“Hairy Black Holes by Spontaneous Symmetry Breaking in ESGB theory"

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Content

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where $\mathcal{G}$ is the GB term

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- ESGB theory evades **the no-hair theorems** which indicates the existence of hairy black holes.

Evasion of (old/novel) No-Hair theorem

Rectifying the no-hair theorem in EsGB [1] [2] [3]

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  No restriction to evade the no-hair theorem for any coupling function of $f(\varphi)$

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We suggest the formation of hairy black holes by spontaneous symmetry breaking [4]

[4] Boris Latosh, Miok Park, Phys.Rev.D 110 (2024) 2, 024012, “Hairy black holes by spontaneous symmetry breaking"

Our Lagrangian

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G}\right], \tag{1}$$

$$\mathcal{L}_\varphi = -\nabla_\alpha\varphi^*\nabla^\alpha\varphi + f(\varphi)\mathcal{G} = T - V, \qquad V = -f(\varphi)\mathcal{G} \tag{2}$$

where $\mathcal{G}$ is the Gauss-Bonnet term and the scalar field coupling function is

$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

Our metric ansatz is

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right). \tag{3}$$

- This Lagrangian respects the global $U(1)$ symmetry

$$\varphi(r) \to e^{i\chi}\varphi(r)$$

- This action allows to have Schwarzschild black holes ($\varphi = const$).

Schwarzschild BH are always stable in ESGB?

Scalar field perturbation

$$\delta\varphi_1(t,r,\theta,\phi) = \sum_{l,m} \frac{\Phi(r)Y_{lm}(\theta,\phi)}{r}\, e^{-i\omega t} \tag{4}$$

$$\Phi''(r_*) - (V_{\rm eff} - \omega^2)\Phi(r_*) = 0, \qquad \mathrm{d}r_* = \frac{1}{\sqrt{AB}}\mathrm{d}r, \tag{5}$$

$$V_{\rm eff}(r) = \frac{l(l+1)A}{r^2} + \frac{1}{2\,r}\left(A'B + AB'\right) - \frac{1}{2} f_{\varphi_1\varphi_1} A\,\mathcal{G}, \tag{6}$$

where l is the angular momentum.

$$\int_{r_h}^{\infty} dr\, \frac{1}{\sqrt{AB}}\, V_{\rm eff}(r) < 0 \qquad \rightarrow \qquad \alpha > \alpha_{\rm Sch.} = \frac{5}{24} \approx 0.2083, \tag{7}$$

$$A(r) = B(r) = 1 - \frac{2M}{r}, \qquad l = 0 \tag{8}$$

Schwarzschild black holes in EsGB

become unstable beyond the critical value of α ($= \alpha_{\rm Sch.}$)

Schwarzschild BH are always stable in ESGB? No

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Hairy Black Holes by SSB

$$V = -f(\varphi)\mathcal{G},$$
$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

- Black holes in the symmetric phase :

$$\langle\varphi\rangle = 0, \qquad \varphi(r) = \frac{1}{\sqrt{2}}\left(\varphi_1(r) + i\,\varphi_2(r)\right) \tag{9}$$

- Black holes in the symmetry-broken phase : the stable minima are determined by

$$\langle\varphi\rangle = v\,e^{i\beta}, \qquad v = \sqrt{\frac{\alpha}{2\,\lambda}} \tag{10}$$

we expand a field around the ground state v by reparameterizing it as follows

$$\varphi(r) = \left(v + \frac{\sigma(r)}{\sqrt{2}}\right)e^{i\theta(r)} \tag{11}$$

Flux near the black hole horizon [5]

The conserved current is given by

$$\partial_\alpha J^\alpha = 0, \qquad J_\alpha = i\, g\, (\varphi^* \partial_\alpha \varphi - \varphi \partial_\alpha \varphi^*).$$

- In the symmetric phase :

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y = \int_\Sigma \Big[g(\varphi_2 \partial_r \varphi_1 - \varphi_1 \partial_r \varphi_2) \Big] \Big[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \Big] = 0 \quad (12)$$

- In the symmetry-broken phase :

$$\theta'(r) = \frac{c_2}{4r^2\sqrt{A(r)B(r)}} \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^{-2}, \quad (13)$$

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y = \int_\Sigma \left[-\, 2\, g \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^2 \theta'(r) \right] \Big[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \Big]$$
$$= -\, 8\pi\, g\, c_2 \quad (14)$$

$c_2 = 0$ is required. The Goldstone boson is trivial.

