How to build a dimension*

(*or, slightly more conservatively, "a new look at multi-gravity and dimensional deconstruction")

Talk based on the paper:
- [2501.16442]

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PASCOS '25, Durham University, 25/7/25

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IP3

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“What does gravitational physics in a compact extra dimension look like to a lower dimensional observer?”

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- Standard multi-gravity *cannot* arise consistently from dimensional deconstruction – so what gives?

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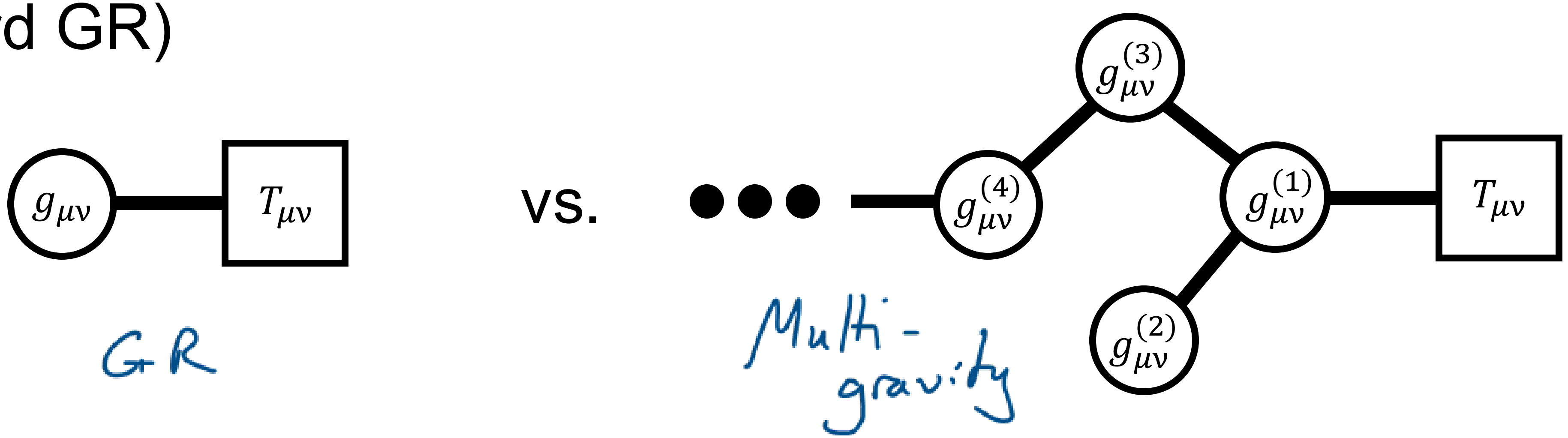
- How do we fix this?

Part 1: multi-gravity

What is multi-gravity and why should we care?

As the name suggests, multi-gravity is a modified theory of gravity involving multiple interacting metric tensors rather than just one (as in standard GR)



A theory containing N metrics describes the nonlinear interactions of:

- 1 massless spin-2 field
- $N - 1$ massive spin-2 fields

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- Most importantly, can recover GR so crucial to test!

Theory of multi-gravity

It's very tricky to couple multiple metrics without introducing ghosts, but it can be done! Need to work with dRGT framework

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N copies of Einstein-Hilbert term

- $I_V = -\sum_{i_1 \dots i_D = 0}^{N-1} \int_{\mathcal{M}_D} \varepsilon_{a_1 \dots a_D} T_{i_1 \dots i_D} \underbrace{e^{(i_1)a_1}} \wedge \dots \wedge e^{(i_D)a_D}$

ghost free interaction terms.

$$g^{(i)}_{\mu\nu} = e^{(i)a}_{\mu} e^{(i)b}_{\nu} \eta_{ab}$$

Interaction structure

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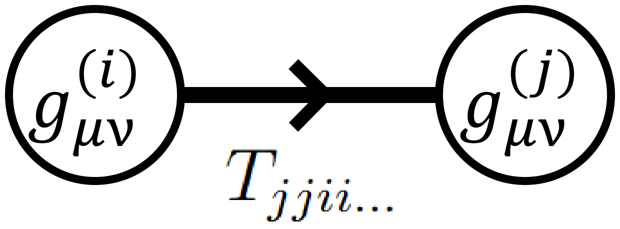
We can express the structure of the interactions diagrammatically, depending on which $S_{i\to j}$ matrices appear in the action:



- T's live on links, characterise interaction strength
- Interactions are oriented in direction of arrows since $S_{i\to j} = S_{j\to i}^{-1}$

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$$g^{(1)}_{\mu\nu} \longrightarrow g^{(2)}_{\mu\nu} \longrightarrow g^{(3)}_{\mu\nu}$$

$$\begin{aligned} I_V = &- \sum_{m=0}^{D} \int \mathrm{d}^D x \sqrt{-\det g_{(1)}}\, D!\, T_{\{2\}^m \{1\}^{D-m}} \frac{1}{m!} \delta^{\mu_1 \ldots \mu_m}_{\nu_1 \ldots \nu_m} (S_{1\to 2})^{\nu_1}{}_{\mu_1} \ldots (S_{1\to 2})^{\nu_m}{}_{\mu_m} \\ &- \sum_{m=0}^{D} \int \mathrm{d}^D x \sqrt{-\det g_{(2)}}\, D!\, T_{\{3\}^m \{2\}^{D-m}} \frac{1}{m!} \delta^{\mu_1 \ldots \mu_m}_{\nu_1 \ldots \nu_m} (S_{2\to 3})^{\nu_1}{}_{\mu_1} \ldots (S_{2\to 3})^{\nu_m}{}_{\mu_m} \end{aligned}$$

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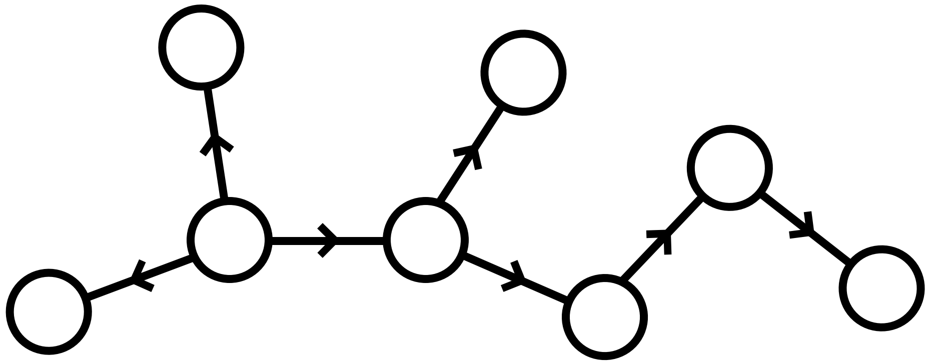

