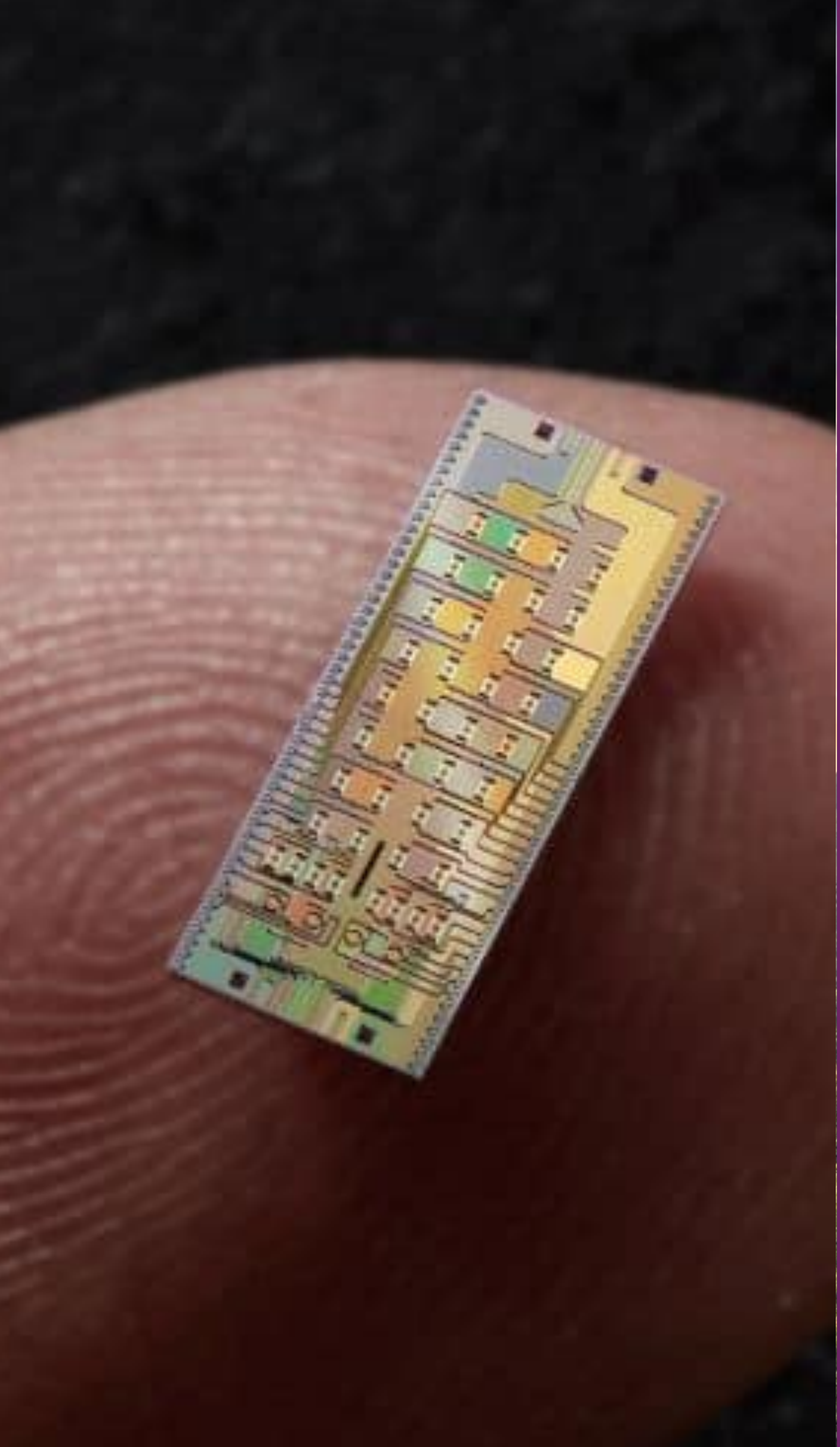
Qumode Tensor Networks for Quantum Field Theories

Simon Williams

PASCOS 2025

[1] S Abel, M Spannowsky and SW, Real-time scattering processes with continuous-variable quantum computers, Phys. Rev. A 112, 012614
[2] S Abel, M Spannowsky and SW, Qumode Tensor Networks for False Vacuum Decay in Quantum Field Theory, arXiv:2506.17388



Durham University

IP3

Qumode Tensor Networks for Quantum Field Theories

Simon Williams

PASCOS 2025

[1] S Abel, M Spannowsky and SW, Real-time scattering processes with continuous-variable quantum computers, Phys. Rev. A 112, 012614
[2] S Abel, M Spannowsky and SW, Qumode Tensor Networks for False Vacuum Decay in Quantum Field Theory, arXiv:2506.17388

Durham University

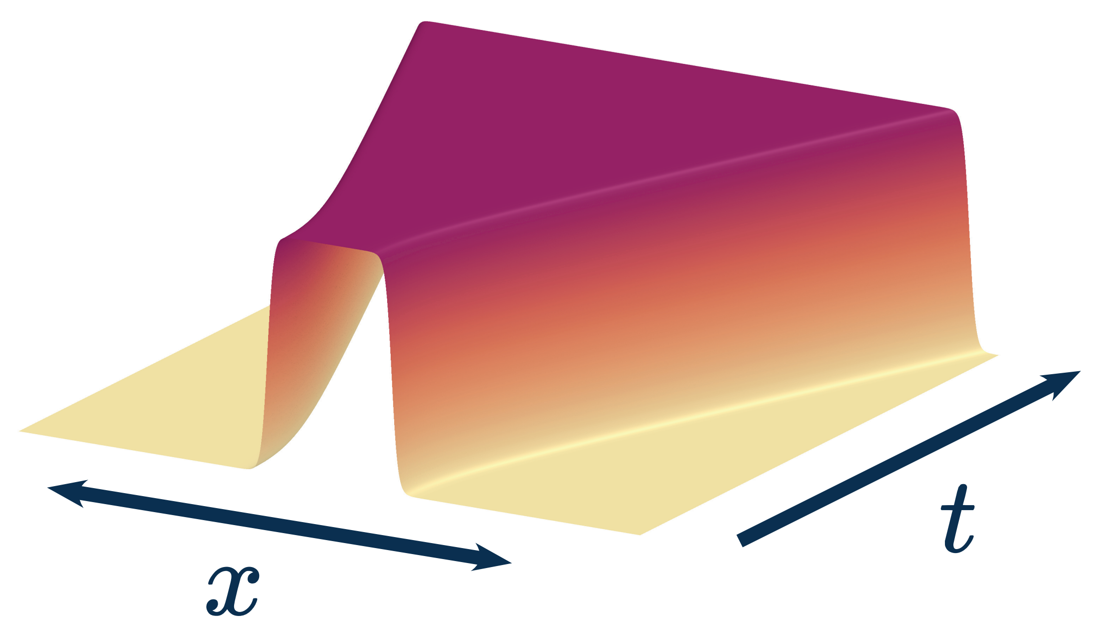
Qumode Tensor Networks for Quantum Field Theories

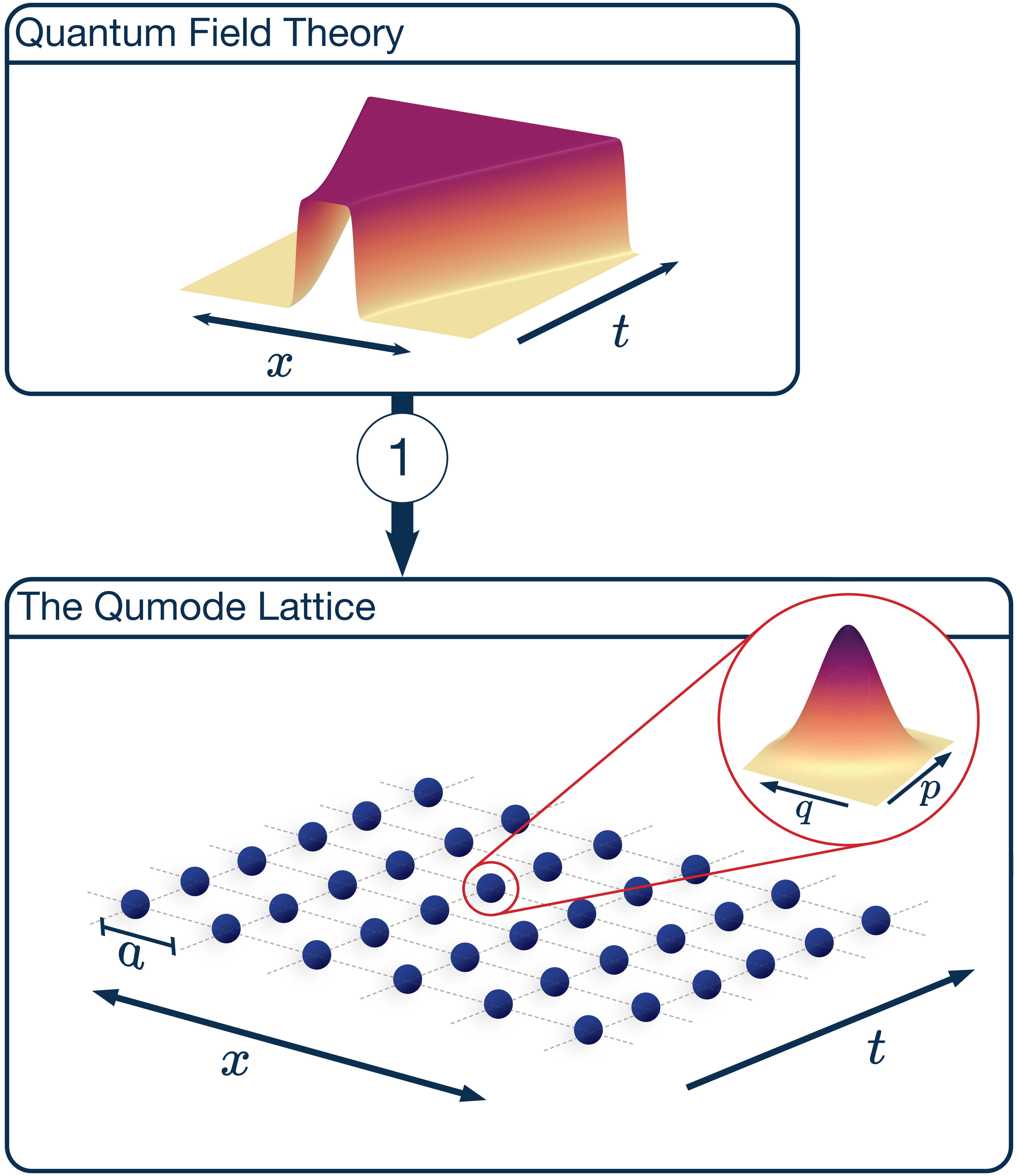
Simon Williams

PASCOS 2025

[1] S Abel, M Spannowsky and SW, Real-time scattering processes with continuous-variable quantum computers, Phys. Rev. A 112, 012614
[2] S Abel, M Spannowsky and SW, Qumode Tensor Networks for False Vacuum Decay in Quantum Field Theory, arXiv:2506.17388

Quantum Field Theory




Quantum Field Theory
x
t
1
The Qumode Lattice
q
p
a
x
t

The qumode lattice - interacting field theory in (1+1)

Interacting Hamiltonian Density:

$$\mathscr{H}(x,t) = \frac{1}{2}\left(\pi(x,t)^2 + (\nabla\varphi(x,t))^2 + \omega^2\varphi(x,t)^2\right) + \mathscr{V}_I(\varphi(x,t))$$

where the potential $\mathscr{V}_I$ is no longer quadratic

Qumode lattice Hamiltonian:

