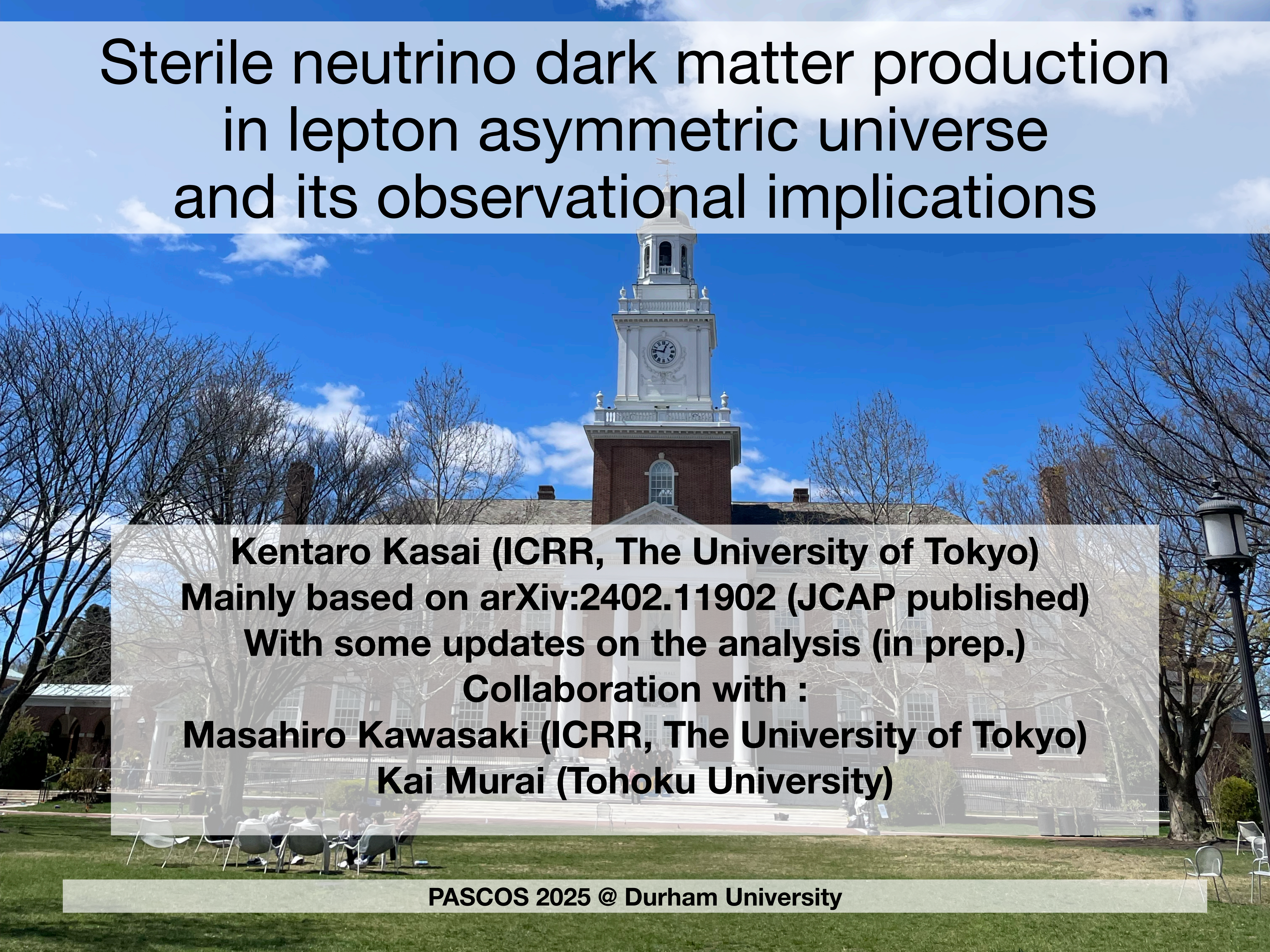


Sterile neutrino dark matter production in lepton asymmetric universe and its observational implications

Kentaro Kasai (ICRR, The University of Tokyo)

Mainly based on arXiv:2402.11902 (JCAP published)

With some updates on the analysis (in prep.)

Collaboration with :

Masahiro Kawasaki (ICRR, The University of Tokyo)

Kai Murai (Tohoku University)

PASCOS 2025 @ Durham University

Today's talk :

Phenomenology with the
"large" primordial neutrino/anti-neutrino asymmetry
$$(n_\nu - n_{\bar{\nu}})/s_{\rm tot} \gtrsim \mathcal{O}(10^{-4}) \gg n_B/s_{\rm tot}$$

- **Cosmological Implications:**

1. Can resonantly produce the **sterile neutrinos as all DM**

X. D. Shi, G. M. Fuller (1998)

→ Comprehensive discussion of the allowed parameter region

- **How is such an asymmetry generated?**

1. Decay of non-topological solitons predicted in MSSM
→Consider the compatible parameter space to be consistent with sterile neutrino DM production

2. Observational implications on the stochastic GW background in this scenario

Large lepton asymmetry : recent observation

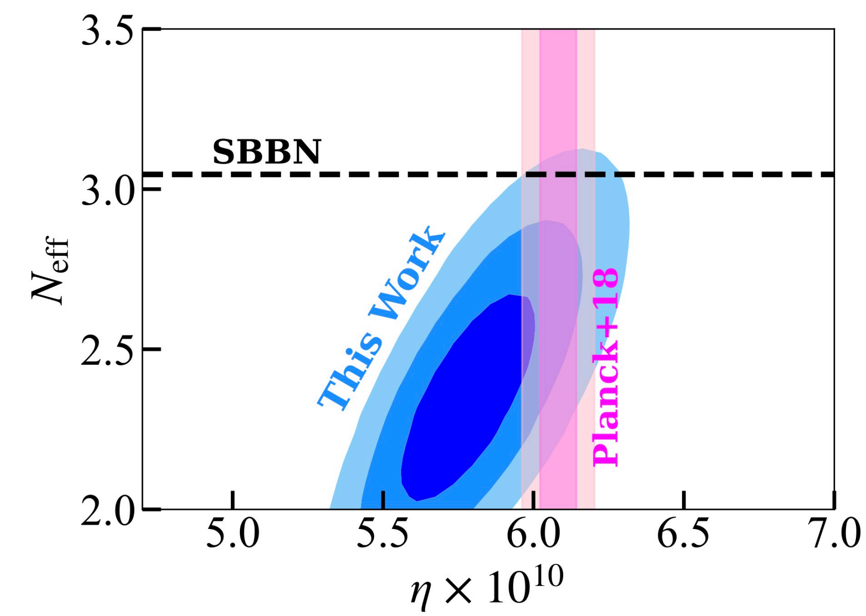
A. Matsumoto et al., Astrophys. J. (2022)

(H. Yanagisawa et al., arXiv : 2506.24050)

Measurements of He4 in metal-poor galaxies determined primordial Helium abundance

$$Y_P \equiv \rho_{\mathrm{H_e}4}/\rho_{\mathrm{B}} = 0.2370^{+0.0034}_{-0.0033}$$

($\sim 1.5\sigma$ smaller than previous results)



(Matsumoto et al.)

Together with the priors of deuterium abundance (Cooke et al., (2018)) and baryon asymmetry (Planck 2018), they determined N_{eff} and $L^{\mathrm{init}}_{\nu_e} \equiv (n_{\nu_e} - n_{\bar{\nu}_e})/s_{\mathrm{tot}}$

$$\rightarrow L^{\mathrm{init}}_{\nu_e} = 1.8^{+1.1}_{-0.7} \times 10^{-3}$$

$$N_{\mathrm{eff}} = 3.11^{+0.34}_{-0.31}$$

(Matsumoto et al.)

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Introduction : Sterile neutrino dark matter

Sterile neutrino : $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge singlet fermion (ν_s)

$$\mathcal{L}_{\nu\mathrm{MSM}} \supset -m_s \overline{\nu_s^c} \nu_s - y_\alpha \overline{\ell_{L\alpha}} \epsilon \phi^* \nu_s + \mathrm{h.c.}$$

Motivated by origin of neutrino mass, etc…..

- The lightest sterile neutrino ((mass)~ $\mathcal{O}(10)$keV) is a **dark matter** candidate.

For review, see A. Boyarsky et al., 1807.07938

- keV-scale sterile neutrino dark matter can be tested by

1. Effect on structure formation with $\sim \mathcal{O}(0.1)$ Mpc
   $\rightarrow$ can be probed by Lyman α forest method J. Baur et al., 1706.03118
2. **X-ray** observations
   via radiative decay $\nu_s \rightarrow \nu_\alpha + \gamma$ via mixing with SM neutrinos

A. Neronov et al., 1607.07328 (2016)

Dodelson-Widrow mechanism

Production of keV-scale sterile neutrinos
via **neutrino oscillation** involving active-sterile states :

Initial : Energy-eigenstate
$$|\nu_1\rangle = \cos\theta\,|\nu_a\rangle - \sin\theta\,|\nu_s\rangle$$

Interaction of $|\nu_a\rangle$ with thermal plasma →

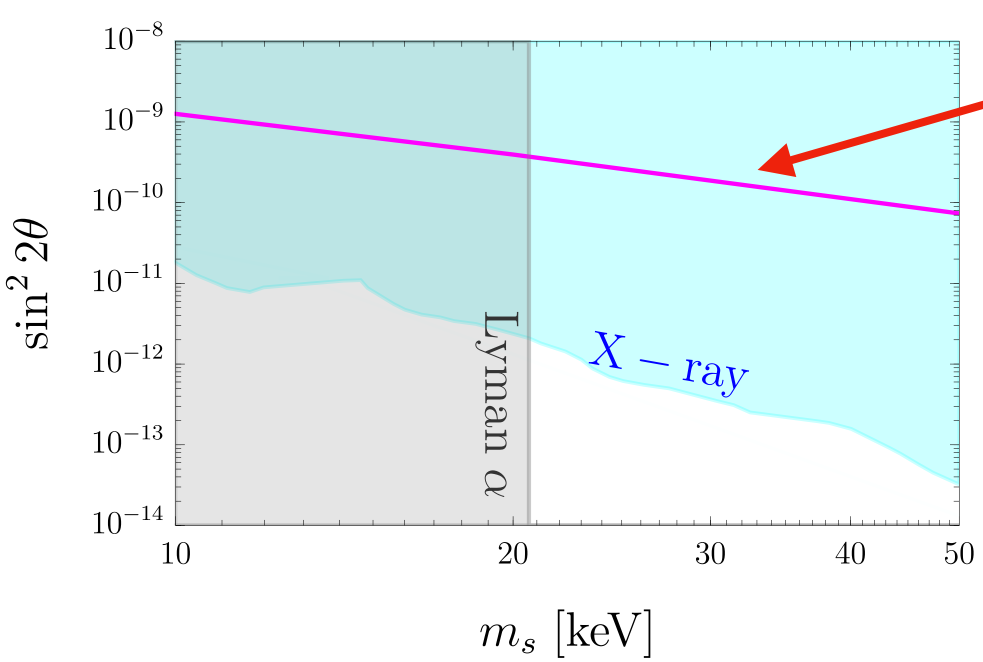
Coherence breaking, $|\nu_s\rangle$ production

$|\nu_a\rangle$: "active" eigenstate $|\nu_s\rangle$: "sterile" eigenstate $|\nu_1\rangle, |\nu_2\rangle$: mass eigenstates

- Contour to explain all DM

S. Dodelson, L. M. Widrow, (1993)

Minimum setup → Free parameters : m_{ν_s}, θ



To explain All DM

→Already excluded
by X-ray and Lyman α observations

Resonant production of sterile neutrinos

Consider lepton asymmetric thermal plasma.