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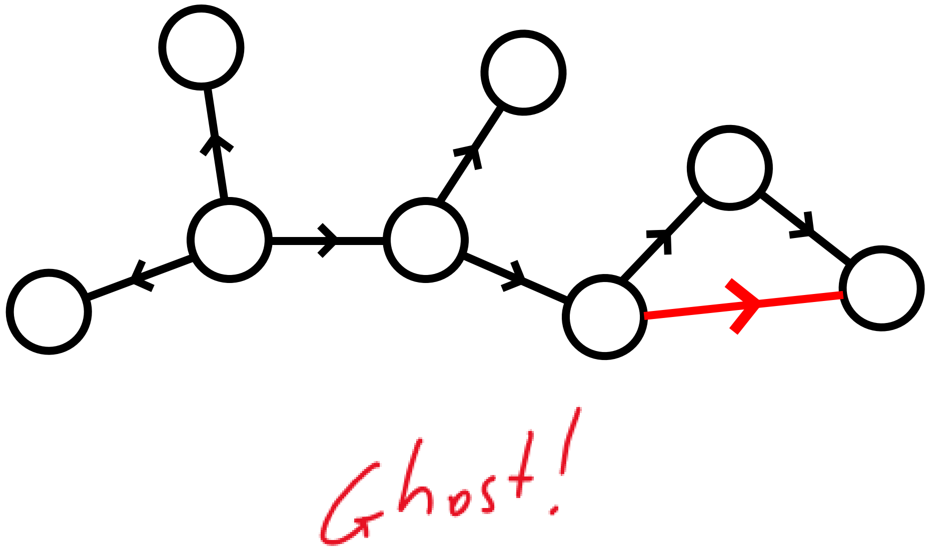

Part 2: dimensional deconstruction

Dimensional deconstruction

The basic idea (for our purposes):

Take some gravitational theory in 5D and discretise the extra dimension on a lattice

Dimensional deconstruction

- Arkani-Hamed, Schwartz, Georgi [hep-th/0302110]
- Deffayet, Mourad [hep-th/0311124]
- de Rham, Matas, Tolley [1308.4136]

More precisely:

- Consider multi-gravity with chain-type interactions

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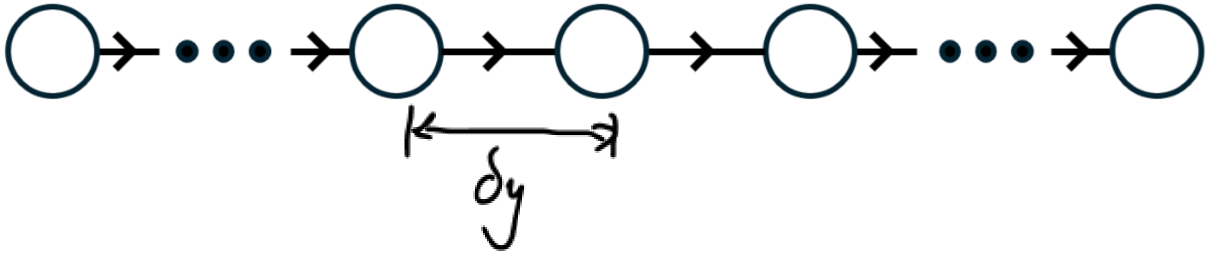

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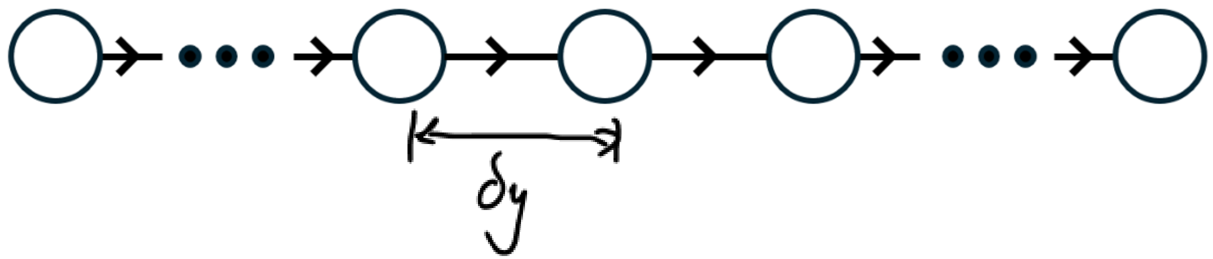

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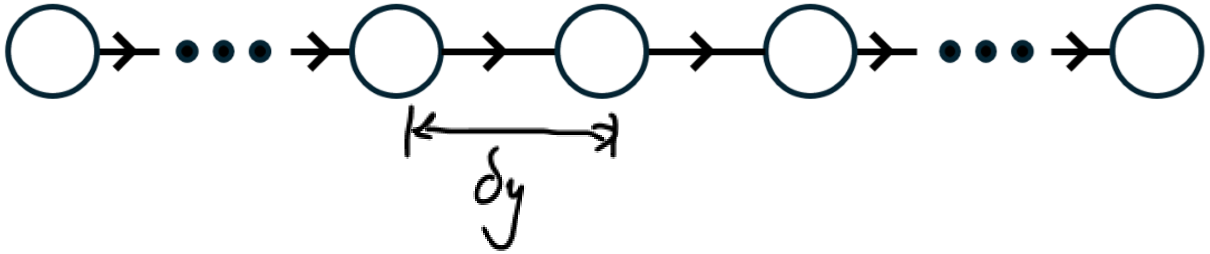

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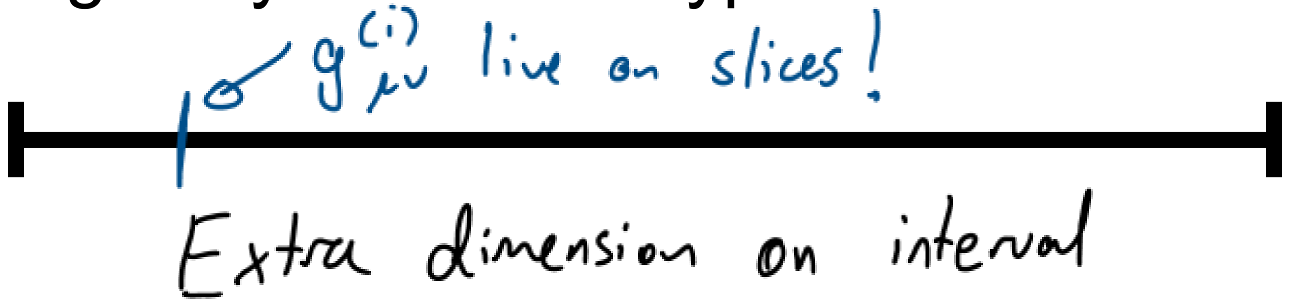