Making the same discretisation as the free theory and expanding terms

$$Ha^{-1} = \sum_n \left[\frac{1}{2}\left(\hat{p}_n(t)^2 + \omega^2\hat{q}_n(t)^2\right) + V_I(\hat{q}_n)\right] - \frac{1}{a^2}\sum_n \hat{q}_{n+1}\hat{q}_n$$

where

$$V_I = \frac{1}{a^2}\hat{q}_n^2 + \mathscr{V}_I(\hat{q}_n)$$

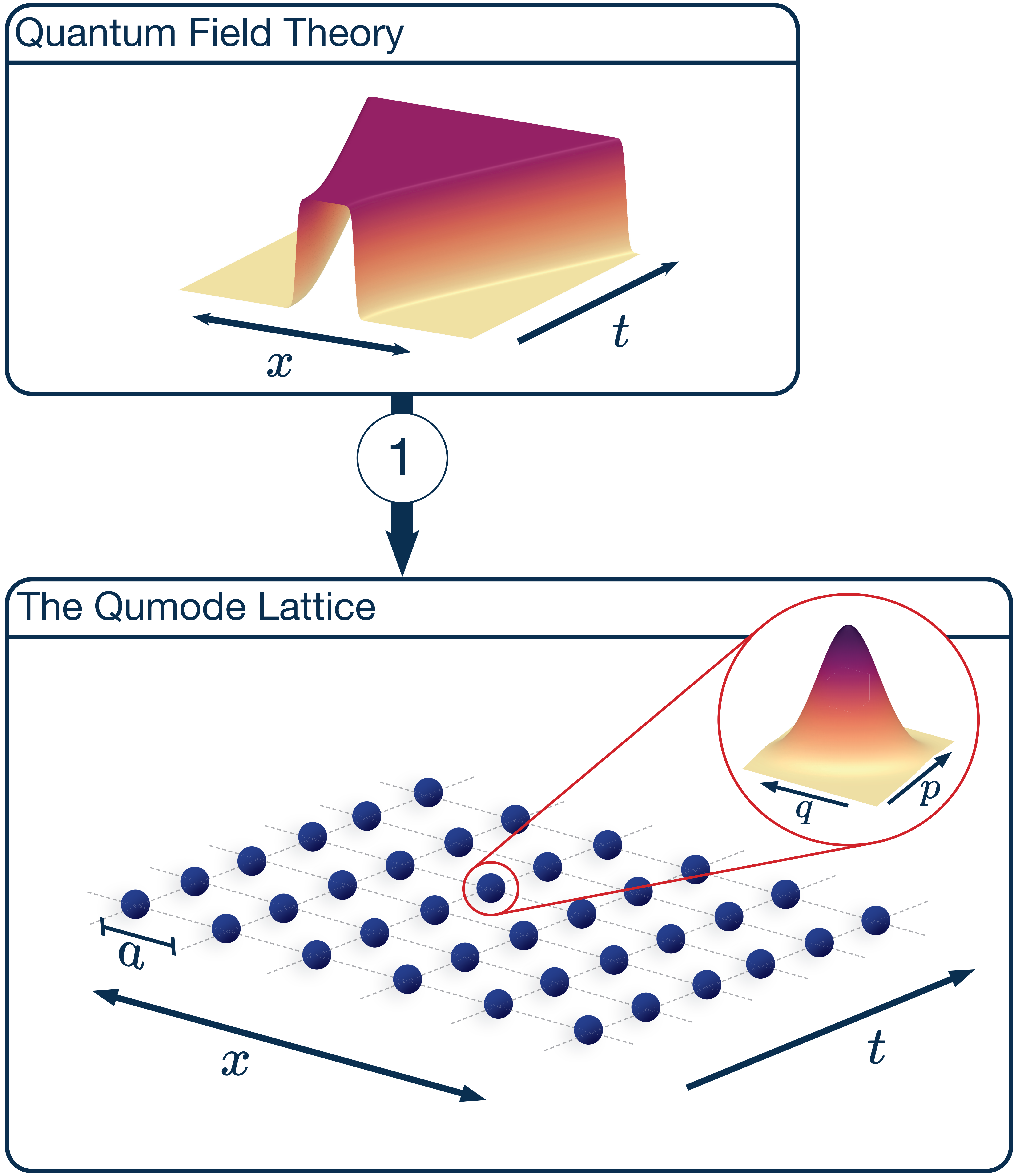
Regardless of the potential, the formulation reduces to a sum of three terms:

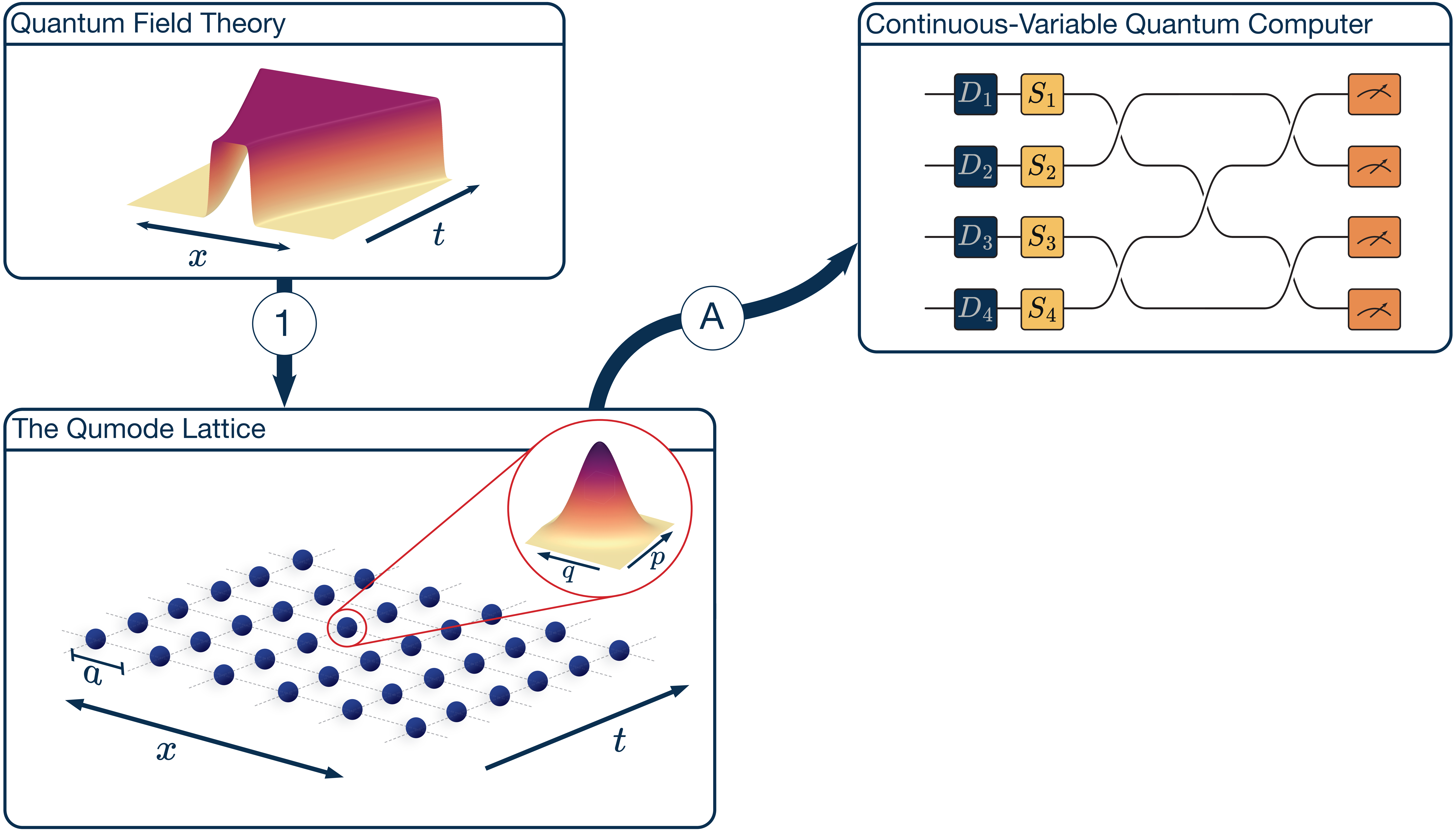
1) The **SHO Hamiltonian**

2) An **arbitrary potential** V_I acting locally on each site

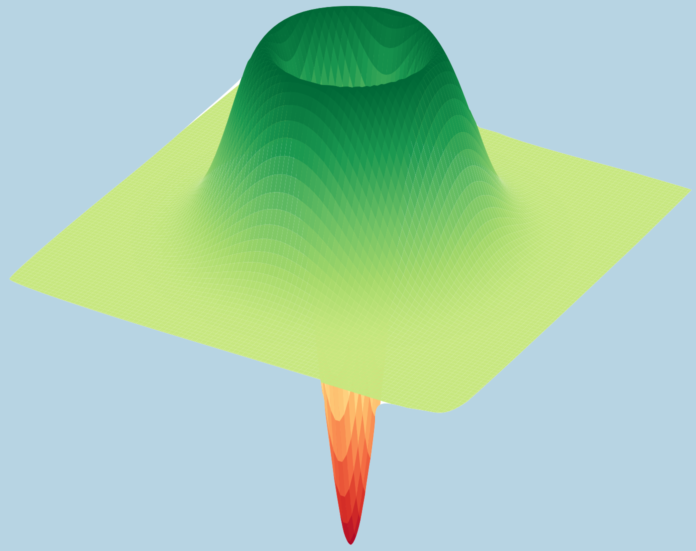
3) A simple **hopping term** which connects nearest neighbour sites

Simulating these three steps will approximate the real-time evolution of the scalar QFT


Quantum Field Theory
x
t
1
The Qumode Lattice
a
x
t
q
p


Quantum Field Theory
x
t
1
The Qumode Lattice
a
q
p
x
t
A
Continuous-Variable Quantum Computer
D_1
S_1
D_2
S_2
D_3
S_3
D_4
S_4

Quantum Computing - With a little help from my photons



Photonic Quantum Devices

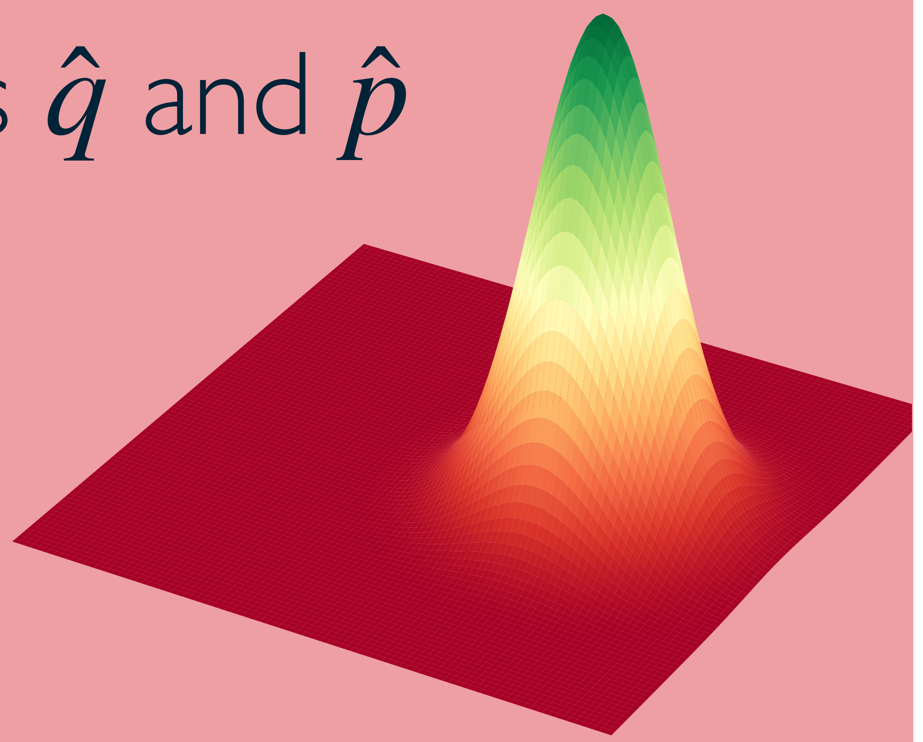
Quantum Computation via quantum optics

Universal computing:

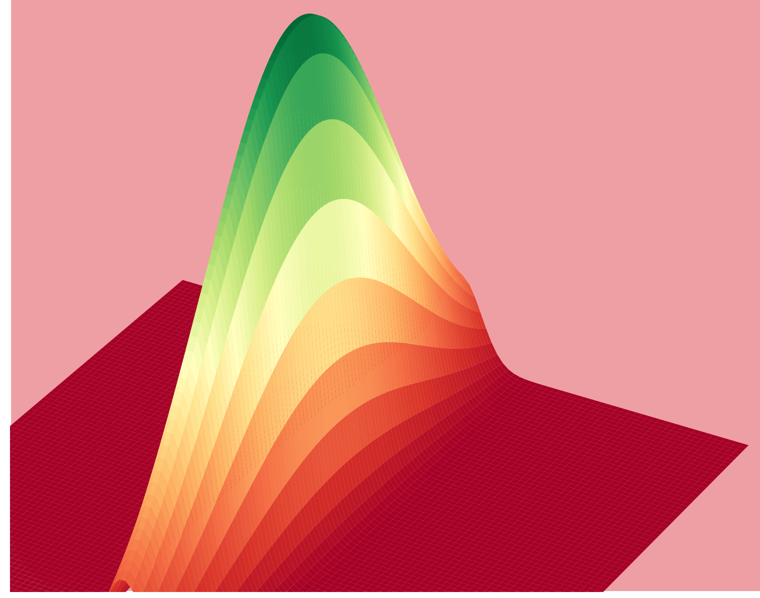
It is impossible to achieve universal computation with only Gaussian operations - "no-go" theorem

Gaussian operations:

Maps Gaussian states to Gaussian states - at most quadratic in quadrature variables $\hat{q}$ and $\hat{p}$



$$D(\alpha) = \exp\left[-i\sqrt{2}\left(\mathfrak{R}(\alpha)\hat{p} - \mathfrak{I}(\alpha)\hat{x}\right)\right]$$

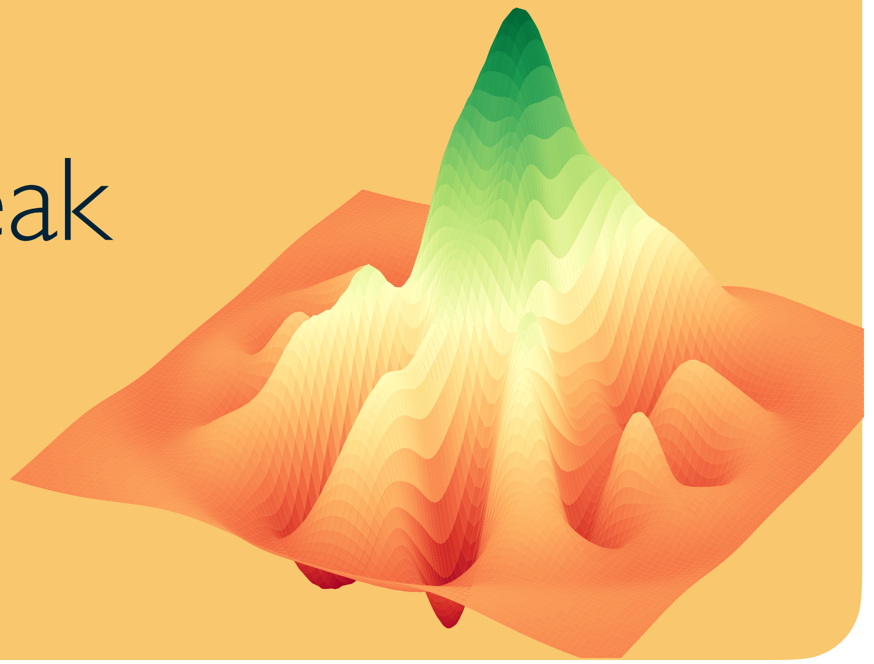


$$S(z) = \exp\left[\frac{1}{2}\left(z^* a^2 - z a^{\dagger 2}\right)\right]$$

Non-Gaussian operations:

Maps Gaussian states to non-Gaussian states - greater than quadratic in quadrature variables $\hat{q}$ and $\hat{p}$

Optical non-linearities are too weak to introduce required non-Gaussianity

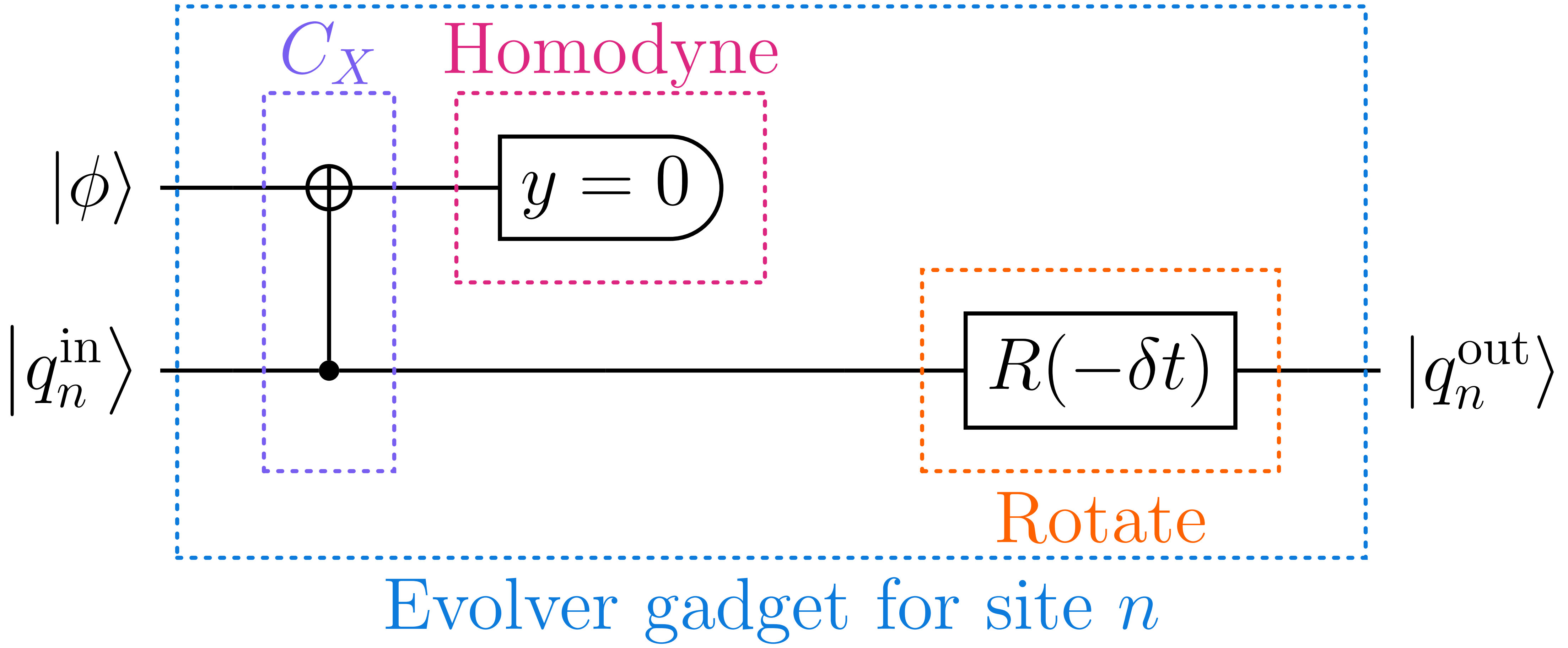


Real-time Simulation on CVQCs - single qumode evolution

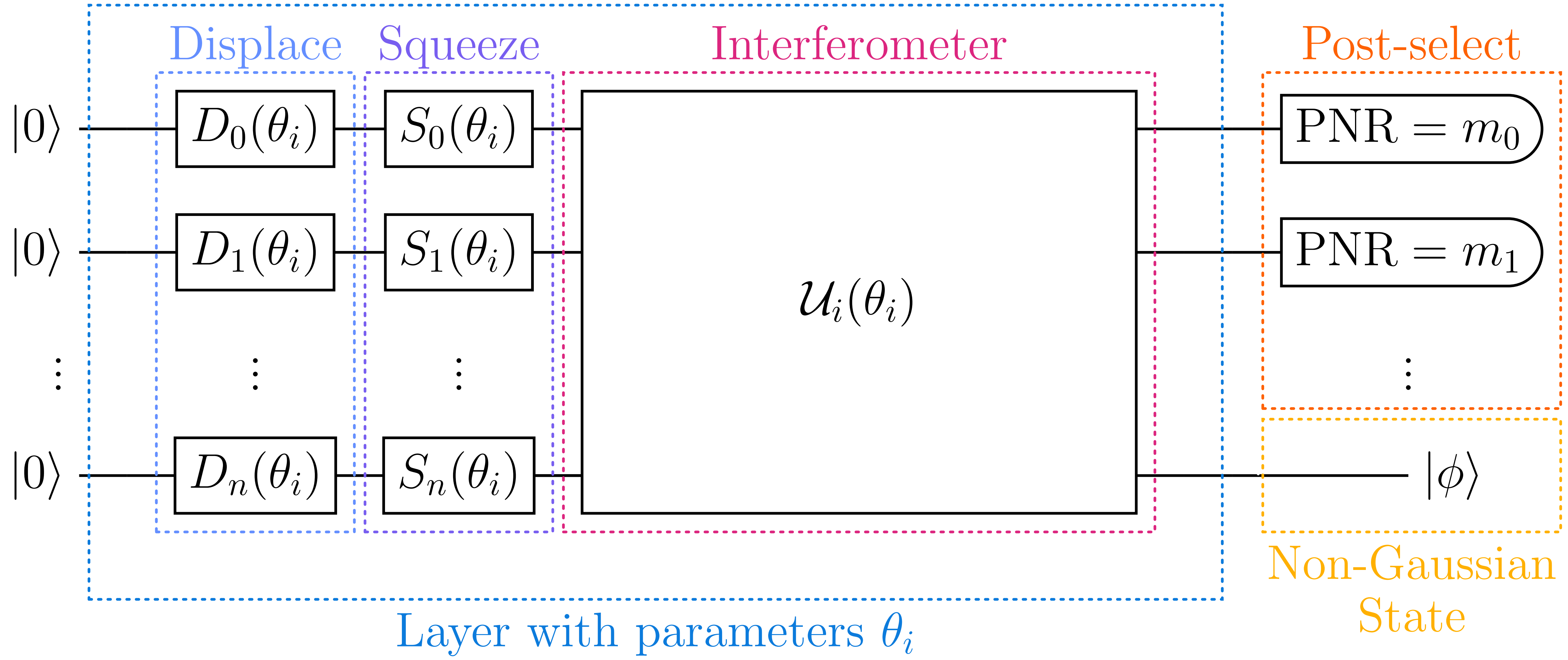
The time-evolution of a single qumode is performed via the diagonal part of $\boldsymbol{H}$

$$\mathcal{U}_{\text{diag}}(\delta t) \;=\; \mathcal{U}_R(-\delta t)\,\mathcal{U}_V(\delta t)$$

On a CVQC device this is performed by the **"evolver-gadget"**



The **SHO contribution** to the Hamiltonian corresponds directly to an $\boldsymbol{R}$ gate on the CVQC, however one must construct a **non-Gaussian gate** operation to implement $\mathcal{U}_V$