[5] special thanks to Prof. Seong Chan Park

Hairy Black Holes by SSB in EsGB with global $U(1)$

We are interested in the situation that

“Scalar fields are about to grow from black holes without scalar hair. Finally it evolves to hairy black holes."

ex. Schwarzschild black hole $\longrightarrow$ *Hairy Black holes*

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- “$V = -f(\varphi)\mathcal{G}$" as an “interacting potential" :
  - effective near the black hole horizon
  - not effective at infinity ($V \to 0$ as $r \to \infty$, since $\mathcal{G} \to 0$ as $r \to \infty$)

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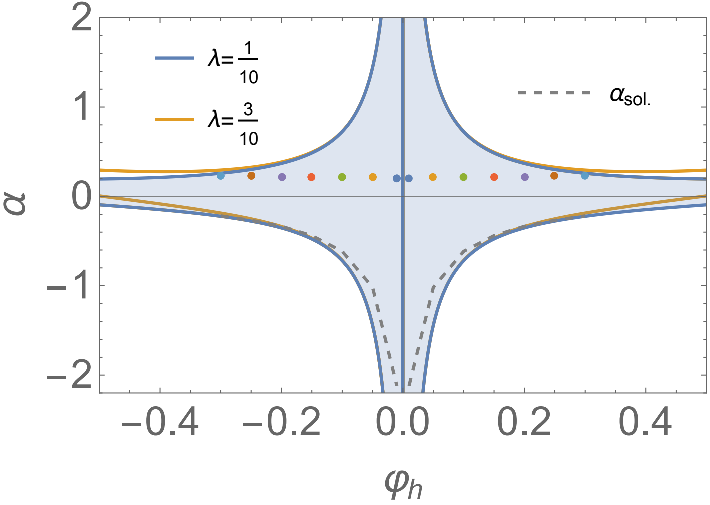
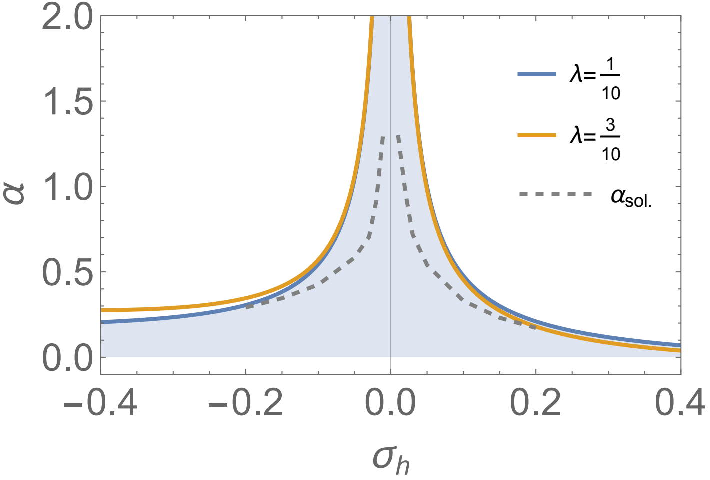
Based on these assumption,

we numerically generated the hairy black holes in symmetric and symmetry-broken phase

Hairy black holes in *symmetric phase*



Hairy black holes in *symmetry broken phase*



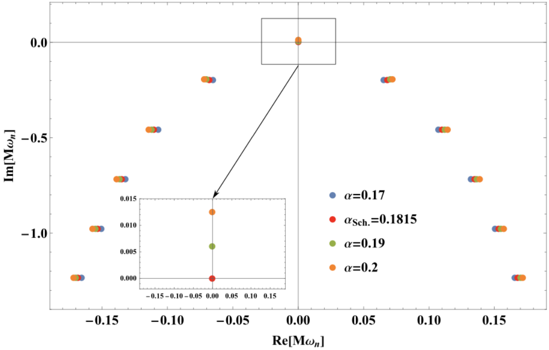
Hairy black holes + Scalar field perturbation

$\rightarrow$ **Quasinormal modes (QNM)** [6]

[6] Young-Hwan Hyun, Boris Latosh, Miok Park, JHEP 08 (2024) 163, “Scalar field perturbation of hairy black holes in EsGB theory"