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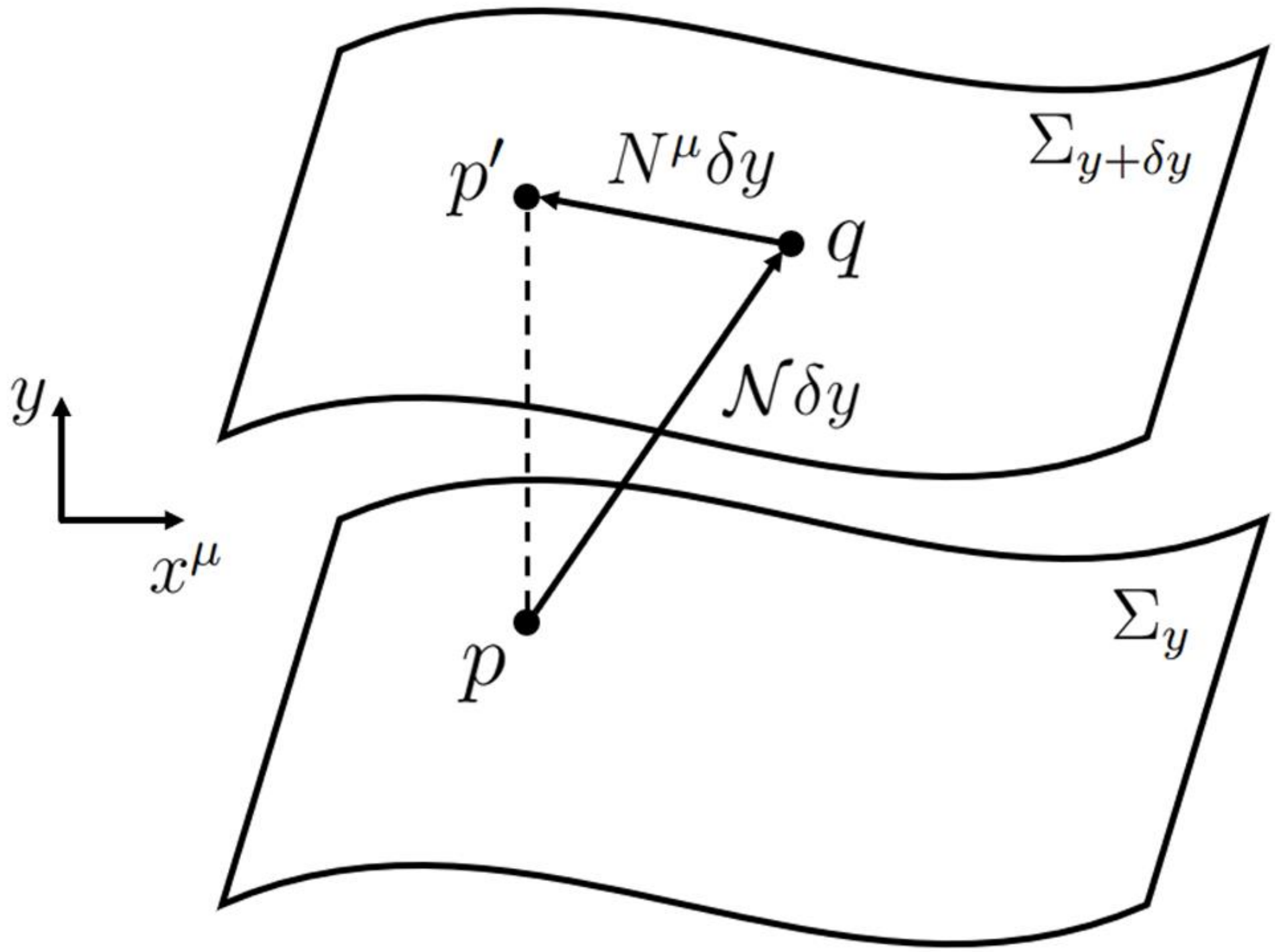
- Each $g_{\mu\nu}^{(i)}$ now is an induced metric on a constant y hypersurface

ADM decomposition

$$\mathrm{d}s^2 = G_{MN}\mathrm{d}x^M \mathrm{d}x^N = g_{\mu\nu}\left(\mathrm{d}x^\mu + N^\mu \mathrm{d}y\right)\left(\mathrm{d}x^\nu + N^\nu \mathrm{d}y\right) + \mathcal{N}^2 \mathrm{d}y^2$$



- $\mathcal{N}$ is the <u>lapse</u>
- N^μ is the <u>shift</u>

Deconstructing standard multi-gravity

We expect 3 kinds of fields in 4D:

$$g_{\mu\nu}^{(i)}(x) = g_{\mu\nu}(x, y_i)$$

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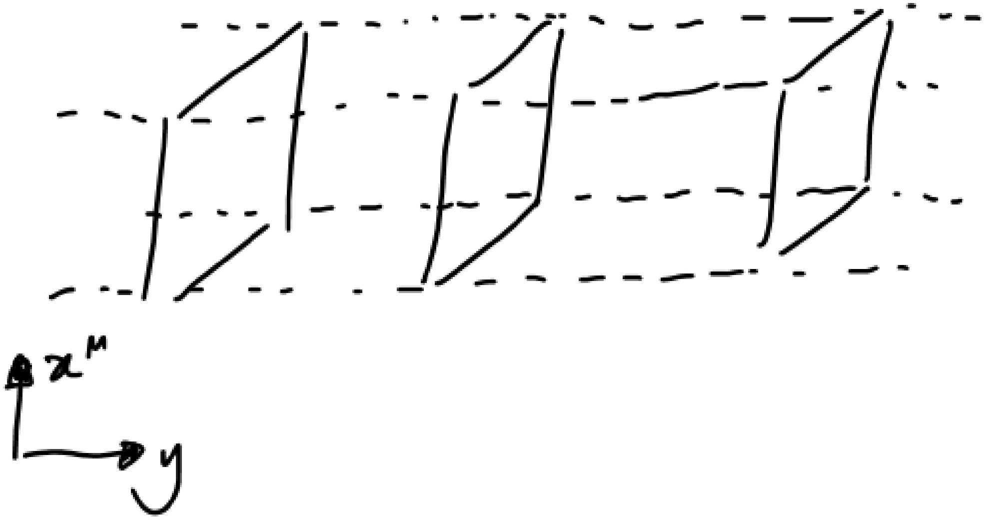

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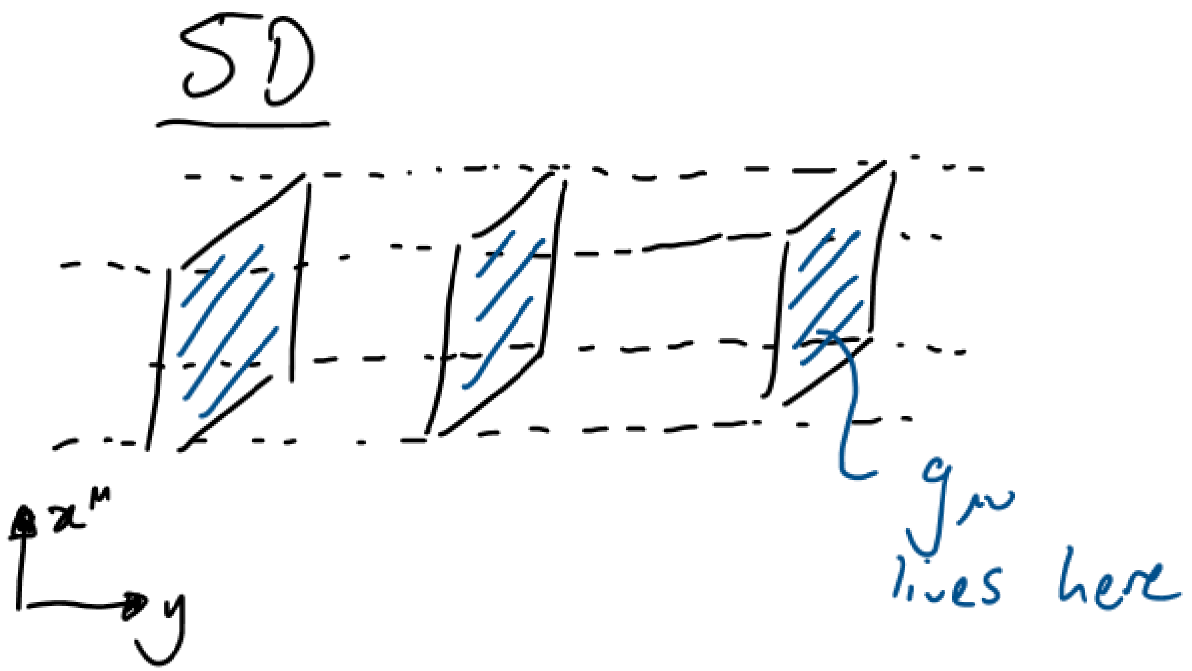