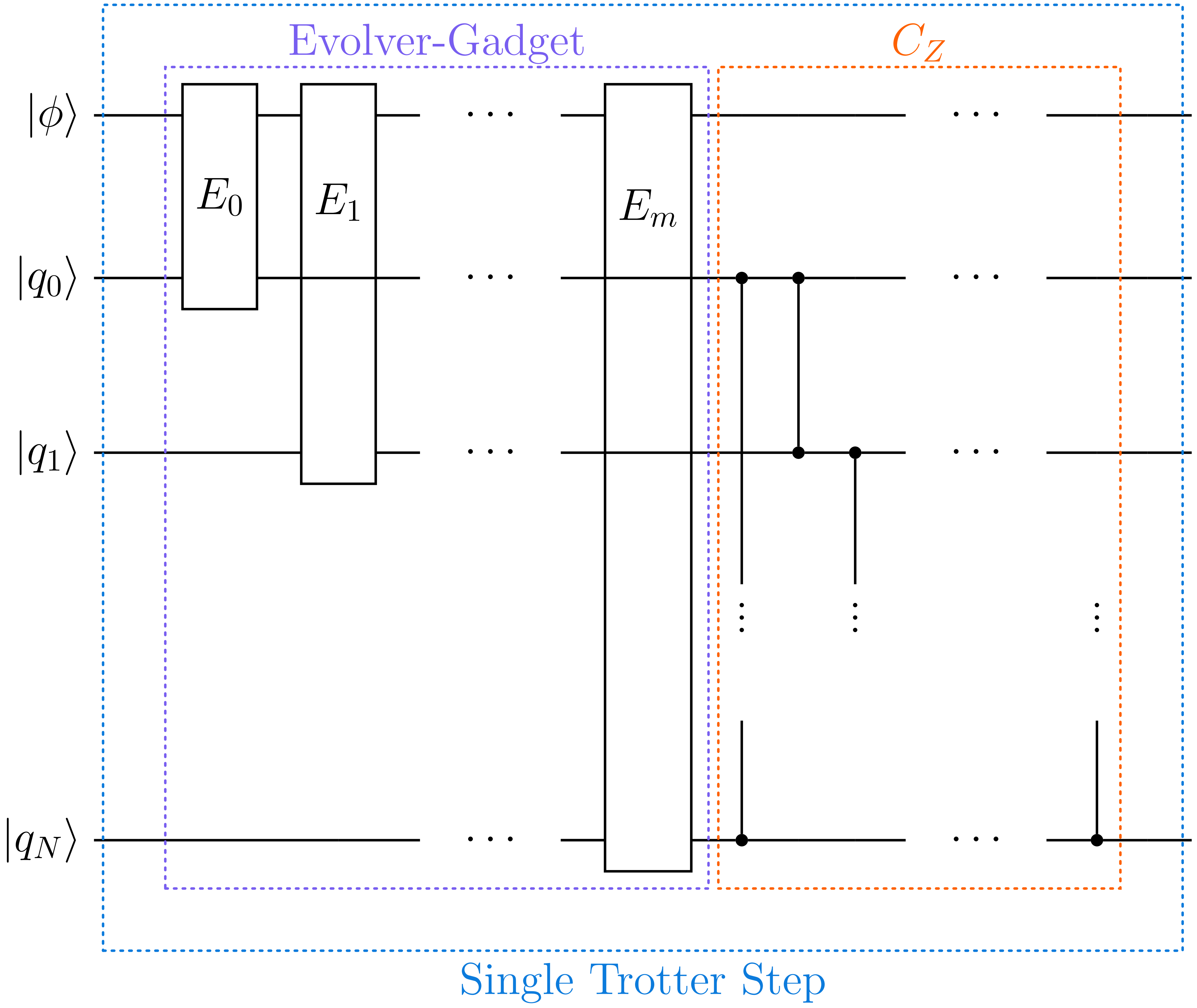
Real-time Simulation on CVQCs

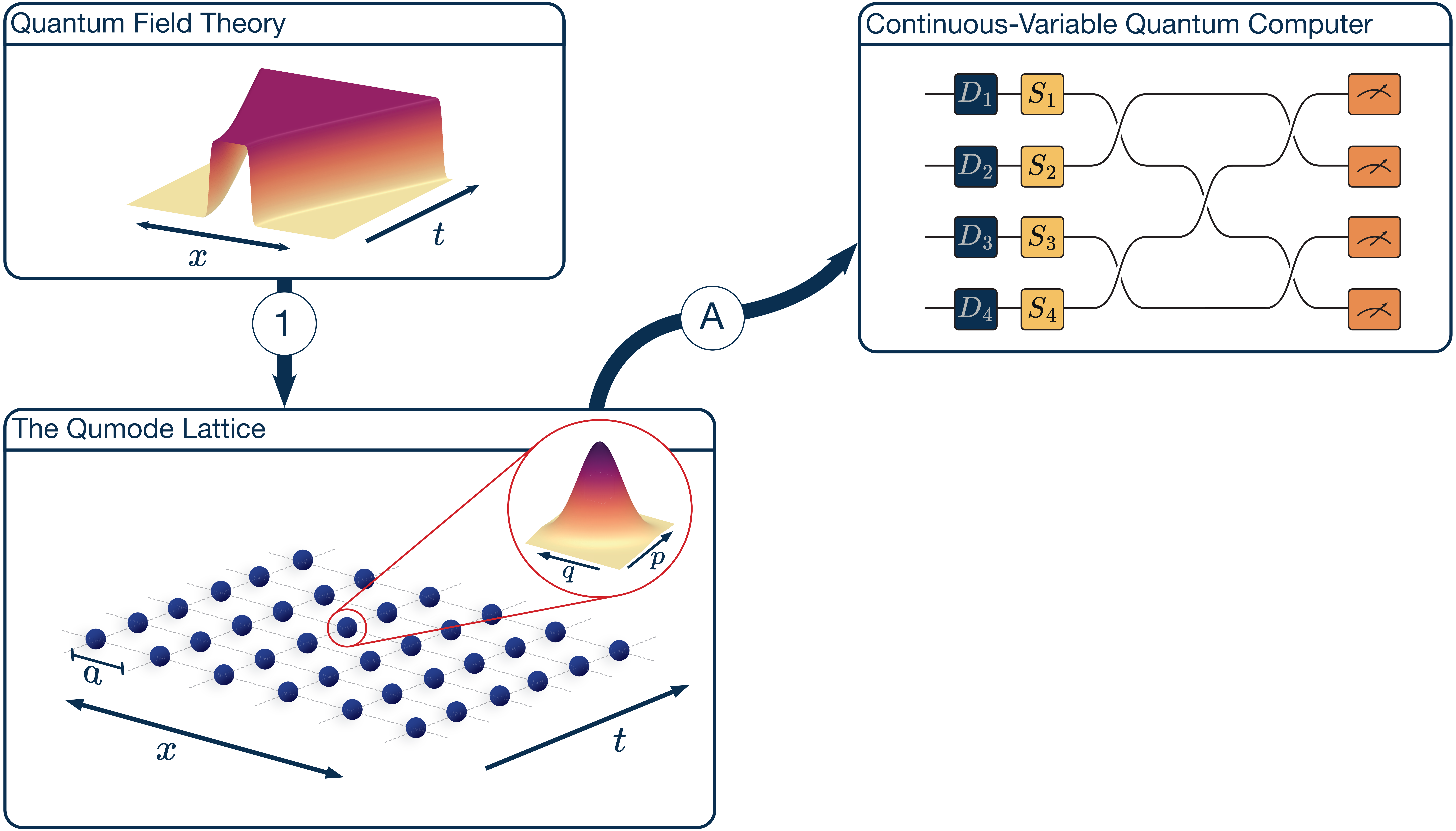
The extension to a full QFT simulation is simply achieved by performing **nearest neighbour hopping** terms:

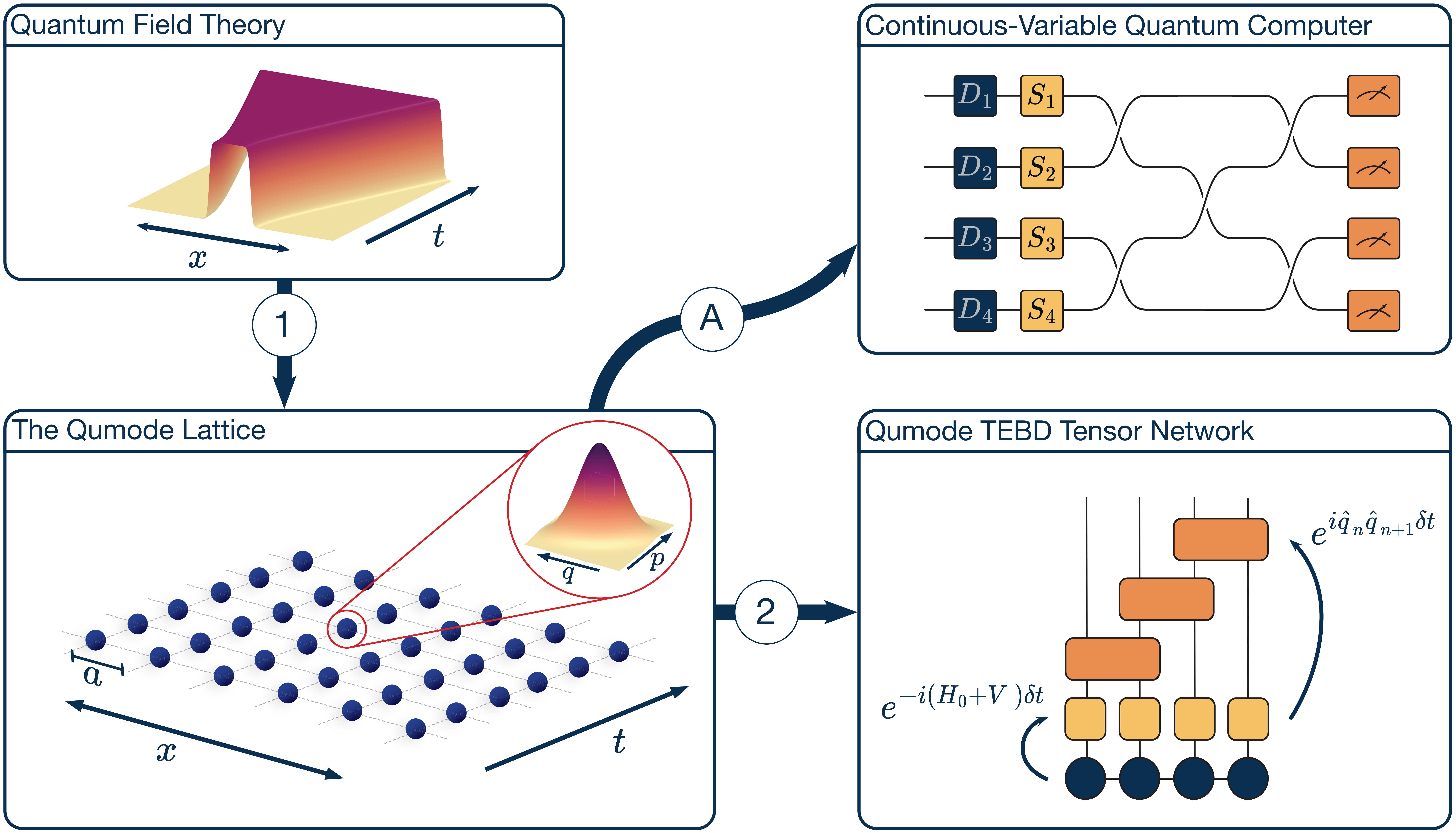
$$\mathcal{U}_{\text{hop}} = \prod_{n=1}^{N} e^{ia^{-2}\,\hat{q}_{n+1}\,\hat{q}_n\,\delta t}$$

The form of the hopping term corresponds directly to a C_z gate on the CVQC device

Combined, the quantum qumode lattice can simulate the **real-time dynamics** without the need of **entanglement truncation** or **field digitisation**




Quantum Field Theory
x
t
1
A
Continuous-Variable Quantum Computer
D_1
D_2
D_3
D_4
S_1
S_2
S_3
S_4
The Qumode Lattice
q
p
a
x
t


Quantum Field Theory
x
t
1
The Qumode Lattice
a
q
p
x
t
A
Continuous-Variable Quantum Computer
D_1 S_1
D_2 S_2
D_3 S_3
D_4 S_4
2
Qumode TEBD Tensor Network
$e^{i\hat{q}_n\hat{q}_{n+1}\delta t}$
$e^{-i(H_0+V)\delta t}$

