Initiative Physique des Infinis
Alliance Sorbonne Université

SORBONNE UNIVERSITÉ

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

Baryogenesis and Preheating in Starobinsky – Higgs inflation

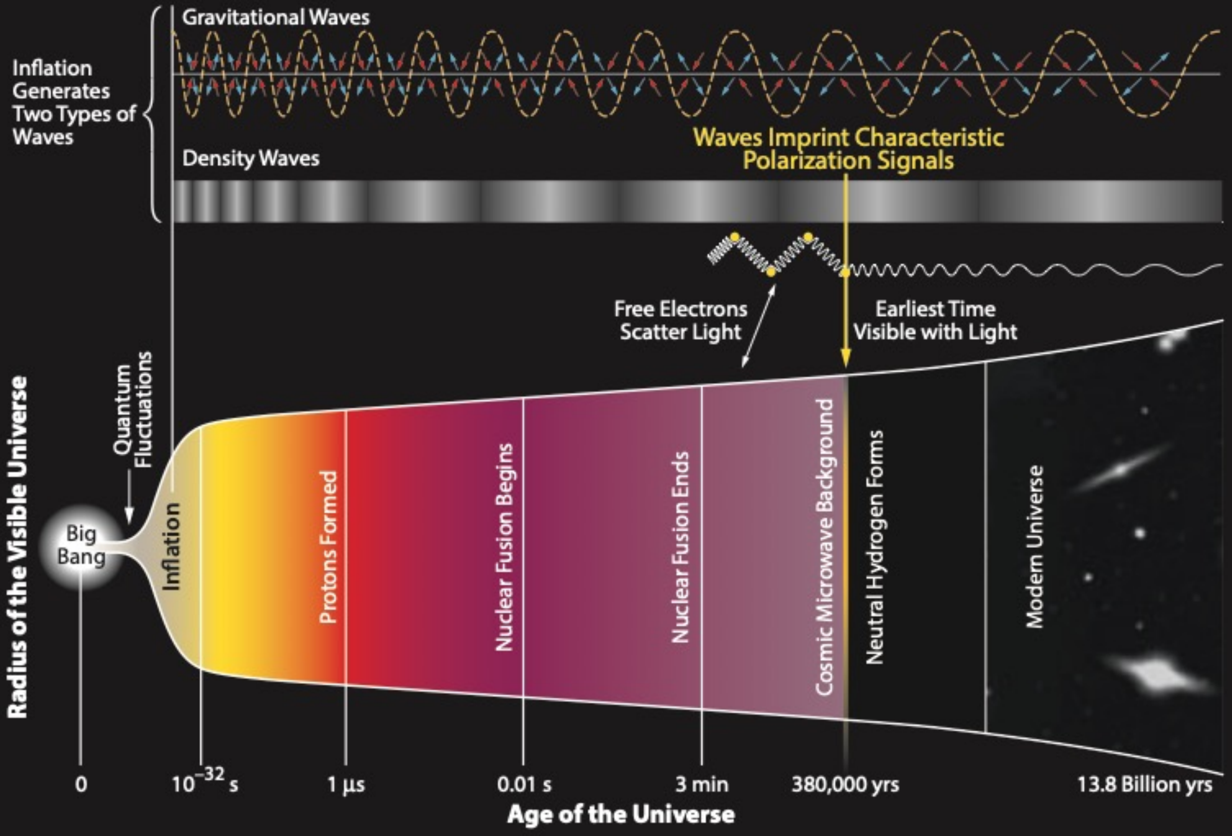
Yann Cado
25 / 07 / 25

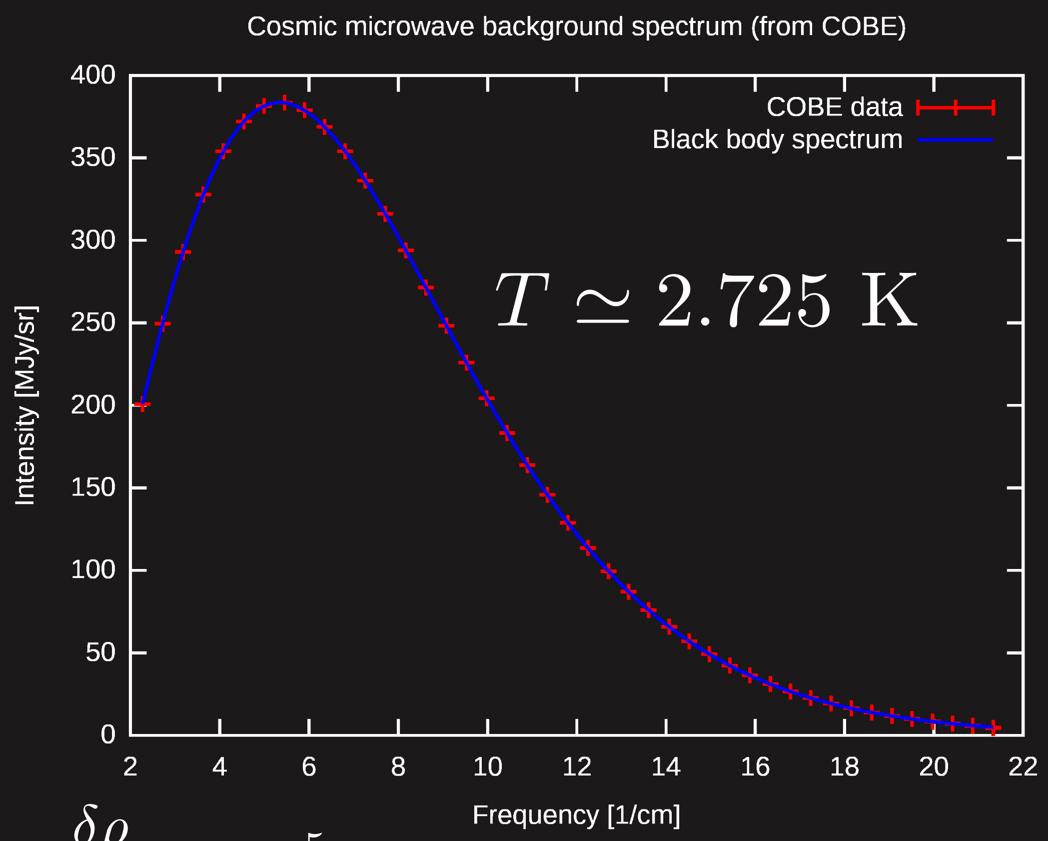
based on 2312.10414 & 2411.11128
in collaboration with C. Englert, T. Modak & M. Quirós

OUTLINE

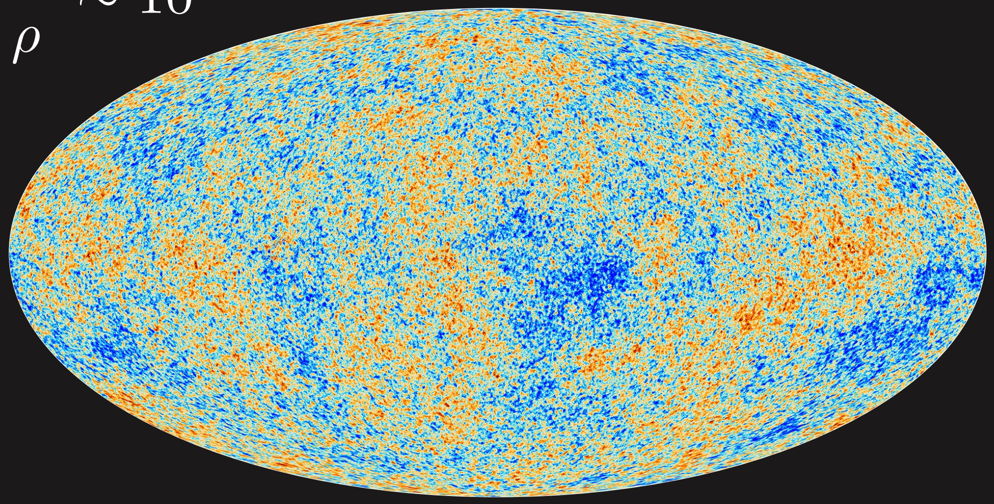
- Inflation
  - Slow roll inflation
  - Higgs Inflation
  - R^2 – Higgs Inflation with Chern-Simons coupling
- Post-inflationary physics
  - Preheating
  - Baryogenesis
- Conclusion

HISTORY OF THE UNIVERSE





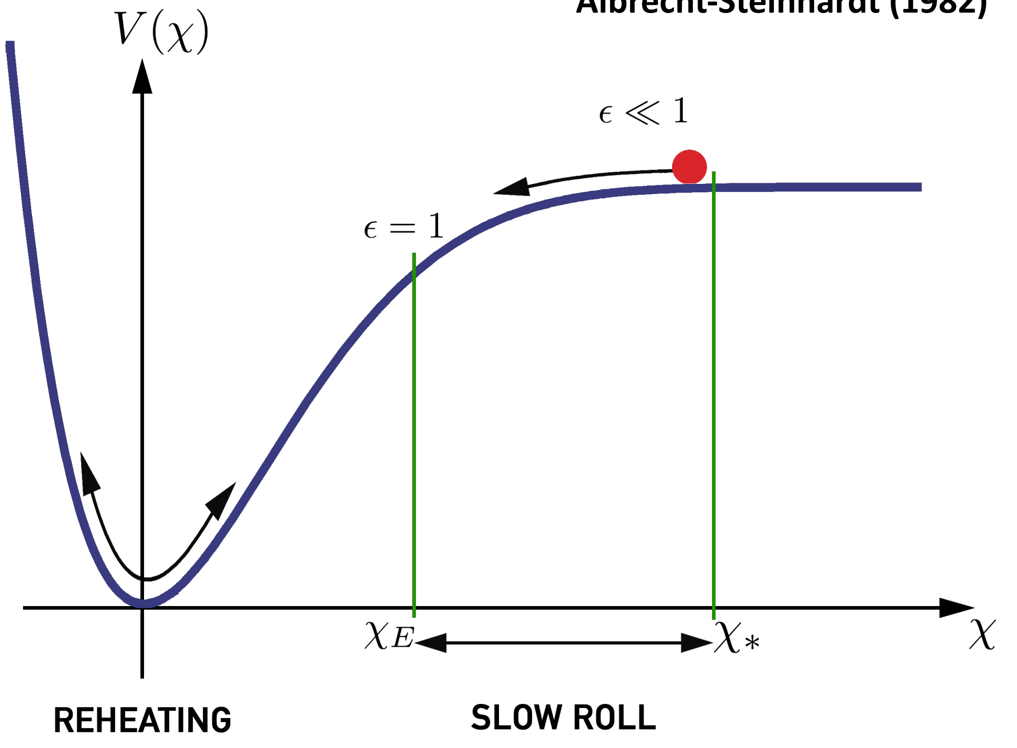
$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \sim 10^{-5}$$



BICEP2 2014 Release Image Gallery
FIRAS CMB Monopole Spectrum (NASA – Wikipedia)
Planck Collaboration (ESA)

Guth (1981), Linde (1982), Albrecht-Steinhardt (1982)

SLOW ROLL INFLATION



Introduce new scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V(\chi) \quad \Rightarrow \frac{\ddot{a}}{a} \simeq H^2(1-\epsilon)$$

Compute slow roll parameters

$$\epsilon(\chi) = \frac{M_{\rm pl}^2}{2}\left(\frac{V'(\chi)}{V(\chi)}\right)^2 \qquad \eta(\chi) = M_{\rm pl}^2 \frac{V''(\chi)}{V(\chi)}$$

The inflaton has **quantum fluctuations** constrained by the CMB anisotropies

Scalar spectral index $\quad n_s \simeq 1 + 2\eta - 6\varepsilon$

Tensor to scalar ratio $\quad r \simeq 16\varepsilon$

Amplitude of scalar fluctuations $\quad A_s = \frac{1}{24\pi^2 M_{\rm P}^4}\frac{V}{\varepsilon}$

evaluated at CMB scale $\chi_* = \chi(N_*)$

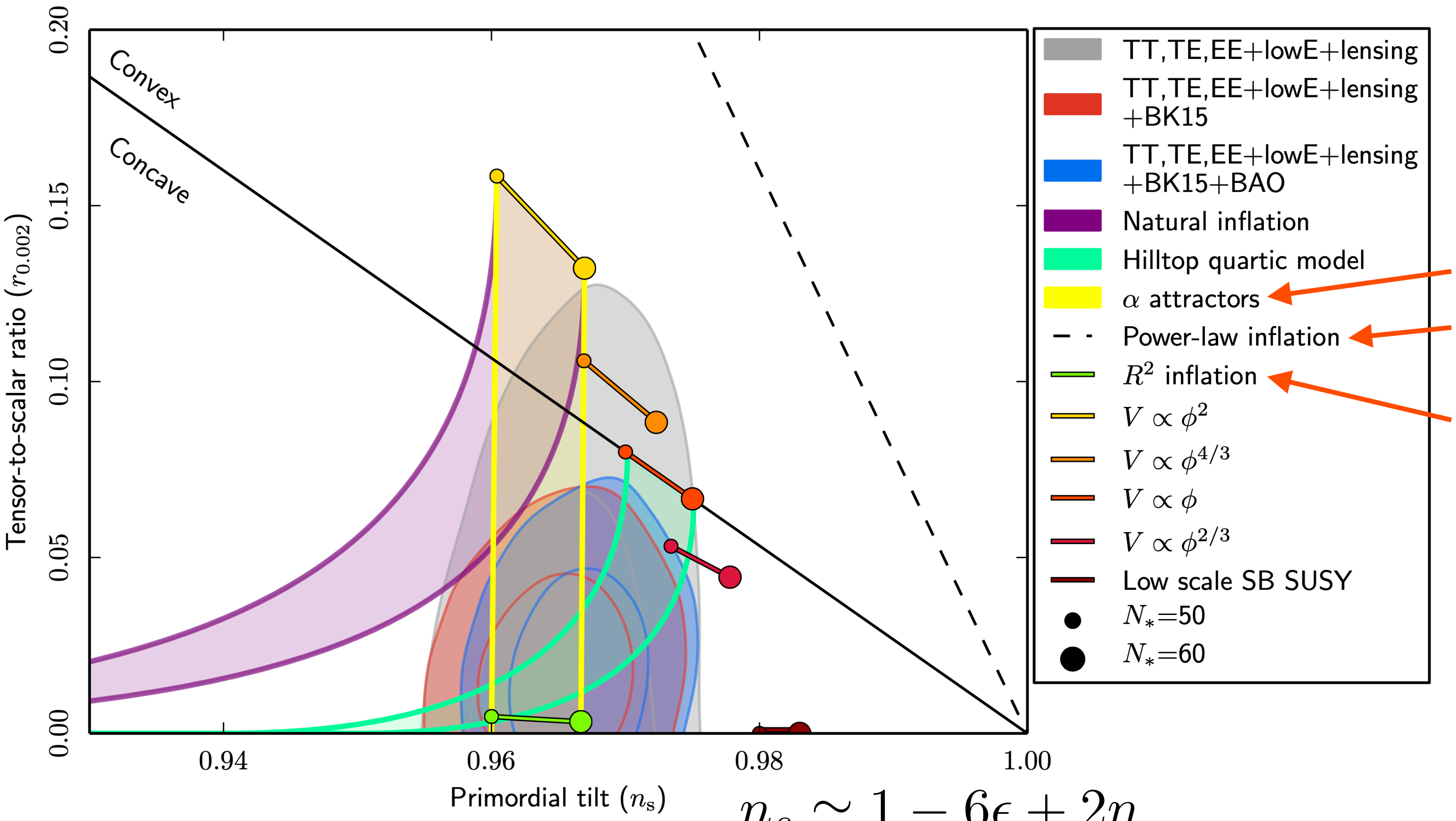
$$N_* = \frac{1}{M_{\rm pl}^2}\int_{\chi_E}^{\chi_*} \frac{V(\chi)}{V'(\chi)}\, d\chi$$