Self-energy of active neutrinos :

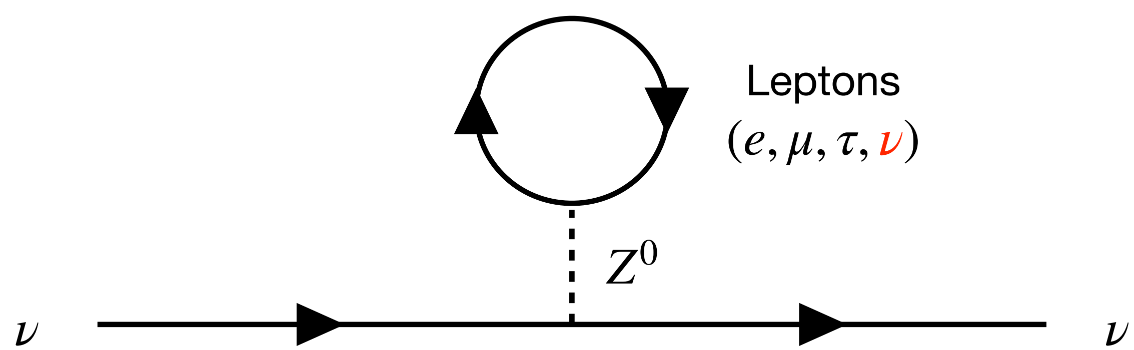
$$V_a \equiv \underbrace{\sqrt{2}G_{\mathrm{F}}\Big(2(n_{\nu_a} - n_{\bar{\nu}_a}) + \sum_{b\neq a}(n_{\nu_b} - n_{\bar{\nu}_b})\Big)}\ - \underline{B_a p T^4} \supset \sqrt{2}G_{\mathrm{F}}(2L_{\nu_a} + \underline{\sum_{b\neq a} L_{\nu_b}})s_{\mathrm{tot}}$$

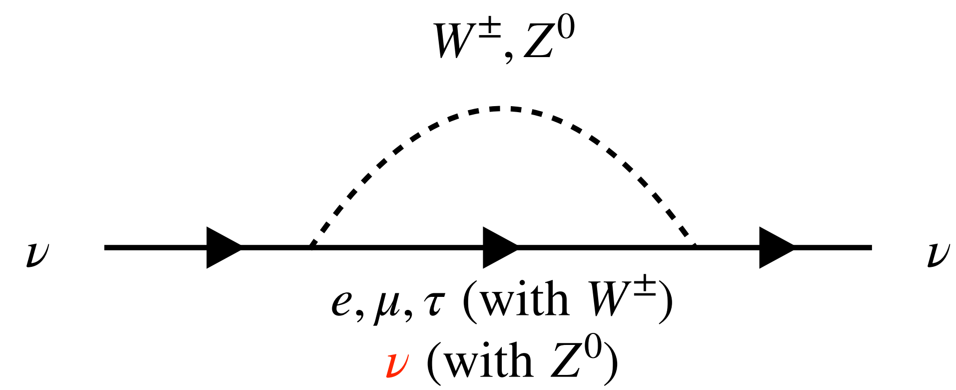
Exists in DW setup

Lepton charge per unit entropy

$B_e \simeq 10.88 \times 10^{-9}\mathrm{GeV}^{-4}$

D. Notzold, G. Raffelt, (1987)





Effective mixing angle :

$$\langle P(\nu_a \to \nu_s)\rangle \propto \frac{\sin^2 2\theta}{\sin^2 2\theta + \underline{\left(\cos\theta + 2p(-2\sqrt{2}\pi^2 G_{\mathrm{F}} s_{\mathrm{tot}}\mathcal{L}/45 + B_a p T^4)/m_{\nu_s}^2\right)^2}}$$

$= 0$: Resonance

X. D. Shi, G. M. Fuller (1998)

Where $\mathcal{L} \equiv 2L_{\nu_a} + \sum_{b\neq a} L_{\nu_b}$

Can explain all dark matter with smaller mixing angle.

Details in formalism on sterile neutrino production

A. D. Dolgov (2002), T. Asaka, M. Shaposhnikov (2005)

Consider only two states for simplicity :

$$i(\partial_t - Hp\partial_p)\rho = \underbrace{[\mathcal{H}, \rho]}_{\text{neutrino oscillation with matter effect}} - \underbrace{i\{\Gamma, \rho - \rho_{\rm eq}\}}_{\text{active neutrino scattering with thermal plasma}}$$

$$\mathcal{H} = \begin{array}{c} \quad\text{active} \qquad\qquad \text{sterile} \\ \begin{pmatrix} -\frac{m_s^2}{4p}\cos 2\theta & \frac{m_s^2}{4p}\sin 2\theta \\ \frac{m_s^2}{4p}\sin 2\theta & \frac{m_s^2}{4p}\cos 2\theta \end{pmatrix} \end{array} + \begin{pmatrix} \underline{V_a} & 0 \\ 0 & 0 \end{pmatrix} ,$$

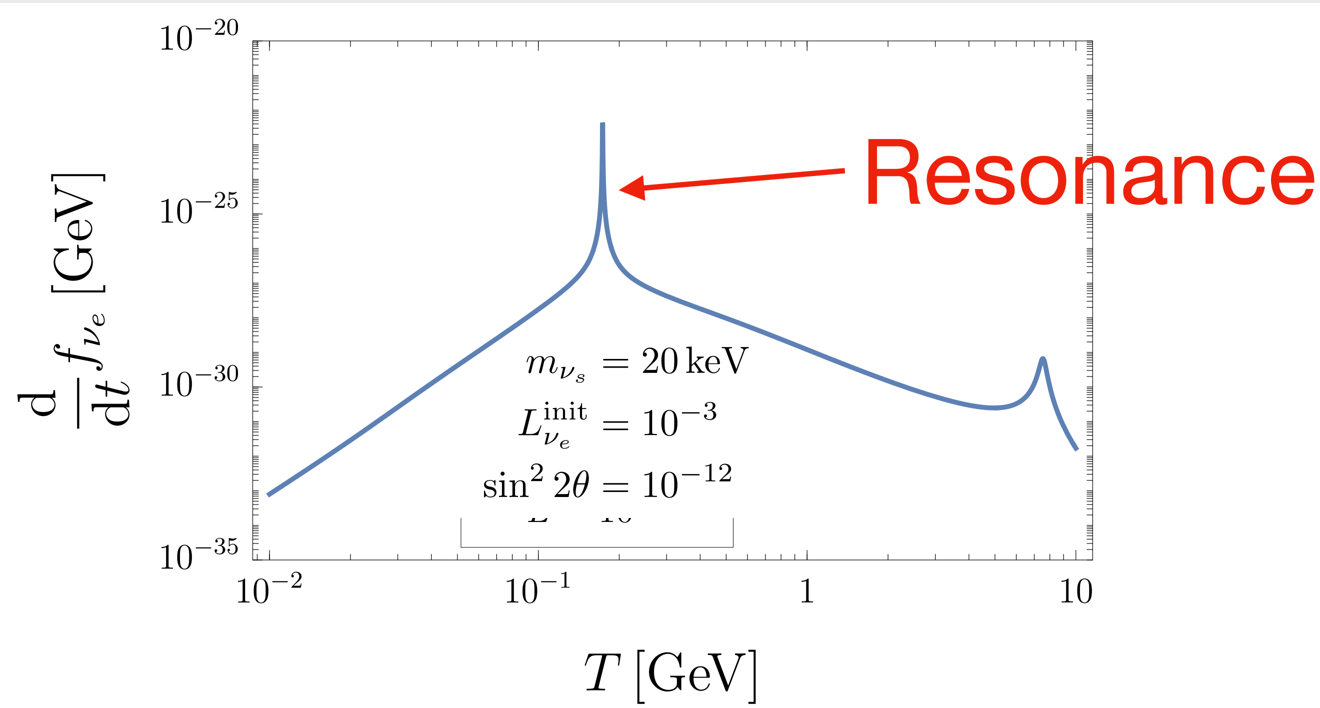
: effective Hamiltonian taking matter effect into account

Thermal self energy of active neutrinos

$$\Gamma = \frac{1}{2}\begin{pmatrix} \Gamma_{\nu_a} & 0 \\ 0 & 0 \end{pmatrix}$$

: collision matrix

- Production rate of a mode with $p \simeq T$:



Numerical setup

- In the presence of positive/negative lepton asymmetry,
  the production from active→sterile/anti-active→sterile causes the resonance

$$V_a \supset +\sqrt{2}G_{\rm F}\left(2(n_{\nu_a}-n_{\bar{\nu}_a})+\sum_{b\neq a}(n_{\nu_b}-n_{\bar{\nu}_b})\right),\quad V_{\bar{a}} \supset -\sqrt{2}G_{\rm F}\left(2(n_{\nu_a}-n_{\bar{\nu}_a})+\sum_{b\neq a}(n_{\nu_b}-n_{\bar{\nu}_b})\right)$$

→During the resonance, the lepton asymmetry also time-evolves:

$$\frac{\mathrm{d}}{\mathrm{d}t}L_a \propto -\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\left(P_{\nu_a\to\nu_s}-P_{\bar{\nu}_a\to\nu_s}\right)$$

P : Conversion rate of a mode

Here, we have defined

$$L_a \equiv \frac{n_{\nu_a}-n_{\bar{\nu}_a}+n_a-n_{\bar{a}}}{s_{\rm tot}}$$

a : charged leptons

Solve simultaneously
the time evolution of L_a + Time evolution of each ν_s modes

Comprehensive parameter scan : The status of our work

M. Laine, M. Shaposhnikov (2008), Bauer et al, (2017)

: Assume efficient flavor oscillation in the Standard Model sector during the ν_s production, which is typically at $T \sim \mathcal{O}(100)$ MeV

However, the flavor oscillation will take place $T \lesssim \mathcal{O}(10)$ MeV

$\rightarrow$ At the resonant production of ν_s DM, we should switch-off the flavor oscillations

Ex.) A. D. Dolgov et al., (2002)
J. Froustey, C. Pitrou, (2024)

This work :

1. Modified the assumption of the flavor oscillations in the Standard Model sector

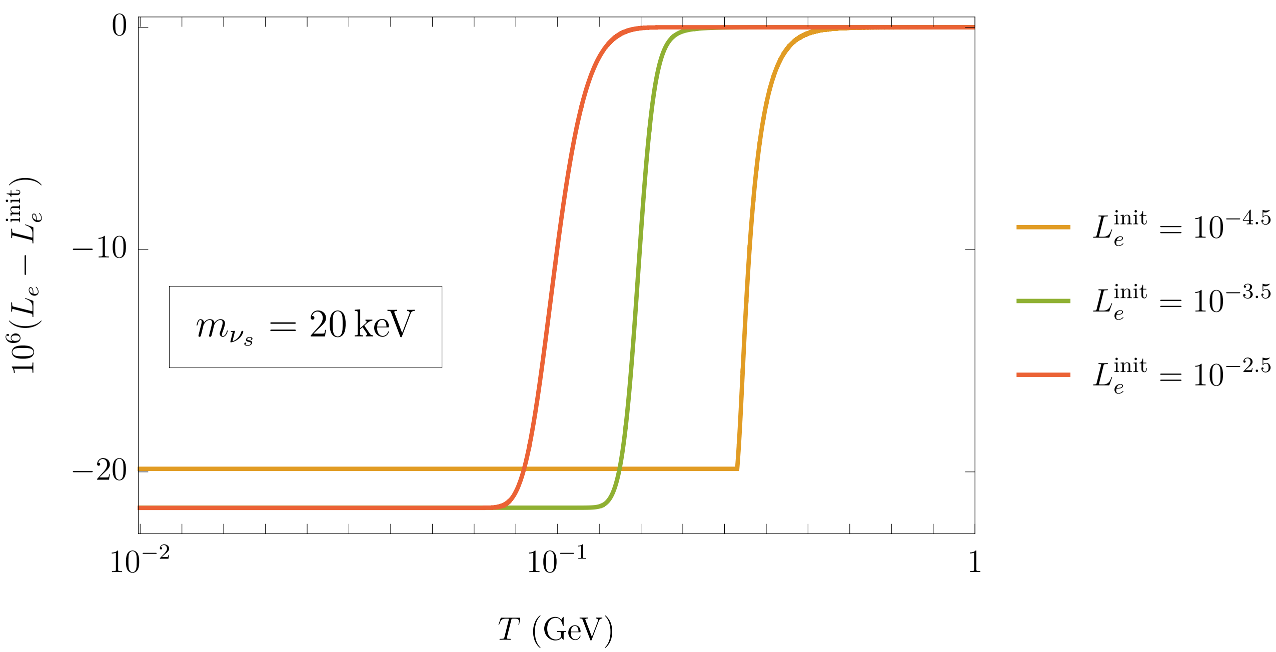
2. Numerical difficulties still remain due to the sharpness of the backreaction from the evolving lepton asymmetry

