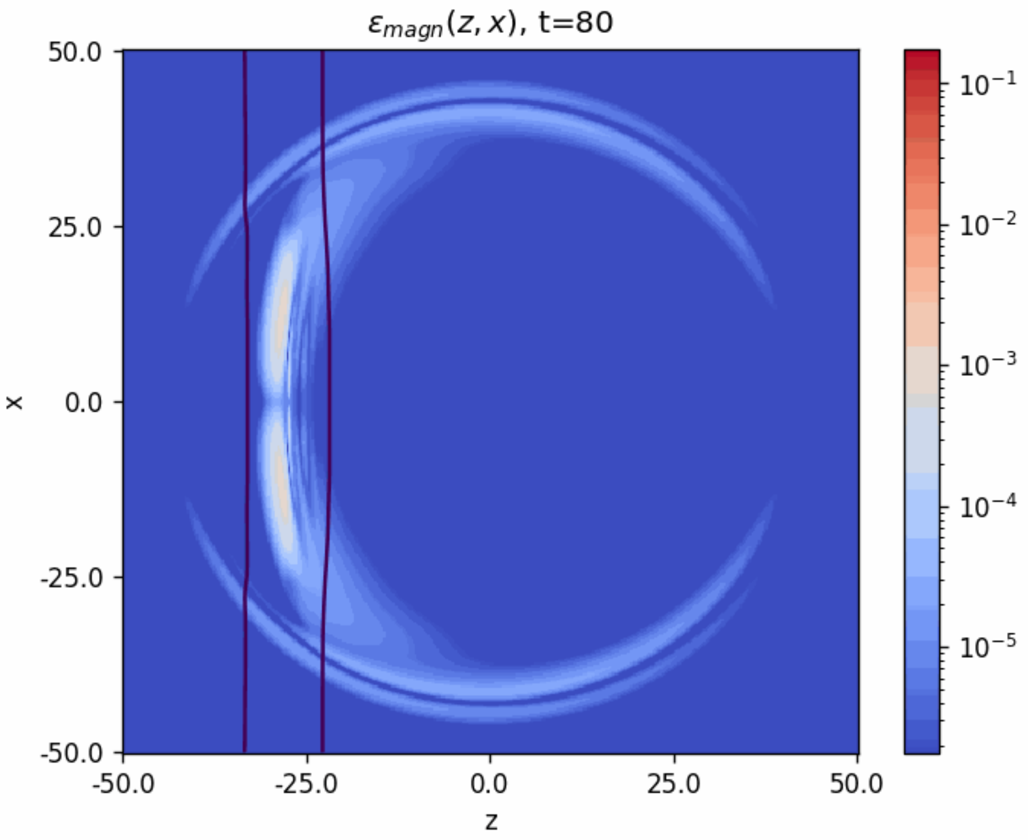

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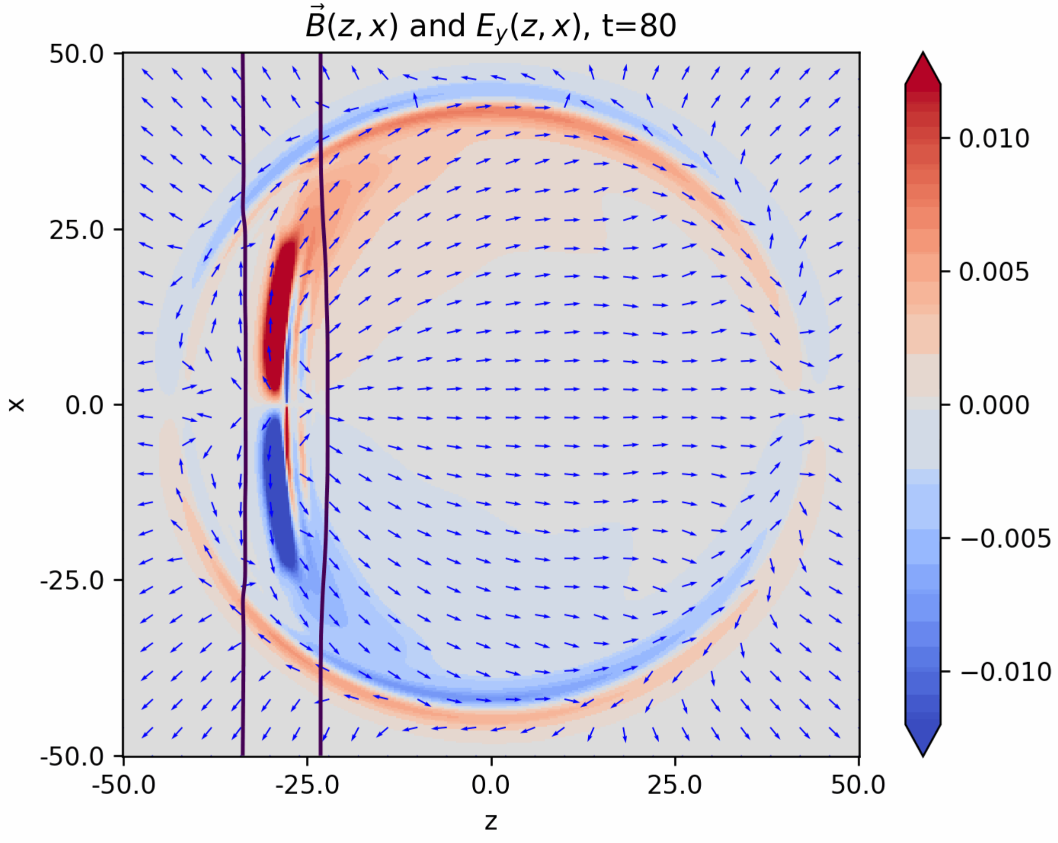