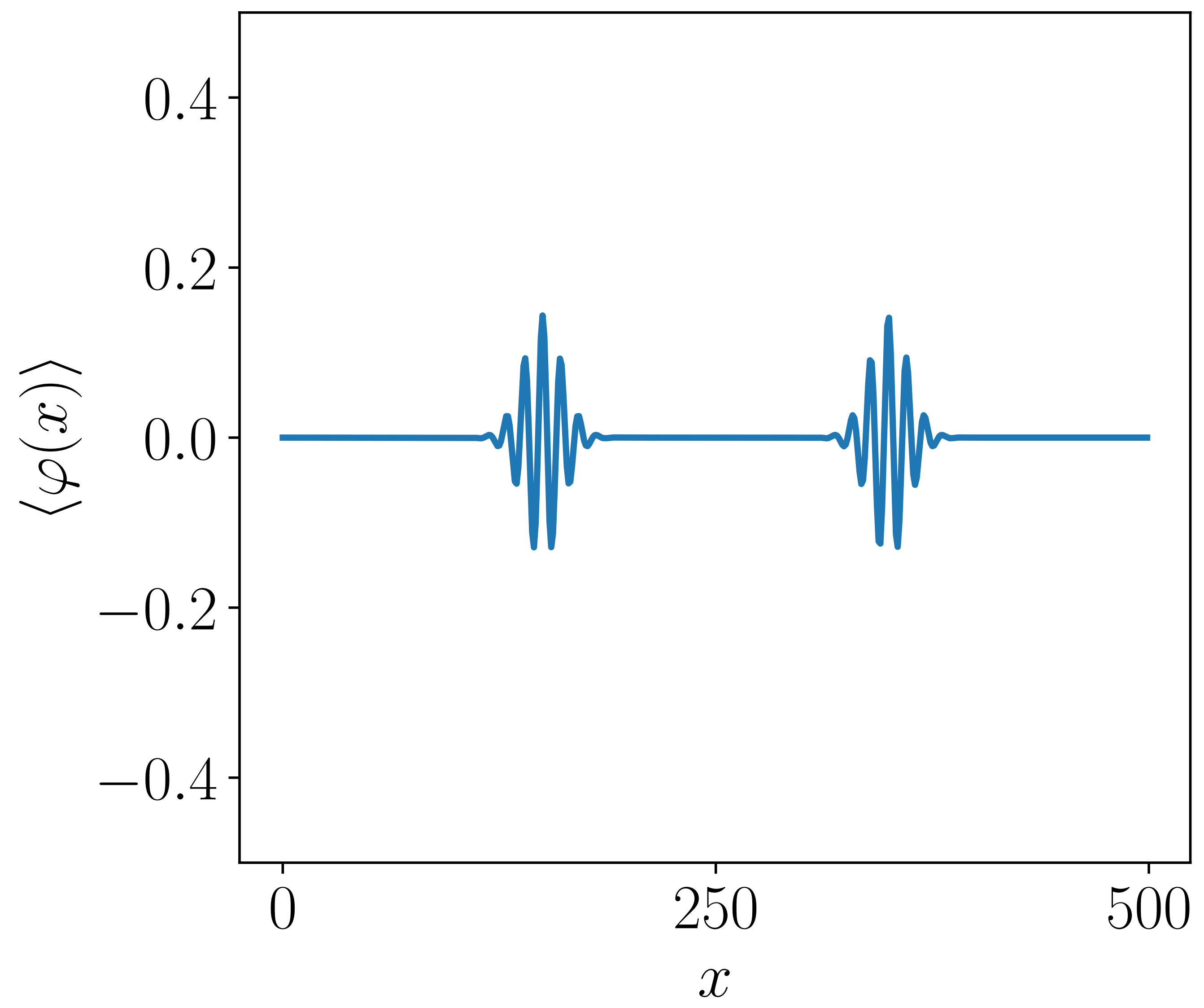
Scattering in φ^4 - initial state preparation

To validate the method, we consider scattering in φ^4 with the potential:

$$\mathscr{V}_I(\varphi) \;=\; \lambda\varphi^4$$

Such that the effective lattice potential is:

$$V_I \;=\; \frac{1}{a^2}\hat{q}_n^2 + \frac{\lambda}{4!}\hat{q}_n^4$$

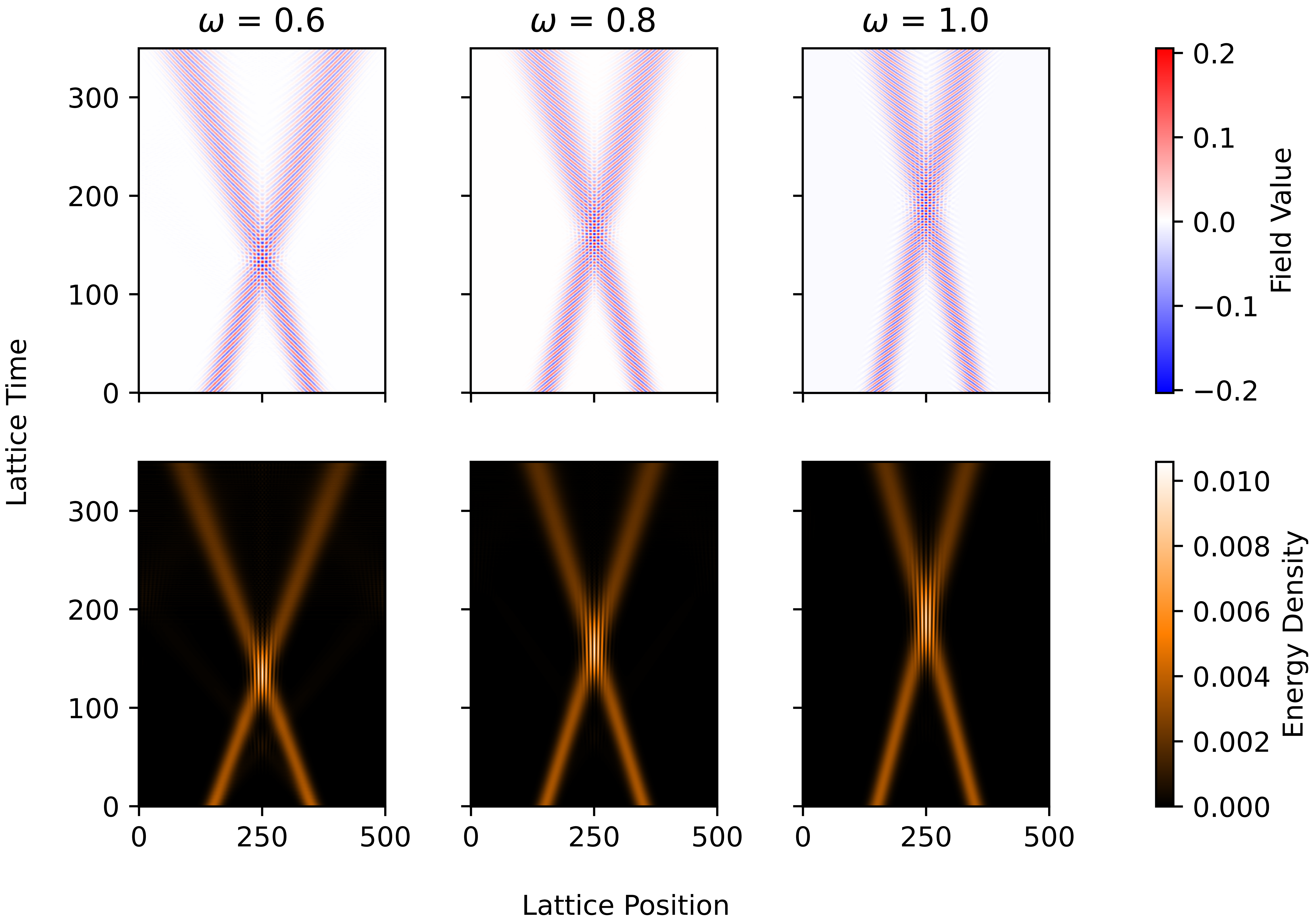


We restrict to **non-relativistic approximation** by imposing $\bar{k} \ll \omega$ and $\sigma \ll \omega$

$$|\psi_n\rangle \;=\; \left(A^0 + \sum_{\alpha=0}^{N-1} \frac{1}{\sqrt{\mathcal{N}_\alpha}} \exp\left(-\frac{(k_\alpha - \bar{k})^2}{2\sigma^2}\right) \exp\left(ik_\alpha(x_n - \bar{x})\right) \hat{a}_n^\dagger\right)|0\rangle$$

We **normalise the wavepackets** by adjusting the zeroth Fock amplitude:

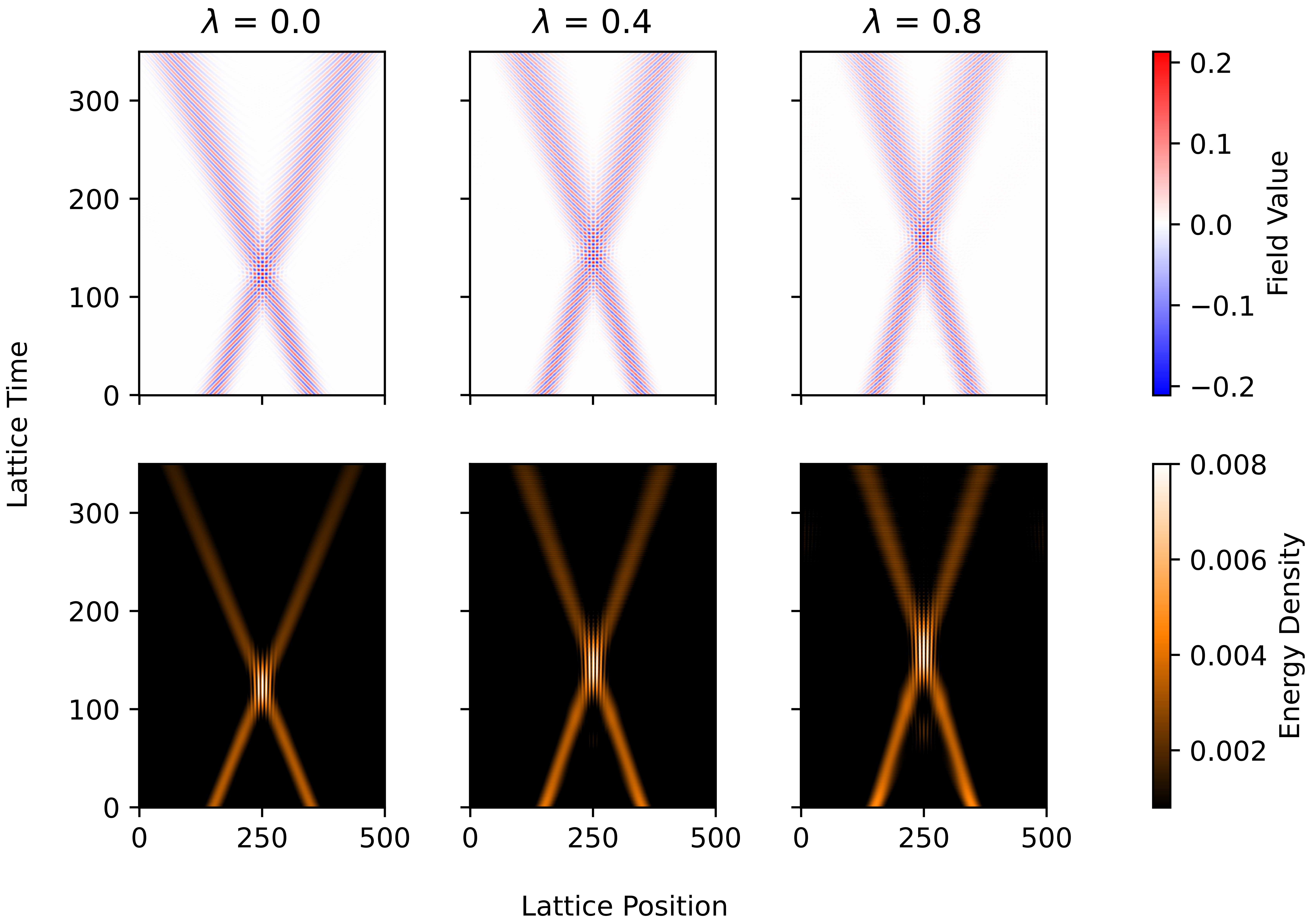
$$\mathcal{N}_\alpha \;=\; 2\omega_\alpha N \qquad\qquad |A^0|^2 \;=\; 1 - \sum_{\alpha=0}^{N-1} \frac{1}{\sqrt{\mathcal{N}_\alpha}} e^{-\frac{4\pi^2\alpha^2}{L^2\sigma^2}}$$