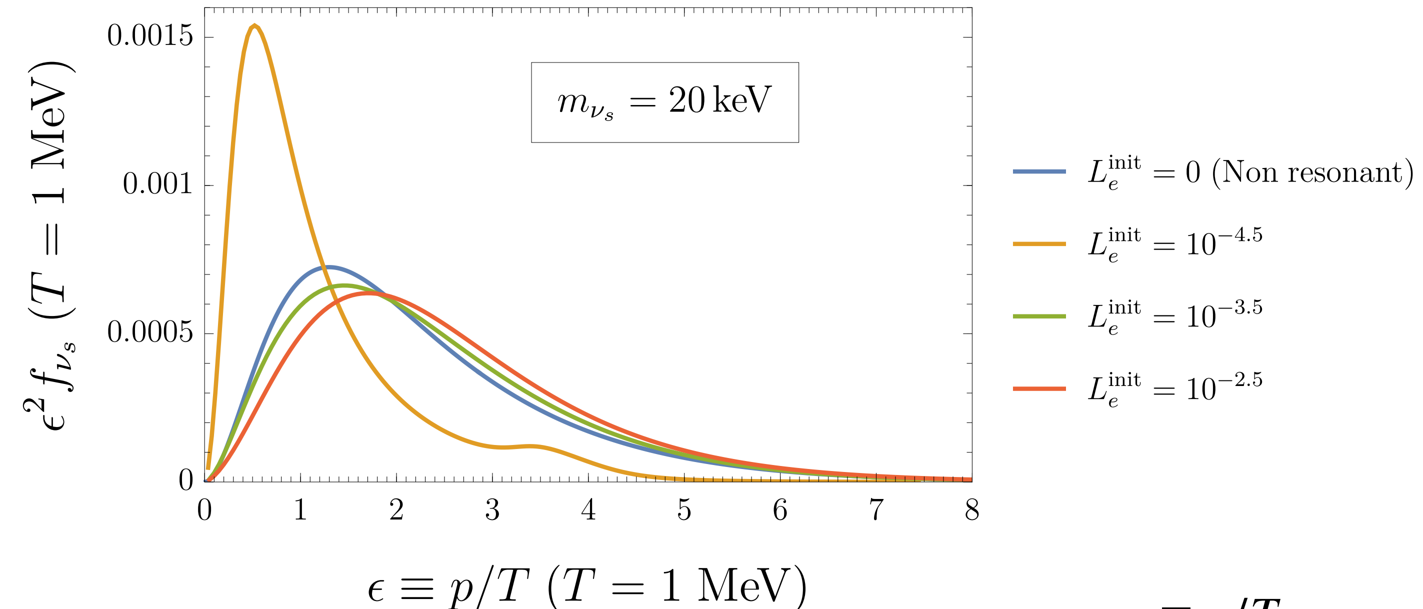
$\rightarrow$ Developed the numerical scheme to accurately capture the resonance

: Dynamical discretization
with locally fine binning around the resonant modes

Numerical Result 1.
Final spectrum of sterile neutrino DM

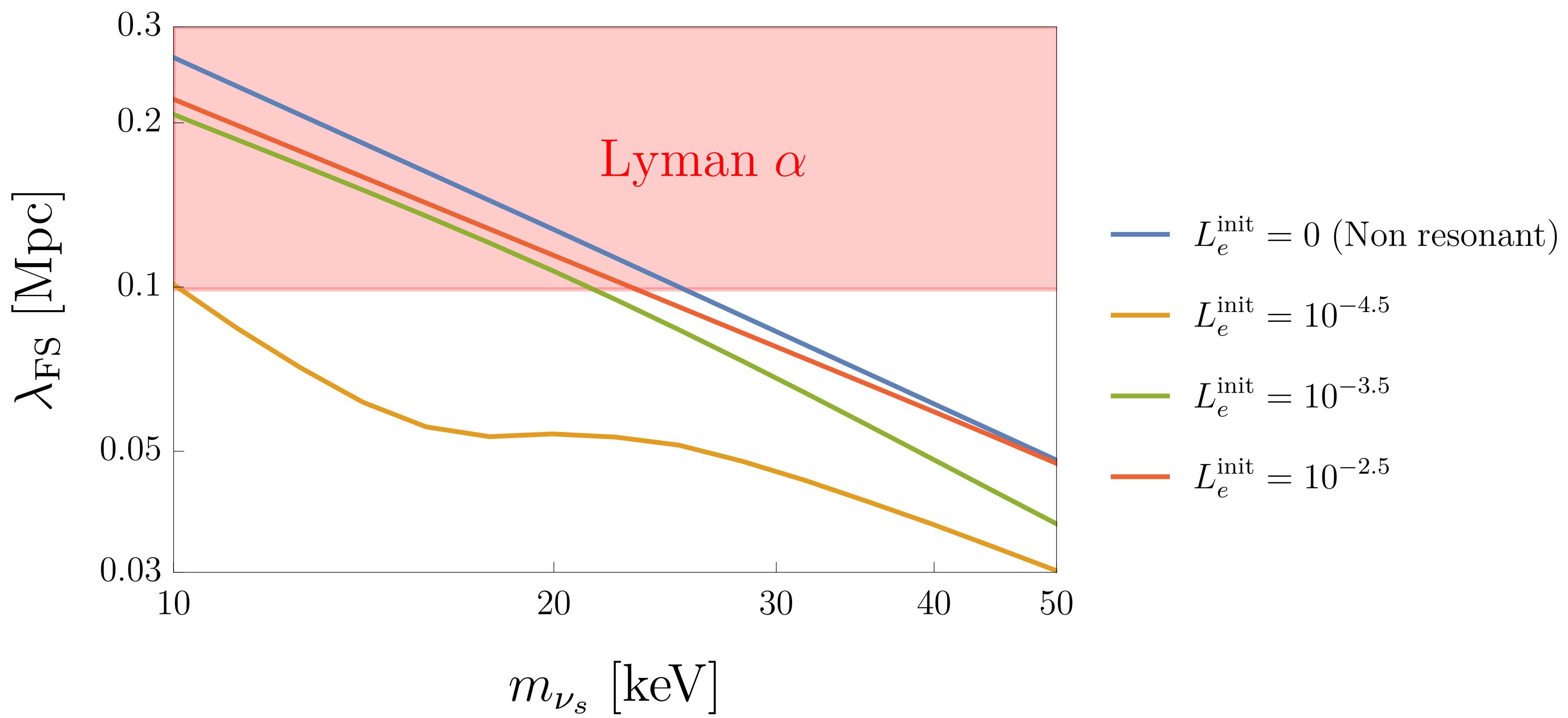
Numerical result (Assume $\nu_e \leftrightarrow \nu_s$ mixing, θ is fixed to explain all DM)





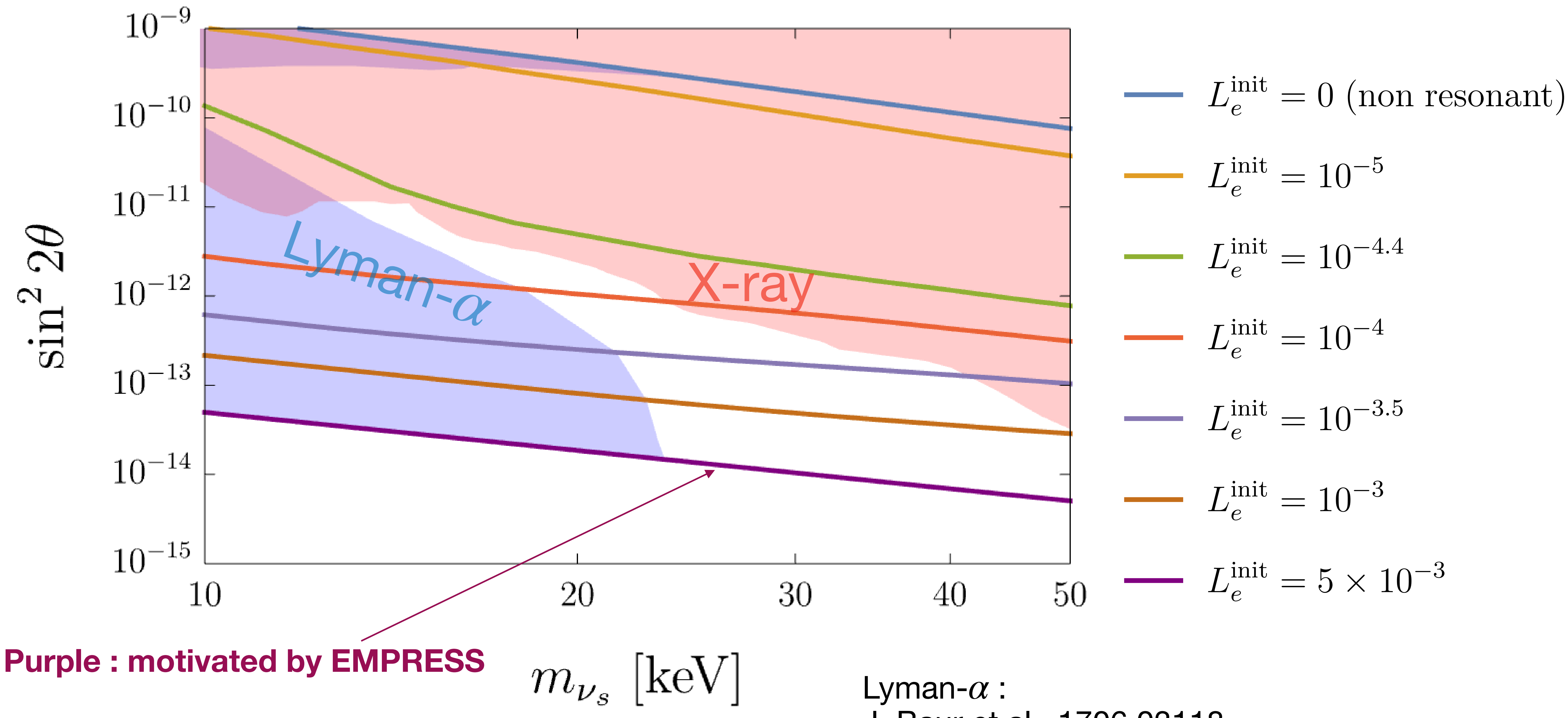
$\epsilon_{\rm phys} \equiv p/T$

Free-streaming length (Effect on the structure formation) :



Numerical Result 2. Observational constraints

m_{ν_s} - θ contour to explain all DM



Lyman-α :
J. Baur et al., 1706.03118

This scenario requires **large neutrino asymmetry** with $L_{\nu_e}^{\rm init} \gtrsim \mathcal{O}(10^{-4})$

Today's talk :

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→ Comprehensive discussion of the allowed parameter region

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Generation of large lepton asymmetry

If the lepton asymmetry is generated before the electroweak phase transition $T_{\rm ew} \sim 100$ GeV, the baryon asymmetry with the same order is produced via sphaleron process.

