Clustering Effects on Primordial Black Hole Merger Rates

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Introduction

Primordial black holes (PBHs)

- Black holes (BHs) formed from overdense regions in the early universe.

Ya. B. Zel'dovich & I. D. Novikov(1966),
S. W. Hawking (1971), B. J. Carr & S. W. Hawking (1974)

- Formation condition: Their density contrast exceeds $\mathcal{O}(1)$.

$$\delta \equiv \frac{\rho - \rho_{\mathrm{BG}}}{\rho_{\mathrm{BG}}} > \mathcal{O}(1)$$

ρ: energy density
ρ_{BG}: background energy density

B. J. Carr (1975), T. Harada et. al.(2022)

- Motivation: GW source

The merger rate of binary BHs is inferred as

$$17.9 - 44 \ \mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1} \quad \text{at a fiducial redshift } z = 0.2$$

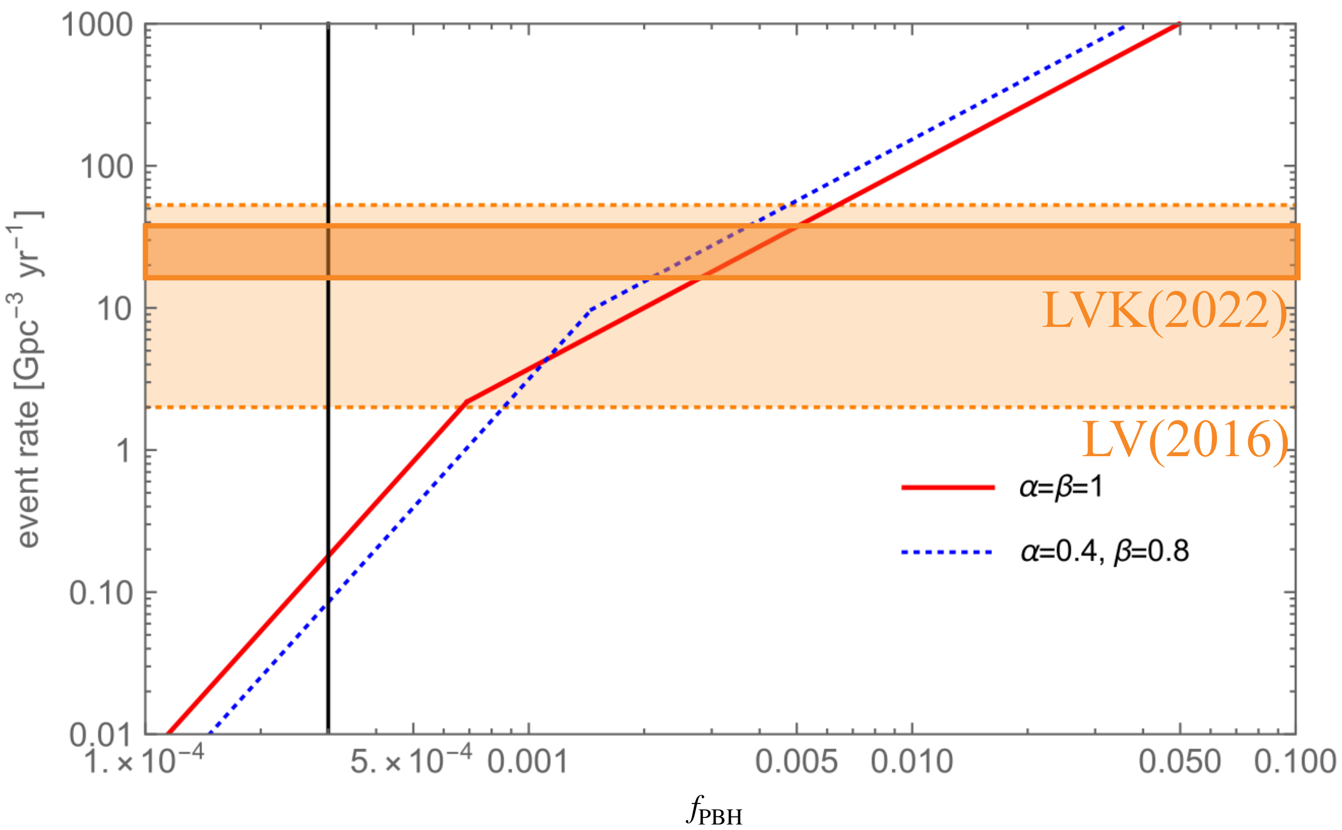
LVK collaborations (2022)

The origin of the source BHs is still under debate.