Initial Configuration – Domain Wall

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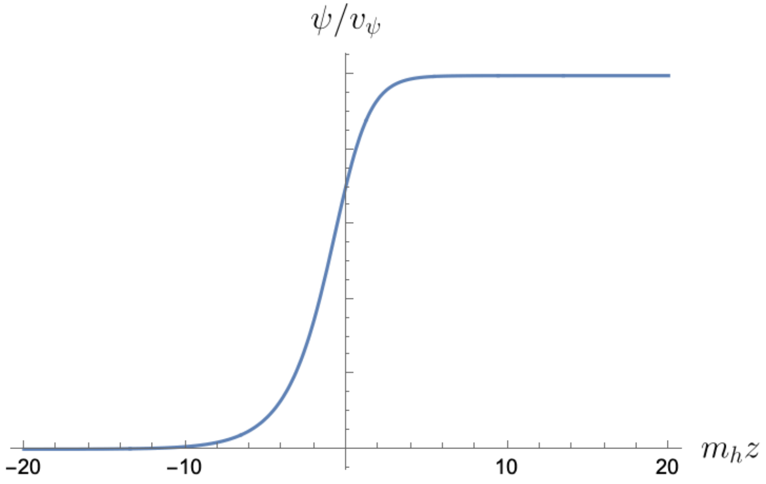
This term needs to be minimized