SLOW ROLL INFLATION

$$A_s^{\mathrm{obs}} \simeq 2.2 \cdot 10^{-9}$$

Observational constraints

$$r = 16\,\epsilon$$



= R^2 when α = 1

Monomial inflation

Starobinsky inflation (one of the best fit)

$$n_s \simeq 1 - 6\epsilon + 2\eta$$

PLANCK results X (2018)

Bezrukov-Shaposhnikov-Magnin-Sibiryakov (2008-2011)

Another realisation : Higgs Inflation

$$\mathcal{L} = -\frac{M_{\rm pl}^2}{2}R - \frac{\xi h^2}{2}R + \frac{1}{2}\partial_\mu h \partial^\mu h - U(h) \qquad U(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

JORDAN FRAME

$$g_{\mu\nu} \to \Theta g_{\mu\nu} \qquad \Theta = \left(1 + \frac{\xi h^2}{M_{\rm pl}^2}\right)^{-1}$$

PERFORM WEIL TRANSFORMATION

EINSTEIN FRAME

$$\mathcal{L} = -\frac{M_{\rm pl}^2}{2}R + \frac{M_{\rm pl}^2}{2} K(\Theta)\partial_\mu \Theta \partial^\mu \Theta - V(\Theta) \qquad V(\Theta) = \Theta^2 U[h(\Theta)]$$

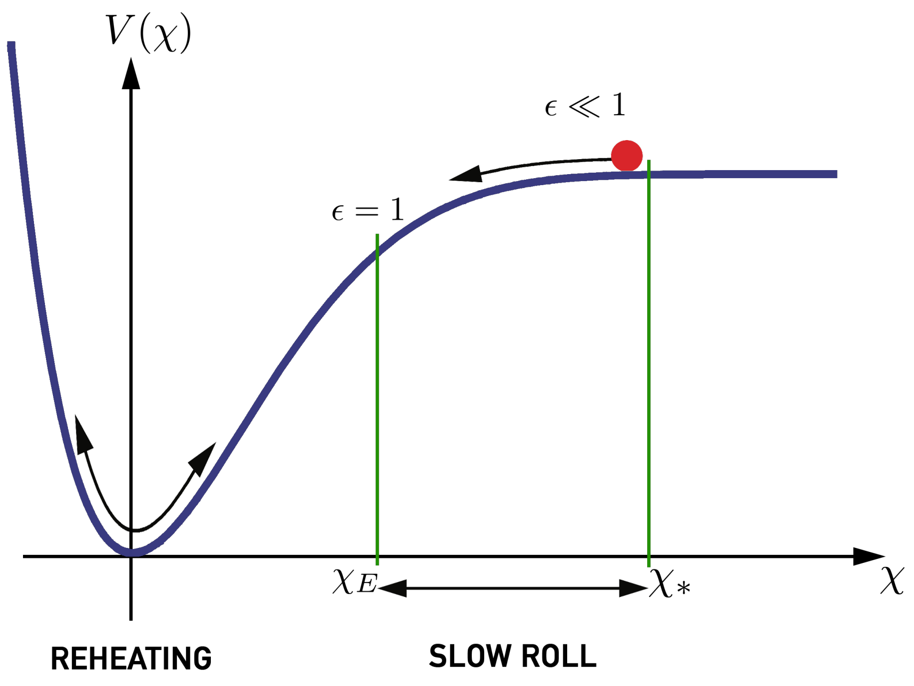
Some success

Define χ such that $\left(\frac{d\chi}{d\Theta}\right)^2 = M_{\rm pl}^2 K(\Theta)$

and get

$$\mathcal{L} = -\frac{M_{\rm pl}^2}{2} R + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - V(\chi)$$

THE TRUE INFLATON



$$V(\chi) \simeq \frac{\lambda M_{\rm pl}^4}{4\xi^2}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\chi}{M_{\rm pl}}\right)\right]^2$$

STAROBINSKY POTENTIAL

Starobinsky (1980)

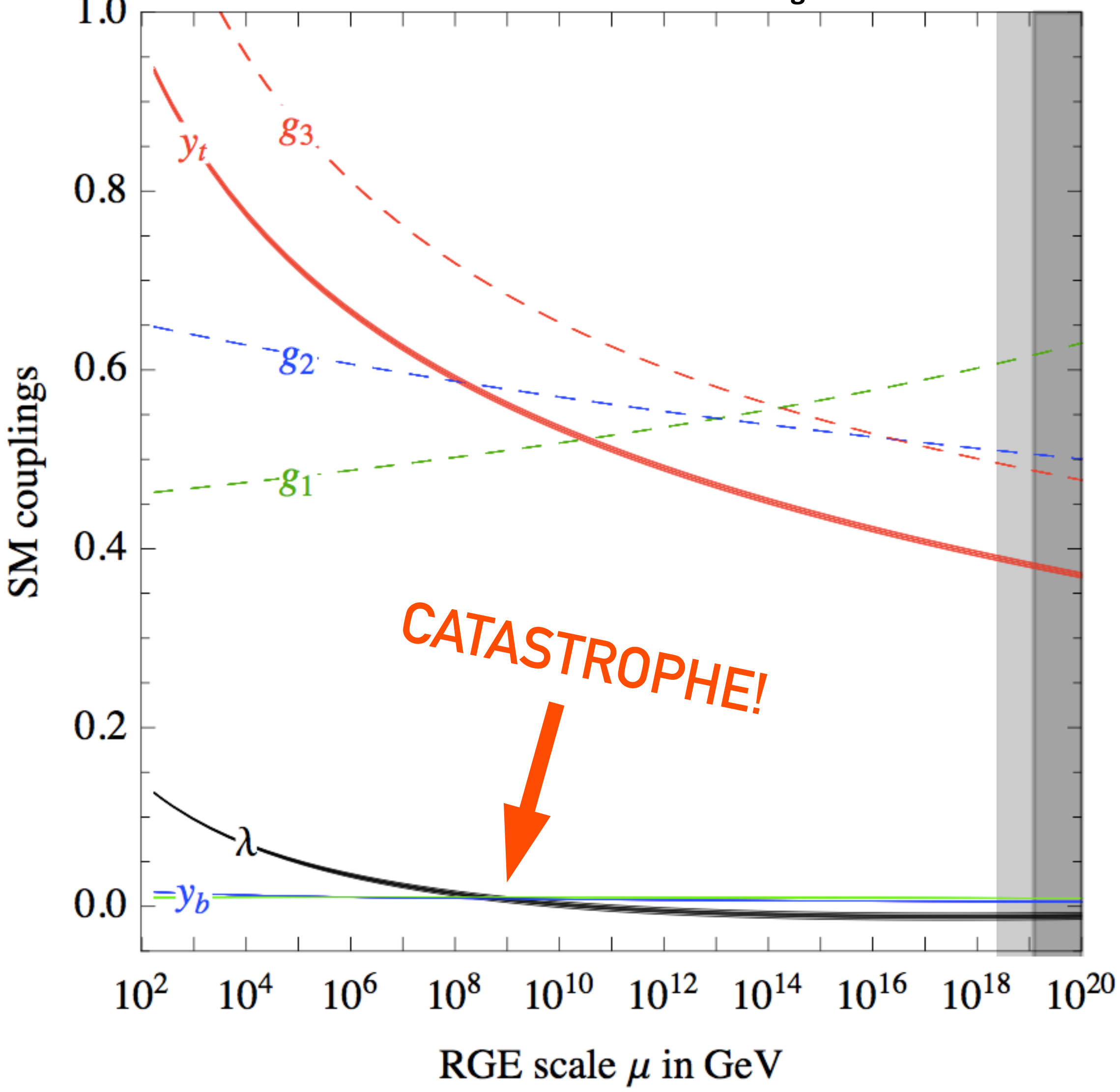
Barbon-Espinosa (2009), Lerner-McDonald (2010)
Burgess-Lee-Trott (2010), Bezrukov-Rubio-Shaposhnikov (2015)
Antoniadis-Guillen-Tamvakis (2021)

Some issues

- Density perturbations fix $\xi \sim 10^4$
- Unitarity of the model still in discussion
- Says nothing about the Higgs vacuum instability

USUALLY FIXED BY THE ADDITION OF ANOTHER SCALAR

Figure from 1205.6497



Bezrukov-Shaposhnikov-Magnin-Sibiryakov (2008-2011)

R^2 – Higgs Inflation with Chern-Simons coupling and EW sector

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}f(R) - g^{\mu\nu}(\nabla_\mu\Phi)^\dagger\nabla_\nu\Phi - V(\Phi,\Phi^\dagger) - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}B_{\mu\nu}B_{\rho\sigma} - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}W^i_{\mu\nu}W^i_{\rho\sigma}\right]$$

$$f(R) = R + \frac{\xi_R}{2M_P^2}R^2 + \frac{2\xi_H}{M_P^2}|\Phi|^2R - \frac{2}{\Lambda^2M_P^2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}B_{\mu\nu}B_{\rho\sigma}R - \frac{2}{\Lambda^2M_P^2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}W^i_{\mu\nu}W^i_{\rho\sigma}R$$

$$\nabla_\mu = D_\mu + ig'\frac{1}{2}Q_{Y_f}B_\mu + ig\,\mathbf{T}\cdot\mathbf{W}_\mu$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_3 + i\phi_4\\ h + i\phi_2\end{pmatrix}$$

$$V(\Phi,\Phi^\dagger) = \lambda|\Phi|^4$$

JORDAN FRAME

$$g_{\mu\nu} \to \Theta^{-1}g_{\mu\nu}, \qquad \Theta = \left.\frac{\partial f(\Psi)}{\partial\Psi}\right|_{\Psi=R}$$

PERFORM WEIL TRANSFORMATION

EINSTEIN FRAME

JORDAN FRAME

$$g_{\mu\nu} \to \Theta^{-1} g_{\mu\nu}, \qquad \Theta = \left.\frac{\partial f(\Psi)}{\partial \Psi}\right|_{\Psi=R}$$

PERFORM WEIL TRANSFORMATION

EINSTEIN FRAME

At quadratic order:

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_{\rm P}^2}{2}R - \frac{1}{2}G_{IJ}g^{\mu\nu}D_\mu\phi^I D_\nu\phi^J - V_E(\phi^I) - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{A\mu\nu}F_{A\rho\sigma} - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{Z\mu\nu}F_{Z\rho\sigma}$$
$$-\frac{1}{2}g^{\mu\rho}g^{\nu\sigma}F^+_{W\mu\nu}F^-_{W\rho\sigma} - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}g^{\mu\nu}\Big(\frac{g_Z^2}{8}h^2 Z_\mu Z_\nu + \frac{g_Z}{2}\left[(D_\mu h)\,\phi_2 - (D_\mu\phi_2)h\right]Z_\nu + \frac{e^2}{4s_W^2}h^2 W^+_\mu W^-_\nu +$$
$$\frac{ie}{2\sqrt{2}s_W}D_\mu h\left[W^-_\nu(\phi_3+i\phi_4) - W^+_\nu(\phi_3-i\phi_4)\right] - \frac{ie}{2\sqrt{2}s_W}\left[W^-_\nu D_\mu(\phi_3+i\phi_4) - W^+_\nu D_\mu(\phi_3-i\phi_4)\right]h\Big),$$

Field space:

$$\phi^I \in \{\phi, h, \phi_2, \phi_3, \phi_4\} \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_3 + i\phi_4 \\ h + i\phi_2\end{pmatrix}$$

Non-vanishing field-space metric elements:

$$G_{\phi\phi} = 1 \qquad G_{hh} = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}$$
$$G_{\phi_i\phi_i} = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}$$

Mass basis:

$$W^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt{2}}, \qquad W^2_\mu = \frac{i}{\sqrt{2}}\left(W^+_\mu - W^-_\mu\right),$$
$$W^3_\mu = s_W A_\mu + c_W Z_\mu, \qquad B_\mu = c_W A_\mu - s_W Z_\mu,$$

$$F_{A\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$
$$F_{Z\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu,$$
$$F^\pm_{W\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu.$$

JORDAN FRAME

$$g_{\mu\nu} \to \Theta^{-1} g_{\mu\nu}, \qquad \Theta = \left.\frac{\partial f(\Psi)}{\partial \Psi}\right|_{\Psi=R}$$

PERFORM WEIL TRANSFORMATION

EINSTEIN FRAME

At quadratic order:

$$S = \int d^4x \sqrt{-g}\Big[\frac{M_{\rm P}^2}{2}R - \frac{1}{2}G_{IJ}g^{\mu\nu}D_\mu\phi^I D_\nu\phi^J - \boxed{V_E(\phi^I)} - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{A\mu\nu}F_{A\rho\sigma} - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{Z\mu\nu}F_{Z\rho\sigma}$$

$$-\frac{1}{2}g^{\mu\rho}g^{\nu\sigma}F^+_{W\mu\nu}F^-_{W\rho\sigma} - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}g^{\mu\nu}\Big(\frac{g_Z^2}{8}h^2 Z_\mu Z_\nu + \frac{g_Z}{2}\left[(D_\mu h)\,\phi_2 - (D_\mu\phi_2)h\right]Z_\nu + \frac{e^2}{4s_W^2}h^2 W^+_\mu W^-_\nu + \ldots$$