To explain large hierarchy between lepton/baryon asymmetry, lepton asymmetry must be produced after the freeze-out of the sphaleron process.

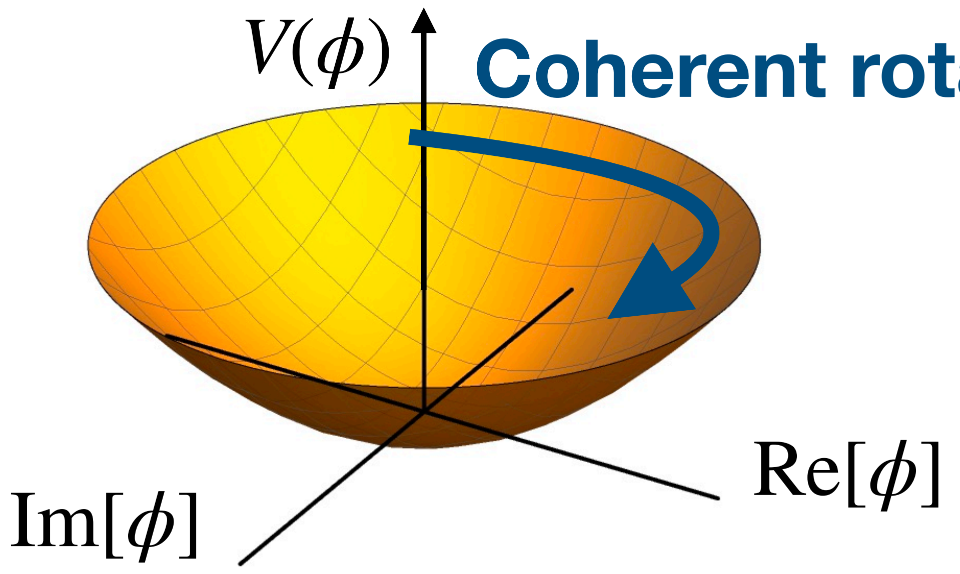
This talk : Affleck-Dine leptogenesis
: $|L_\nu| \gtrsim \mathcal{O}(10^{-4})$, which is also compatible with the resonant ν_s production, can be generated.

Our setup : Affleck-Dine leptogenesis

Consider a dynamics of leptonic scalar field in MSSM (denoted as ϕ)

I. Affleck, M. Dine, (1984)

M. Dine, L. Randall, S. D. Thomas, (1994)



Lepton number $\leftrightarrow$ global $U(1)$ sym.

Rotation of ϕ

$\downarrow$

lepton number generation.

If potential satisfies $\min\left[\frac{2V(\varphi)}{\varphi^2}\right]_{\varphi\neq 0} < \omega < V''(\varphi=0)$

, the coherent rotation has an spatial instability

$\rightarrow$forms spherical solitons (=**Q-ball**)

S. R. Coleman, (1985)

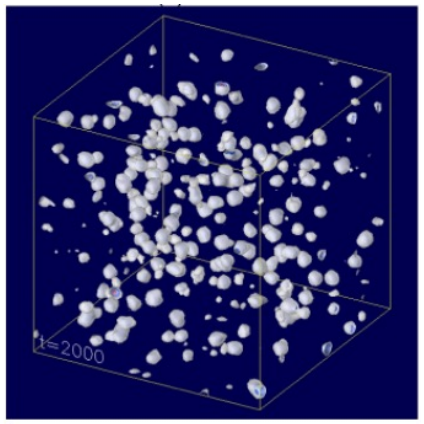


Figure: T. Hiramatsu et al., (2010)

Why Q-balls ?

1. Stable against the sphaleron process

M. Kawasaki, F. Takahashi, M. Yamaguchi, (2002)

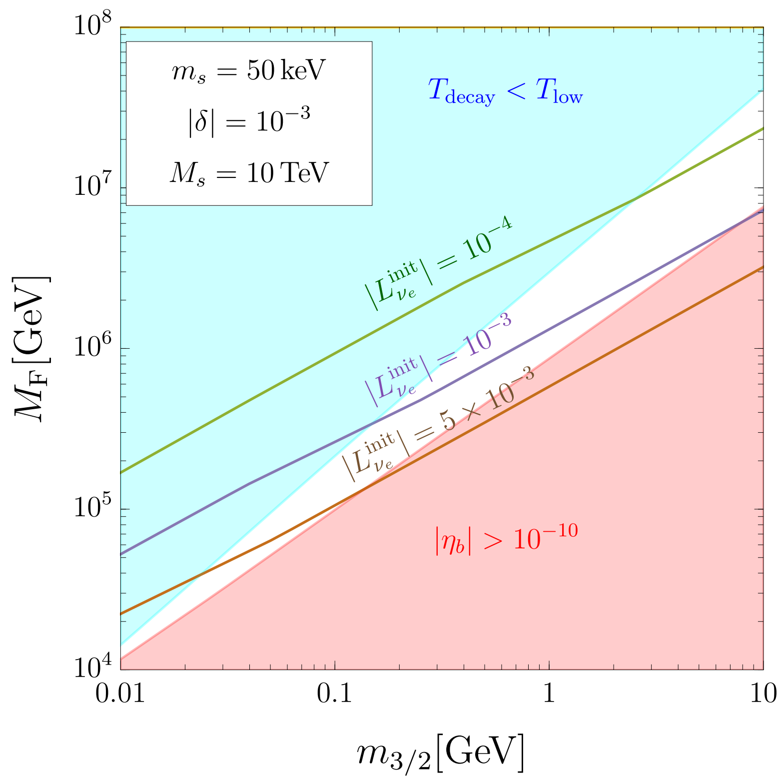
2. Can decay into neutrinos via quantum effect

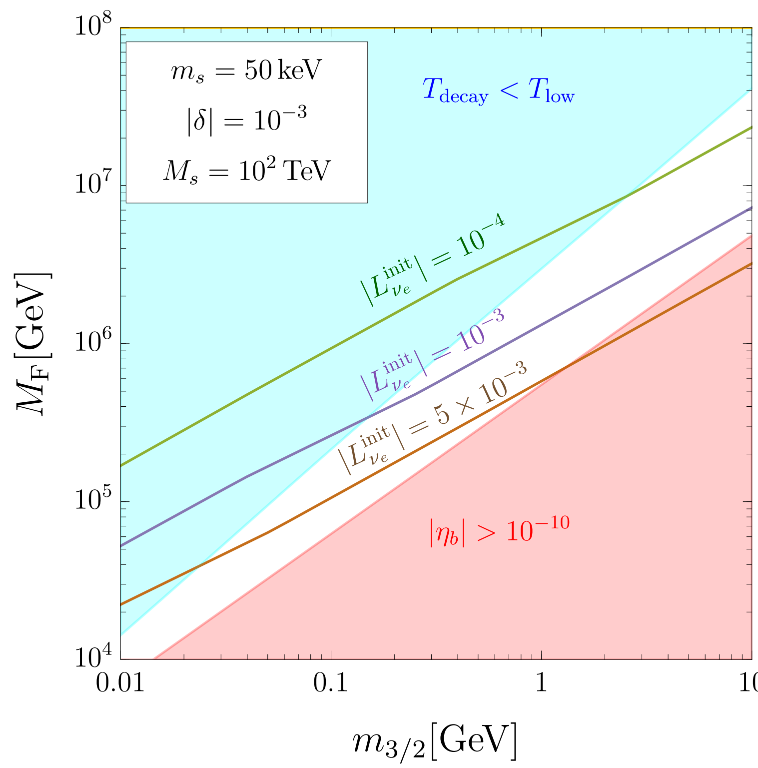
$\rightarrow$ Once (decay rate) $\sim$(Hubble rate), they instantaneously decay and generate a lepton asymmetric plasma

A. G. Cohen et al., (1986)

Generation of large lepton asymmetry

We consider Q-ball domination before the decay into SM neutrinos within Gauge-mediated SUSY breaking scenario