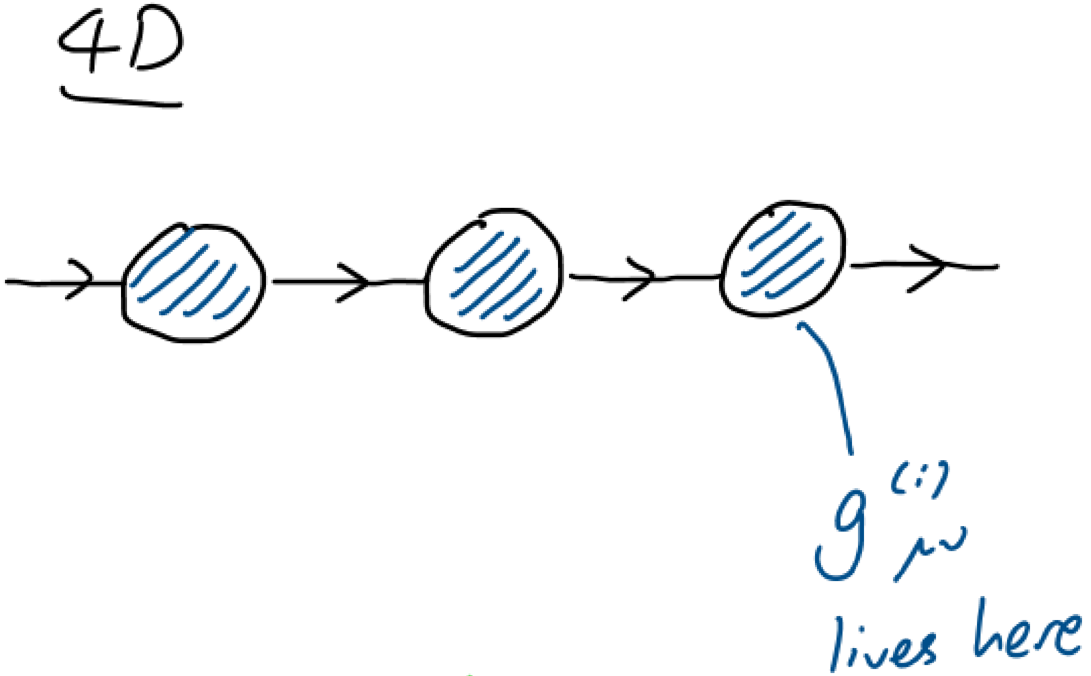

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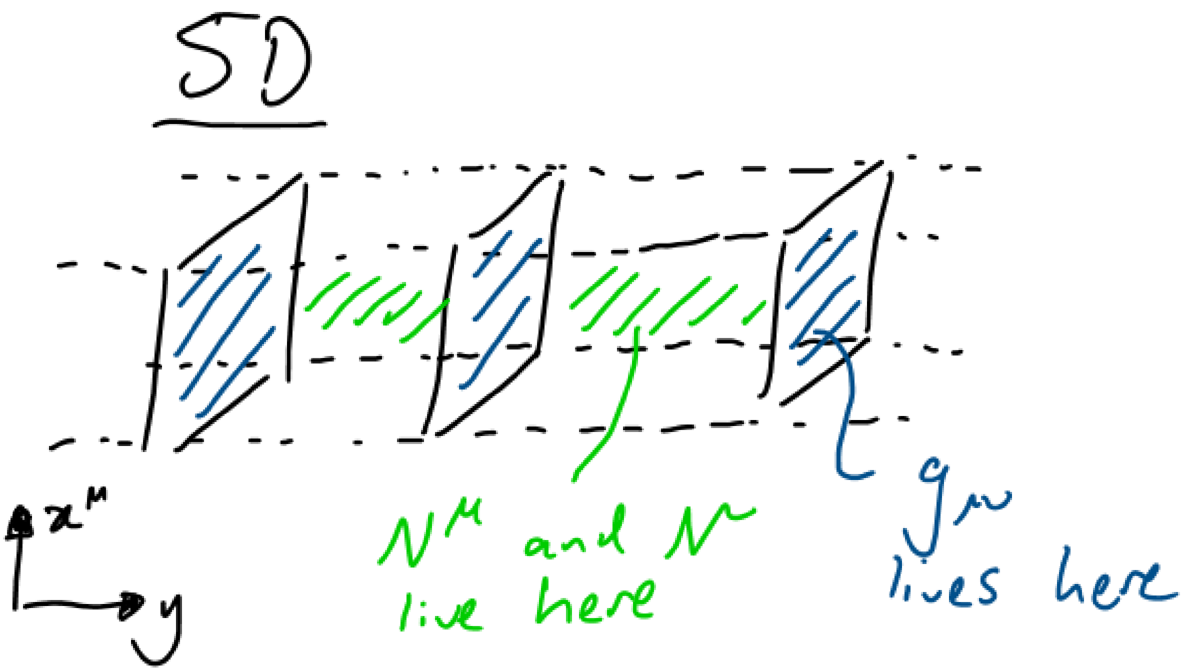