PBH merger rate of LVK region

M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama (2016)

- Assumption
- monochromatic mass distribution
- uniform spatial distribution
- external contribution
  = nearest external PBH only



- Binary formation rate of the latest LVK inference is achieved by

$$f_{\rm PBH} = (2-4)\times 10^{-3}$$

$f_{\rm PBH}$: PBH-to-DM ratio

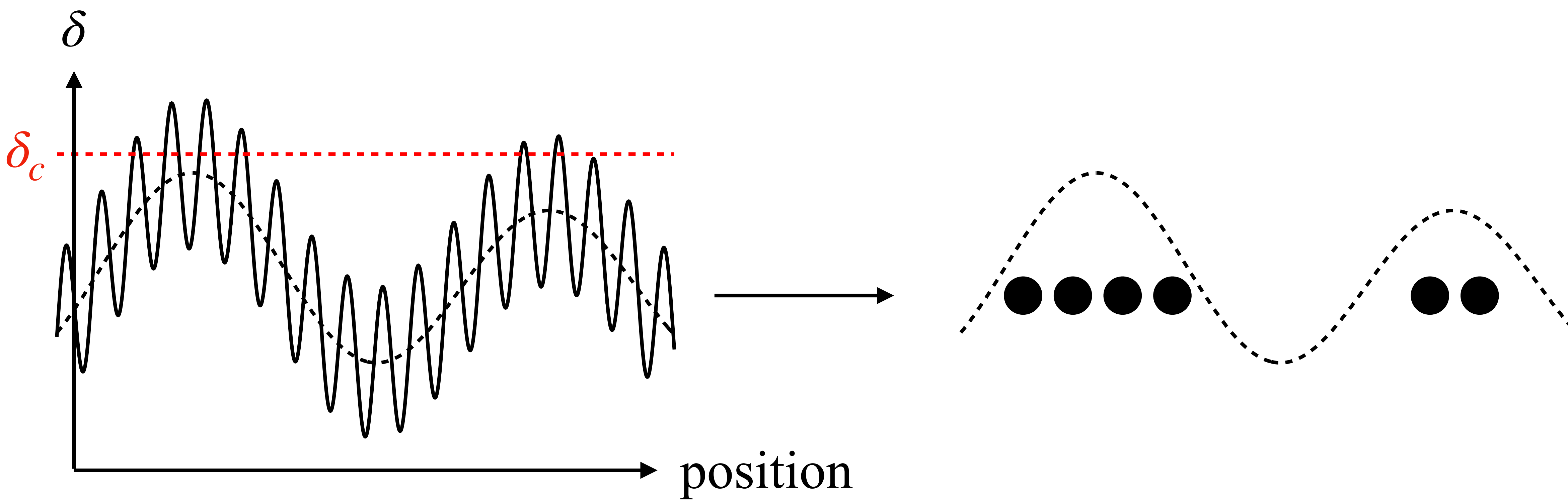
- CMB anisotropy puts constraints on accretion

$$f_{\rm PBH} \lesssim 10^{-2} \text{ (conservative)}$$

Y. Ali-Haïmoud &
M. Kamionkowski (2017)

PBH clustering

- Analysis beyond the uniform distribution is important.
- Inflation generates fluctuations in wide scale range $\rightarrow$ superposition



- Some concrete PBH models predict “strong clustering”

  (e.g. PBHs from axion bubbles, PBHs from Affleck-Dine mechanisms)