G_{QNM} : Schwarzschild black holes

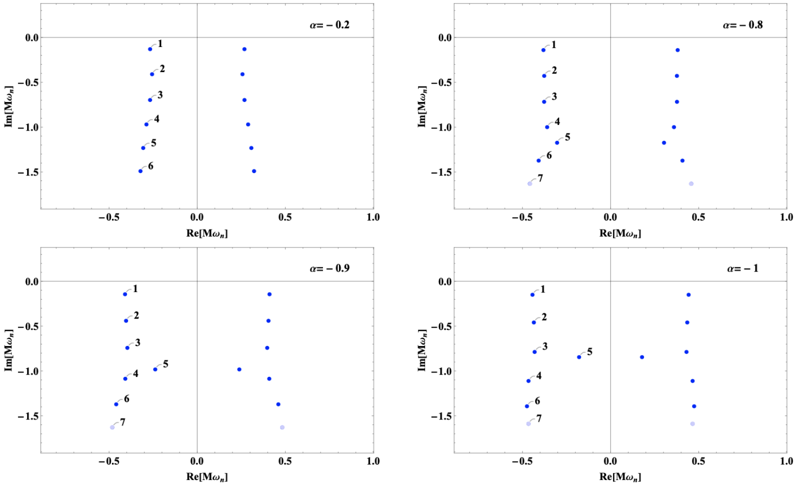
$$\delta\varphi(v,z,\theta,\phi) = \sum_{l,m} \Phi(z) Y_{lm}(\theta,\phi) e^{-i\omega v}, \quad \omega = \omega_{\rm R} + i\omega_{\rm I}$$



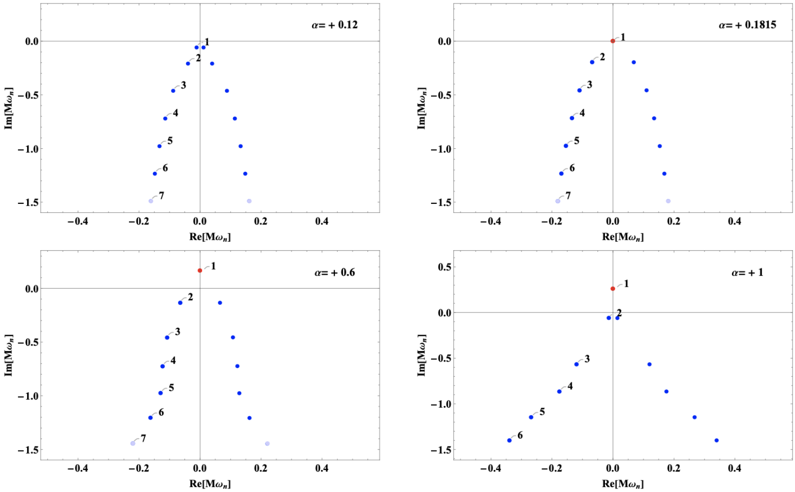
	Re$[M\omega_n]$	Im$[M\omega_n]$
α=0.17	$\pm$ 0.06528	-0.19795
	$\pm$ 0.10718	-0.45845
	$\pm$ 0.13196	-0.71851
	$\pm$ 0.15043	0.97682
$\alpha_{\rm Sch.} \approx 0.1815$	0	0.00002
	$\pm$ 0.06826	-0.19618
	$\pm$ 0.11012	-0.45776
	$\pm$ 0.13483	-0.71820
	$\pm$ 0.15324	-0.97669
α=0.19	0	0.00600
	$\pm$ 0.07013	-0.19489
	$\pm$ 0.11208	-0.45728
	$\pm$ 0.13677	-0.71798
	$\pm$ 0.15513	-0.97662
α=0.20	0	0.01254
	$\pm$ 0.07198	-0.19340
	$\pm$ 0.11415	-0.45675
	$\pm$ 0.13881	-0.71775
	$\pm$ 0.15712	-0.97654

Figure 4 & Table 1: QNMs of Schwarzschild black holes for serval values α

G_{QNM} : Symmetric Phase with $\alpha < 0$



G_{QNM} : Symmetric Phase with $\alpha > 0$



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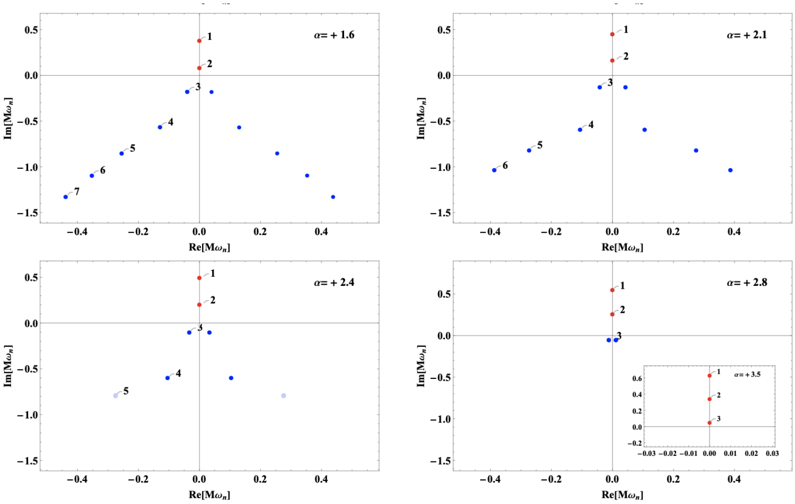
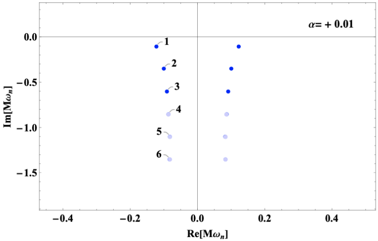
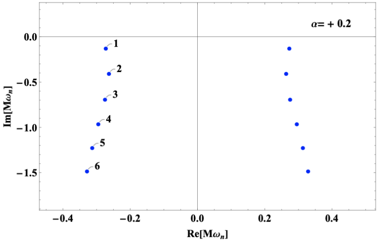