$$\rightarrow \psi^\dagger\sigma^a\psi \sim \phi^a$$

The profile of the domain wall is

$$|\psi| = \frac{v_\psi}{\sqrt{1+e^{-m_h z}}}$$



Initial Configuration – Magnetic Monopole

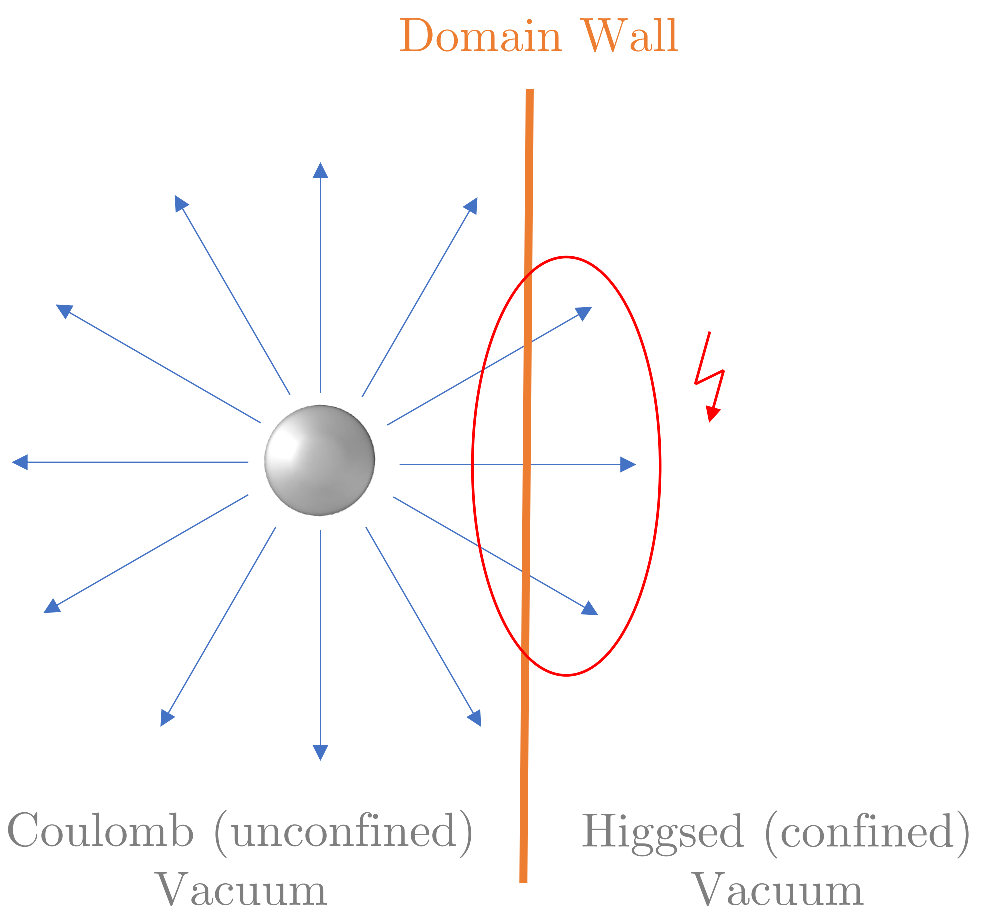
Ansatz for ’t Hooft-Polyakov magnetic monopole:

$$W_i^a = \varepsilon_{aij} \frac{r_j}{r^2} \frac{1}{g} (1 - K(r))$$

$$W_t^a = 0$$

$$\phi^a = \frac{r^a}{r^2} \frac{1}{g} H(r)$$

Magnetic field needs to be repelled from the domain wall!



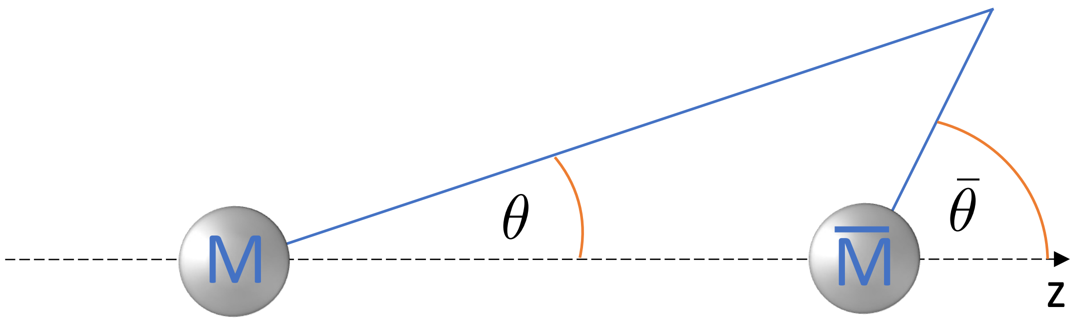
Initial Configuration – Magnetic Monopole

Ansatz for a monopole-antimonopole configuration[1]:

$$\hat{\phi}_1 = \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\cos\left(\varphi - \alpha/2\right) + \sin\theta\sin\alpha\sin\left(\varphi - \alpha/2\right)$$

$$\hat{\phi}_2 = \left(\sin\bar{\theta}\cos\theta - \sin\theta\cos\bar{\theta}\cos\alpha\right)\sin\left(\varphi - \alpha/2\right) - \sin\theta\sin\alpha\cos\left(\varphi - \alpha/2\right)$$

$$\hat{\phi}_3 = -\cos\theta\cos\bar{\theta} - \sin\theta\sin\bar{\theta}\cos\alpha$$



α: twist parameter

The gauge field is given by

$$W^a_\mu \sim \varepsilon_{abc}\hat{\phi}^b\partial_\mu\hat{\phi}^c$$

[1]T. Vachaspati, G. B. Field (1994) / A Saurabh, T. Vachaspati (2018)