Merger rate for clustered PBHs

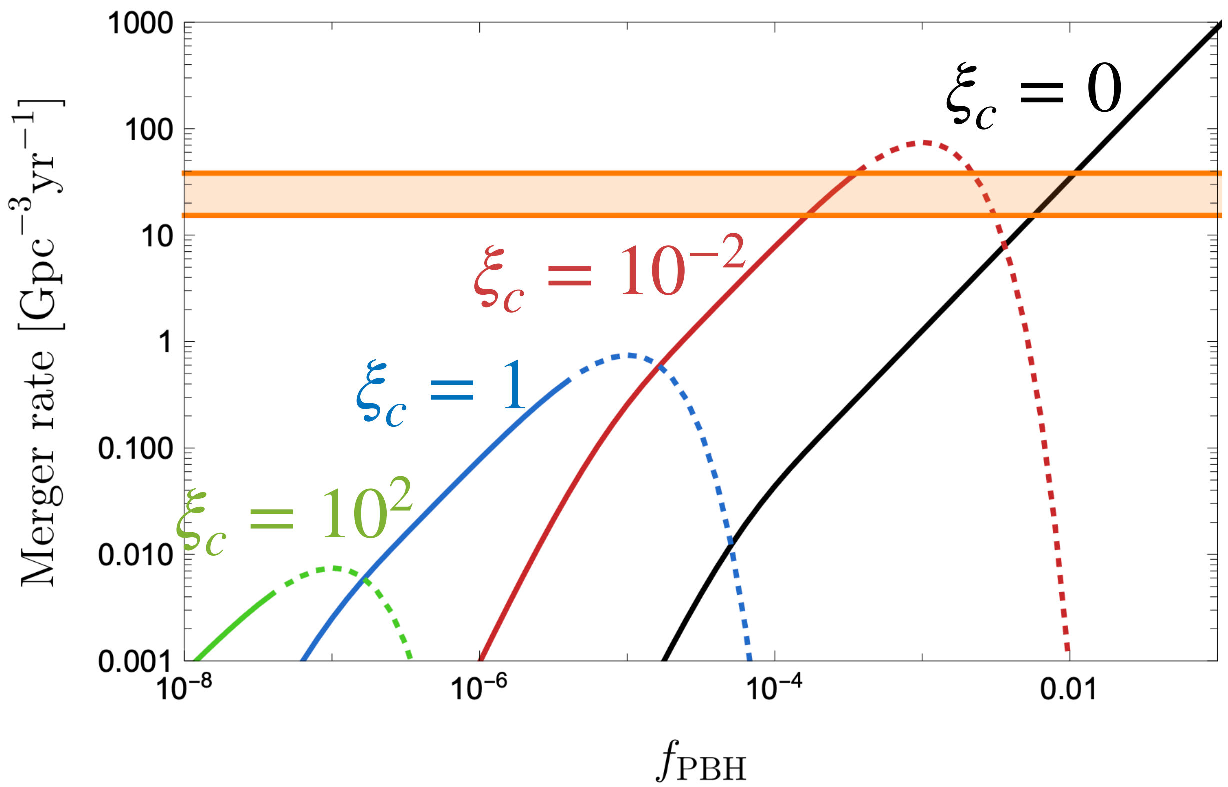
M. Kawasaki, K. Murai, and H. Nakatsuka (2021)

- CMB constraints on isocurvature perturbations from PBH clustering

- Merger rate calculation for power-law correlation functions of PBHs under the assumptions of

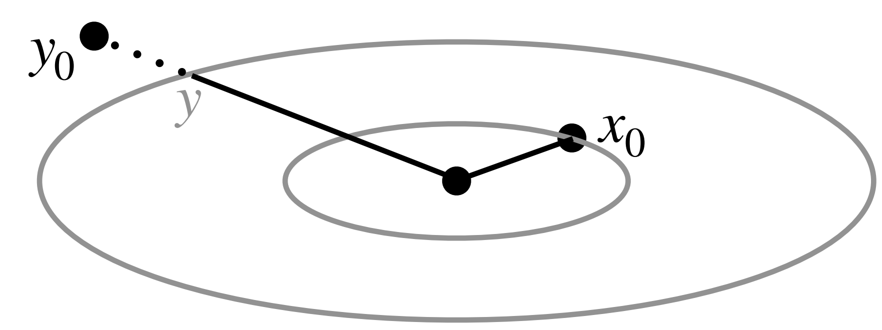
- a monochromatic mass distribution

- external contribution
  = nearest external PBH only



In fact, the eccentricity which corresponds one-to-one with the angular momentum is given by