Figure 6: QNMs with positive values of α for $\varphi_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

G_{QNM} : Symmetry-Broken Phase

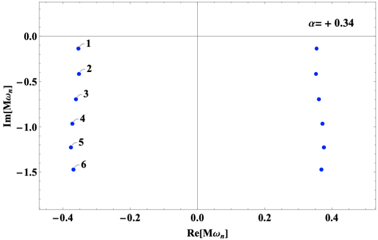


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.12229	-0.10701
2	± 0.10025	-0.35077
3	± 0.09104	-0.60312
4	± 0.08662	-0.85448
5	± 0.08226	-1.10205
6	± 0.08295	-1.35246

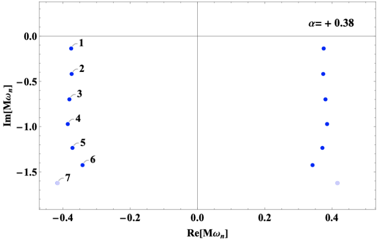


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.27261	-0.13084
2	± 0.26325	-0.40969
3	± 0.27500	-0.69474
4	± 0.29492	-0.96587
5	± 0.31297	-1.22847
6	± 0.32848	-1.4865

G_{QNM} : Symmetry-Broken Phase



	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.35393	-0.13610
2	± 0.35217	-0.41623
3	± 0.36116	-0.69570
4	± 0.37179	-0.96582
5	± 0.37610	-1.22716
6	± 0.36866	-1.47193

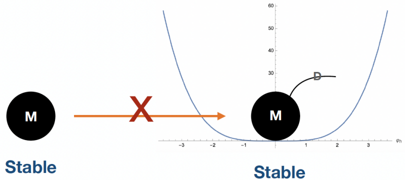


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.37538	-0.13731
2	± 0.37399	-0.41842
3	± 0.38069	-0.69878
4	± 0.38549	-0.97137
5	± 0.37156	-1.23482
6	± 0.34159	-1.42465
7	± 0.41624	-1.62298

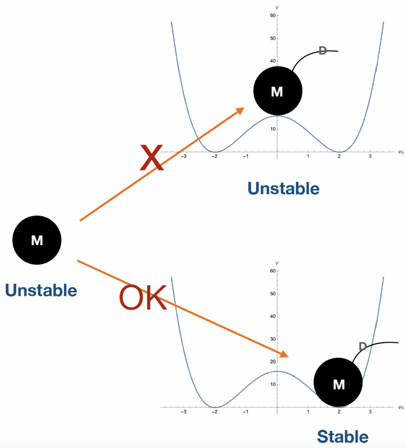
Figure 11 & Table 3: QNMs with various values of α for $\sigma_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

Summary

When $\alpha < 0$

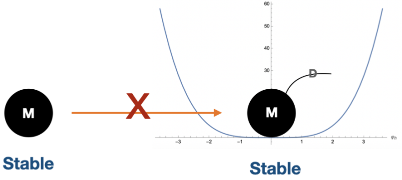


When $\alpha > 0$

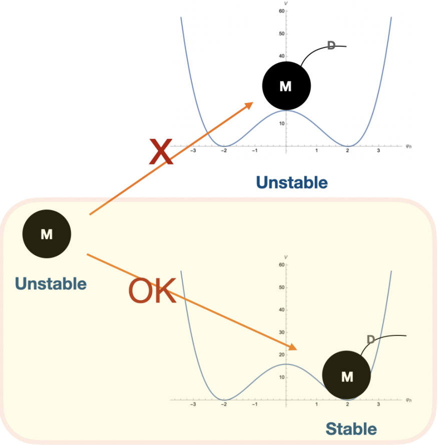


Summary

When $\alpha < 0$



When $\alpha > 0$



Our Claim :

Phase transition will occur and form black hole's hairs in symmetry-broken phase!

Future works

1. quasinormal modes with metric perturbations
2. dynamical evolution from non-hairy black holes to hairy black holes
3. hairy black holes by SSB with $U(1)$
4. thermodynamic stability
5. relation with Love number

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Thank you!