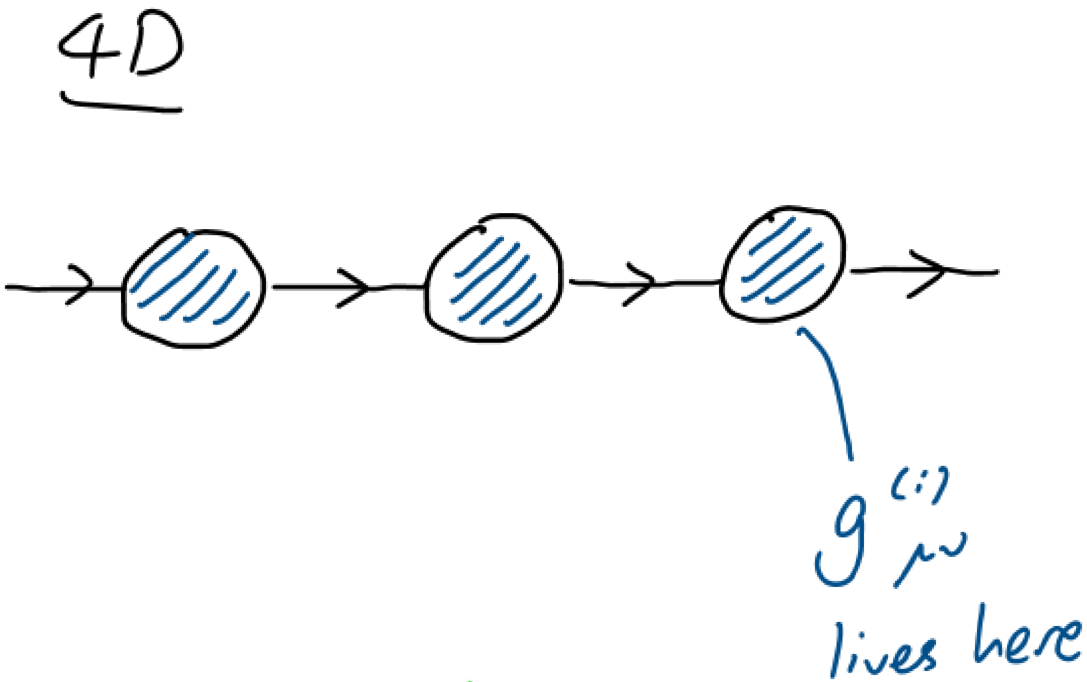

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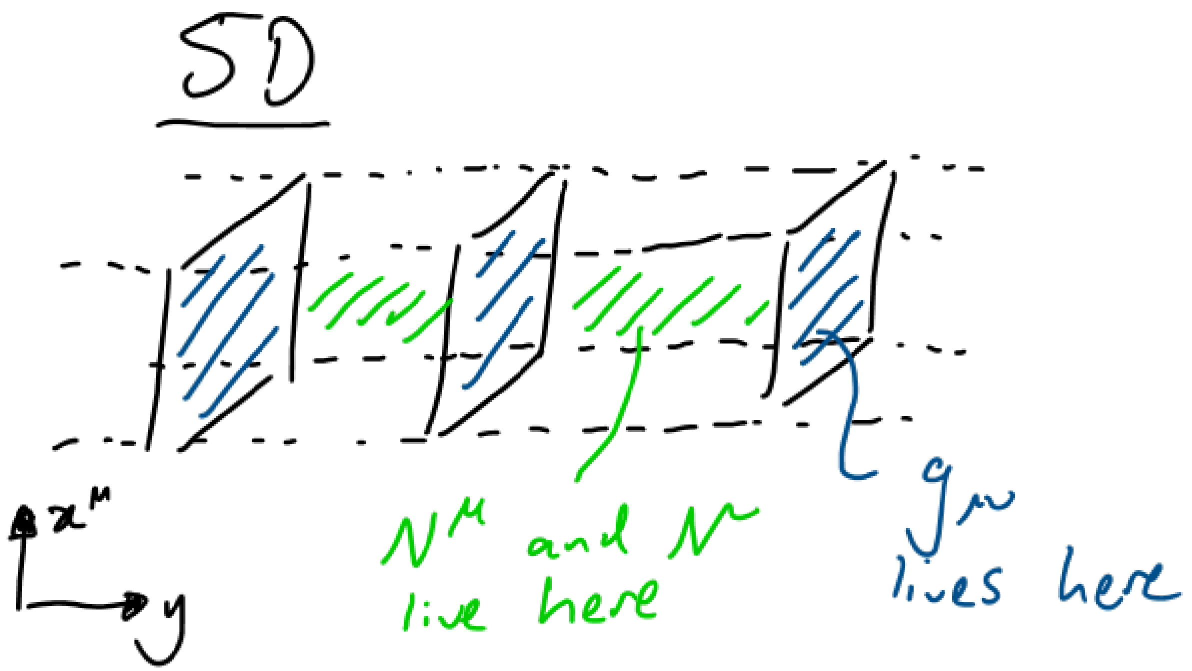

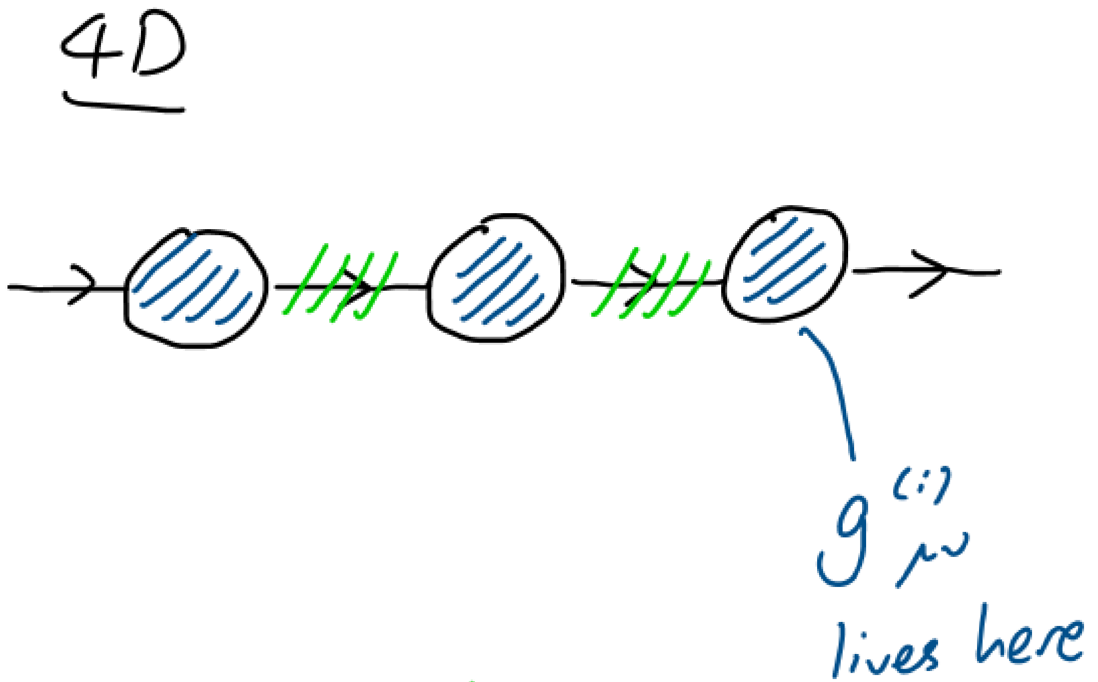

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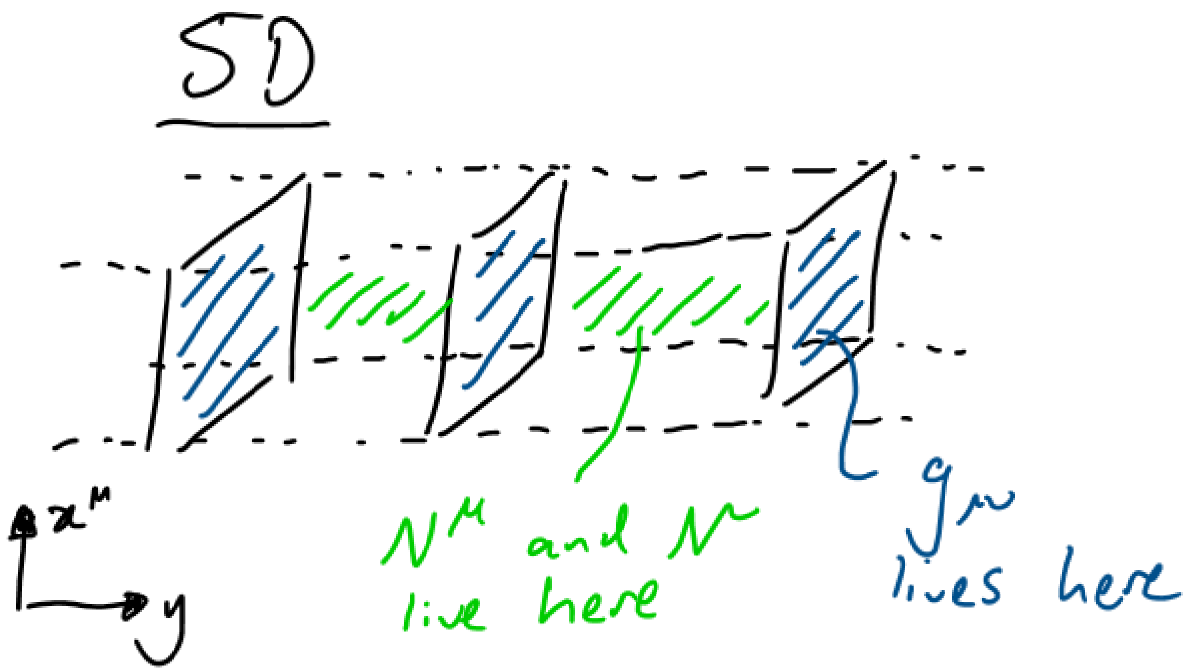