Potential:

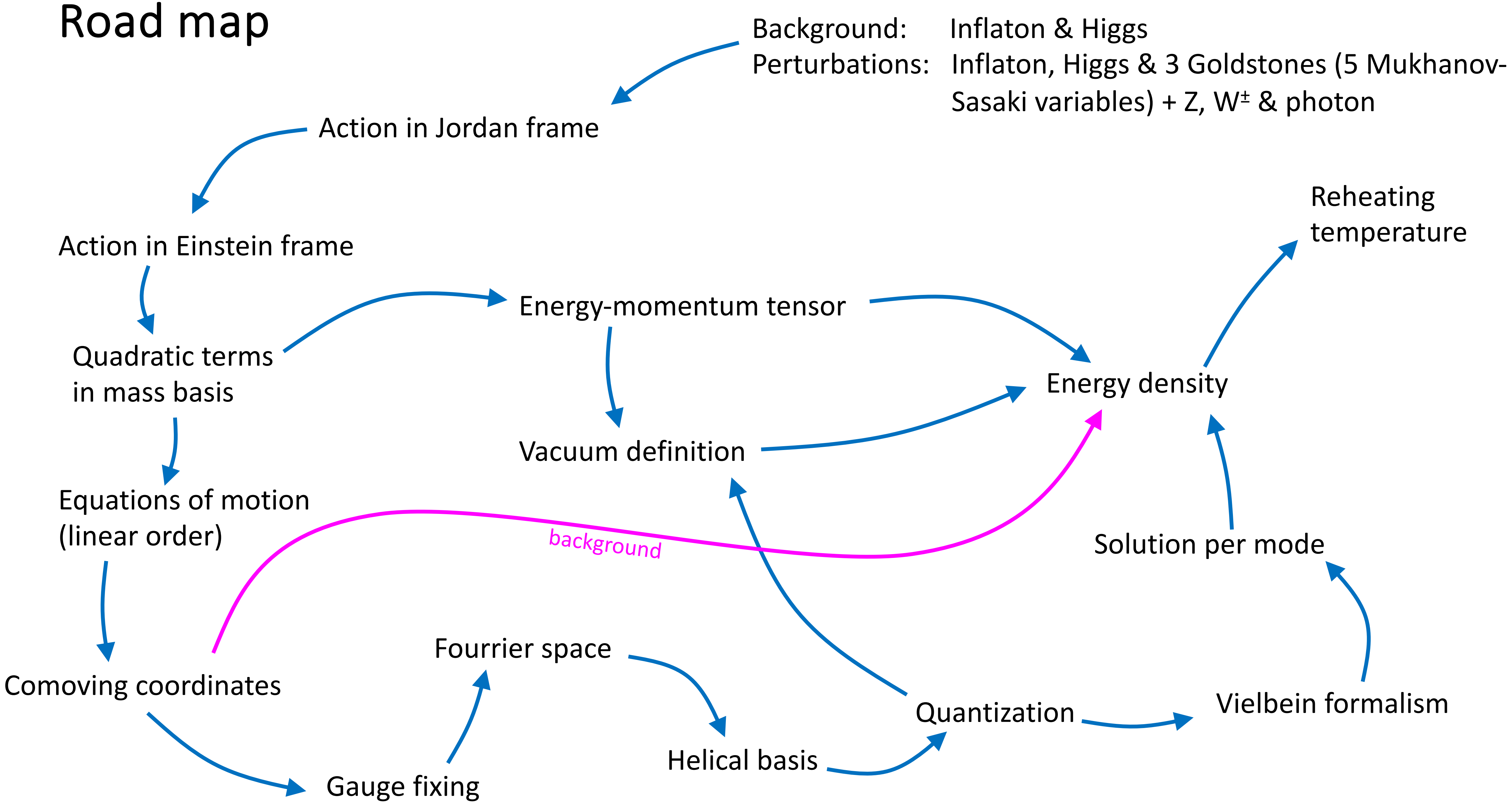
$$V_E(\phi^I) = V_0(\phi^I) + \frac{2M_{\rm P}^2}{\xi_R\Lambda^2}F(\phi^I)e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}\left(F_{A\mu\nu}\widetilde{F}^{\mu\nu}_A + F_{Z\mu\nu}\widetilde{F}^{\mu\nu}_Z + 2F^+_{W\mu\nu}\widetilde{F}^{-\mu\nu}_W\right)$$

$$F(\phi^I) = 1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}} - \frac{\xi_H}{M_{\rm P}^2}\left(h^2 + \sum_{i=2}^4 \phi_i^2\right)e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}},$$

$$V_0(\phi^I) = \frac{\lambda}{4}\left(h^2 + \sum_{i=2}^4\phi_i^2\right)^2 e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}} + \frac{M_{\rm P}^4}{4\xi_R}F^2(\phi^I),$$

$$\widetilde{F}^{\mu\nu}_A = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}F_{A\rho\sigma}, \quad \widetilde{F}^{\mu\nu}_Z = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}F_{Z\rho\sigma}, \quad \widetilde{F}^{\pm\mu\nu}_W = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}F^{\pm}_{W\rho\sigma}.$$

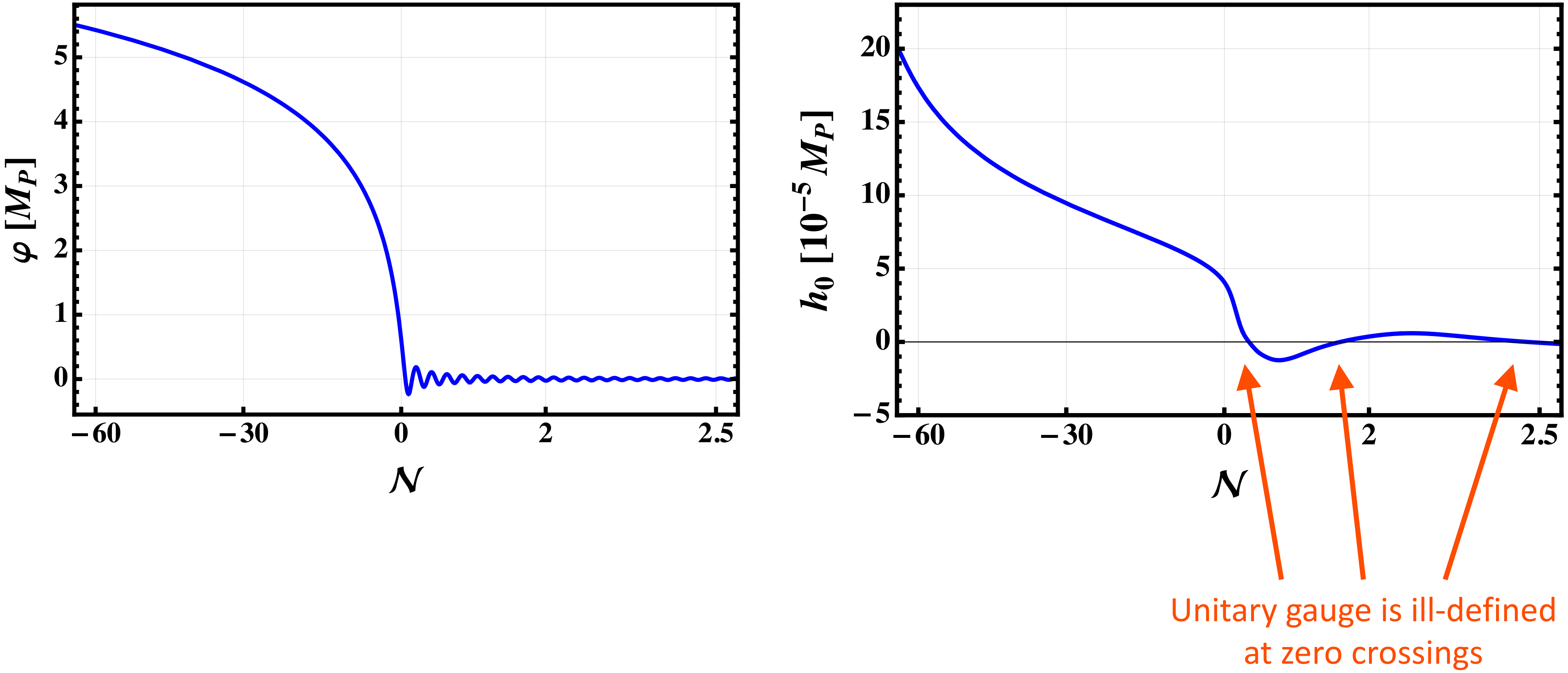
Road map



Background: Inflaton & Higgs
Perturbations: Inflaton, Higgs & 3 Goldstones (5 Mukhanov-Sasaki variables) + Z, W± & photon
Action in Jordan frame
Action in Einstein frame
Quadratic terms in mass basis
Equations of motion (linear order)
Comoving coordinates
Gauge fixing
Fourrier space
Helical basis
Quantization
Vielbein formalism
Solution per mode
Energy-momentum tensor
Vacuum definition
Energy density
background
Reheating temperature

Background dynamics

$$\mathcal{D}_t \dot{\varphi}^I + 3H\dot{\varphi}^I + G^{IJ}\frac{\partial V_0(\varphi^I)}{\partial \varphi^J} = 0 \qquad \varphi^I = \{\varphi, h_0\}$$



$$\xi_R = 2.35 \times 10^9$$

$$\xi_H = 10^{-3}$$

$$\mathcal{N} \equiv \ln \frac{a(t)}{a(t_{\rm end})}$$

COULOMB GAUGE

Higgs and inflaton fluctuations

Preheating happens when $\rho_{\rm inf} = \rho^{(q)}_{(X)}$

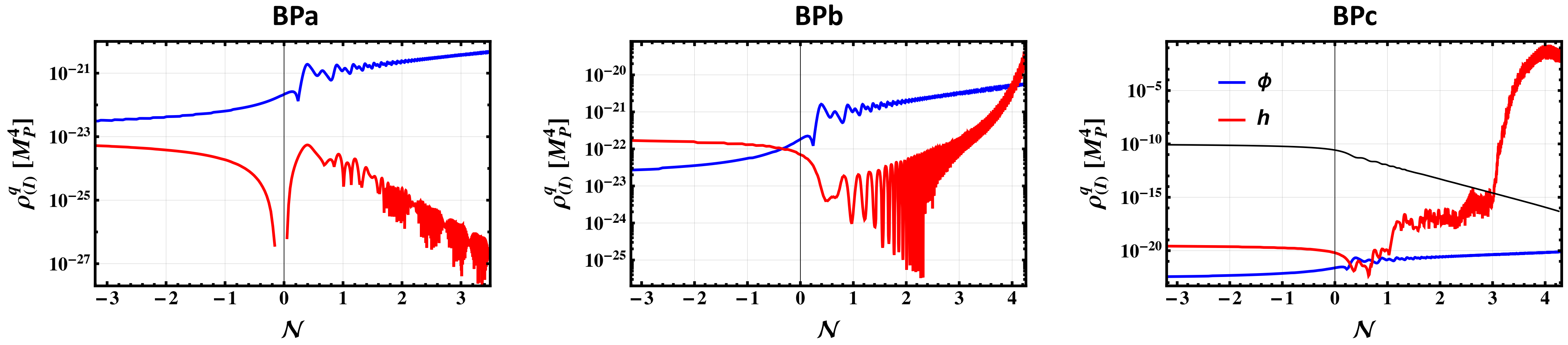
Reheating temperature

$$\rho_{\rm inf}(a_{\rm rh}) \equiv \rho_{\rm rh} = \frac{g_{\rm rh}\pi^2}{30}\, T_{\rm rh}^4$$

3 benchmark points

BP	ξ_R	ξ_H	$\varphi(t_{\rm in})\ [M_{\rm P}]$	$h_0(t_{\rm in})\ [M_{\rm P}]$
a	2.35×10^9	10^{-3}	5.5	2×10^{-4}
b	2.55×10^9	1	5.5	8.94×10^{-4}
c	2.2×10^9	10	5.4	5.00×10^{-3}