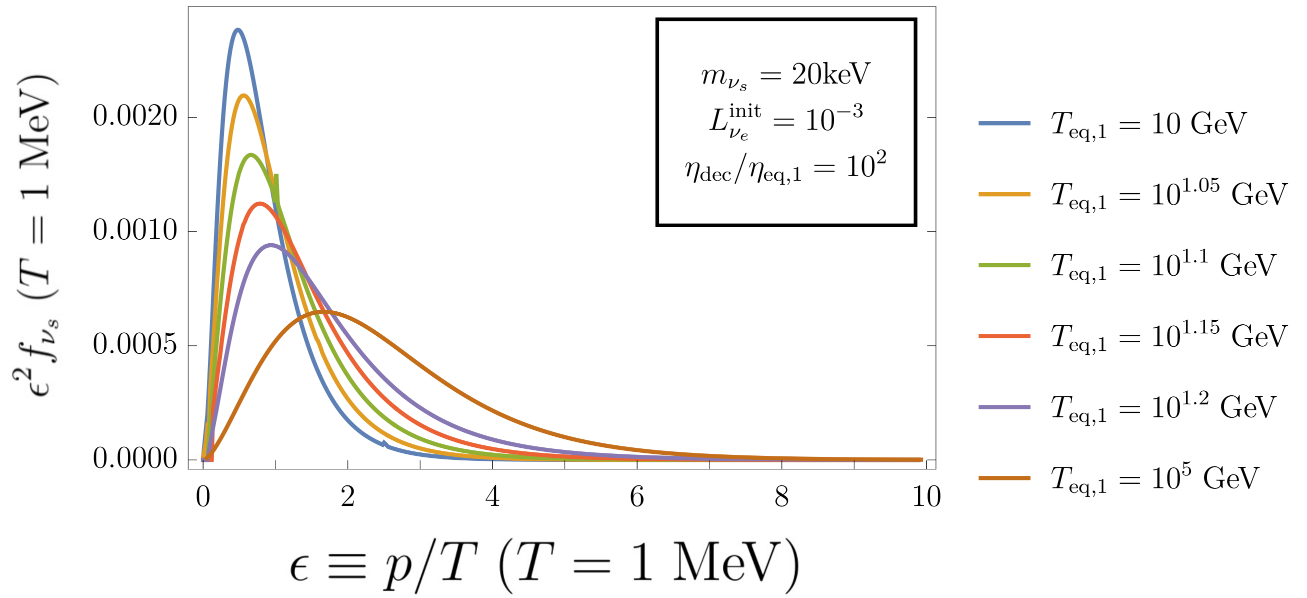




Blue region :
$T_{\rm decay} < T_{\rm resonance}$

Red region :
Baryon overproduction

- The case where the ν_s resonance occurs during the lepton number injection

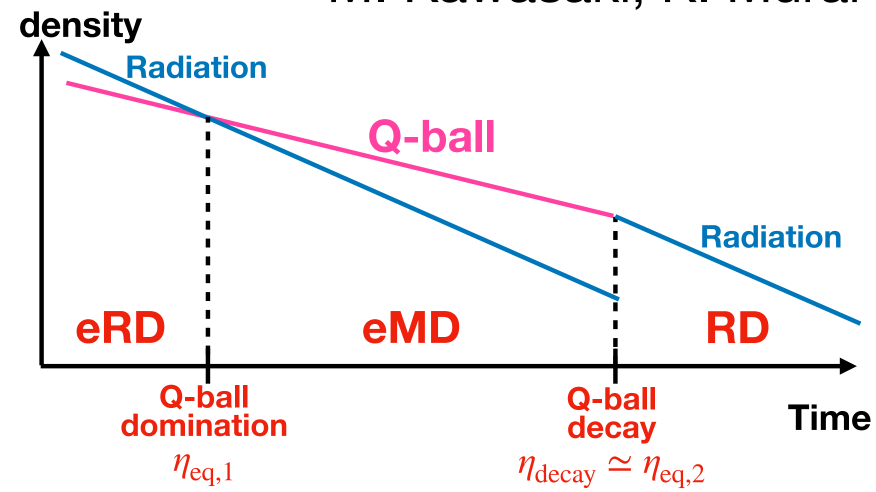


“Eq,1” : first matter-radiation equality

Enhancement of second-order scalar-induced gravitational waves at Q-ball decay

K. Inomata et al. (2019)
M. Kawasaki, K. Murai (2023)

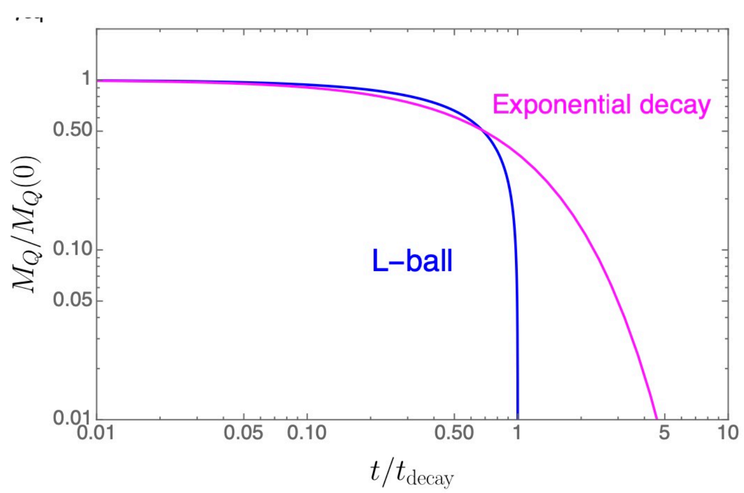
In our setup, Q-balls can dominate over the energy density of the universe before the decay.



- Energy of Q-balls

$$\Gamma \equiv -\frac{1}{Q}\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{4}{5}\frac{1}{t_{\rm dec} - t},$$

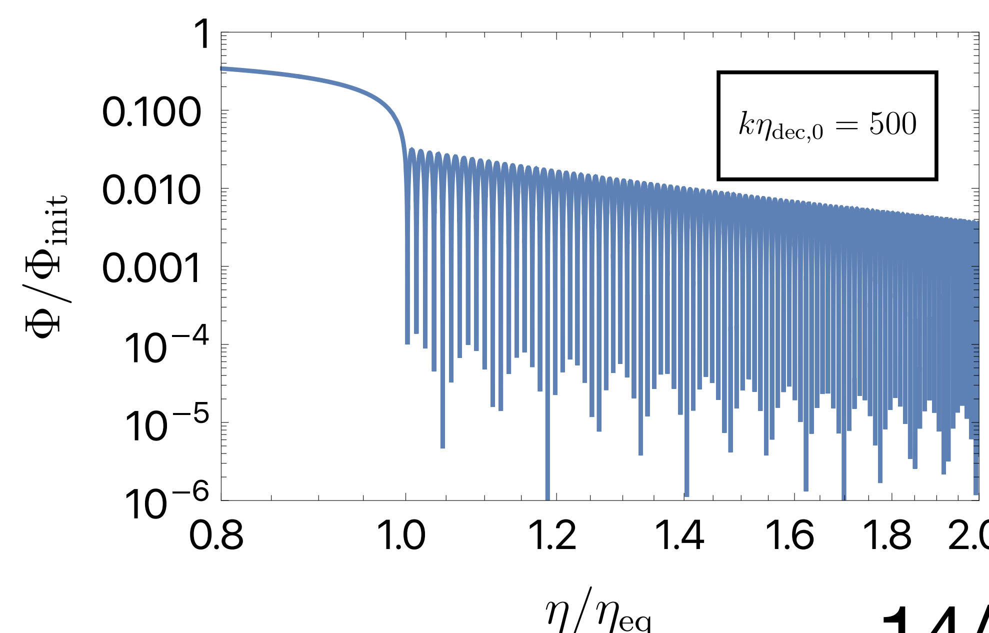
$$M_{\rm Q} = M_{\rm Q}(0)\left(1 - \frac{t}{t_{\rm dec}}\right)^{3/5}$$



- Time evolution of Φ

The amplitudes do not have sufficient time to decay right before the oscillation

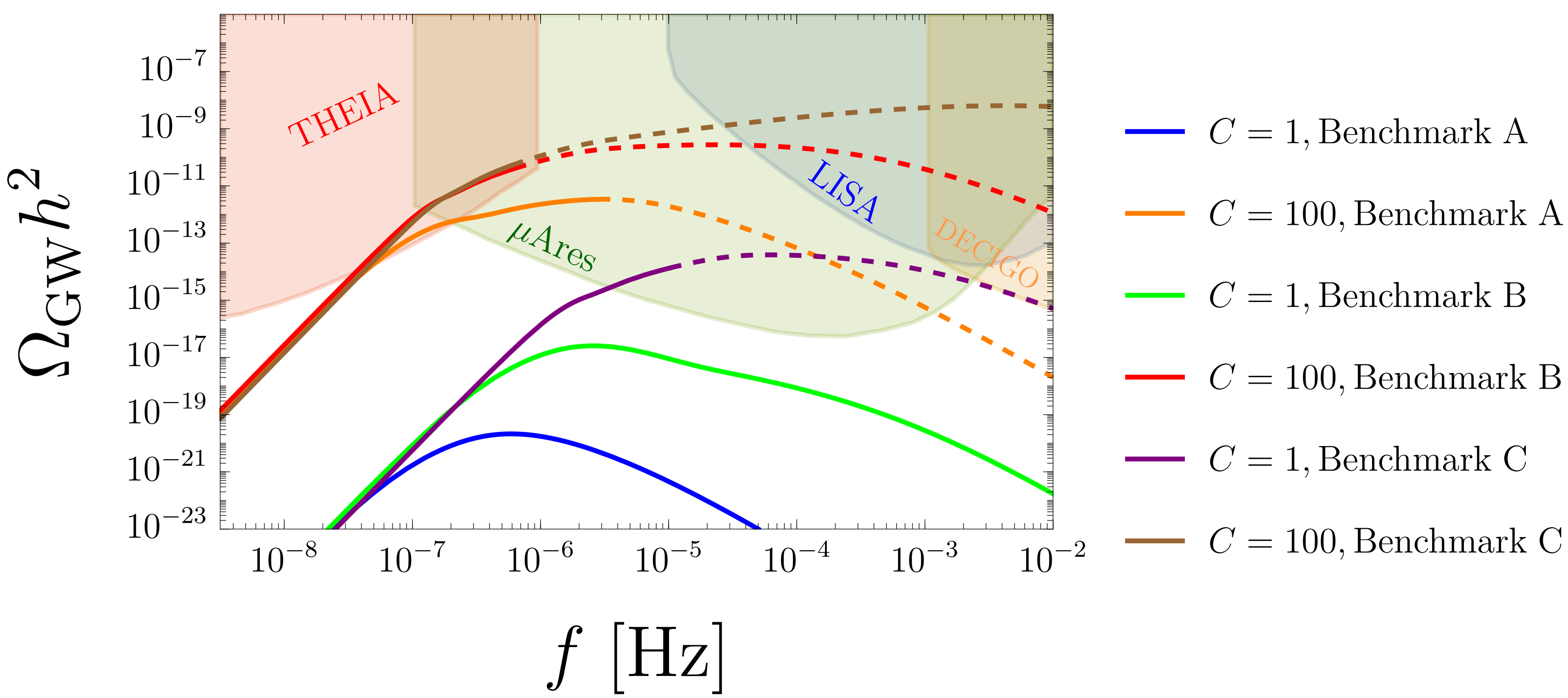
→ The **large amplitude of oscillation lead to the GW production** through <u>second-order effect</u>



Enhancement of second-order scalar-induced gravitational waves at Q-ball decay

Assume initial flat curvature power spectrum $P_\zeta(k) = C^2 A_s$ ($A_s = 2.1 \times 10^{-9}$)
with scales smaller than CMB scale.

With sufficiently subhorizon scale $k/H_{\rm dec} \gg \mathcal{O}(100)$, the density perturbations exceed $O(1)$ during Q-ball dominated era,
and the linearity of the scalar perturbations breaks down
→ Use the result of N-body simulations, and obtain the metric perturbations by Poisson equations J. A. Peacock and S. J. Dodds, (1994) M. Kawasaki, K. Murai, (2022)



	$m_{3/2}$ [GeV]	$M_{\rm F}$ [GeV]	$T_{\rm R}$ [GeV]	M_s [GeV]	$\lvert\eta_{\rm b}\rvert$	$\lvert L_{\nu_e}^{\rm init}\rvert$
A	0.5	1.3×10^6	1.0×10^2	1.7×10^4	8.7×10^{-11}	4.0×10^{-4}
B	0.4	1.0×10^6	1.0×10^3	2.0×10^5	8.7×10^{-11}	4.8×10^{-4}
C	1.0	2.9×10^6	1.5×10^4	6.6×10^5	8.7×10^{-11}	2.2×10^{-4}

Summary and the discussion

1. We revisited the calculation of resonant production scenario of sterile neutrino DM and confirmed that initial lepton asymmetry is required to be $L_e \gtrsim \mathcal{O}(10^{-4})$ and $m_s \gtrsim 10$ keV to evade the current X-ray constraints.

2. We showed that Affleck-Dine leptogenesis can successfully explain resonant production scenario of sterile neutrinos as all DM.

3. We estimated the power spectrum of background GW from Q-ball decay effect and showed that this scenario can be tested by future LISA/DECIGO/μAres/THEIA experiments.

Supplemental materials

Numerical treatment to accurately capture the resonance and the backreaction

Time-dependent transformation of the momentum bins :