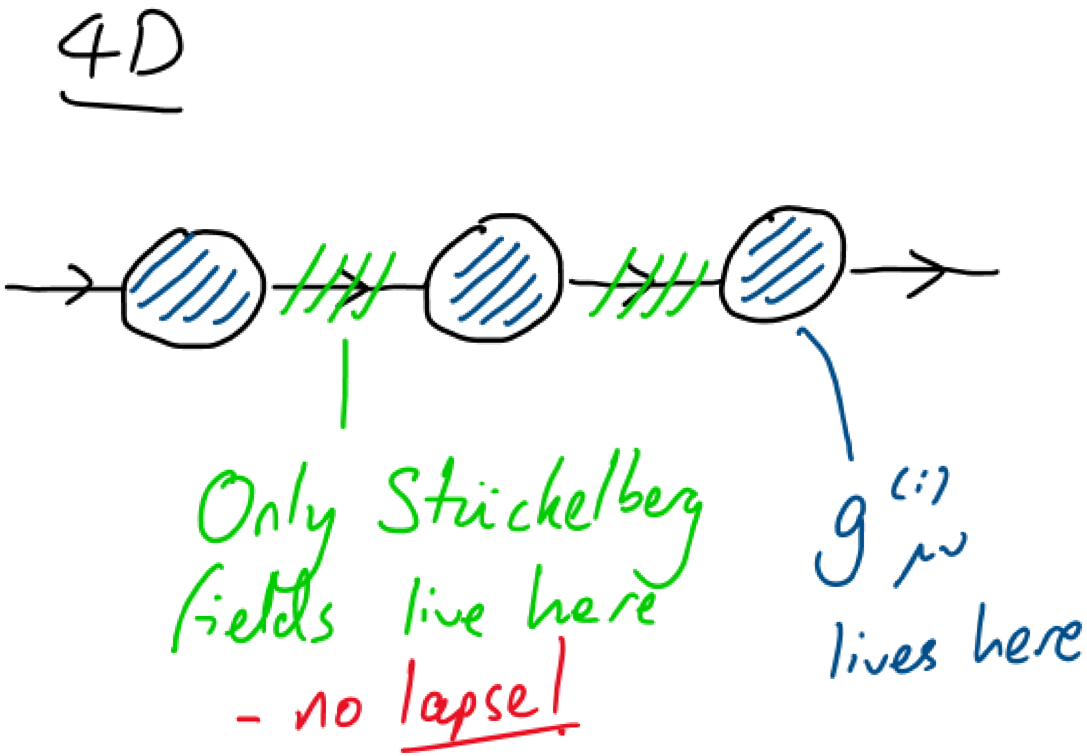

A more precise statement of the problem

The 5D GR action in 4+1 ADM variables is:

$$I_{(5)} = \int_{\mathcal{M}} \mathrm{d}^4x\mathrm{d}y\sqrt{-g}\mathcal{N}\left[\frac{M^3_{(5)}}{2}\left(R_{(4)} + K^2 - K_{\mu\nu}K^{\mu\nu}\right) - 2\Lambda_5\right] + \int_{\partial\mathcal{M}} M^3_{(5)}K$$

where the extrinsic curvature is:

$$K_{\mu\nu} = \frac{1}{2\mathcal{N}}\mathcal{L}_{D_y}g_{\mu\nu} = \frac{1}{2\mathcal{N}}\eta_{ab}\left(e_\mu{}^a\mathcal{L}_{D_y}e_\nu{}^b + e_\nu{}^b\mathcal{L}_{D_y}e_\mu{}^a\right)$$

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Only when we fix $\mathcal{N} = 1$ do we get the replacement:

$$K_{\mu\nu} \to \frac{1}{2}\left(e_{\mu a}^{(i)}\frac{e_\nu^{(i+1)a} - e_\nu^{(i)a}}{\delta y} - e_{\nu a}^{(i)}\frac{e_\mu^{(i+1)a} - e_\mu^{(i)a}}{\delta y}\right)$$

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$$S_{\mu\nu} = e^{(i)}_{\mu a} e^{(j)a}_{\nu}.$$

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- No analogue of diff invariance along the extra dimension
- No analogue of the extra dimensional Hamiltonian constraint, which gave crucial structure to 5D GR
- Can be linked directly to massive/multi-gravity's low strong coupling scale

$$\Lambda_{\rm SC} = (m_1^2 M_{\rm Pl})^{\frac{1}{3}} \quad \Lambda_{\rm SC} \sim \left(\frac{M_{(5)}}{L}\right)^{\frac{1}{2}}$$

- de Rham, Matas, Tolley [1308.4136]

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But – key ingredients are still present: the obvious question is, "If not standard multi-gravity, then what?"

Part 3: an improved deconstruction procedure

Keeping the lapse

Keeping the lapse in the discretisation procedure leads to a 4D theory of the form:

$$I_K = \sum_{i=0}^{N-1} \int \mathrm{d}^4 x \sqrt{-\det g_{(i)}}\, \mathcal{N}_i \left(\frac{M_{(4)}^2}{2} R_{(i)} - 2\Lambda_4 \right)$$

$$I_V = - \sum_{i,j,k,l=0}^{N-1} \int_{\mathcal{M}_4} \varepsilon_{abcd} T_{ijkl} \left(\boldsymbol{\mathcal{N}} \right) e^{(i)a} \wedge e^{(j)b} \wedge e^{(k)c} \wedge e^{(l)d}$$

Keeping the lapse

Keeping the lapse in the discretisation procedure leads to a 4D theory of the form:

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Standard multi-gravity non-minimally coupled to a collection of scalar fields – scalar-tensor multi-gravity (STMG) – still ghost free!