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&\quad + \sin\theta\sin\alpha\sin\left(\varphi - \alpha/2\right) \\
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&\quad - \sin\theta\sin\alpha\cos\left(\varphi - \alpha/2\right) \\
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$$

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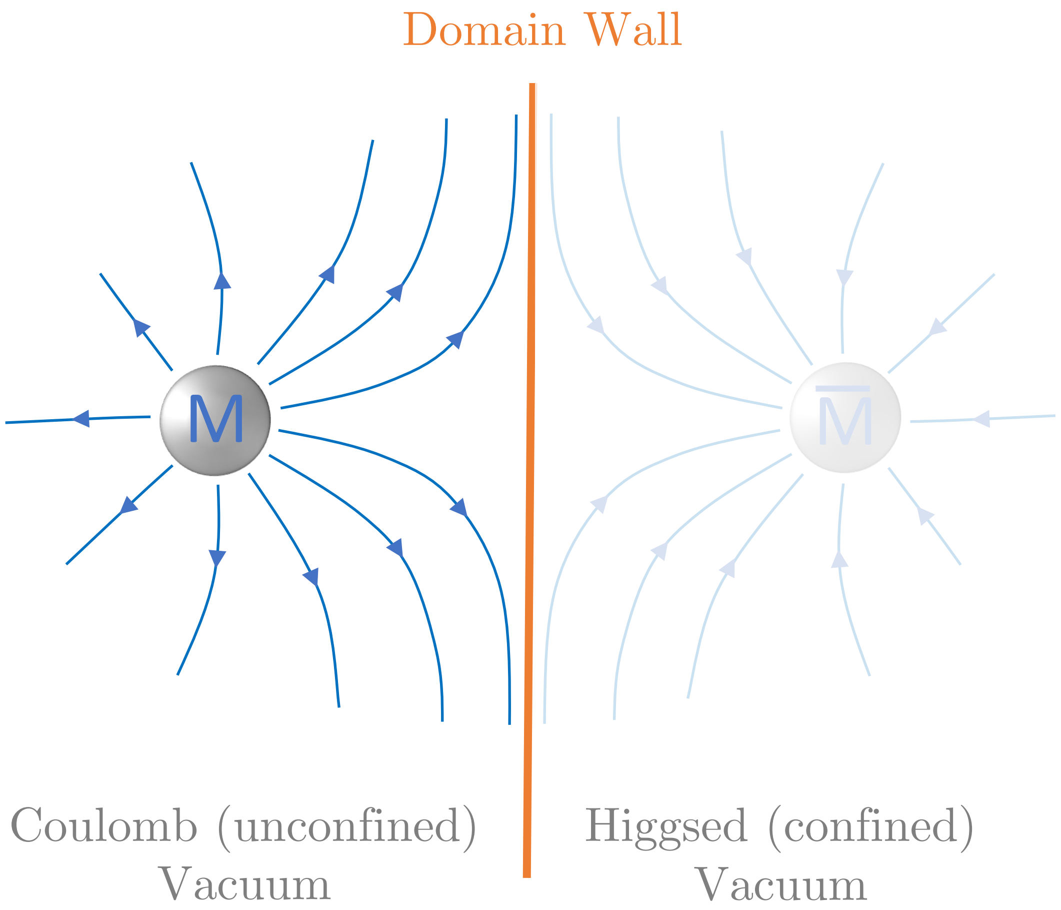
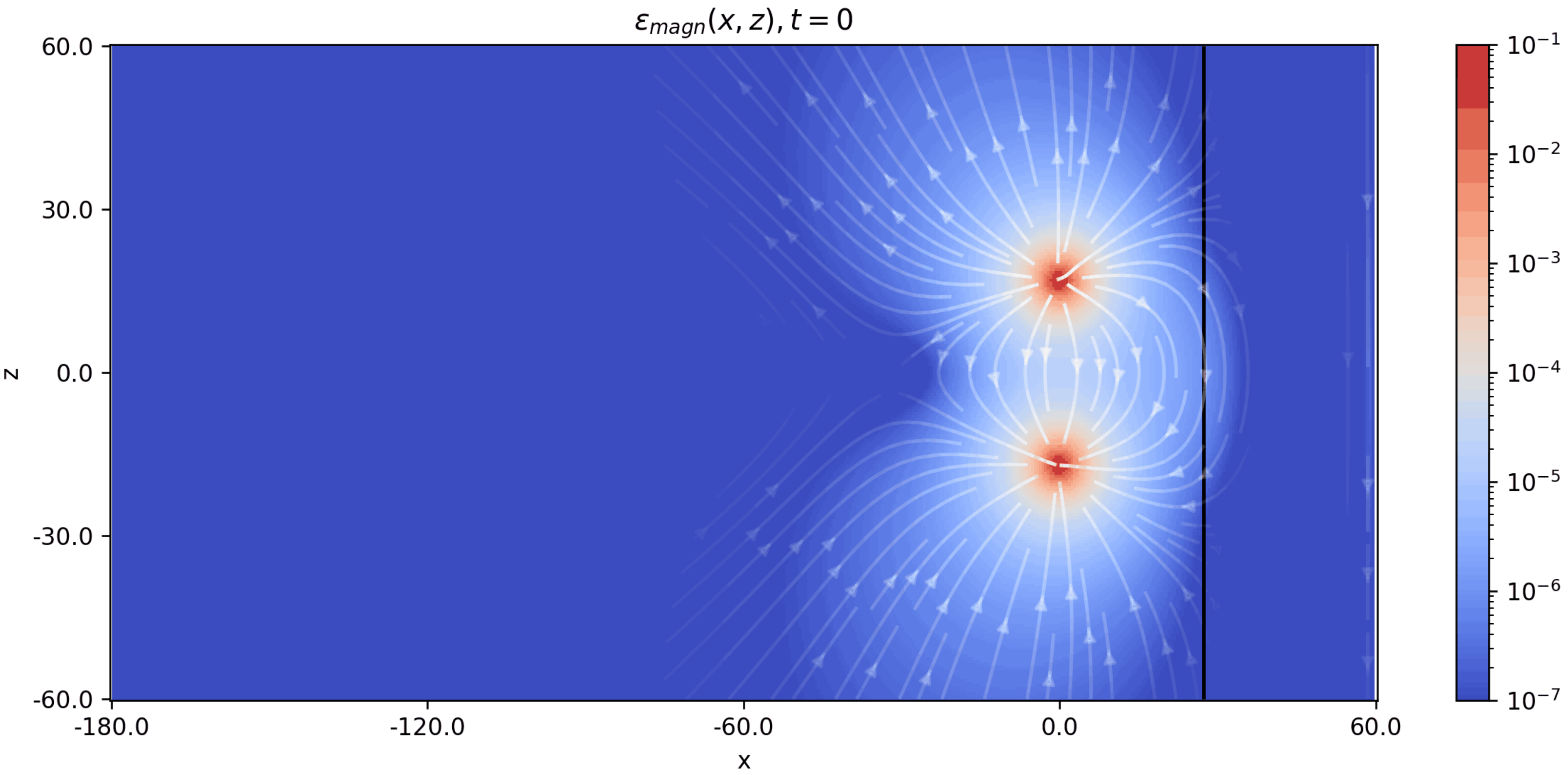