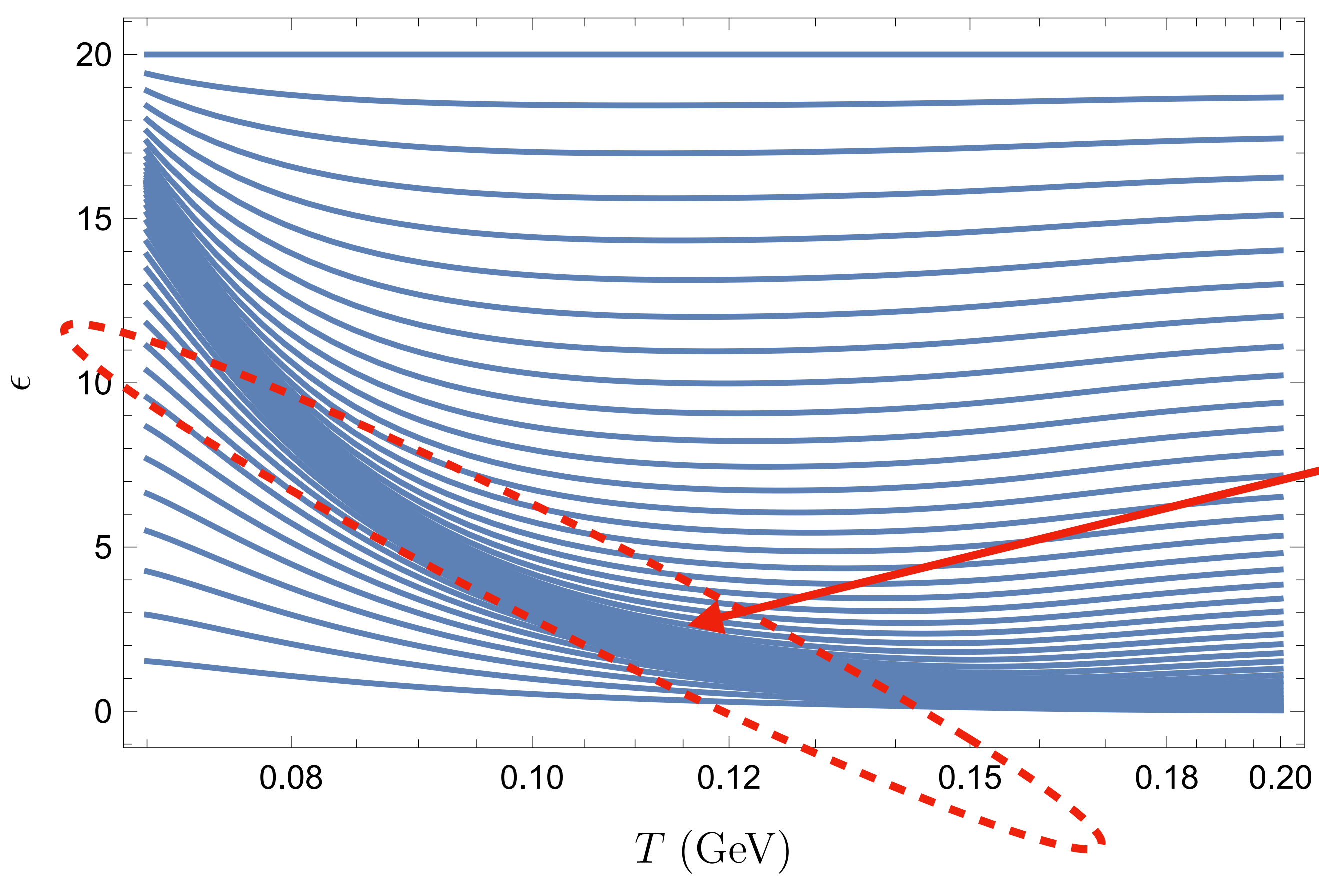
$$u(\epsilon) \equiv \frac{\epsilon - \epsilon_{\rm min}}{\epsilon_{\rm max} - \epsilon_{\rm min}} \qquad u = u_{\rm res}(T) + \alpha\,(v - v_{\rm res}(T)) + \beta(T)\,(v - v_{\rm res}(T))^3$$

Where $\epsilon_{\rm res}$: Resonant mode

$\alpha = 10^{-3}$: Controls the local mesh resolution around the resonance

Original basic variables of QKE : $(\epsilon, T) \to$ New variable : (v, T)

Discritization of $0 < v < 1$ linearly with $N_{\rm bin} \sim 500$
automatically ensures the fine resolution around the resonant mode



Blue lines : $v = \text{const.}$

Regions under resonance

Stationary-point approximations

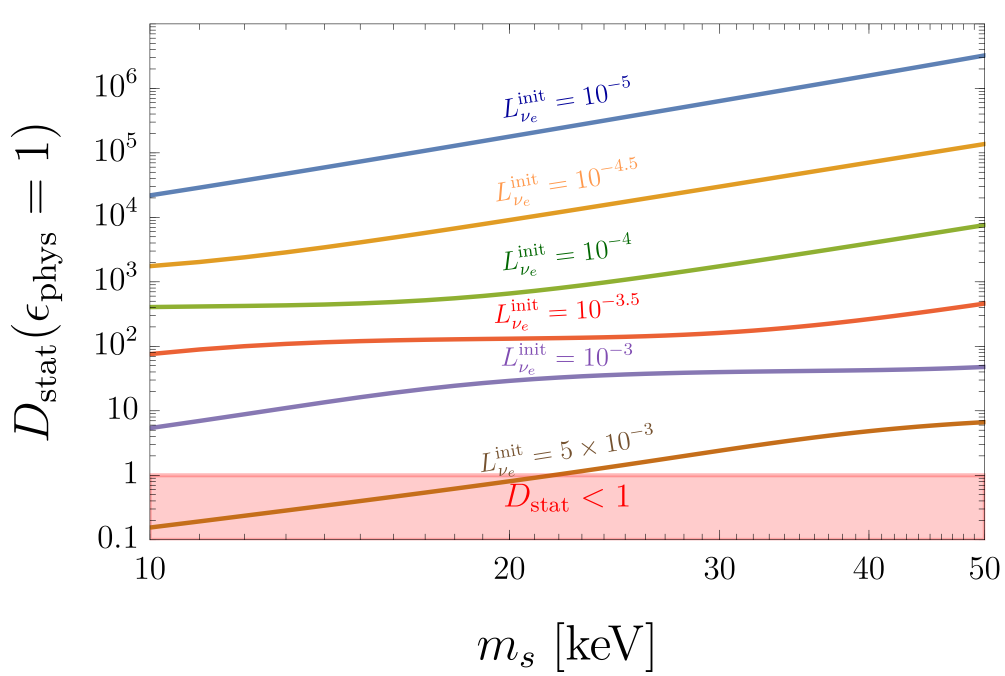
Approximate the off-diagonal component of the density matrix by stationary-solutions:

$$\rho_{as} \simeq \rho_{as,\mathrm{stat}} \equiv -\frac{\sin 2\theta}{2\cos 2\theta - \frac{2p}{m_{\nu_s}^2}(2V_a - i\Gamma_{\nu_a})}\rho_{aa}.$$

For the above treatment to be valid :

$$\delta t_{\mathrm{res}} \equiv \left.\frac{\mathrm{d}t}{\mathrm{d}T}\right|_{T_{\mathrm{res}}} \times \frac{\Gamma_{\nu_a}(T_{\mathrm{res}})T_{\mathrm{res}}}{3V_a(\epsilon_{\mathrm{phys}}, T_{\mathrm{res}})} \gg \Gamma_{\nu_a}^{-1} \Leftrightarrow D_{\mathrm{stat}} \equiv \delta t_{\mathrm{res}} \cdot \Gamma_{\nu_a} \gg 1$$

δt_{res} : timescale of the resonance of a given mode



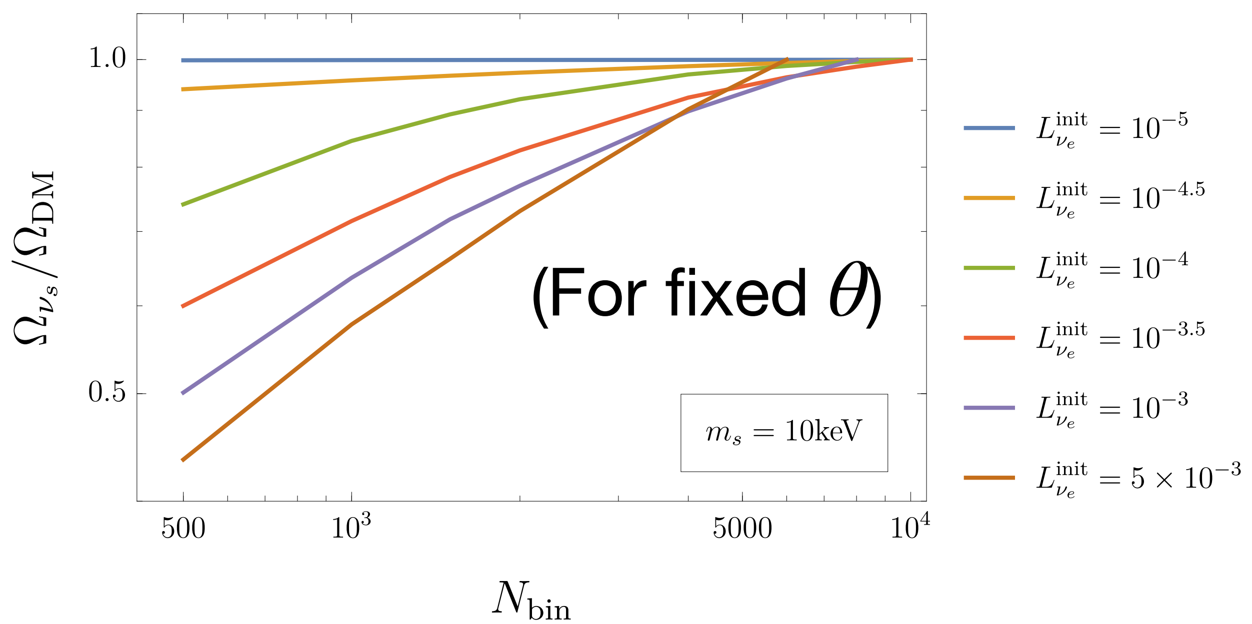
2. Numerical treatment to accurately capture the resonance and the backreaction

- The decrease of L_{ν_e} due to production of a resonant mode backreacts to the production rate of other nearby modes
  $\rightarrow$ **strong mode coupling**
- The resonance peaks are **very sharp**
  $\rightarrow$ Very fine resolution is required for mode discretization!!

Ex.) If we discritize the modes linearly.....

$$\epsilon \equiv \left(\frac{g_{*,s}(T_i)}{g_{*,s}(T)}\right)^{1/3} \frac{p}{T}$$

$$\epsilon = \epsilon_{\max} \cdot i/N_{\rm bin},\ \epsilon_{\max} = 20$$



$\rightarrow$ Strong dependence of the final abundance on bin number $N_{\rm bin}$ (For $L_{\nu_e}^{\rm init} \gtrsim 10^{-4}$)

$\leftrightarrow$ Only $N_{\rm bin} = 200$ is adopted by J. Ghiglieri, M. Laine, 1506.06752

$\rightarrow$ **Our strategy** : Time-dependent discretization s.t. the modes around the resonance are most finely discritized

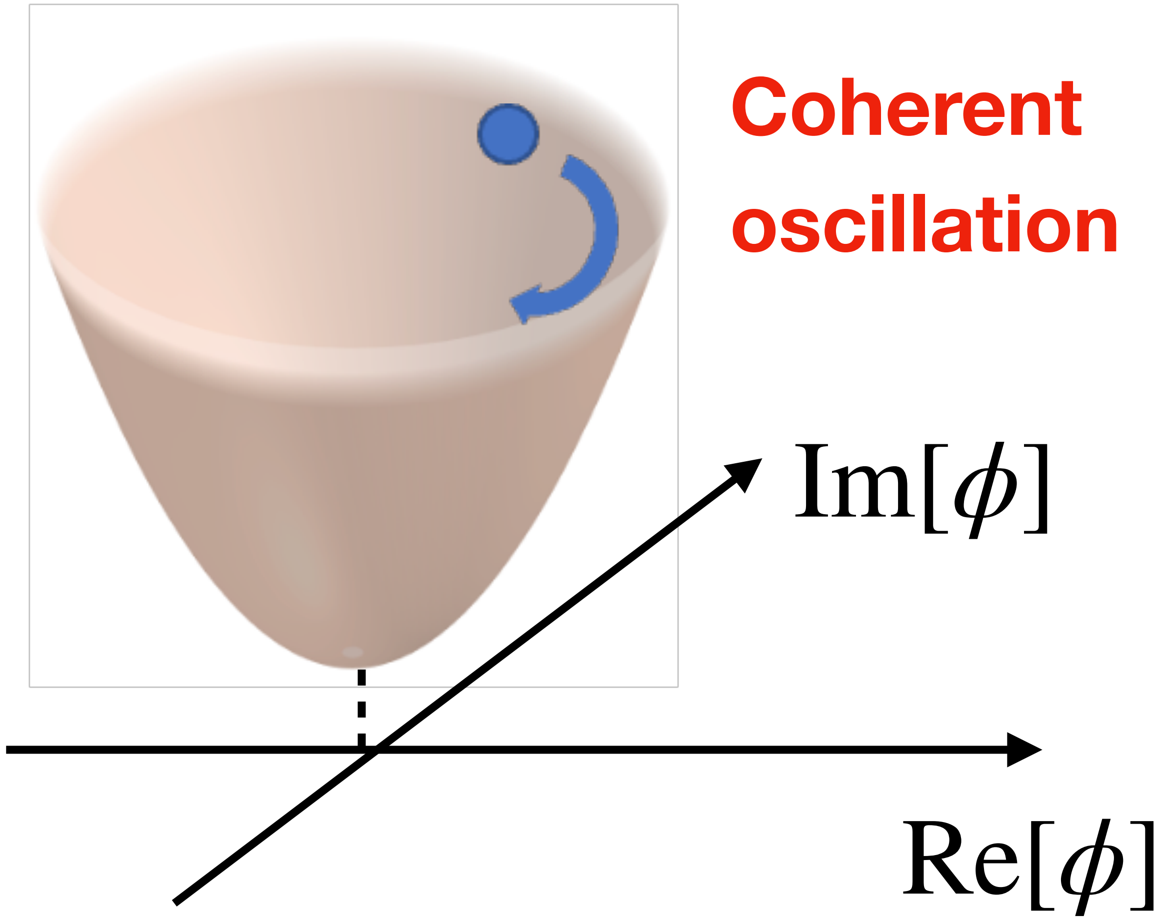
Affleck-Dine (AD) leptogenesis

- Potential

I. Affleck, M. Dine (1985), M. Dine, L. Randoll, S. D. Thomas (1996)

$$V(\phi) \simeq m_{3/2}^2|\phi|^2 - cH^2|\phi|^2 + |\lambda|^2\frac{|\phi|^{2(n-1)}}{M_{\rm Pl}^{2n-6}} + \lambda a_{\rm M}\frac{m_{3/2}\phi^n}{nM_{\rm Pl}^{n-3}} + {\rm h.c.}$$