$$e = \sqrt{1 - \left(\frac{x_0}{y_0}\right)^6}$$



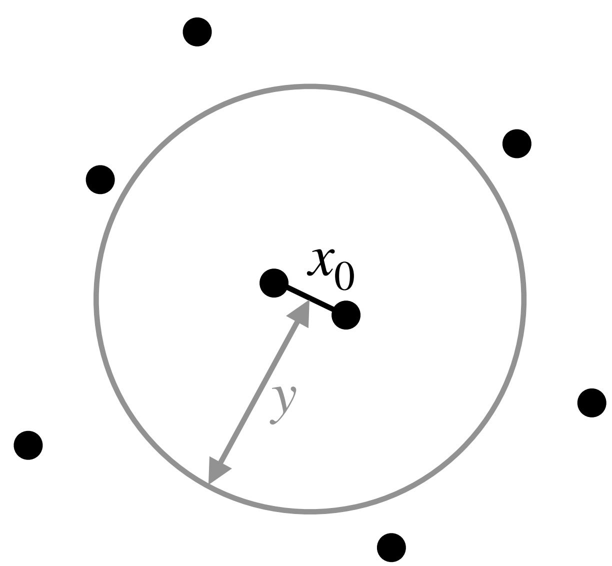
x_0, y_0: comoving distance

PBH merger rate WITHOUT clustering

Recent calculation technique

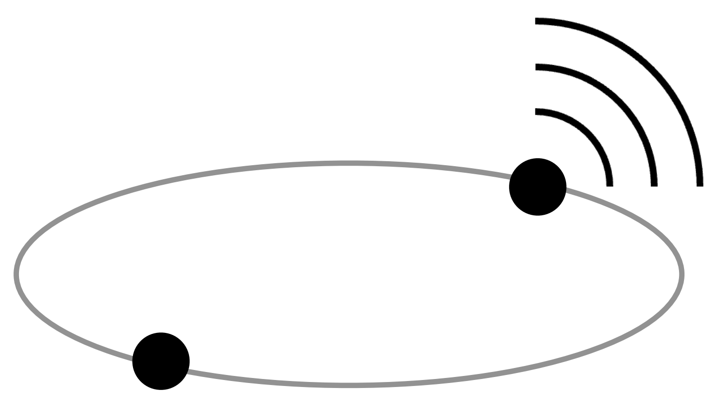
M. Raidal, V. Vaskonen, and H. Veermäe (2024)

- Assumption
  - monochromatic mass distribution
  - Poisson spatial distribution
- Focus : Early two-body channel of PBH merger



1. Binary formation

Excluding three-body system formation ($|\boldsymbol{x}| < y$)



2. Binary shrinking by GW emission

3. Merger

Binary formation

M. Raidal, V. Vaskonen, and H. Veermäe (2024)

- Ignoring external forces, equation of motion in the relative coordinate is

$$\frac{\boldsymbol{F}}{\mu} = \boxed{-\frac{GM}{r^3}\boldsymbol{r}} + \boxed{\frac{\ddot{a}}{a}\boldsymbol{r}}$$

Gravity $\propto a^{-2}$ Hubble flow $\propto a^{-3}$ (RD)

$\propto a^{-2}$ (MD)

G : Newton constant

$M = 2m$: binary mass

- Binary formation (Gravity>Hubble flow) occurs at

$$a_{\rm dc} = \frac{a_{\rm eq}}{\delta_{\rm pair}} \quad \text{during RD era}$$

$a_{\rm eq}$: matter-radiation equality

- Binary formation condition is

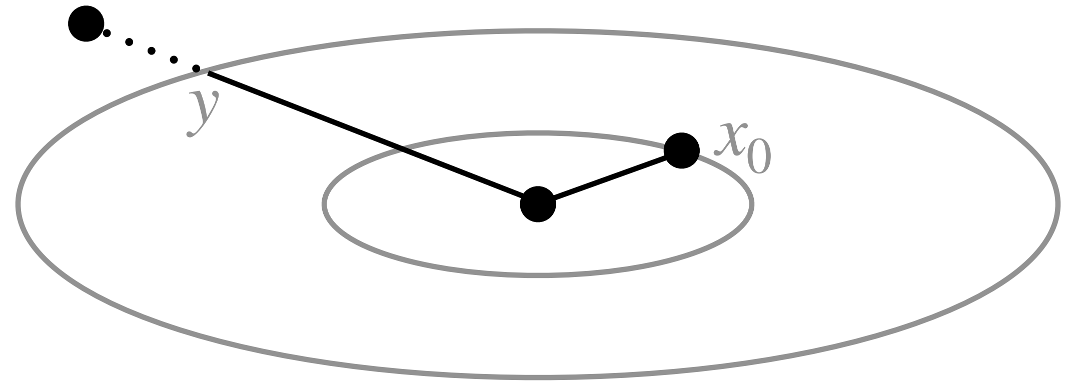
$$\delta_{\rm pair} := \frac{M/2}{\rho_{M,0} V(x_0)} > 1$$