We simulate scattering in φ^4 theory, initialising the wavepackets with $\sigma = 0.09$, $k_\alpha = 0.3$ and a Trotter time step of $\delta t = 0.01$

The lattice is constructed from **500 qumodes** each of which is discretised with $M = 200$

The coupling is fixed at $\lambda = 0.2$ and the masses are varied



We simulate scattering in φ^4 theory, initialising the wavepackets with $\sigma = 0.09$, $k_\alpha = 0.3$ and a Trotter time step of $\delta t = 0.01$

The lattice is constructed from **500 qumodes** each of which is discretised with $M = 200$

The mass is fixed at $\omega = 0.6$ and the couplings are varied

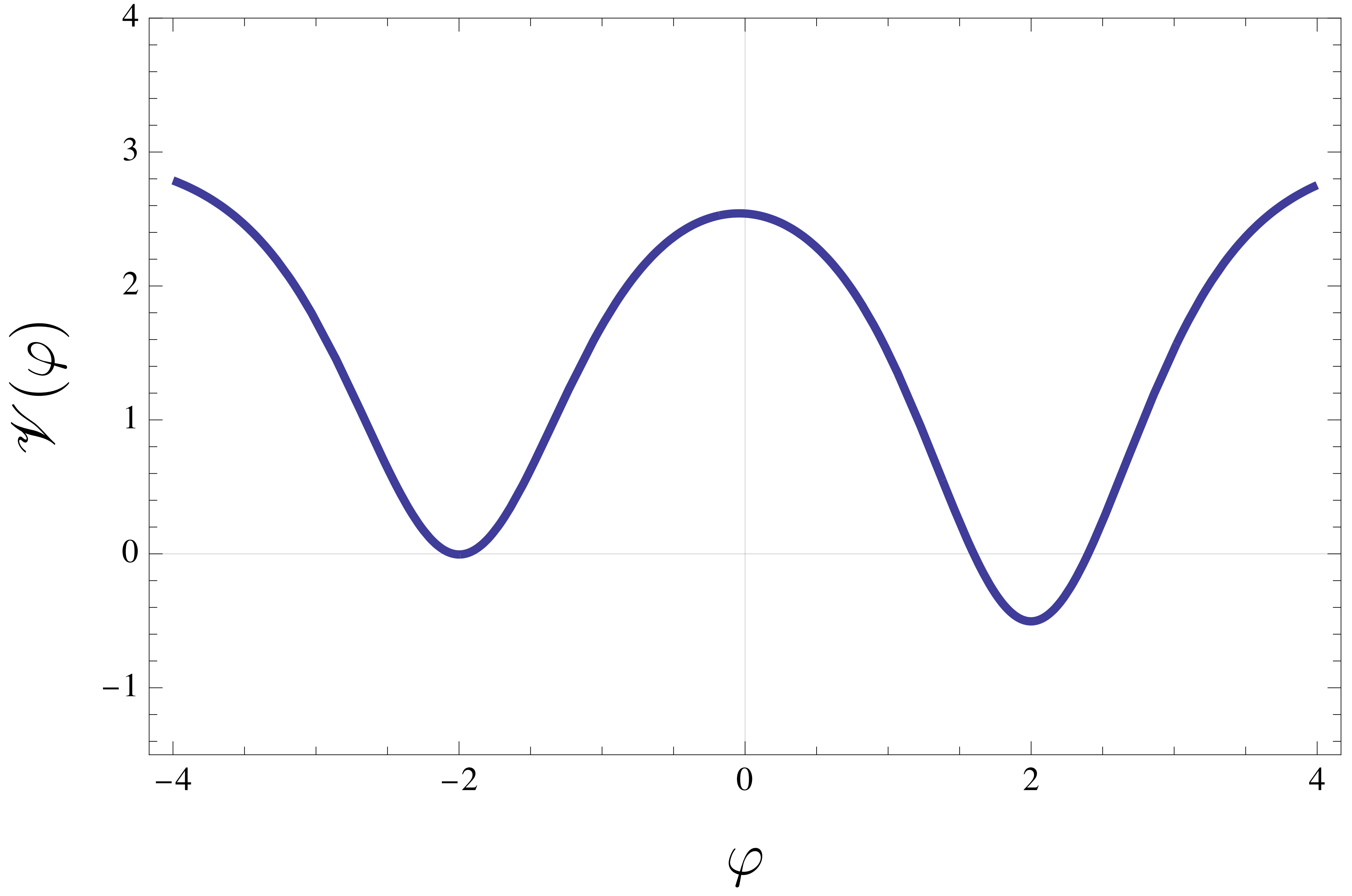
False Vacuum Decay in (1+1)-Dimensional Scalar QFT

It is possible to consider potentials that destabilise the vacuum locally and allow **non-perturbative tunnelling processes**

Subcritical bubbles: Pöschl-Teller

$$\begin{aligned}\mathscr{V}(\varphi) &= \frac{\alpha^2\gamma(\gamma+1)}{2}\tanh^2(\alpha(\varphi-\varphi_{\rm fv})) \\ &\quad + \left(\frac{\alpha^2\gamma(\gamma+1)}{2}+\varepsilon\right)\operatorname{sech}^2(\alpha(\varphi-\varphi_{\rm tv}))\end{aligned}$$

with $\varphi_{tv} = -\varphi_{fv} = 2, \alpha = 1, \gamma = 3/2$ and $\varepsilon = 0.25$



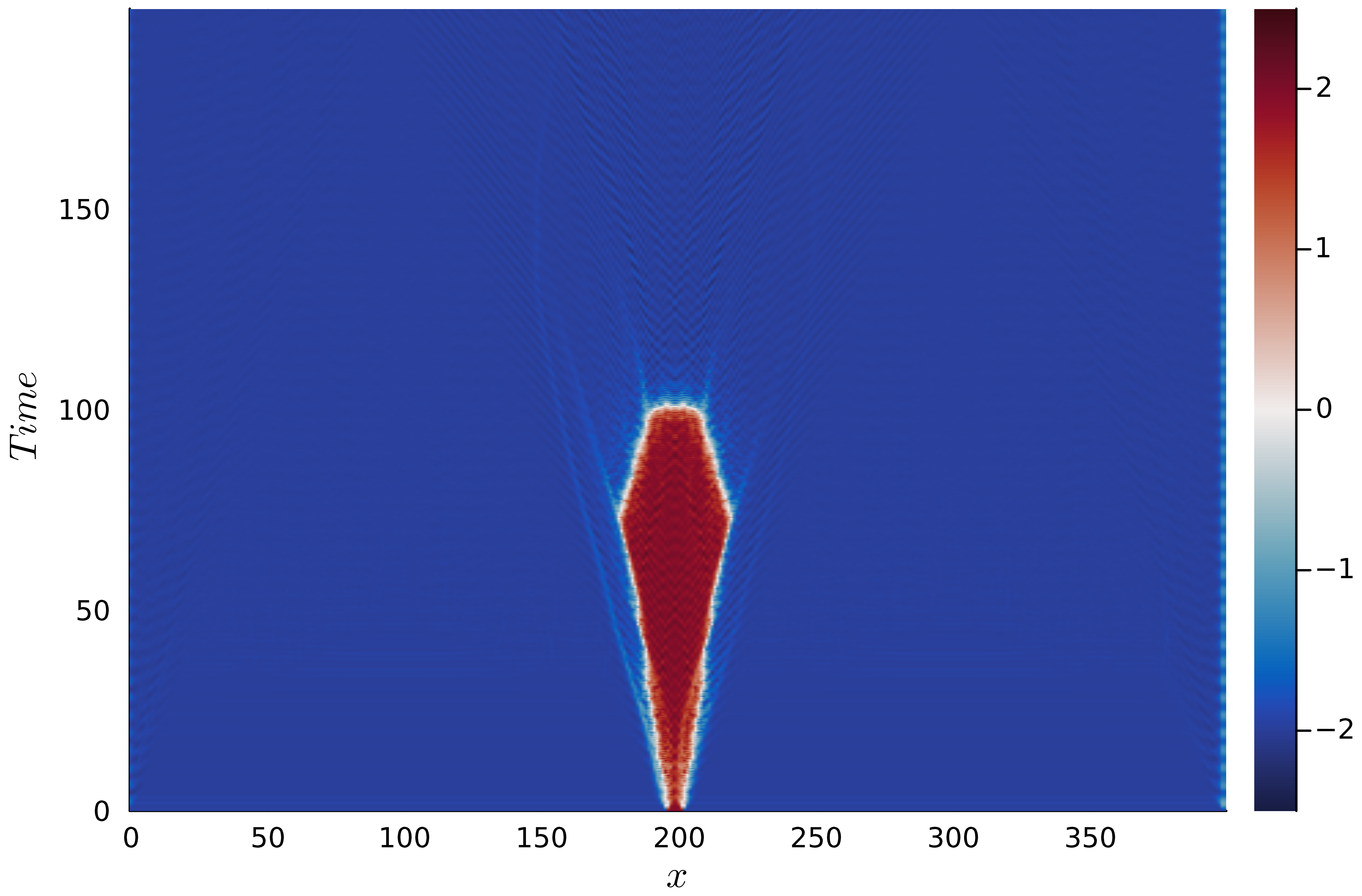
False Vacuum Decay in (1+1)-Dimensional Scalar QFT

It is possible to consider potentials that destabilise the vacuum locally and allow **non-perturbative tunnelling processes**

Subcritical bubbles: Pöschl-Teller

$$\mathcal{V}(\varphi) = \frac{\alpha^2\gamma(\gamma+1)}{2}\tanh^2(\alpha(\varphi-\varphi_{\rm fv})) + \left(\frac{\alpha^2\gamma(\gamma+1)}{2}+\varepsilon\right)\text{sech}^2(\alpha(\varphi-\varphi_{\rm tv}))$$

with $\varphi_{tv} = -\varphi_{fv} = 2, \alpha = 1, \gamma = 3/2$ and $\varepsilon = 0.25$



False Vacuum Decay in (1+1)-Dimensional Scalar QFT

Critical bubbles:

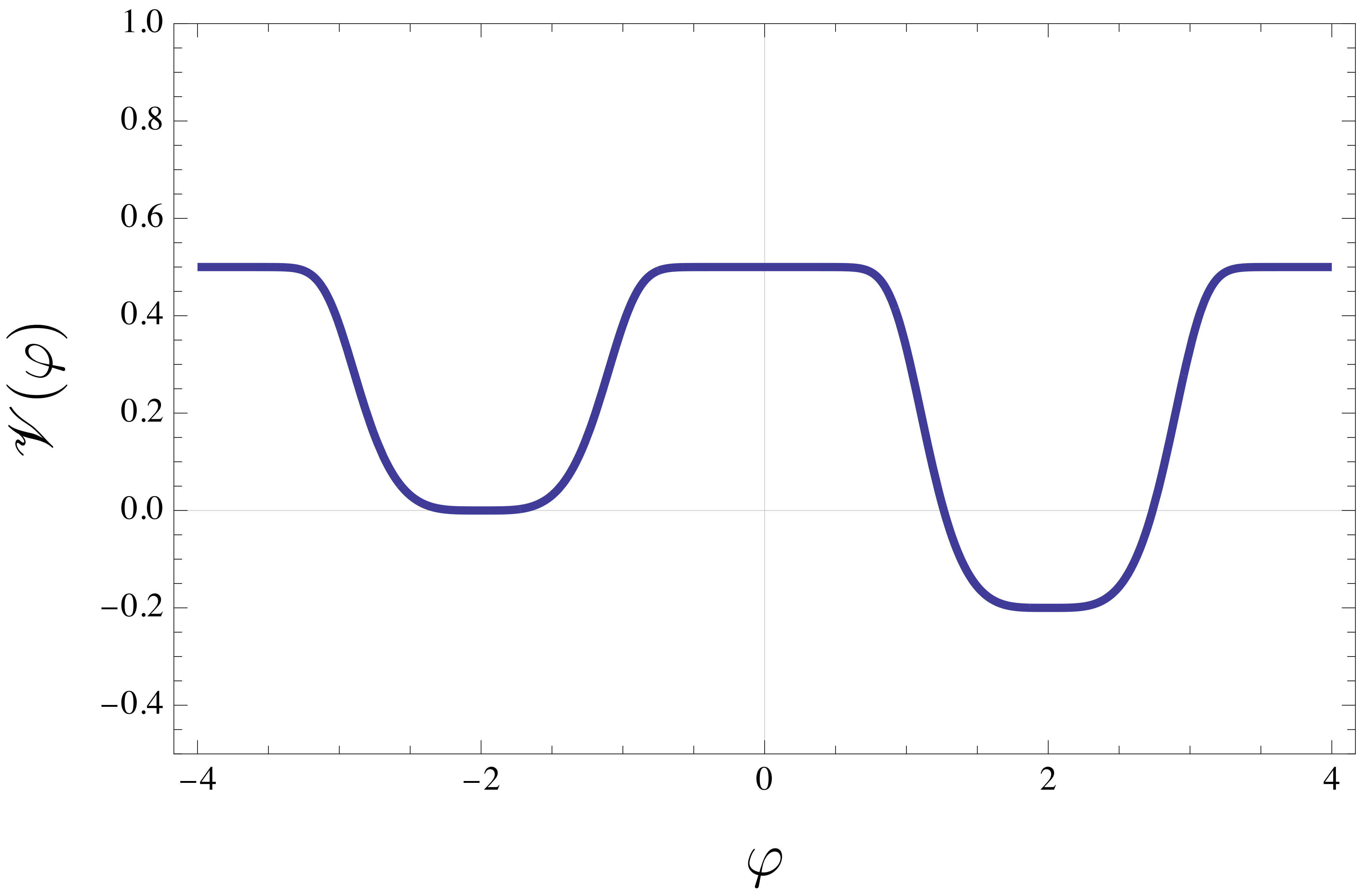
$$\mathscr{V}(\varphi) = \frac{\lambda}{\ell!}\tanh(\mu^{-\ell}(\varphi-\varphi_{\rm fv})^{\ell}) + \left(\frac{\lambda}{\ell!}+\varepsilon\right)\left(\tanh(\mu^{-\ell}(\varphi-\varphi_{\rm tv})^{\ell})-1\right)$$

with $\varphi_{tv} = -\varphi_{fv} = 2, \lambda = 1, l = 2$ and $\varepsilon = 0.1$

Bubble wall mass:

$$m_{\rm wall} \approx \frac{2\varepsilon d_{\rm bounce}}{v_{\rm term}^2} \approx 7.7$$

where $d \approx 20$ and $v^2 \approx 0.72$. Same order of magnitude of analytical calculations : $m \approx 2.74$



False Vacuum Decay in (1+1)-Dimensional Scalar QFT

Critical bubbles:

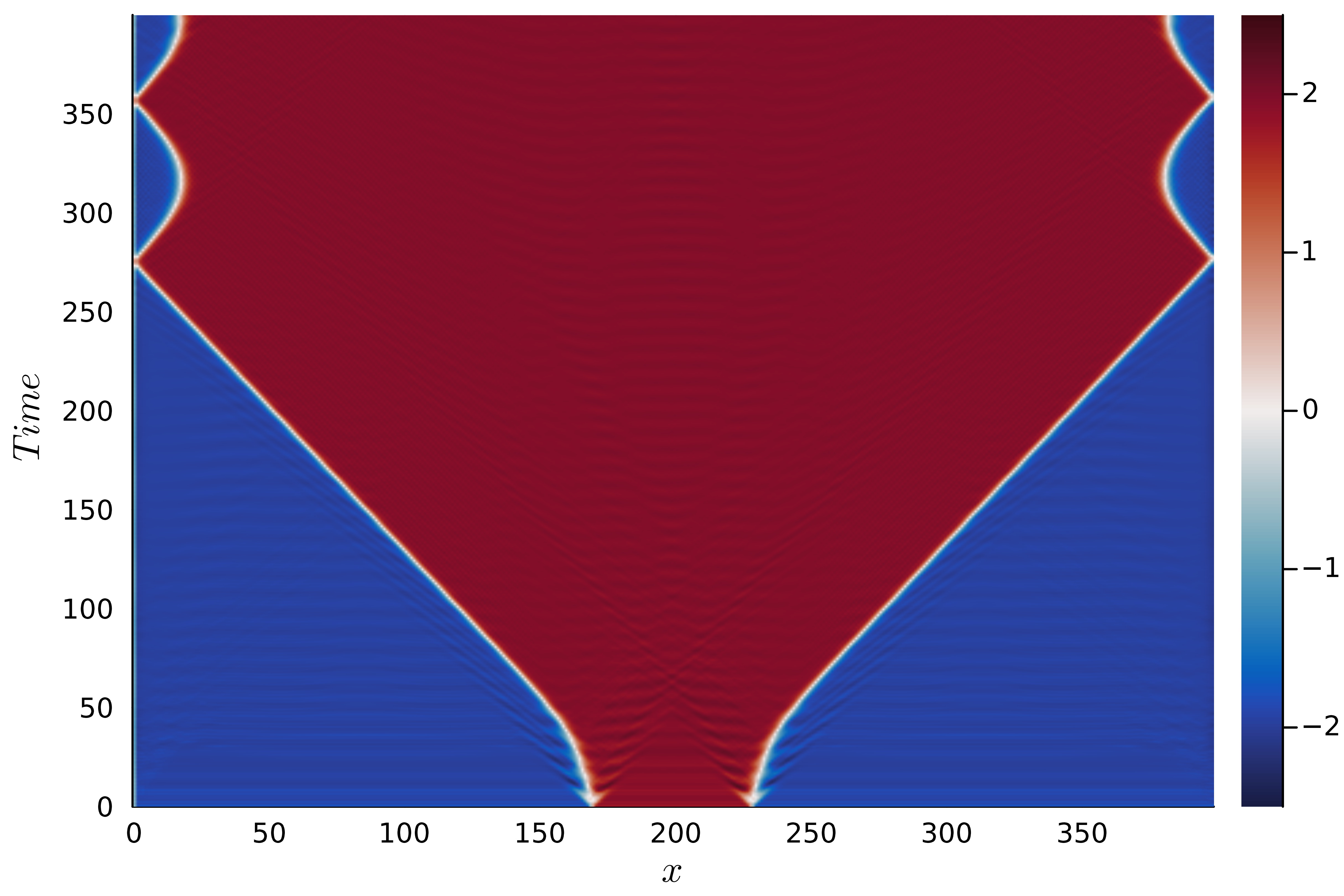
$$\mathscr{V}(\varphi) = \frac{\lambda}{\ell!}\tanh(\mu^{-\ell}(\varphi-\varphi_{\rm fv})^{\ell}) + \left(\frac{\lambda}{\ell!}+\varepsilon\right)\left(\tanh(\mu^{-\ell}(\varphi-\varphi_{\rm tv})^{\ell})-1\right)$$

with $\varphi_{tv} = -\varphi_{fv} = 2, \lambda = 1, l = 2$ and $\varepsilon = 0.1$

Bubble wall mass:

$$m_{\rm wall} \approx \frac{2\varepsilon d_{\rm bounce}}{v_{\rm term}^2} \approx 7.7$$

where $d \approx 20$ and $v^2 \approx 0.72$. Same order of magnitude of analytical calculations : $m \approx 2.74$