Inflaton & Higgs perturbations



Goldstones & gauge fields fluctuations

Z & W$^{\pm}$ bosons

BP	ξ_R	ξ_H	$\varphi(t_{\rm in})$ $[M_{\rm P}]$	$h_0(t_{\rm in})$ $[M_{\rm P}]$
a	2.35×10^9	10^{-3}	5.5	2×10^{-4}
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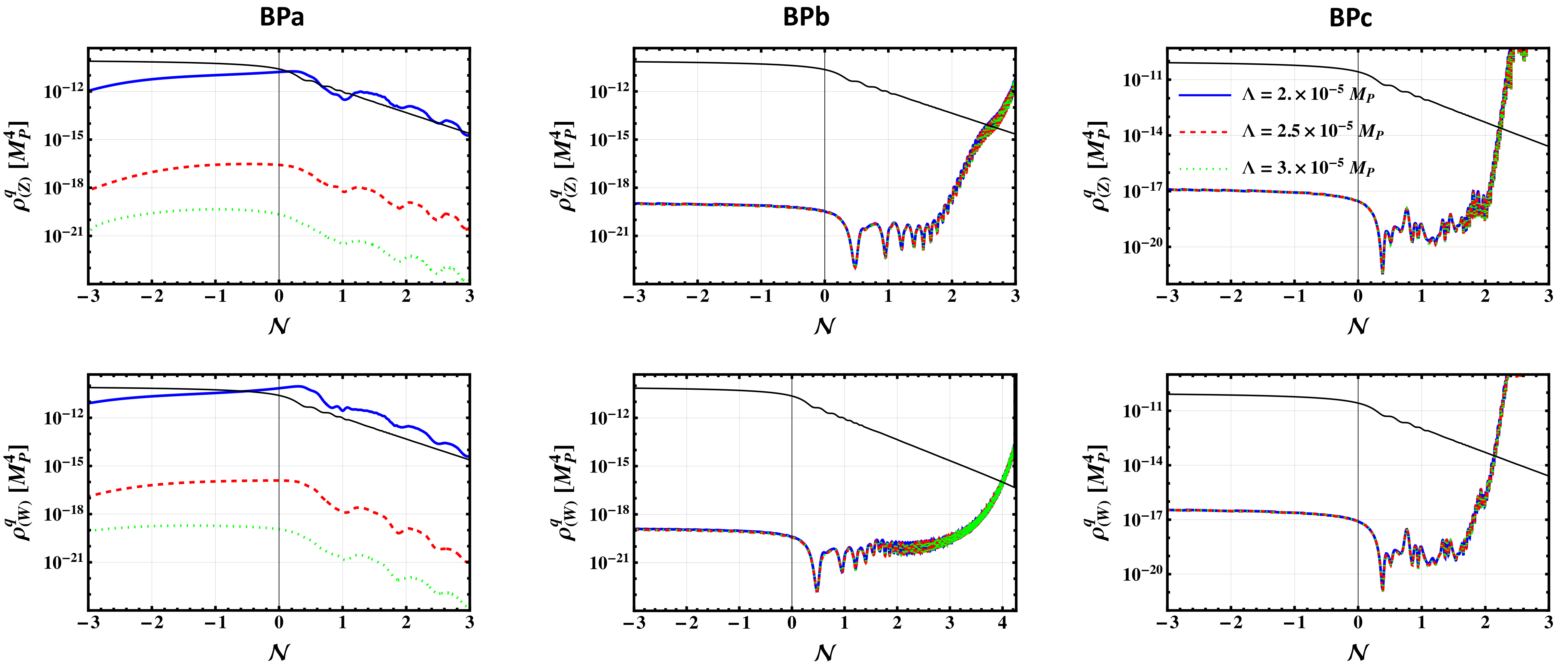
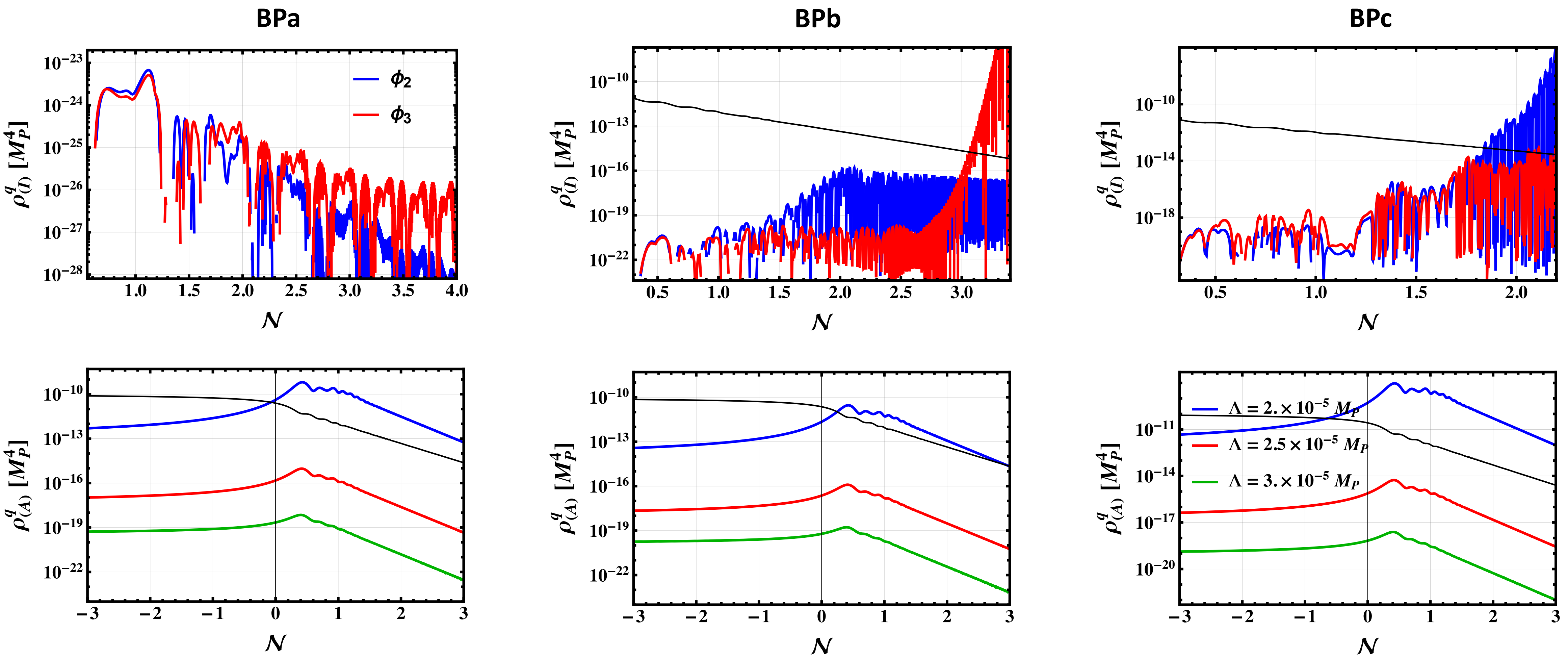


Goldstones & gauge fields fluctuations

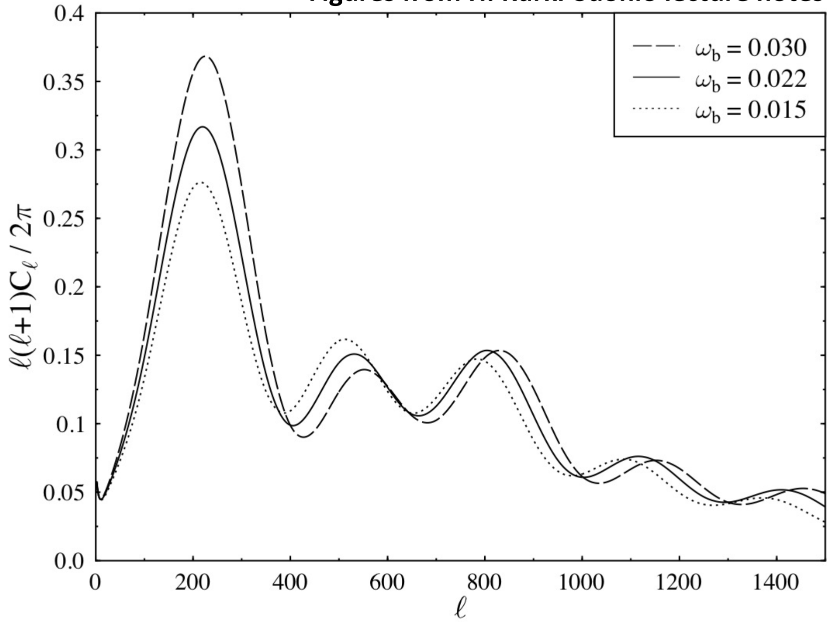
Goldstone bosons & photon

BP	preheating field(s)	$\mathcal{N}_{\rm rh}$	$a_{\rm rh}$	$\rho_{\rm rh}\ [M_{\rm P}^4]$	$T_{\rm rh}\ [\rm GeV]$
a	–	–	–	–	–
b	ϕ_3, ϕ_4	3.05	21	5×10^{-14}	5×10^{14}
c	ϕ_2	1.83	6.2	10^{-13}	6×10^{14}



Figures from H. Kurki-Suonio lecture notes

BARYOGENESIS



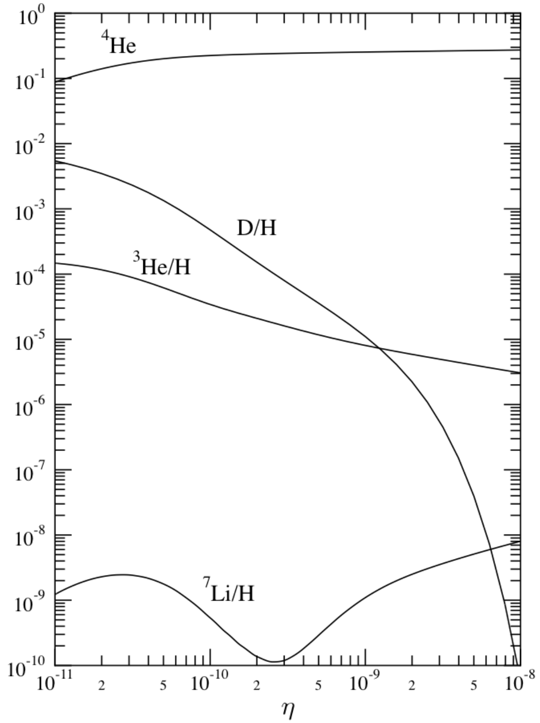
- No matter vs antimatter patches in the Universe
  - No γ rays from annihilation
  - Universe is isotropic and homogeneous

- Relative size of Doppler peaks in CMB are sensitive to

$$\eta = \frac{n_{\rm baryon}}{n_{\rm photon}} \qquad \eta_{\rm CMB} = 10^{-10} \cdot (6.14 \pm 0.25)$$

- BBN predicts observed abundance of lightest elements

for $\eta_{\rm BBN} = 10^{-10} \cdot \begin{cases} 6.28 \pm 0.35 \\ 5.92 \pm 0.56 \end{cases}$



→ We need a dynamical process that violates B, C/CP and thermal equilibrium

Sakharov (1967)

ELECTROWEAK BARYOGENESIS

't Hooft (1976)
Rubakov-Shaposhnikov (1996)
Anber-Sabancilar (2015)
Kamada-Long (2016)

Chern-Simons coupling

$$\mathcal{L} \supset f(\phi) B_{\mu\nu} \tilde{B}^{\mu\nu} \qquad F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\langle \mathbf{E}\cdot\mathbf{B}\rangle = 2\frac{d}{d\tau}\langle \mathbf{A}\cdot\mathbf{B}\rangle \qquad \mathcal{H} = \lim_{V\to\infty}\frac{1}{V}\int_V d^3x\,\langle \mathbf{A}\cdot\mathbf{B}\rangle$$

HYPERHELICITY

Chiral anomaly in the SM:

$$\Delta N_B = \Delta N_L = N_g\left(\Delta N_{\rm cs} - \frac{g_Y^2}{16\pi^2}\,\Delta\mathcal{H}_Y\right)$$

BARYON ASYMMETRY

$$\eta_B = \frac{255}{592}\frac{g_Z^2}{\pi^3\sqrt{10 g_*}}\frac{\mathcal{H}_Y}{(T_{\rm rh}a_{\rm rh})^3}\frac{f_{\theta_W}}{M_{\rm P}}\frac{T}{\gamma_{W\rm sph}}\bigg|_{T=135\ \rm GeV} \simeq 9\cdot 10^{-11}$$

Depends on the (p)reheating details

R^2 – Higgs Inflation with Chern-Simons coupling and EW sector

$$S=\int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}f(R)-g^{\mu\nu}(\nabla_\mu\Phi)^\dagger\nabla_\nu\Phi-V(\Phi,\Phi^\dagger)-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}B_{\mu\nu}B_{\rho\sigma}-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}W^i_{\mu\nu}W^i_{\rho\sigma}\right]$$

$$f(R)=R+\frac{\xi_R}{2M_P^2}R^2+\frac{2\xi_H}{M_P^2}|\Phi|^2R-\frac{2}{\Lambda^2M_P^2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}B_{\mu\nu}B_{\rho\sigma}R-\frac{2}{\Lambda^2M_P^2}\frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}W^i_{\mu\nu}W^i_{\rho\sigma}R$$

$$\nabla_\mu=D_\mu+ig'\frac{1}{2}Q_{Y_f}B_\mu+ig\,\mathbf{T}\cdot\mathbf{W}_\mu$$

$$\Phi=\frac{1}{\sqrt{2}}\begin{pmatrix}\phi_3+i\phi_4\\ h+i\phi_2\end{pmatrix}$$

$$V(\Phi,\Phi^\dagger)=\lambda|\Phi|^4$$

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PERFORM WEIL TRANSFORMATION

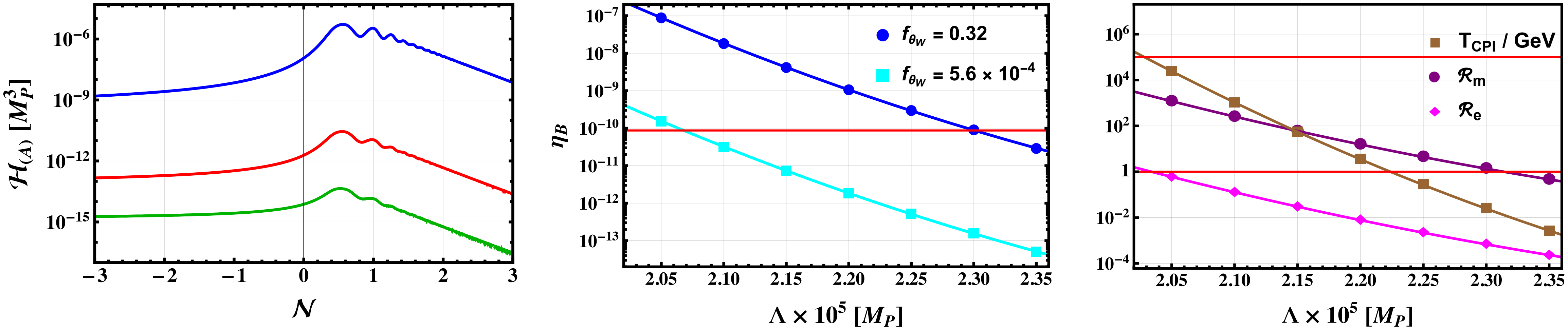
EINSTEIN FRAME

BARYOGENESIS

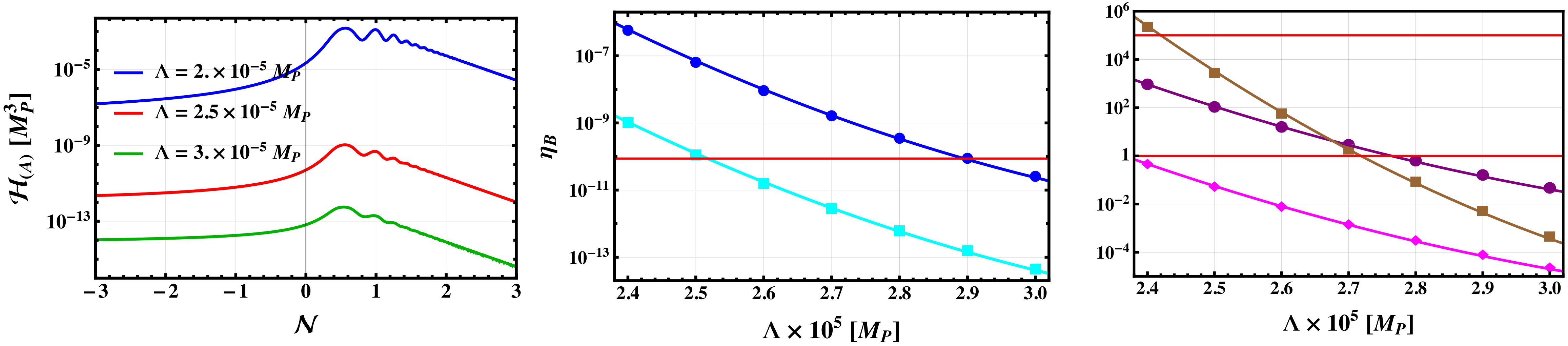
Parameter space

BP	preheating field(s)	$\mathcal{N}_{\rm rh}$	$a_{\rm rh}$	$\rho_{\rm rh}\ [M_{\rm P}^4]$	$T_{\rm rh}$ [GeV]
a	–	–	–	–	–
b	ϕ_3, ϕ_4	3.05	21	5×10^{-14}	5×10^{14}
c	ϕ_2	1.83	6.2	10^{-13}	6×10^{14}