$V(x_0)$: comoving volume of the initial separation x_0

$\rho_{M,0}$: BG density of total matter

Merger rate calculation

M. Raidal, V. Vaskonen, and H. Veermäe (2024)

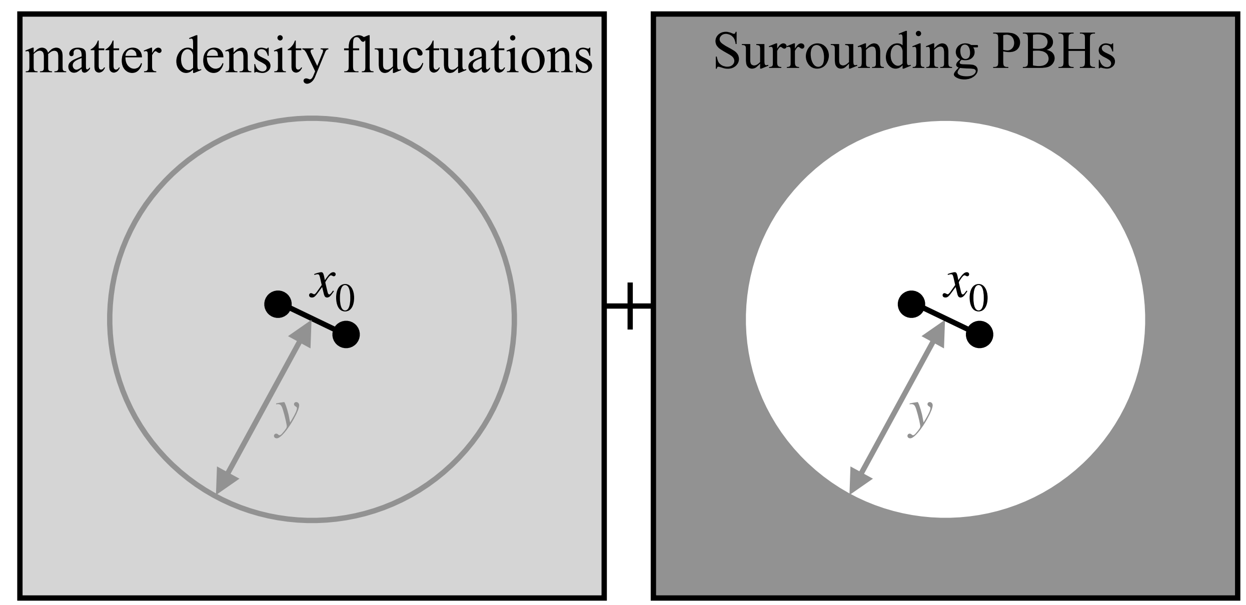


- Probability of the early two-body channel

$$\int d^3x_0\, \bar{n} e^{-V(y)}$$

- Probability distribution of angular momentum j for given initial positions

$$1 = \int dj \frac{dP(j\,|\,x_0, y)}{dj}$$



- Binary formation rate

$$R_b(t) = \int d^3x_0\, \bar{n} e^{-V(y)} \int dj\, \frac{dP(j\,|\,x_0, y)}{dj}\, \delta(t - t_{\rm dc})$$

- Merger rate

coalescence time

$$R_{\rm merger}(t) = \int_0^\infty d^3x_0\, \bar{n} e^{-V(y)} \int dj\, \frac{dP(j\,|\,x_0, y)}{dj}\, \delta(t - t_{\rm dc} - \boxed{\tau(x_0, j)})$$

Merger rate recalculation

M. Raidal, V. Vaskonen, and H. Veermäe (2024)

- Merger rate

$$R_{\text{merger}}(t) = \int_0^{\boxed{\infty}} d^3x_0\, \bar{n} e^{-V(y)} \int dj\, \frac{dP(j\,|\,x_0, y)}{dj}\, \delta(t - t_{\text{dc}} - \tau(x_0, j))$$

It is integrated over $x_0 \in (0,\ \infty)$ to obtain analytic formula in their paper.

- The binary formation during radiation-dominated era requires

$$\delta_{\text{pair}} := \frac{m}{\rho_{M,0} V(x_0)} > 1 \qquad \rightarrow x_{\max} : \text{upper limit of } x_0$$

Merger rate should be integrated over $x_0 \in (0,\ x_{\max})$.

- This upper limit will lowers the merger rate especially for small f_{PBH}.

Recalculation

Our work (in preparation)

- Blue line :