Recovering 5-dimensional GR

I used the example of 5D brane cosmology to demonstrate that in the continuum limit ($N \to \infty,\ \delta y \to 0$):

Deconstructed STMG theory:

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$$\mathcal{N}_i\left[M_{(4)}^2 G_{\mu\nu}^{(i)} + 2\Lambda_4 g_{\mu\nu}^{(i)}\right] + W_{\mu\nu}^{(i)}(\boldsymbol{\mathcal{N}}) = T_{\mu\nu}^{(i)} + M_{(4)}^2\left[\nabla_\mu^{(i)}\nabla_\nu^{(i)}\mathcal{N}_i - g_{\mu\nu}^{(i)}\Box^{(i)}\mathcal{N}_i\right]$$

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5D GR:

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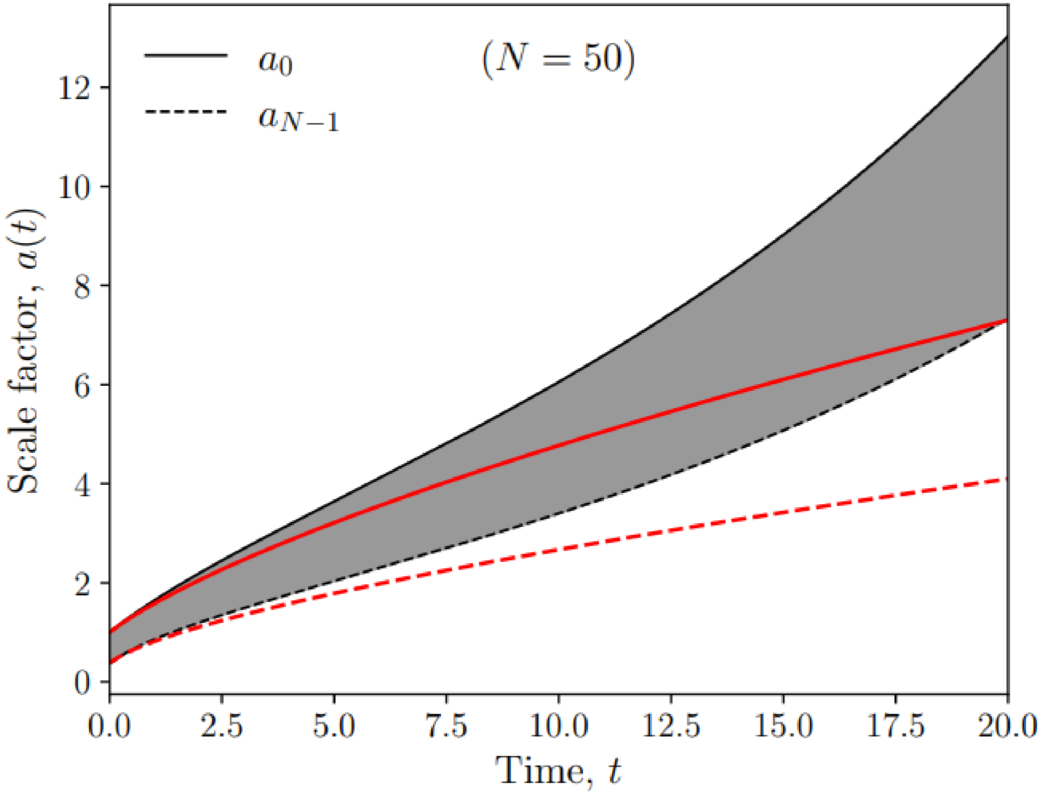

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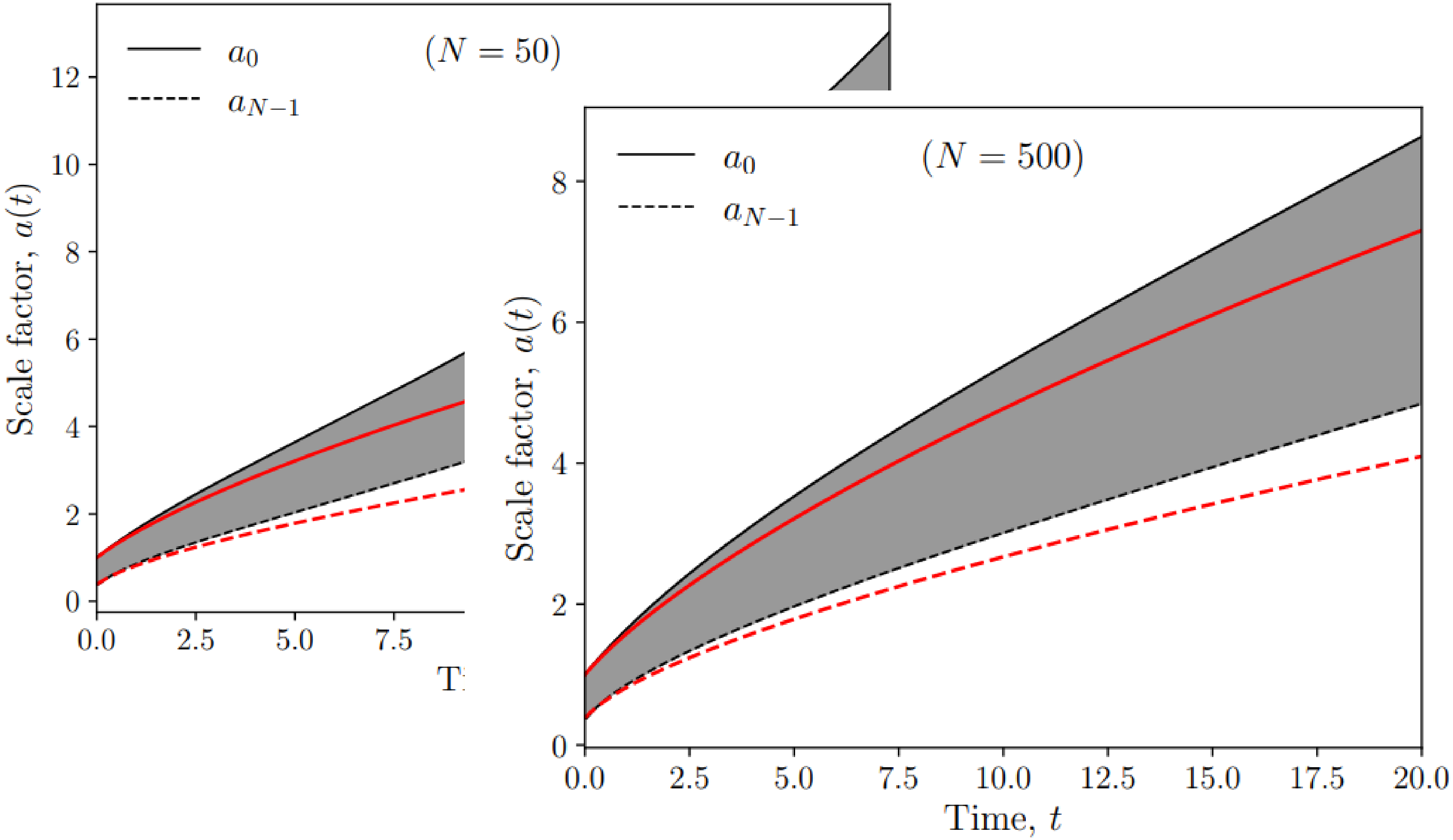