BPb



BPc



TO CONCLUDE

- R^2-Higgs inflation is one of the best-fit models of Planck data.
- We adoped the doubly-covariant formalism
  (with respect to field-space and space-time transformations)
- We took into account the full electroweak sector
- We include all perturbations at the linear order with all effective mass components
  (potential, nontrivial field-space manifold, coupled metric perturbations & expansion of the background spacetime)
- Preheating happens in the Goldstone, Z & $W^{\pm}$ sectors
- With an additionnal dim-6 coupling $\Lambda^{-2}B_{\mu\nu}\tilde{B}^{\mu\nu}R$, the model predicts the right amount of baryon asymmetry in the present Universe, for $\Lambda \simeq 2.5 \times 10^{-5} M_{\rm pl}$
- Backreaction, decay, scattering & non-linearity must be studied in more details as so far no lattice code can simulate non-Abelian gauge fields in a non-minimally coupled inflation model


THANK YOU!
NASA/ESA/CSA James Webb Space Telescope, heart of M74

Back-up slides

COSMOLOGY

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

EINSTEIN

+

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right)$$

HOMOGENEITY + ISOTROPY

=

FRIEDMANN

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Eq. of state: $p = w\rho \quad \Rightarrow \quad \rho \propto \dfrac{1}{a(t)^{3(1+w)}}$

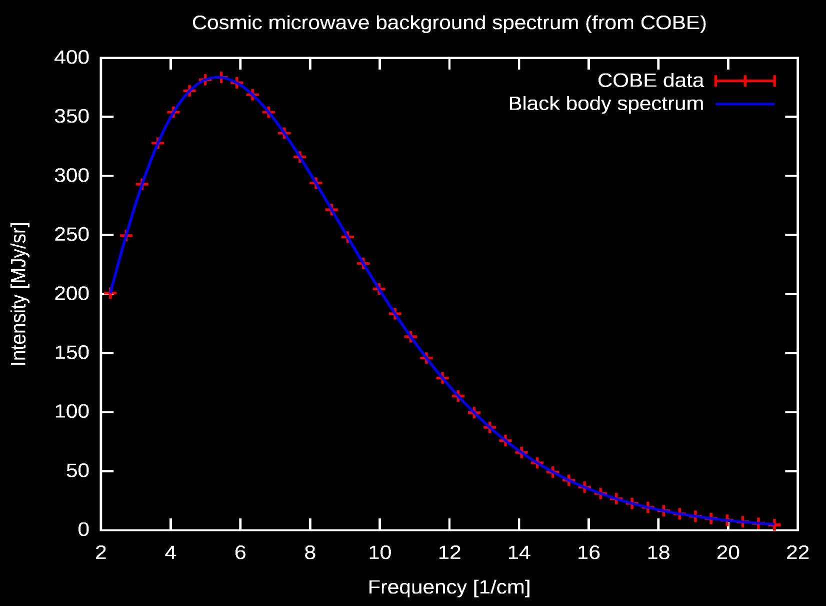
Matter: $\rho \propto a^{-3} \Rightarrow a \propto t^{2/3}$

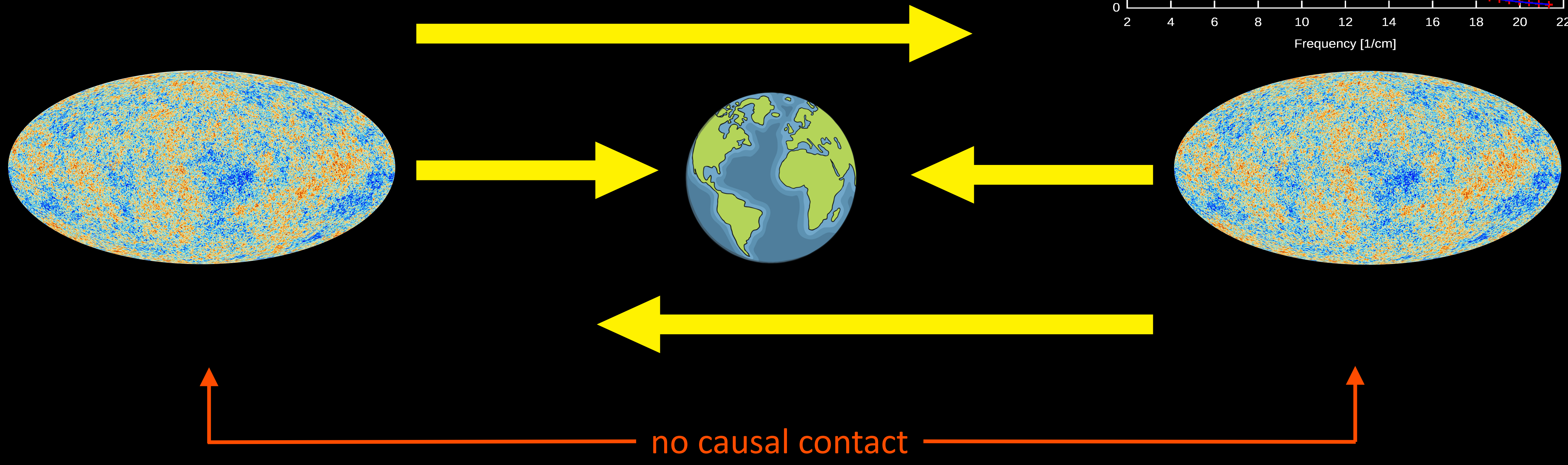
Radiation: $\rho \propto a^{-4} \Rightarrow a \propto t^{1/2}$

Λ (w=-1): $\rho \propto a^{0} \Rightarrow a \propto e^{\Lambda t/\sqrt{3}}$

COSMOLOGY

Homogeneity of the Universe in question





AGE OF THE CMB : 13.8 BILLIONS YEARS

SLOW ROLL INFLATION

Solution: At some point in the past, the whole observable Universe (that we see today) was in causal contact. It then expands by a factor $10^{21\text{-}26}$ in $10^{-36\text{-}32}$ seconds.

=> INFLATION PARADIGM

We need an accelerated expansion (to get a shrinking Hubble horizon)

$$\ddot{a} = -\frac{\dot{a}}{2M_{\rm P}}\sqrt{\frac{\rho}{3}}\,(1+3w) \qquad \ddot{a} > 0 \Rightarrow w < -1/3 \qquad M_{\rm P} \equiv \frac{1}{\sqrt{8\pi G}} \simeq 2.435 \cdot 10^{18}\ \text{GeV}$$

If the Universe energy density is dominated by the potential energy of a homogeneous scalar field, the Universe is Λ-dominated (de Sitter).

PROOF:

SLOW ROLL INFLATION

Introduce a new real scalar field (the inflaton) in the theory $\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$

Energy-momentum tensor: $T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L}_\phi$

A homogeneous and isotropic Universe is a perfect fluid $T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$

Hence $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ and $w_\phi = \dfrac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$

$$V(\phi) \gg \frac{1}{2}\dot{\phi}^2 \Rightarrow w_\phi \simeq -1 \Rightarrow a(t) \propto e^{Ht}$$

SLOW ROLL INFLATION

Guth (1981), Linde (1982), Albrecht-Steinhardt (1982)

In the slow-roll approximation

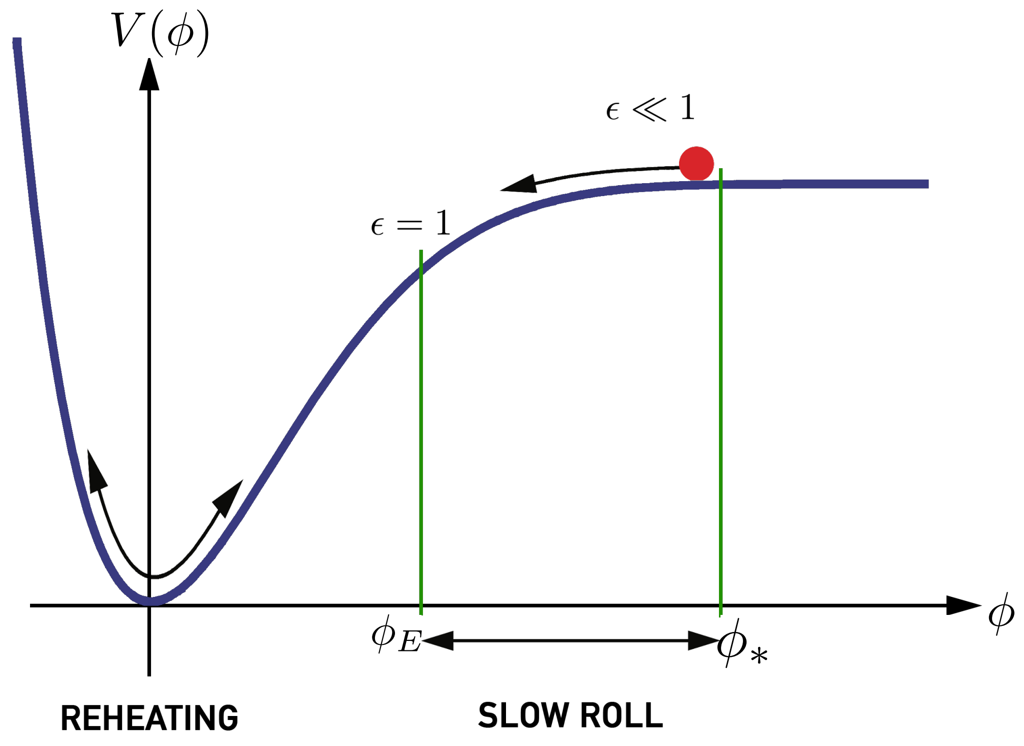
$$3H\dot{\phi} \simeq -V'(\phi) \qquad 3M_{\rm P}^2 H^2 \simeq V(\phi)$$

we can define the slow-roll parameters

$$\varepsilon \simeq \frac{M_{\rm P}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta \simeq M_{\rm P}^2 \frac{V''(\phi)}{V(\phi)}.$$

and rewrite the Friedmann equation as

$$\frac{\ddot{a}}{a} \simeq H^2(1-\varepsilon)$$



The inflaton has **quantum fluctuations** constrained by the CMB anisotropies

Scalar spectral index $n_s \simeq 1 + 2\eta - 6\varepsilon$

Tensor to scalar ratio $r \simeq 16\varepsilon$

Amplitude of scalar fluctuations $A_s = \frac{1}{24\pi^2 M_{\rm P}^4}\frac{V}{\varepsilon}$

evaluated at CMB scale $\phi_* = \phi(N_*)$

$$N_* = \frac{1}{M_{\rm P}^2}\int_{\phi_E}^{\phi_*} \frac{V(\phi)}{V'(\phi)}\, d\phi \simeq 50 \text{ - } 60$$

Background dynamics

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{\rm inf}}{3M_{\rm P}^2}$$

$$\dot{H} = -\frac{1}{2M_{\rm P}^2}\left(G_{IJ}\dot{\varphi}^I\dot{\varphi}^J\right)$$

$$\rho_{\rm inf} = \frac{1}{2}G_{IJ}\dot{\varphi}^I\dot{\varphi}^J + V_0(\varphi^I)$$

$$n_s \simeq 1 + 2\eta - 6\varepsilon$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{G_{IJ}\dot{\varphi}^I\dot{\varphi}^J}{2M_{\rm P}^2H^2}$$

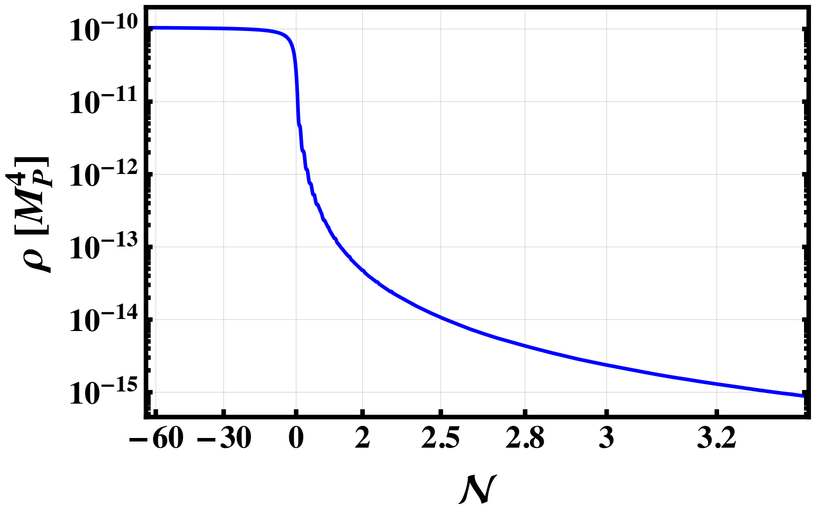
$$\eta = 2\varepsilon - \frac{\dot{\varepsilon}}{2\varepsilon H}$$