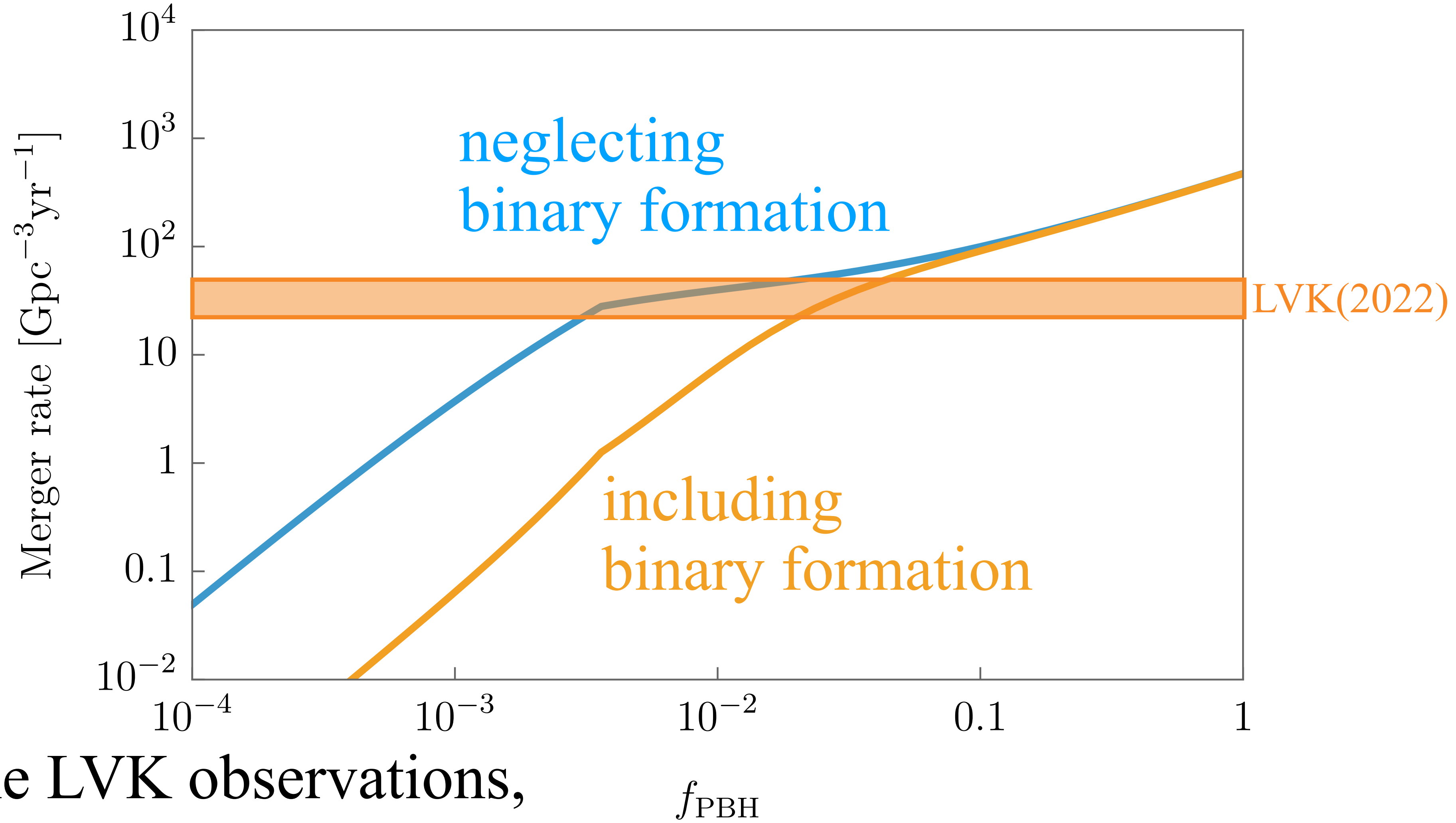
Result of

M. Raidal, V. Vaskonen, and H. Veermäe (2024)

integrating over $x_0 \in (0, \infty)$

- Orange line :

Our revised result with the binary formation condition, integrating over $x_0 \in (0, x_{\max})$



To match the LVK observations,

$f_{\rm PBH} \simeq (2-4)\times 10^{-2}$: comparable to the constraints $f_{\rm PBH} \lesssim 10^{-2}$

PBH merger rate WITH clustering

Correlation function

Our work (in preparation)