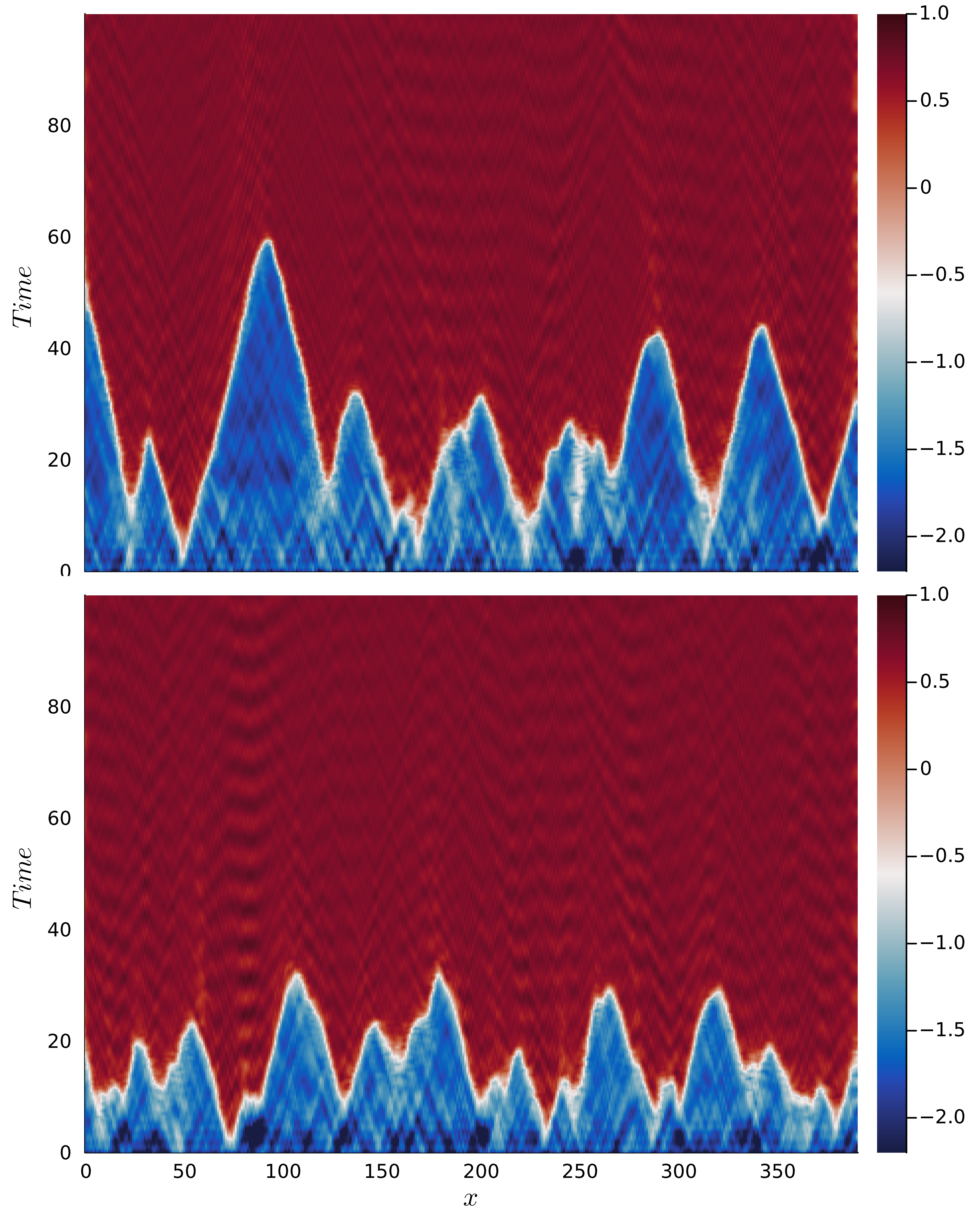
Phase transitions in (1+1)D

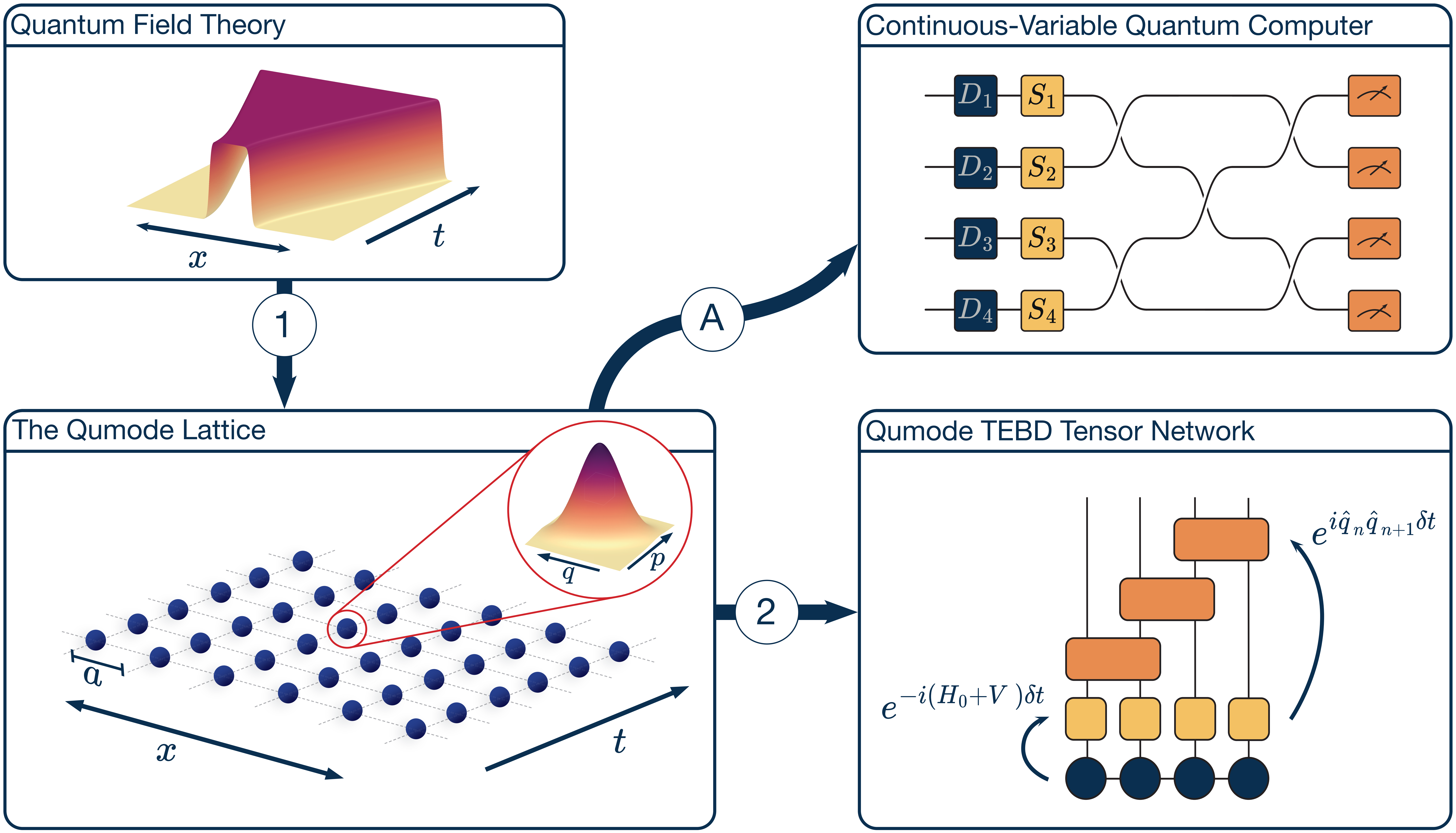
We now consider **phase transitions** in the system as a whole using the $\tanh \varphi^l$ potential with $\varphi_{tv} = 0.5$, **approximating the QFT background** with a sampling method

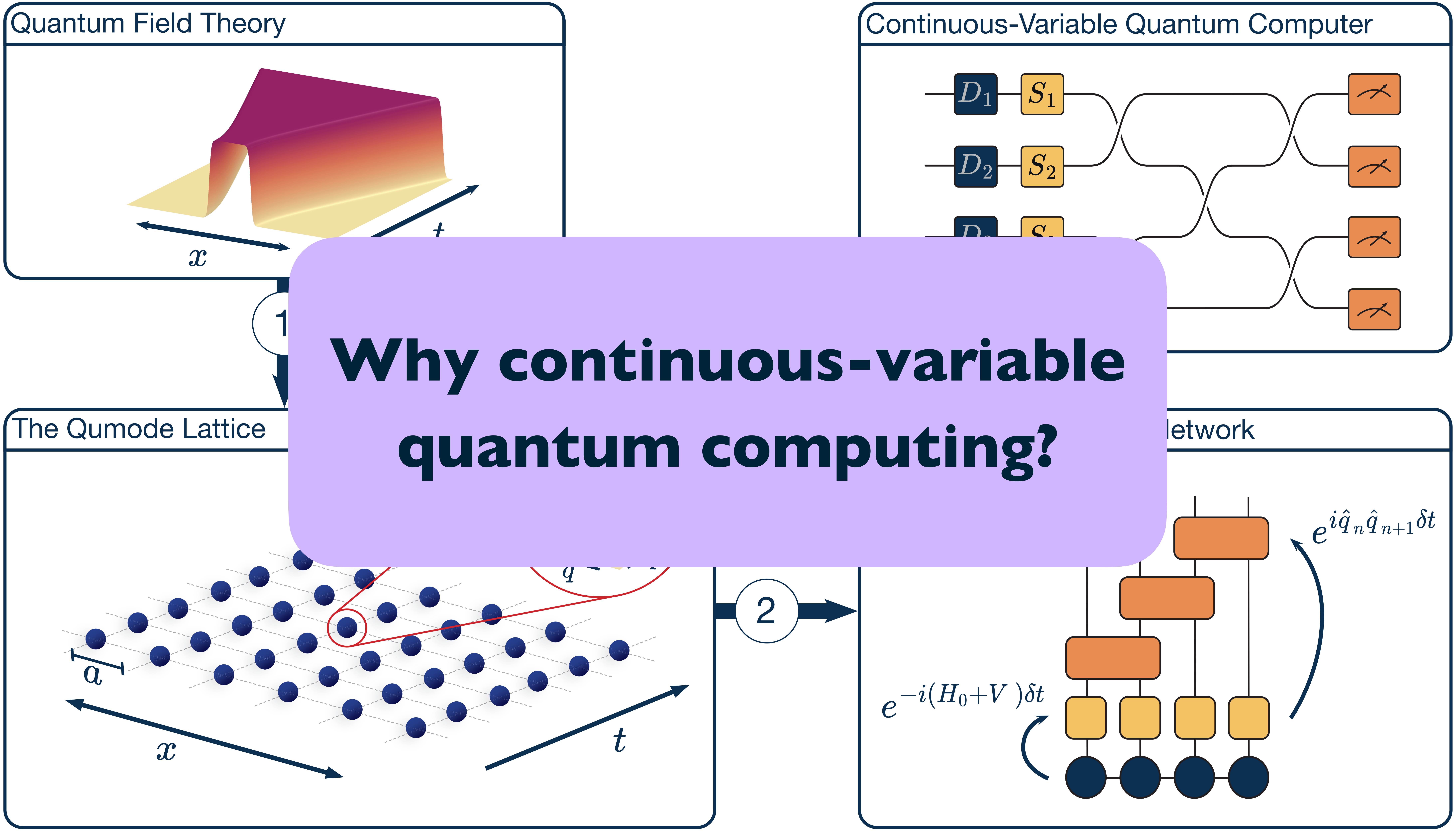
The upper plot is produced with $\varepsilon = 0.4$ leading to a critical bubble radius of $r_c \approx 5$

The lower plot is produced with $\varepsilon = 0.5$ leading to a critical bubble radius of $r_c \approx 2$, considerably **speeding up** the phase transition

Individual bubbles coalesce to complete the whole phase transition in the system.

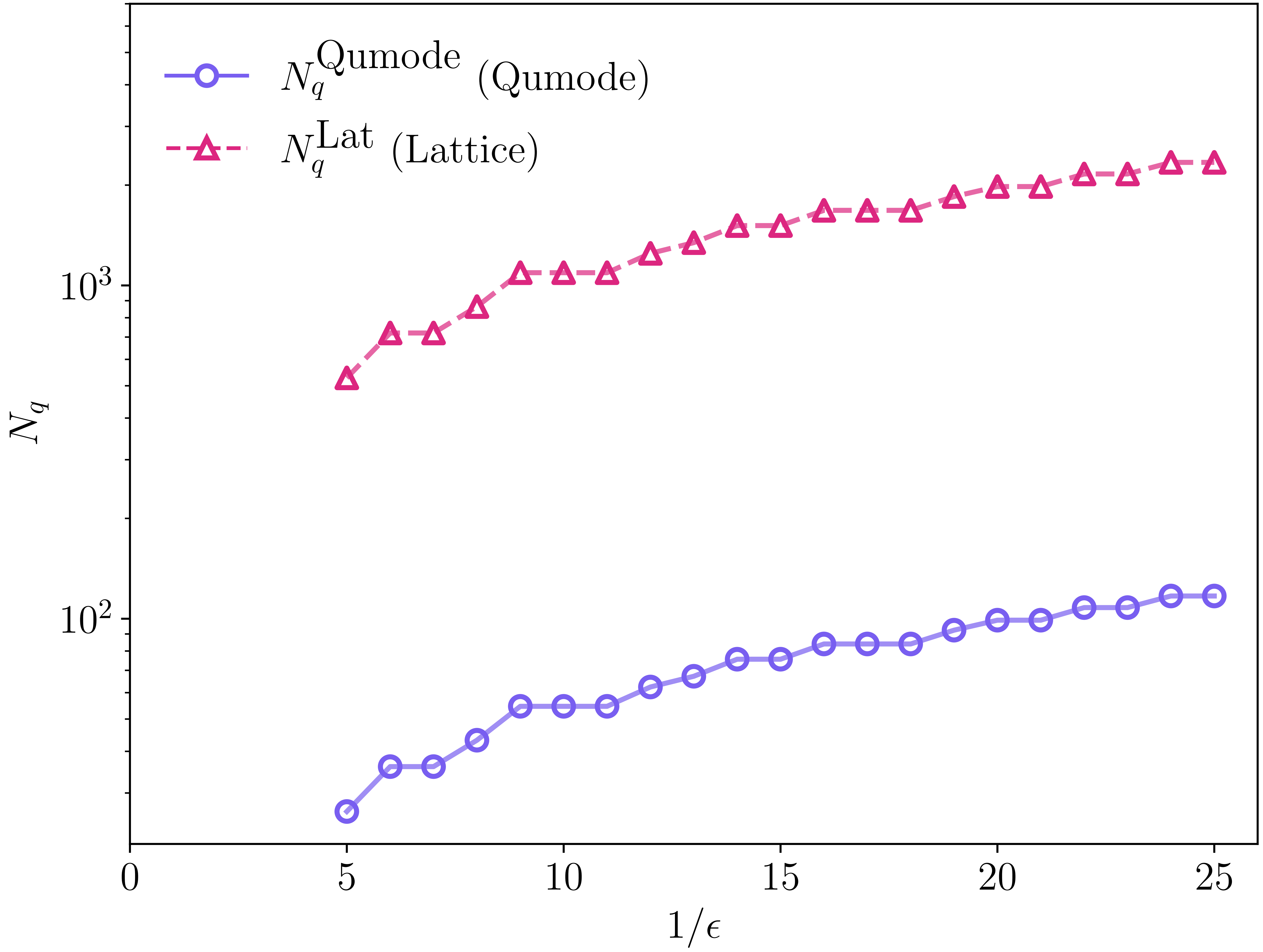



Quantum Field Theory
x
t
1
The Qumode Lattice
a
x
t
q
p
A
Continuous-Variable Quantum Computer
D_1
D_2
D_3
D_4
S_1
S_2
S_3
S_4
2
Qumode TEBD Tensor Network
$e^{i\hat{q}_n\hat{q}_{n+1}\delta t}$
$e^{-i(H_0+V)\delta t}$



Why continuous-variable quantum computing?

Advantages of the quantum qumode lattice



The qumode lattice **does not require** the field value to be **digitised** at each lattice site and thus **reduced the quantum resources** required to simulate QFTs on quantum hardware

The qumode lattice can be **efficiently simulated** using the **qumode tensor network framework**, allowing for **large scale simulations** of (1+1)-dimensional QFTs.

The method has been validated by capturing the underlying physics of **scattering configurations** and **false vacuum decay in (1+1)-dimensional scalar field theories**

Advancements in photonic hardware will be pivotal in unlocking the potential of this approach, enabling the study of **increasingly complex and computationally demanding quantum systems**