$$\xi_R = 2.35 \times 10^9$$

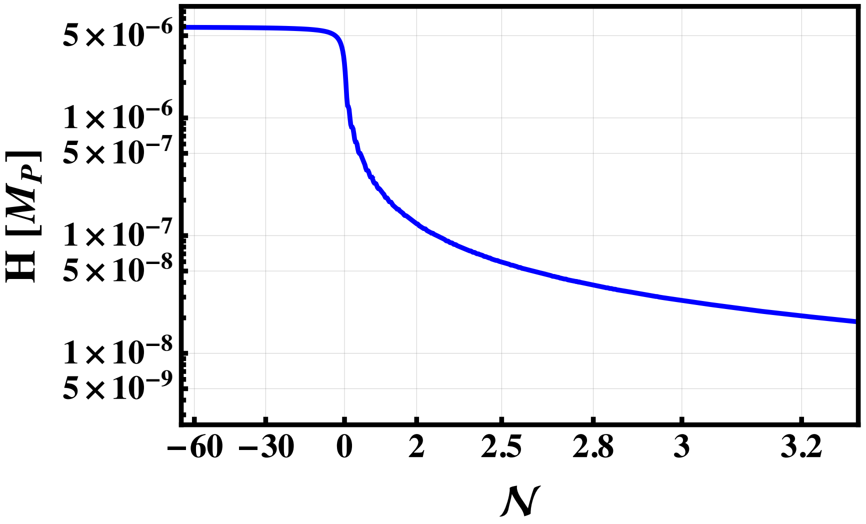
$$\xi_H = 10^{-3}$$

$$\mathcal{N} \equiv \ln\frac{a(t)}{a(t_{\rm end})}$$

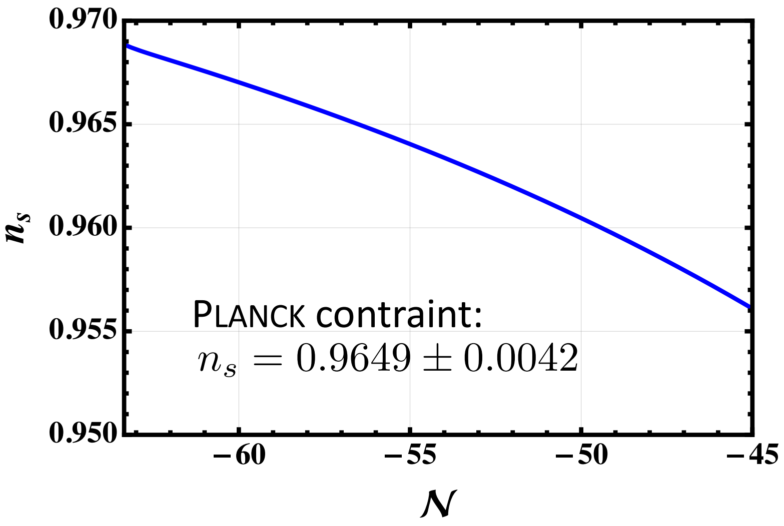
Inflaton energy density



Hubble parameter



Spectral index



Fluctuations

$$
\begin{aligned}
&\Box\phi^K + \Gamma^K_{IJ}\, g^{\alpha\nu} D_\alpha \phi^I D_\nu \phi^J - G^{KM} V_{E,M} \\
&+ G^{KM} e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_\mathrm{P}}} \left(\sqrt{\frac{2}{3}}\frac{1}{M_\mathrm{P}}\right) g^{\alpha\nu}(\partial_\alpha\phi)\left(\frac{g_Z}{2}\delta^3_M h Z_\nu + \frac{ie}{2\sqrt{2}s_W}\left[W^-_\nu(\delta^4_M + i\delta^5_M) - W^+_\nu(\delta^4_M - i\delta^5_M)\right] h\right) \\
&- G^{KM} e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_\mathrm{P}}} g^{\alpha\nu}\left(\frac{g_Z}{2}\; D_\alpha\left(\delta^3_M h Z_\nu\right) + \frac{ie}{2\sqrt{2}s_W} D_\alpha\left[\left(W^-_\nu(\delta^4_M + i\delta^5_M) - W^+_\nu(\delta^4_M - i\delta^5_M)\right] h\right)\right) \\
&- G^{KM}\left[e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_\mathrm{P}}} g^{\mu\nu}\delta^3_M\left(\frac{g_Z}{2}(D_\mu h)Z_\nu\right) + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_\mathrm{P}}} g^{\mu\nu}\delta^4_M\left(\frac{ie}{2\sqrt{2}s_W} D_\mu h\left(\,W^-_\nu - W^+_\nu\right)\right)\right. \\
&\left. + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_\mathrm{P}}} g^{\mu\nu}\delta^5_M\left(\frac{ie}{2\sqrt{2}s_W} D_\mu h\left(\,iW^-_\nu + iW^+_\nu\right)\right)\right] = 0
\end{aligned}
$$

$$\phi^I(x^\mu) = \varphi^I(t) + \delta\phi^I(x^\mu)$$

$$\phi^I \in \{\phi, h, \phi_2, \phi_3, \phi_4\}$$

GAUGE-INDEPENDENT
MUKHANOV-SASAKI VARIABLES

$$Q^\phi = \delta\varphi + \frac{\dot{\varphi}}{H}\psi$$

$$Q^h = \delta h + \frac{\dot{h}_0}{H}\psi$$

$$Q^{\phi_i} = \phi_i, \qquad (i = 2, 3, 4)$$

Higgs and inflaton fluctuations

In the conformal time metric $ds^2 = a^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu$

Rescaled variables $X^I(x^\mu) \equiv a(t)Q^I(x^\mu)$

Equations of motion for the Mukhanov-Sasaki variables: $\mathcal{D}_\tau^2 X^I - \left[\nabla^2 - a^2 m_{\mathrm{eff},I}^2(\tau)\right] X^I = 0$

$$m_{\mathrm{eff},I}^2 = G^{(I)J}(\mathcal{D}_{(I)}\mathcal{D}_J V_E) - \mathcal{R}^{(I)}{}_{JK(I)}\dot{\varphi}^J\dot{\varphi}^K - \frac{1}{M_\mathrm{P}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\varphi}^{(I)}\dot{\varphi}_{(I)}\right) - \frac{R_E}{6}$$

no sum on *(I)*

(potential + nontrivial field-space manifold + coupled metric perturbations + expansion of the background spacetime)

The quantized fluctuations $\hat{\tilde{\pi}}^I(\tau,\mathbf{k}) = \partial_\tau \hat{\tilde{X}}^I(\tau,\mathbf{k})$ with $\left[\hat{\tilde{X}}^I(\tau,\mathbf{k}), \hat{\tilde{\pi}}^J(\tau,\mathbf{q})\right] = i(2\pi)^3\delta^{IJ}\delta^{(3)}(\mathbf{k}+\mathbf{q})$

can be decomposed in momentum space as $\hat{\tilde{X}}^I(\tau,\mathbf{k}) = \sum_m \left[u_m^I(\tau,k)\hat{a}_m(\mathbf{k}) + u_m^{I*}(\tau,k)\hat{a}_m^\dagger(-\mathbf{k})\right]$

The mode functions can be parametrized as $u_m^I(\tau,k) = t_{(m,I)}(\tau,k)e_m^I(\tau)$ ← vielbeins of the field-space metric

complex scalar functions

Higgs and inflaton fluctuations

We are left with

$$v''_{1k} + \omega^2_{(\phi)}\, v_{1k} \simeq 0$$
$$y''_{2k} + \omega^2_{(h)}\, y_{2k} \simeq 0$$

where

$$t_{(1,\phi)}(\tau, k) = v_{1k}(\tau)$$
$$t_{(2,h)}(\tau, k) = y_{2k}(\tau)$$

$$\omega^2_{(I)}(\tau, k) = \left(k^2 + a^2 m^2_{\mathrm{eff},(I)}(\tau)\right)$$

From the energy momentum tensor for the fields fluctuation (for the linearized theory)

$$T^{(\phi h)}_{\mu\nu} = G_{IJ}(\mathcal{D}_\mu X^I)(\mathcal{D}_\nu X^J) + \eta_{\mu\nu}\left[-\frac{1}{2}\eta^{\alpha\beta}G_{IJ}(\mathcal{D}_\alpha X^I)(\mathcal{D}_\beta X^J) - \frac{a^2}{2}\left(\mathcal{M}_{IJ} - \frac{1}{6}G_{IJ}R_E\right)X^I X^J\right]$$

the energy densities for the inflaton and Higgs fluctuations per mode read

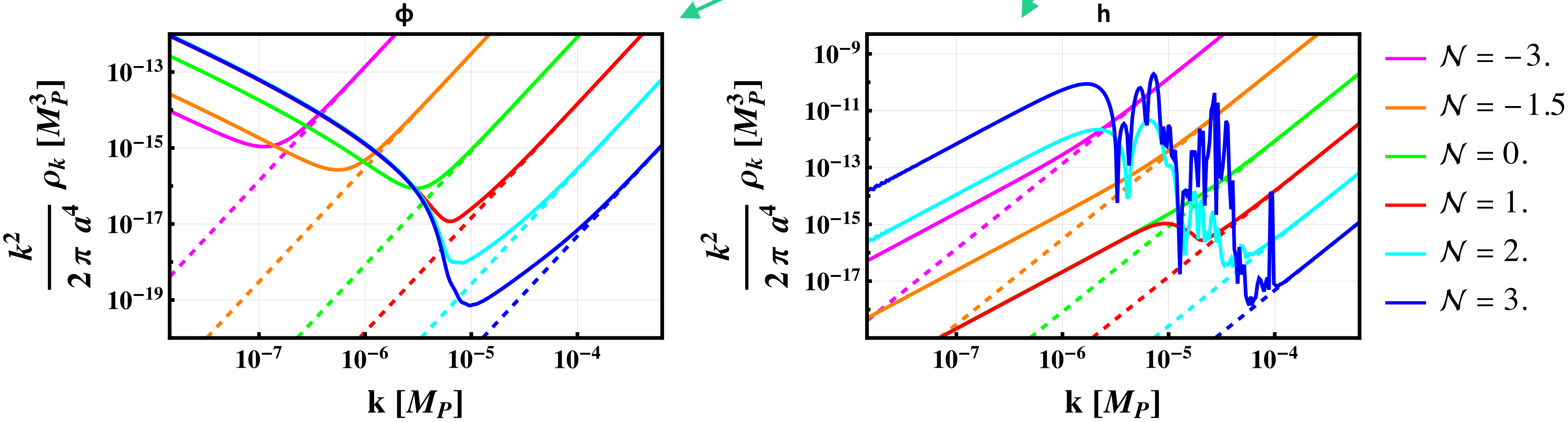
$$\rho_{(\phi)} = \frac{1}{a^2}\int \frac{k^2}{4\pi^2}dk\left[|\dot{v}_{1k}|^2 + \left(\frac{k^2}{a^2} + \left|m^2_{\mathrm{eff},(\phi)}(t)\right|\right)|v_{1k}|^2\right]$$
$$\rho_{(h)} = \frac{1}{a^2}\int \frac{k^2}{4\pi^2}dk\left[|\dot{y}_{2k}|^2 + \left(\frac{k^2}{a^2} + \left|m^2_{\mathrm{eff},(h)}(t)\right|\right)|y_{2k}|^2\right]$$

Higgs and inflaton fluctuations

Vacuum subtraction $\rho^{q}_{(I)} = \rho_{(I)} - \rho^{\mathrm{BD}}_{(I)}$

$$\rho^{\mathrm{BD}}_{(I)} = \int dk \, \boxed{\frac{k^2}{2\pi^2 a^4} \rho^{\mathrm{BD}}_{k,(I)}} = \frac{1}{a^4} \int dk \, \frac{k^3}{4\pi^2}$$

Use Bunch-Davies solutions for the fields $v^{\mathrm{BD}}_{1k} = \frac{1}{\sqrt{2k}} e^{-ik\tau}$ $\quad y^{\mathrm{BD}}_{2k} = \frac{1}{\sqrt{2k}} e^{-ik\tau}$



Goldstones & gauge fields fluctuations

We can redo the same computation starting from the Goldstone Mukhanov-Sasaki fluctuations & gauge fields EoMs:

$$\Upsilon(\tau) = \frac{\varphi'}{\sqrt{6}M_{\rm P}} - \frac{a'}{a} - \frac{h_0'}{h_0}$$

$$\mathcal{D}_\tau^2 X^{\phi_2} - \left[\nabla^2 - a^2 m^2_{\text{eff},(\phi_2)}(\tau)\right] X^{\phi_2} + a\, g_Z h_0 \left(\Upsilon Z_0 - Z_0'\right) = 0$$

$$\mathcal{D}_\tau^2 X^{\phi_3} - \left[\nabla^2 - a^2 m_{\text{eff},(\phi_3)}(\tau)\right] X^{\phi_3} + a\, \frac{ie}{\sqrt{2}s_W}\, h_0 \left[\Upsilon\left(W_0^- - W_0^+\right) - \left(W_0^{-\prime} - W_0^{+\prime}\right)\right] = 0$$

$$\mathcal{D}_\tau^2 X^{\phi_4} - \left[\nabla^2 - a^2 m_{\text{eff},(\phi_4)}(\tau)\right] X^{\phi_4} + a\, \frac{ie}{\sqrt{2}s_W}\, h_0 \left[\Upsilon\left(iW_0^- + iW_0^+\right) - \left(iW_0^{-\prime} + iW_0^{+\prime}\right)\right] = 0$$

$$g^{\mu\alpha} g^{\nu\beta} D_\alpha F_{Z\mu\nu} + \frac{8M_{\rm P}^2}{\xi_R \Lambda^2} \partial_\alpha \left(F(\phi^I) e^{\sqrt{\frac{2}{3}}\,\phi/M_{\rm P}}\right) \widetilde{F}_Z^{\alpha\beta} - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}} g^{\mu\beta} \left(\frac{g_Z^2}{4} h^2 Z_\mu + \frac{g_Z}{2}\left(\phi_2 D_\mu h - h D_\mu \phi_2\right)\right) = 0,$$

$$g^{\mu\alpha} g^{\nu\beta} D_\alpha F^{\pm}_{W\mu\nu} + \frac{8M_{\rm P}^2}{\xi_R \Lambda^2} \partial_\alpha \left(F(\phi^I) e^{\sqrt{\frac{2}{3}}\,\phi/M_{\rm P}}\right) \widetilde{F}_W^{\pm\alpha\beta} - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}} g^{\mu\beta} \left(\frac{e^2}{4s_W^2} h^2 W_\mu^{\pm} \pm \frac{ie}{2\sqrt{2}s_W} D_\mu h \left(\phi_3 \pm i\phi_4\right) \mp \frac{ie}{2\sqrt{2}s_W} D_\mu(\phi_3 \pm i\phi_4) h\right) = 0,$$