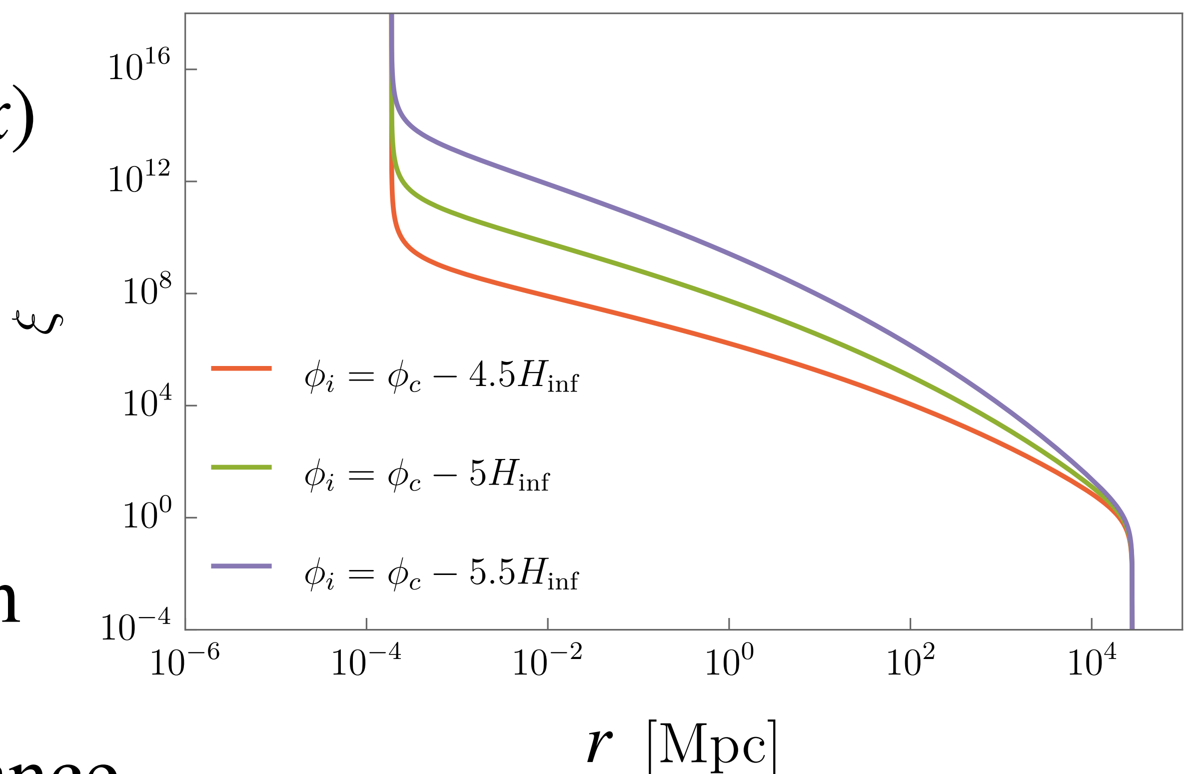
- power spectrum of PBH density perturbations

$$P_{\rm PBH} = P_{\rm Poisson} + \int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \xi(x)$$

- PBH correlation functions

$$\xi(r) = \xi_c \left(\frac{r}{1\ {\rm Mpc}}\right)^{-\alpha}$$

r : distance from the PBH at the origin



Example :
Axion bubble PBH (K. Kasai, et. al. (2023))

- Spatial dependence of PBH abundance

$$n_{\rm PBH}(r) \simeq \bar{n}(1 + \xi(r))$$

- Modifications in the merger rate calculation :
  1. Realization probability of early two-body channel
  2. Angular momentum contributions from surrounding PBHs

Improved binary formation condition

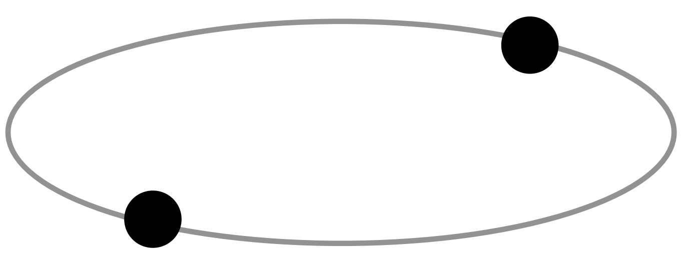
Our work (in preparation)

- As a GR-motivated generalization, we consider EoM for mass excess

$$\ddot{r} - \frac{\ddot{a}}{a}r + \frac{GM}{r^2} = 0 \quad \rightarrow \quad \ddot{r} - \frac{\ddot{a}}{a}r + \frac{G}{r^2}\left(M - 2 \times \frac{4\pi}{3}\rho_{\rm PBH} r^3\right) = 0$$

- This equation can be solved numerically if the initial density contrast is given

$$\delta_{\rm 2B} := \frac{M/2 - \rho_{\rm PBH} V(x_0)}{\rho_{M,0} V(x_0)}$$



- New binary formation condition : binary formation $\dot{r} = 0$ occurs before the matter-radiation equality.