ϕ : Leptonic flat direction in MSSM (=**AD field**) Ex.) $L_1L_2\bar{e}_2$
$m_{3/2}$: "gravitino" mass, H : Hubble parameter, $a_M = \mathcal{O}(1)$, $c(>0) = \mathcal{O}(1)$



Lepton number $\leftrightarrow$ global $U(1)$ sym.

Rotation of ϕ
↓
lepton number generation.

Resultant lepton number :

$$n_{\rm L} = i\left(\phi^*\dot{\phi} - \dot{\phi}^*\phi\right) \simeq \dot{\theta}|\phi|^2$$

$$\simeq \epsilon m_{3/2}\varphi_{\rm osc}^2$$

ϵ : efficiency parameter
determined by $\arg(a_M), \lambda$, etc.

Formation of “Q-ball”s

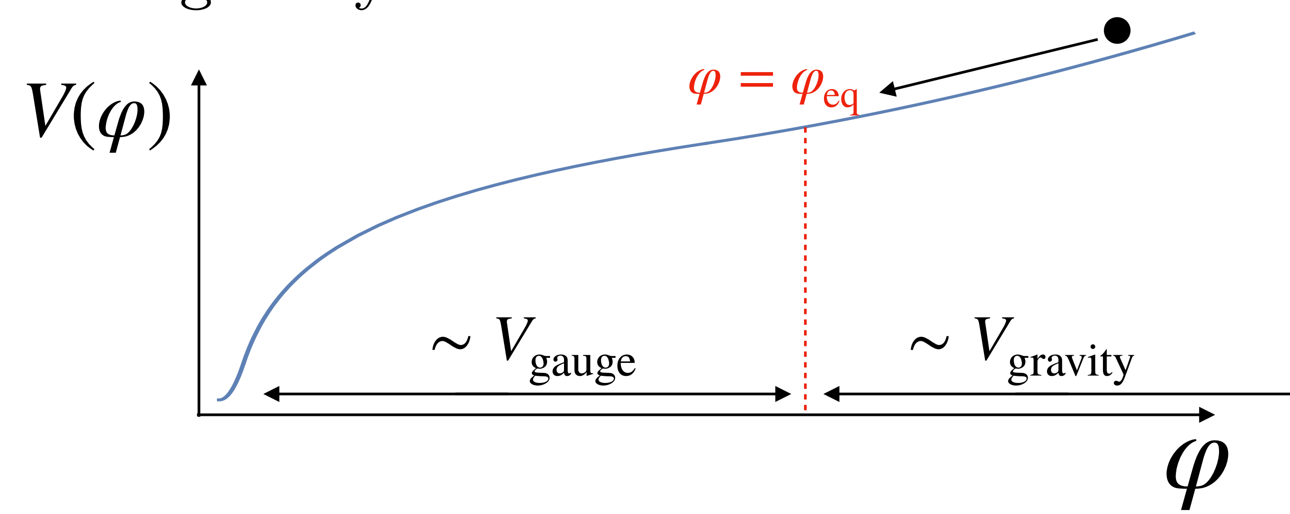
Our setup

SUSY breaking potential : $V = V_{\mathrm{gauge}} + V_{\mathrm{gravity}}$

where
$$V_{\mathrm{gauge}} = M_{\mathrm{F}}^4 \left(\ln \frac{|\phi|^2}{M_S^2} \right)^2 ,$$
$$V_{\mathrm{gravity}} = m_{3/2}^2 \left[1 + K \ln \left(\frac{|\phi|^2}{M_{\mathrm{Pl}}^2} \right) \right] |\phi|^2$$

$M_{\mathrm{F}} \equiv \sqrt{\langle F \rangle}$: SUSY breaking scale

$m_{3/2}$: Gravitino mass



After coherent AD field oscillation, V_{gauge} begins to dominate and the AD field configuration has spatial instability and forms Q-balls.

Instability band of the field fluctuations: $R_Q \sim \dfrac{\varphi_{\mathrm{eq}}}{M_F^2}$

→ Properties of Q-balls : $\omega_Q = \sqrt{2}\pi\zeta M_F Q^{-1/4}, \; Q = \beta \left(\dfrac{\varphi_{\mathrm{eq}}}{M_{\mathrm{F}}} \right)^4$

β, ζ : numerical constants

Inside the Q-ball, ϕ has a VEV of $\varphi \gg \mathcal{O}(100)$ GeV

→ Gauge boson has a huge mass due to the coupling with ϕ

→ Stable against sphaleron process, no baryon number is generated

Decay of Q-balls

Q-balls are classically stable, but decay into light fermions with masses $m_f < \omega$ through quantum effect.

If final states are fermions, decay process is bounded by Pauli-blocking effect of the final states

→upper bound of the decay rate is given by number of possible states of outgoing flow of the fermions

A. Cohen et al. (1986), M. Kawasaki, M. Yamada (2012)

In our case ($\phi =$ sleptons), Q-balls decay into SM neutrinos via $\phi\phi \to \nu\nu$ process (via Zino/Higgsino exchange)

Saturated decay takes place in realistic situations

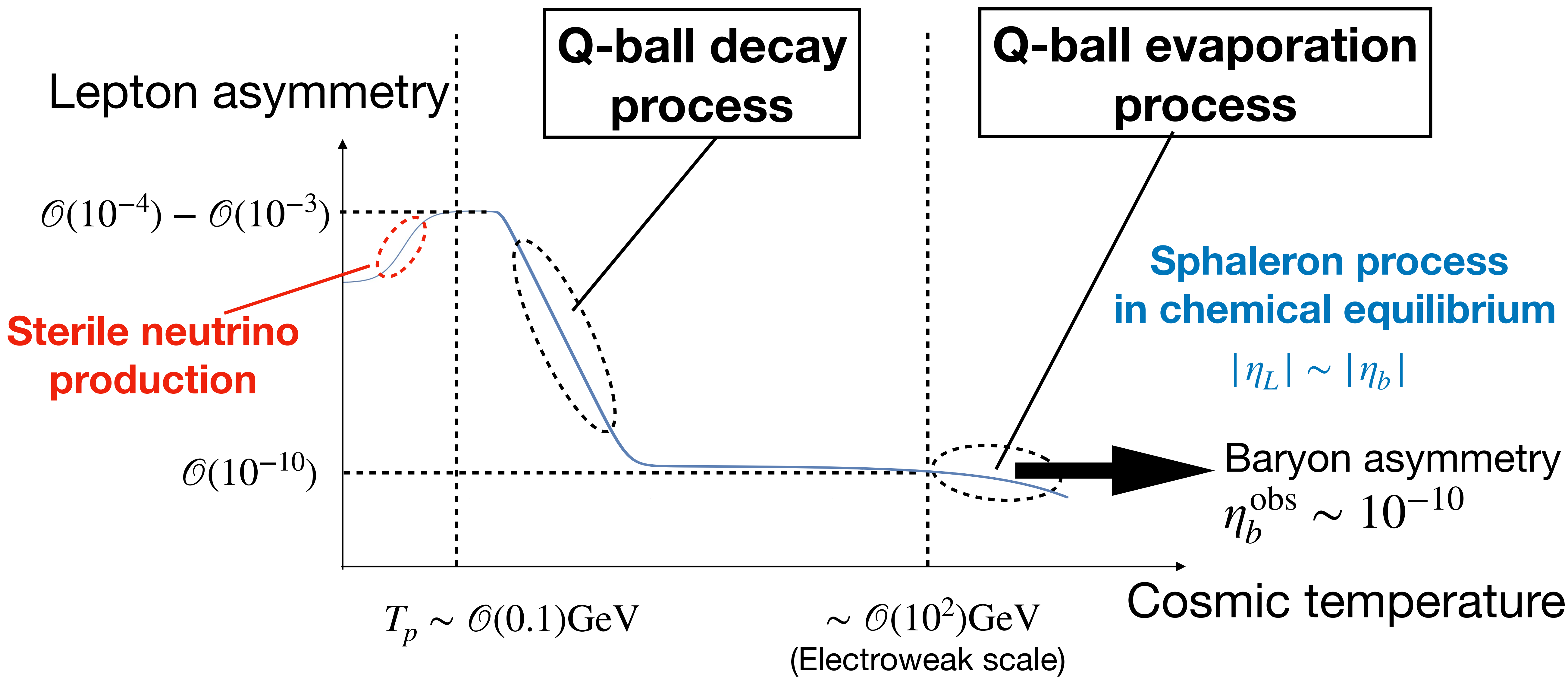
→decay rate is given by

$$\Gamma_Q \simeq \frac{1}{Q}\frac{\omega_Q^3}{\pi}\left(\frac{N_l}{3}\right)R_Q^2$$

N_l : Number of neutrino species

Large lepton asymmetry from Q-ball decay

Once (decay rate) $\sim H$ is satisfied, they instantaneously decay and lepton asymmetric thermal plasma is generated.