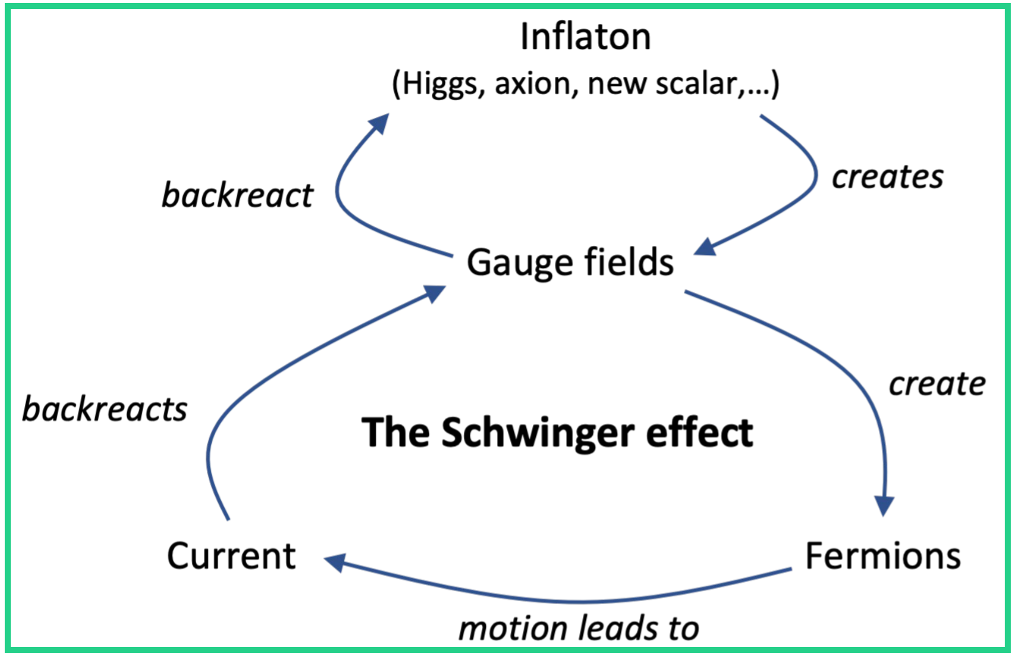
$$g^{\mu\alpha} g^{\nu\beta} D_\alpha F_{A\mu\nu} + \frac{8M_{\rm P}^2}{\xi_R \Lambda^2} \partial_\alpha \left(F(\phi^I) e^{\sqrt{\frac{2}{3}}\,\phi/M_{\rm P}}\right) \widetilde{F}_A^{\alpha\beta} = 0$$

THE SCHWINGER EFFECT

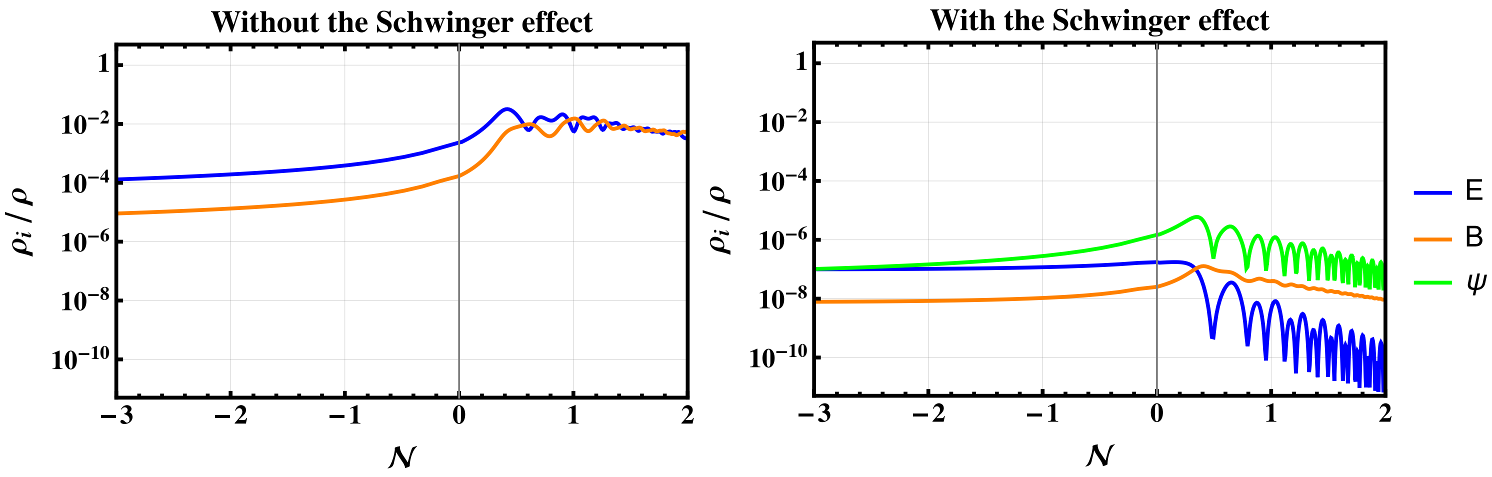
Cohen-McGady (2008)
Domcke-Mukaida (2018)
Kitamoto-Yamada (2022)

The comoving conductivity in the case of one Dirac fermion *f* with mass m_f and charge Q_f under a U(1) group with coupling *g*:

$$\sigma_f^{\mathrm{c}} = \frac{|g\,Q_f|^3}{6\pi^2}\,\frac{a}{H}\,\sqrt{2\rho_B}\,\coth\left(\pi\sqrt{\frac{\rho_B}{\rho_E}}\right)\exp\left(-\frac{\pi m_f^2}{\sqrt{2\rho_E}\,|g\,Q_f|}-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right),\quad \sigma_{\mathrm{c}} = \sum_f \sigma_f^{\mathrm{c}}$$



Explosive gauge fields production leads to fermion/antifermion pairs creation and their currents backreact on the produced gauge fields.



$\xi_R = 2.35\times10^9,\ \xi_H = 10^{-3},\ \Lambda = 2\times10^{-5}M_{\mathrm{P}}$

$$\rho_\psi = \lim_{V\to\infty}\frac{\sigma_{\mathrm{c}}}{V}\int_V d^3x\,\frac{\langle \mathbf{A}\cdot\mathbf{E}\rangle}{a^4} = \frac{\sigma_{\mathrm{c}}}{a^4}\int_{k_{\min}}^{k_{\mathrm{c}}} dk\,\frac{k^2}{2\pi^2}\frac{d}{d\tau}\left(|A_+|^2+|A_-|^2\right)$$

THE SCHWINGER EFFECT

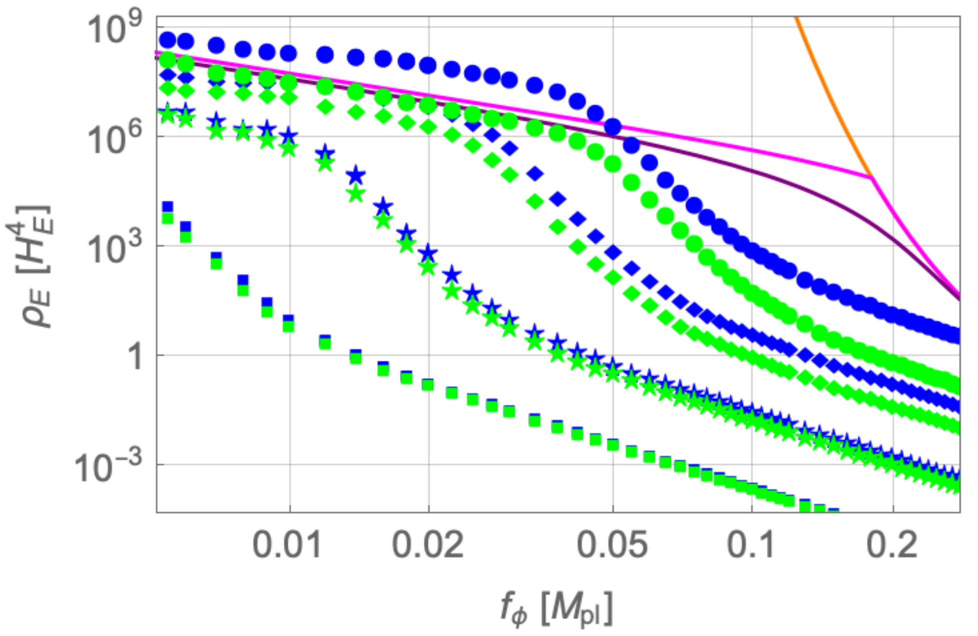
Numerical study for single field inflation

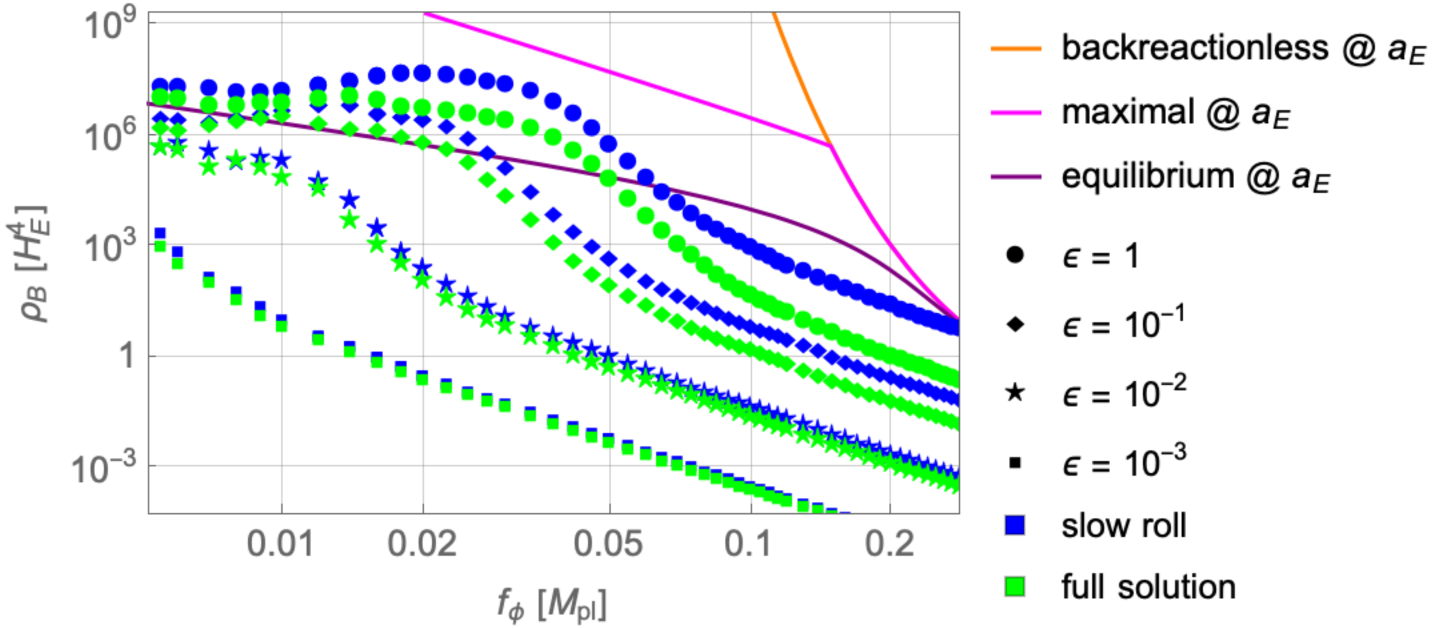
$$S \supset -\int d^4x \, \frac{\phi}{4f_\phi} \, Y_{\mu\nu}\tilde{Y}^{\mu\nu}$$