[1]T. Vachaspati, G. B. Field (1994) / A Saurabh, T. Vachaspati (2018)

Multiple Confinement Slingshots (maximal Twist)



M. B., G. Dvali, J. S. Valbuena-Bermúdez, M. Zantedeschi (to be published)

Confinement Slingshot – Gravitational Waves

The gravitational radiation spectrum can be calculated by Weinberg's formula:

$$\frac{\mathrm{d}E}{\mathrm{d}\Omega\,\mathrm{d}\omega} = \frac{G\,\omega^2}{2\pi^2}\Lambda_{ij,lm}(\hat{\mathbf{k}})T^{ij*}(\mathbf{k},\omega)T^{lm}(\mathbf{k},\omega)\,, \tag{1}$$

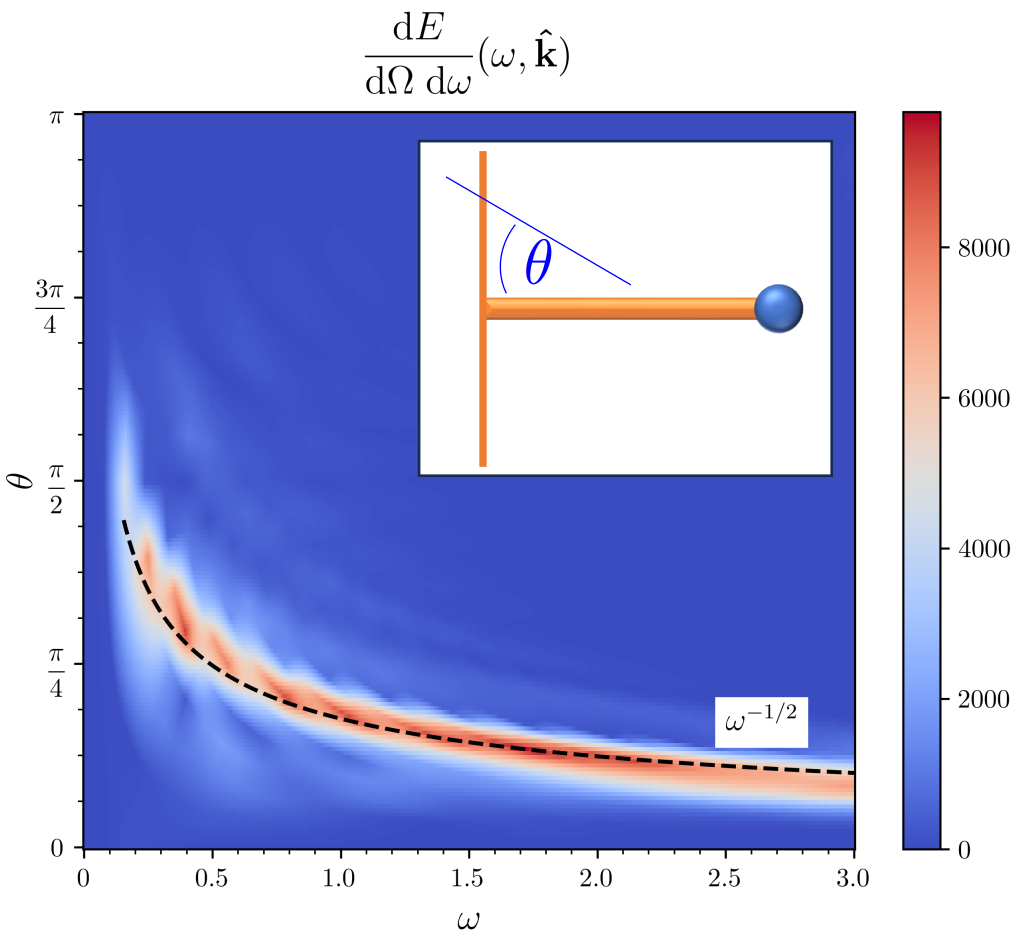
with

$$\Lambda_{ij,lm}(\hat{\mathbf{k}}) \equiv P_{il}(\hat{\mathbf{k}})P_{jm}(\hat{\mathbf{k}}) - \frac{1}{2}P_{ij}(\hat{\mathbf{k}})P_{lm}(\hat{\mathbf{k}})\,, \tag{2}$$

where $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\hat{k}_j$.