  The numerical result shows that the required density contrast is

$\delta_{\rm 2B} > 0.14$: earlier binary formation than previously estimated $a_{\rm 2B} < a_{\rm dc}$

Results

Our work (in preparation)

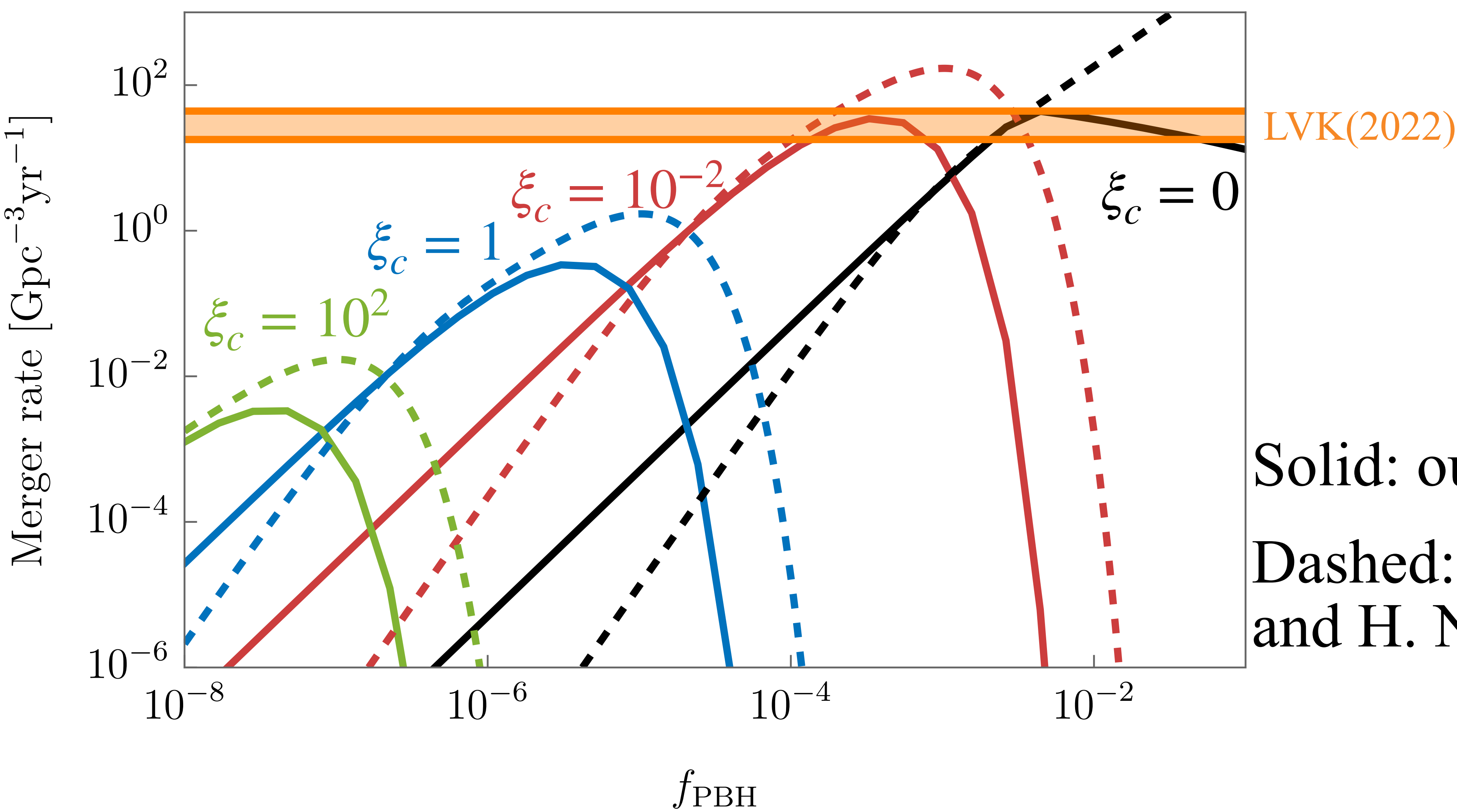
- PBH correlation

$$\xi(r) = \xi_c \left(\frac{r}{1\ \mathrm{Mpc}} \right)^{-\alpha}$$

Here we set $\alpha = 1.5$ as a typical value.

- New binary formation condition

$$\delta_{2\mathrm{B}} > 0.14$$



Solid: our result

Dashed: M. Kawasaki, K. Murai, and H. Nakatsuka (2021)

Take-home message

- PBHs formed from inflationary fluctuations can exhibit clustering. Some concrete models predict strong clustering, which significantly alters merger rate estimates.

- The predicted merger rate is sensitive to physical assumptions, such as binary formation.

- The exponential suppression at large PBH abundance comes from the rarity of early two-body configurations. This suggests that three-body or many-body channels may dominate in that regime.

  $\longrightarrow$ future work

Thank you for your attention!