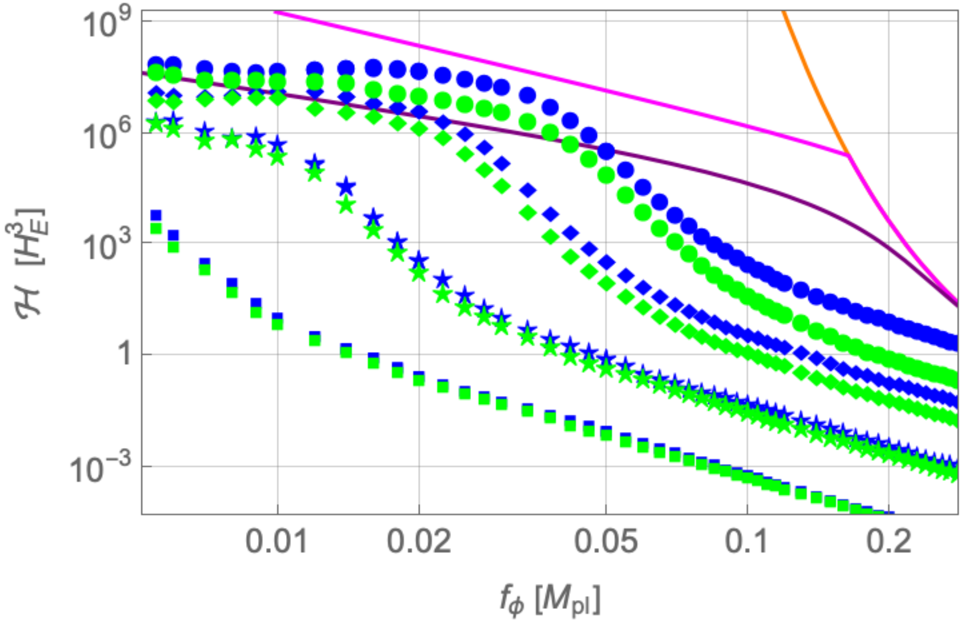
α-ATTRACTOR

HILLTOP

symmetric phase



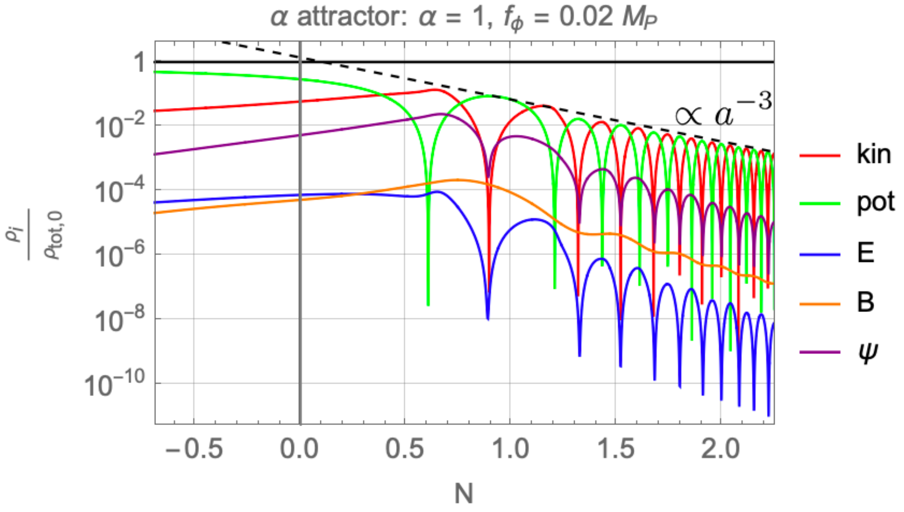


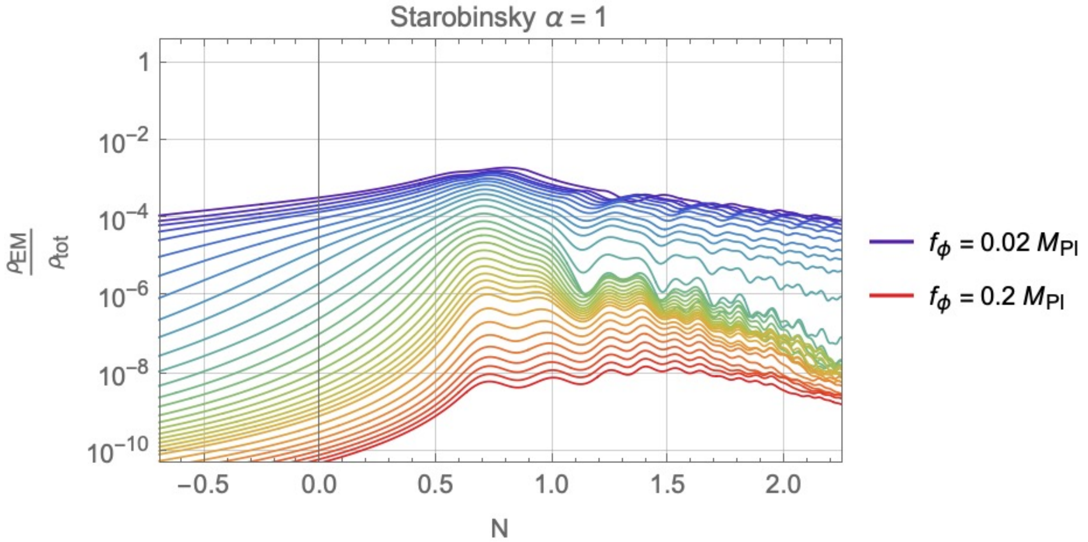


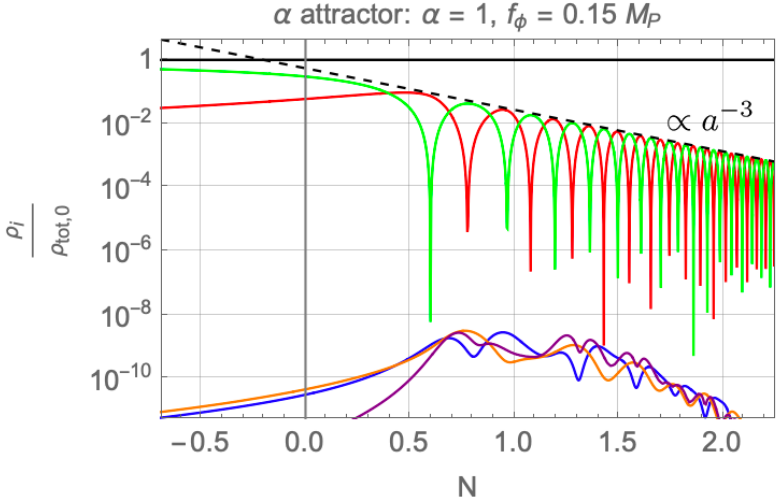
THE SCHWINGER EFFECT

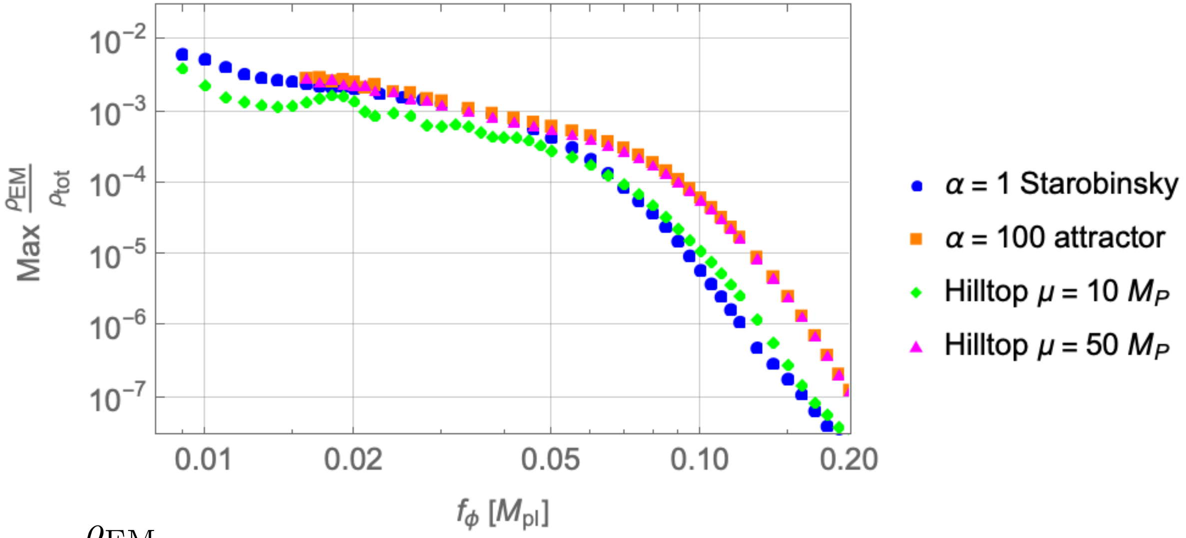
Side effect: no gauge preheating

$$S \supset -\int d^4x \, \frac{\phi}{4f_\phi} \, Y_{\mu\nu}\tilde{Y}^{\mu\nu}$$









Gauge preheating happens if $\frac{\rho_{\rm EM}}{\rho_{\rm tot}} \gtrsim 0.8$

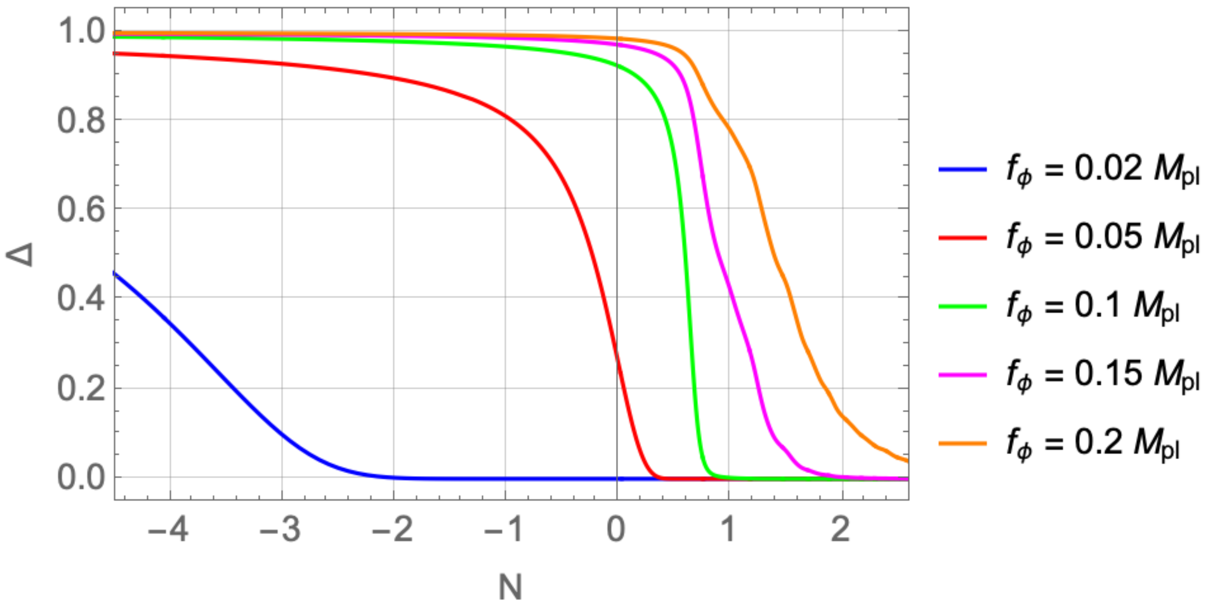
Cuissa-Figueroa (2019)

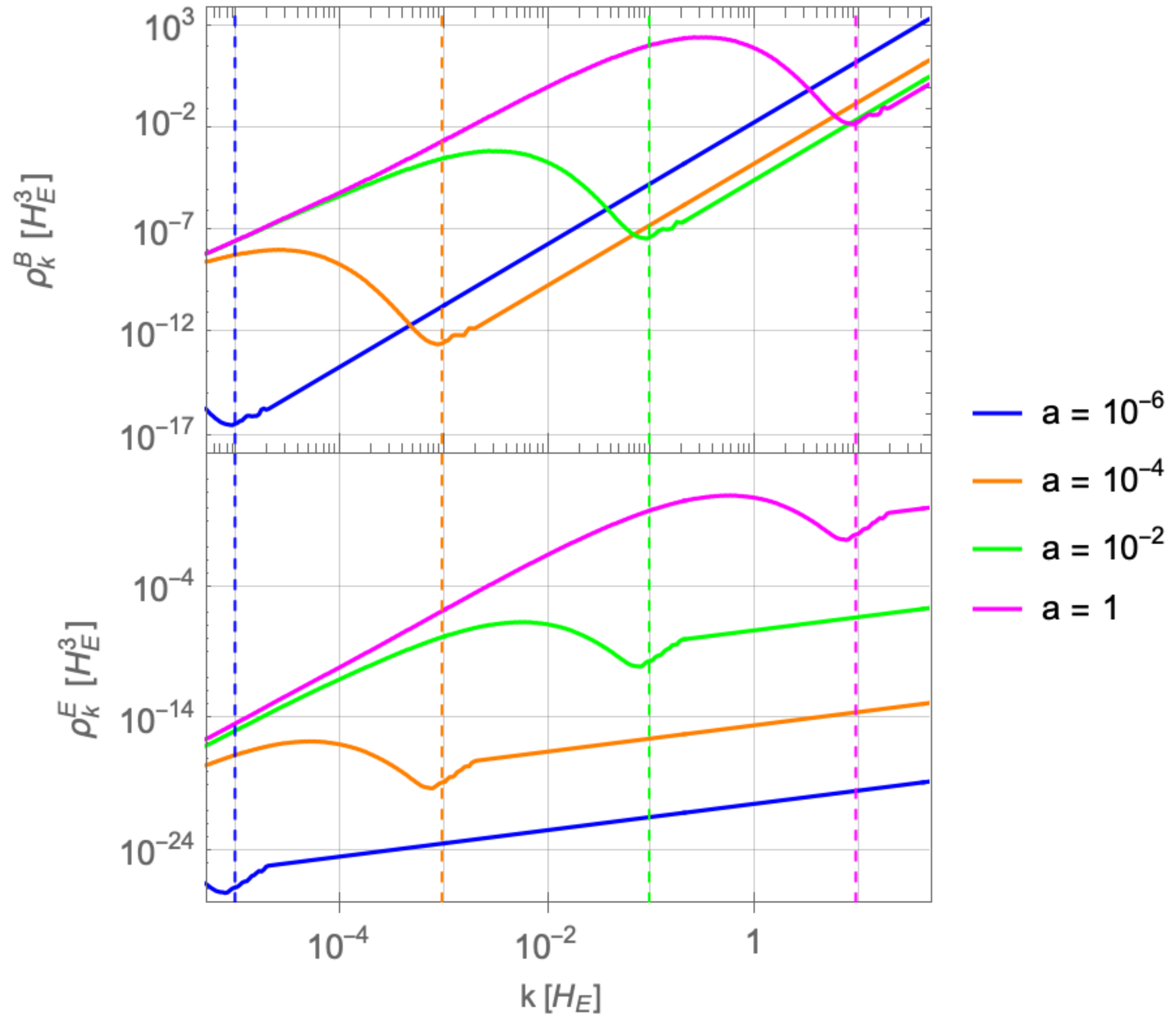
SCHWINGER EFFECT

Bunch-Davies vacuum

$$A_\lambda(\tau, k) = \sqrt{\frac{\Delta(t)}{2k}} e^{-ik\tau} \qquad (\tau \to -\infty)$$

$$\Delta(t) = \exp\left(-\int_{-\infty}^{t} \hat{\sigma}(t')\, dt'\right) \qquad \hat{\sigma} = \sigma/a$$





Boyarsky-Frohlich-Ruchayskiy (2012)
Akamatsu-Yamamoto (2013)
Kamada-Long (2016)
Domcke-Harling-Morgante-Mukaida (2019)

From helicity to baryon asymmetry

- At EWPT, there is a competition between weak sphaleron washout and B asymmetry from decaying helicity (SM chiral anomaly).

$$\eta_B = \frac{255}{592} \frac{g_Z^2}{\pi^3 \sqrt{10 g_*}} \frac{\mathcal{H}_Y}{(T_{\rm rh} a_{\rm rh})^3} \frac{f_{\theta_W}}{M_{\rm P}} \frac{T}{\gamma_{W \rm sph}} \bigg|_{T=135~\rm GeV} \simeq 9 \cdot 10^{-11}$$

$$f_{\theta_W} = -\sin(2\theta_W) \frac{{\rm d}\theta_W}{{\rm d}\ln T}\bigg|_{T=135~\rm GeV}$$

$$5.6 \cdot 10^{-4} \lesssim f_{\theta_W} \lesssim 0.32$$

- Helicity must nevertheless survive from inflaton to EW scale.

$$\mathcal{R}_m \approx 2\,\alpha_Y \frac{c_\sigma}{c_\nu} \frac{\rho^q_{B_Y}}{\rho_{\rm rh}} \left(\frac{\ell^q_{B_Y} T_{\rm rh}}{a_{\rm rh}}\right)^2 > 1$$

- Chiral plamsa instability (CPI) must be avoided.

$$T_{\rm CPI} \approx \frac{4\alpha_Y^5}{\pi^4 c_\sigma} \log(\alpha_Y^{-1}) \left(\frac{2133}{481}\right)^2 \frac{\mathcal{H}_Y^2}{H(t_{\rm end})\, T_{\rm rh}^4 \sqrt{a_{\rm rh}^9\, a(t_{\rm end})}} \lesssim 10^5~\rm GeV$$