S. Weinberg (1972)

Confinement Slingshot – Gravitational Waves



The angle of emission depends on the frequency

Most of the radiation is emitted in the direction of acceleration

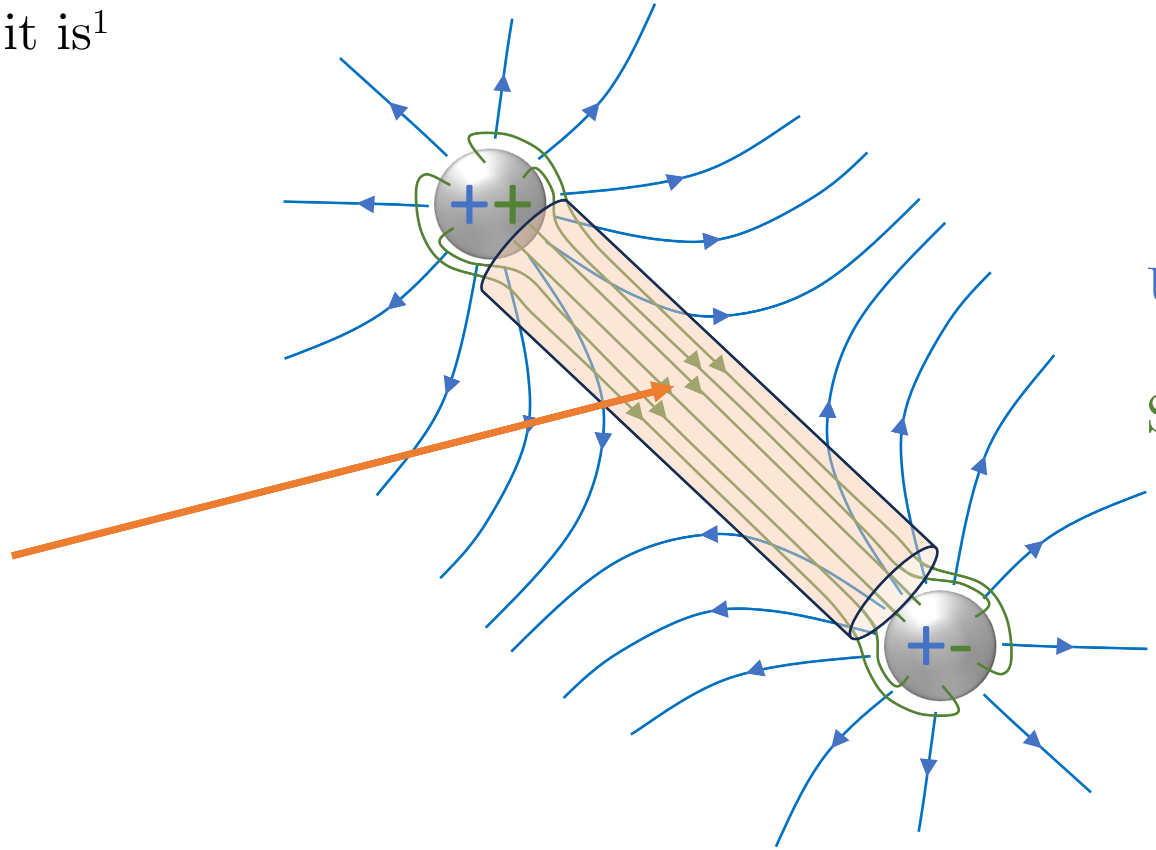
→ Again similar to two monopoles connected by a string [1]

[1] L. Leblond, B. Shlaer, X. Siemens (2009)

Monopole Confinement – Outlook

In SU(2) it's not possible two connect two monopoles of same charge

But in an SU(3) theory it is[1]



[1] Y. Ng, T.W.B. Kibble, T. Vachaspati (2008)

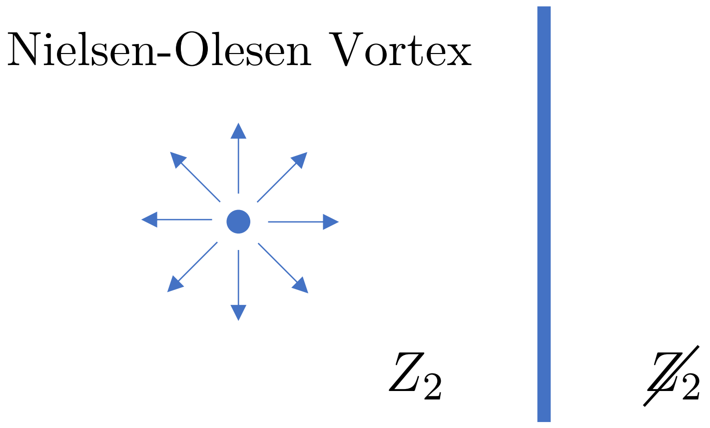
Slingshot for Vortices/Strings

We consider an $U(1)$ gauge theory with two complex fields ϕ, χ and the following potential

$$U(\phi,\chi) = \lambda_\phi(|\phi|^2 - v_\phi^2)^2 + \lambda_\chi(|\chi|^2 - v_\chi^2)^2|\chi|^2 + \beta\phi^*\chi^2 + c.c.$$

Breaking Pattern: $U(1) \xrightarrow{\phi} Z_2 \xrightarrow{\chi} 1$

2+1 Dimensions



3+1 Dimensions