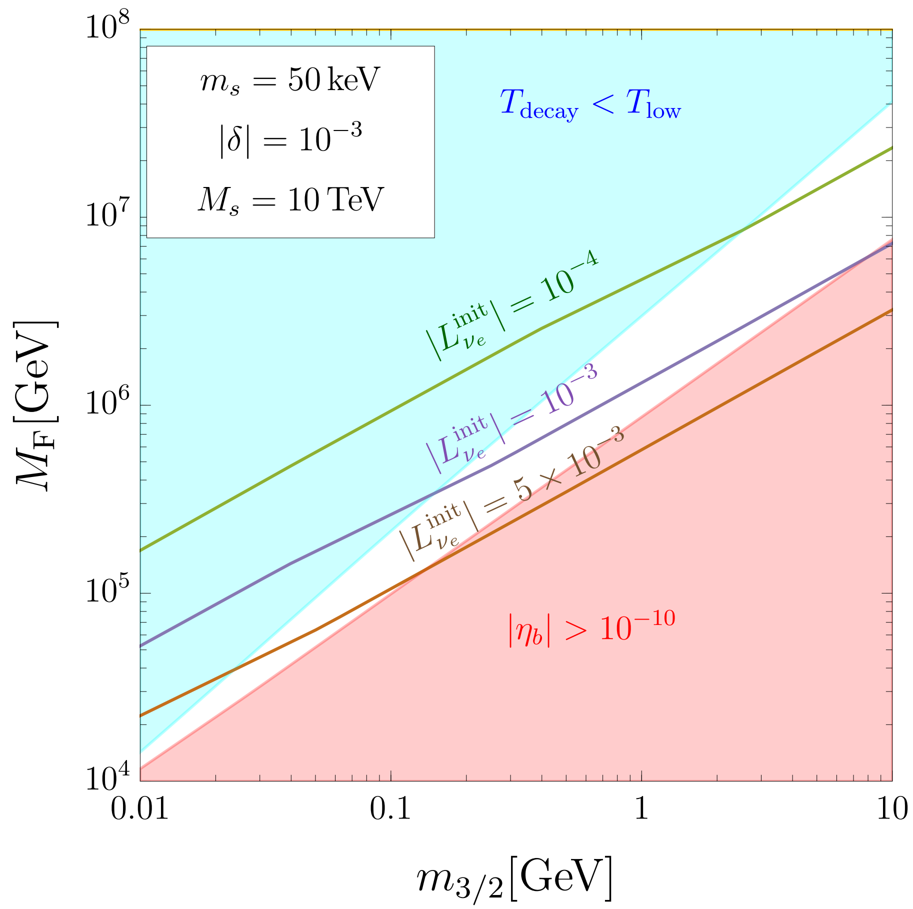


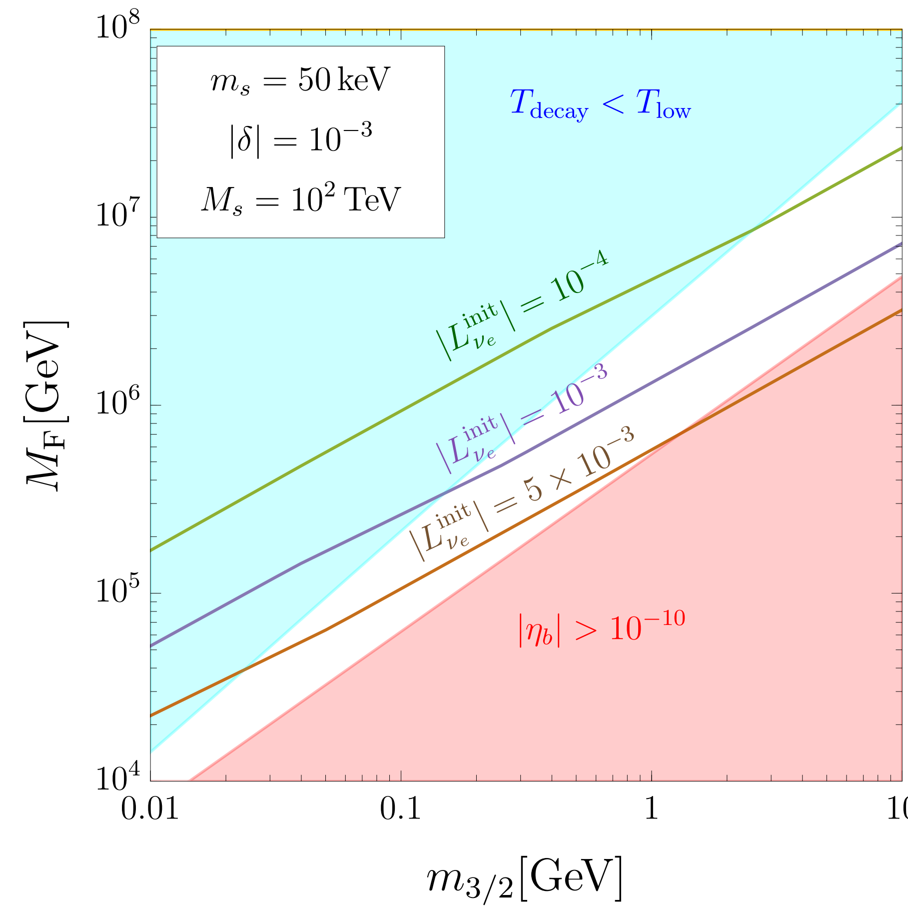
Large lepton asymmetry from Q-ball decay

KK, M. Kawasaki, K. Murai, 2402.11902

We estimate the allowed parameter space in Q-ball decay scenario which is compatible with Shi-Fuller mechanism.

(i.e. Q-ball decay producing lepton asymmetry $\gtrsim \mathcal{O}(10^{-4})$ occurs <u>above the resonance temperature of sterile neutrino production $\sim \mathcal{O}(0.1)\mathrm{GeV}$</u>)





Enhancement of second-order scalar-induced gravitational waves at Q-ball decay

- **Scalar-induced gravitational wave**

D. Bauman et al. (2007)

Take conformal-Newtonian gauge :

$$\mathrm{d}s^2 = a^2(\eta)\left[-\left(1+2\Phi^{(1)}+2\Phi^{(2)}\right)+2V_i^{(2)}\mathrm{d}\eta\mathrm{d}x^i+\left[\left(1-2\Phi^{(1)}-2\Phi^{(2)}\right)\delta_{ij}+\frac{1}{2}h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right]$$

Second-order perturbation of Einstein tensor and stress tensor:

$$G_j^{(2)i} = \frac{1}{4}a^{-2}(h_j''^{(2)i}+2\mathcal{H}h_j'^{(2)i}-\Delta h_j^{(2)i}) + (\text{second order terms with respect to } \Phi_{,i})$$

$$T_j^{(2)i} \supset \left(\rho^{(0)}+P^{(0)}\right)v^{(1)i}v_j^{(1)}$$

EOM for second-order GW:

$$\leftrightarrow h_{ij}''^{(2)} + 2\mathcal{H}h_{ij}'^{(2)} - \Delta h_{ij}^{(2)} = -4\hat{\mathcal{T}}_{ij}^{lm}S_{lm},$$

where $S_{lm} \equiv 3(1+c_s^2)\mathcal{H}^2 v_l^{(1)}v_m^{(1)} + (\text{second order terms with respect to } \Phi)$

Since $v_i^{(1)} = \dfrac{-2\mathcal{H}\Phi_{,i}^{(1)} - 2\Phi_{,i}'^{(1)}}{(1+c_s^2)\mathcal{H}^2},$

$$h''^{(2)} + 2\mathcal{H}h'^{(2)} - \Delta h^{(2)} = \mathcal{O}(\Phi^2)$$

Second order GW can be induced by time derivative of Φ

Enhancement of second-order scalar-induced gravitational waves at Q-ball decay

We consider the time evolution of Φ before and after Q-balls decay and estimate the resultant GW spectrum

- During eMD :

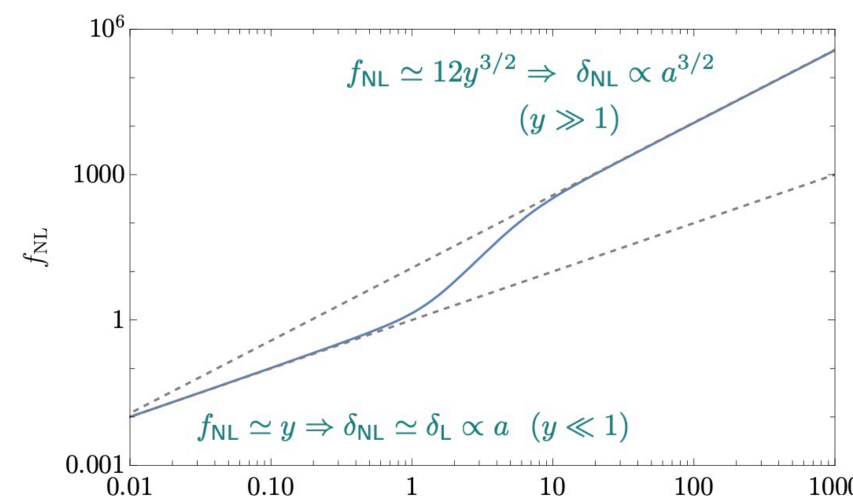
  At linear level, Φ is conserved

  However, the small-scale density perturbations grows to non-linear

  →Use the result of N-body simulations of <u>density perturbations</u> in MD era, and relate them to Φ <u>by Poisson eq.</u>

$$\delta^2_{\mathrm{m,NL}}(k_{\mathrm{NL}}) = f_{\mathrm{NL}}[\delta^2_{\mathrm{m,L}}(k_{\mathrm{L}})],$$

$$k_{\mathrm{L}} = [1+\delta^2_{\mathrm{m,NL}}(k_{\mathrm{NL}})]^{-1/3} k_{\mathrm{NL}}$$



Poisson eq : $\quad k^2_{\mathrm{NL}}\Phi = \mathcal{H}^2 \delta_{\mathrm{m,NL}}(k_{\mathrm{NL}})$

- Transition period :

  When $\; 3a^2|\ddot{\Phi}| \ll k^2\Phi,\;$ Φ decays proportional to M_{Q}

  After that, Φ begins to oscillate satisfying

$$\Phi'' + 4\mathcal{H}\Phi' + \frac{k^2}{3}\Phi = 0$$

Discussions : Beyond the monochromatic approximation of the Q-ball mass distribution

The mass distribution of the Q-balls have finite width

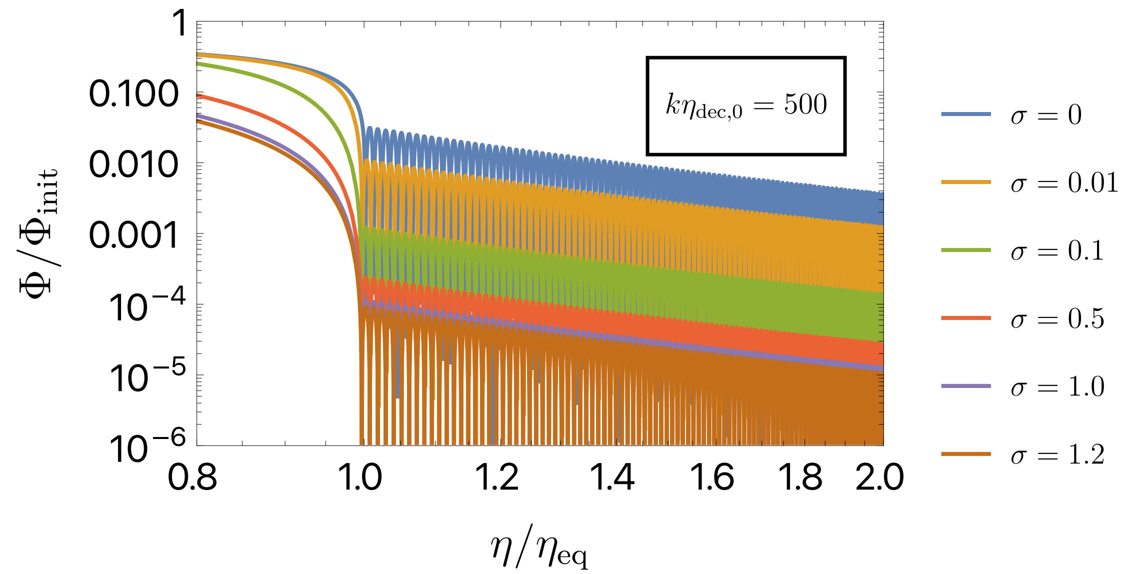
There will be a cut-off of Q-ball size because they cannot be formed in super-horizon scale at the formation

S. Kasuya, M. Kawasaki, (2002)

→We model the charge distribution of Q-balls as

$$\frac{\mathrm{d}E}{\mathrm{d}\ln Q} \propto Q^{1/\sigma}\Theta(Q_{max} - Q)$$

→Due to the finite σ, the eMD-RD transition becomes more gradual
→The oscillation amplitude of ϕ gets more suppressed



→This suppression becomes strong in the high-frequency region
→will alter the GW spectrum in high k