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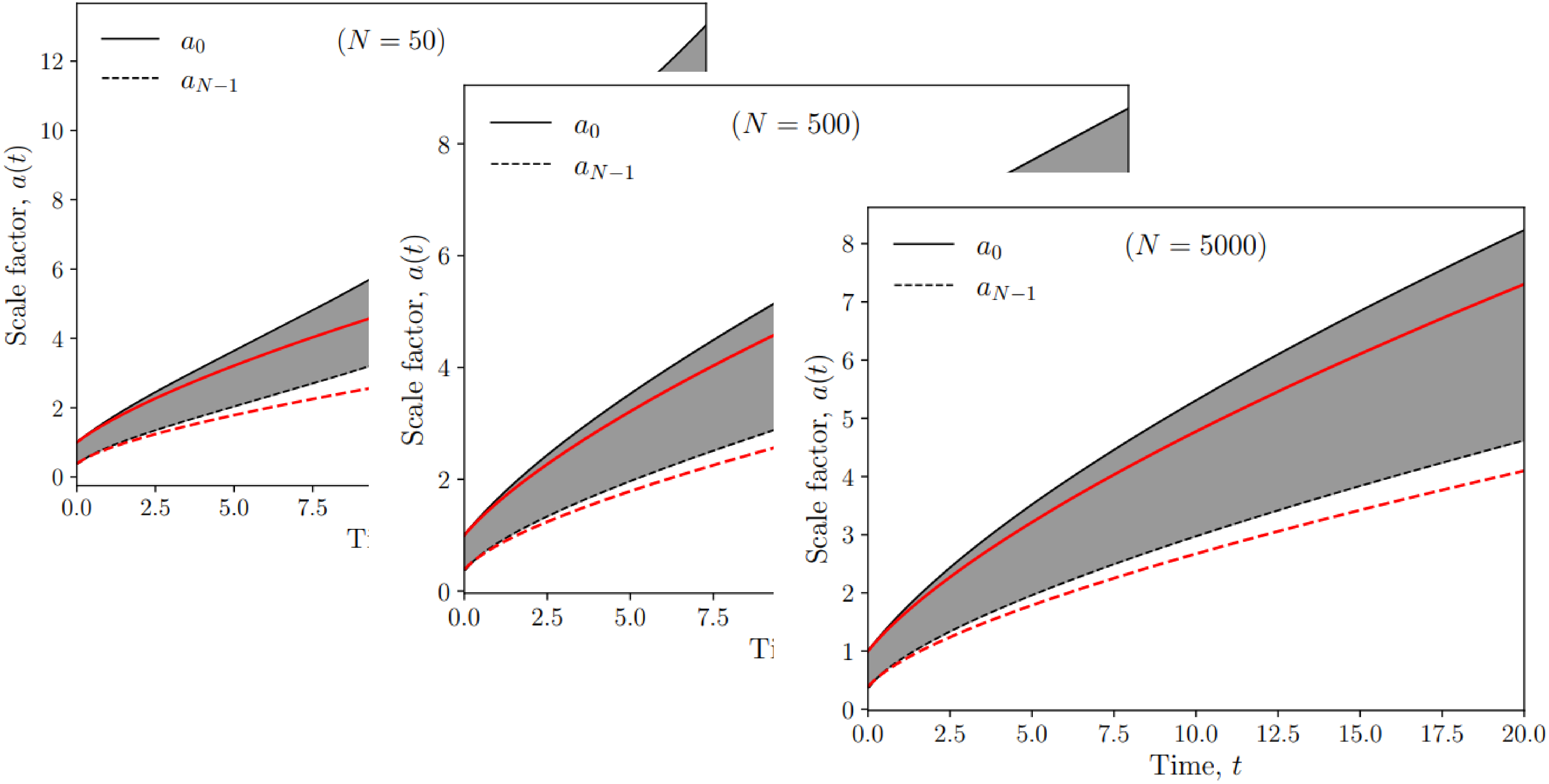

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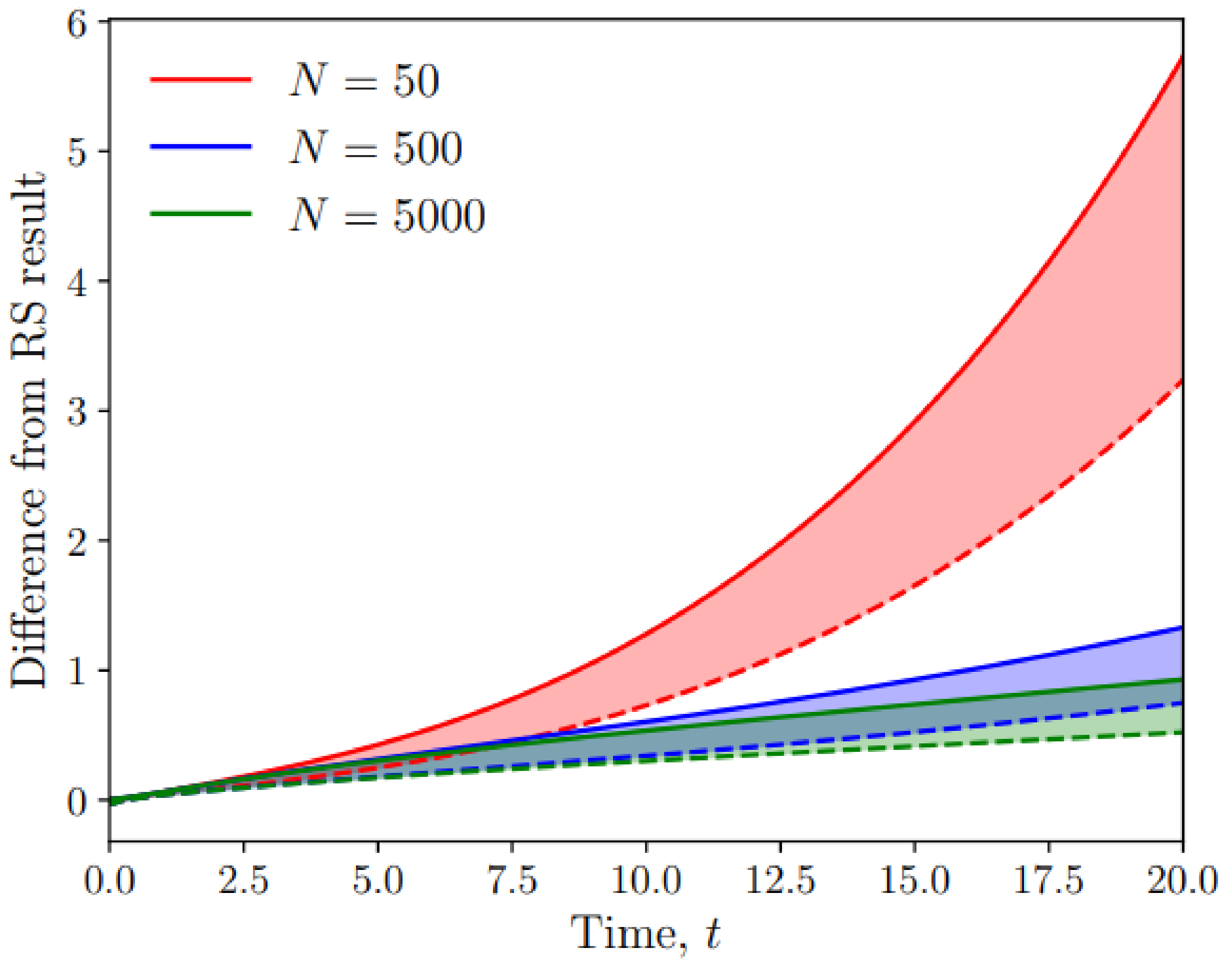
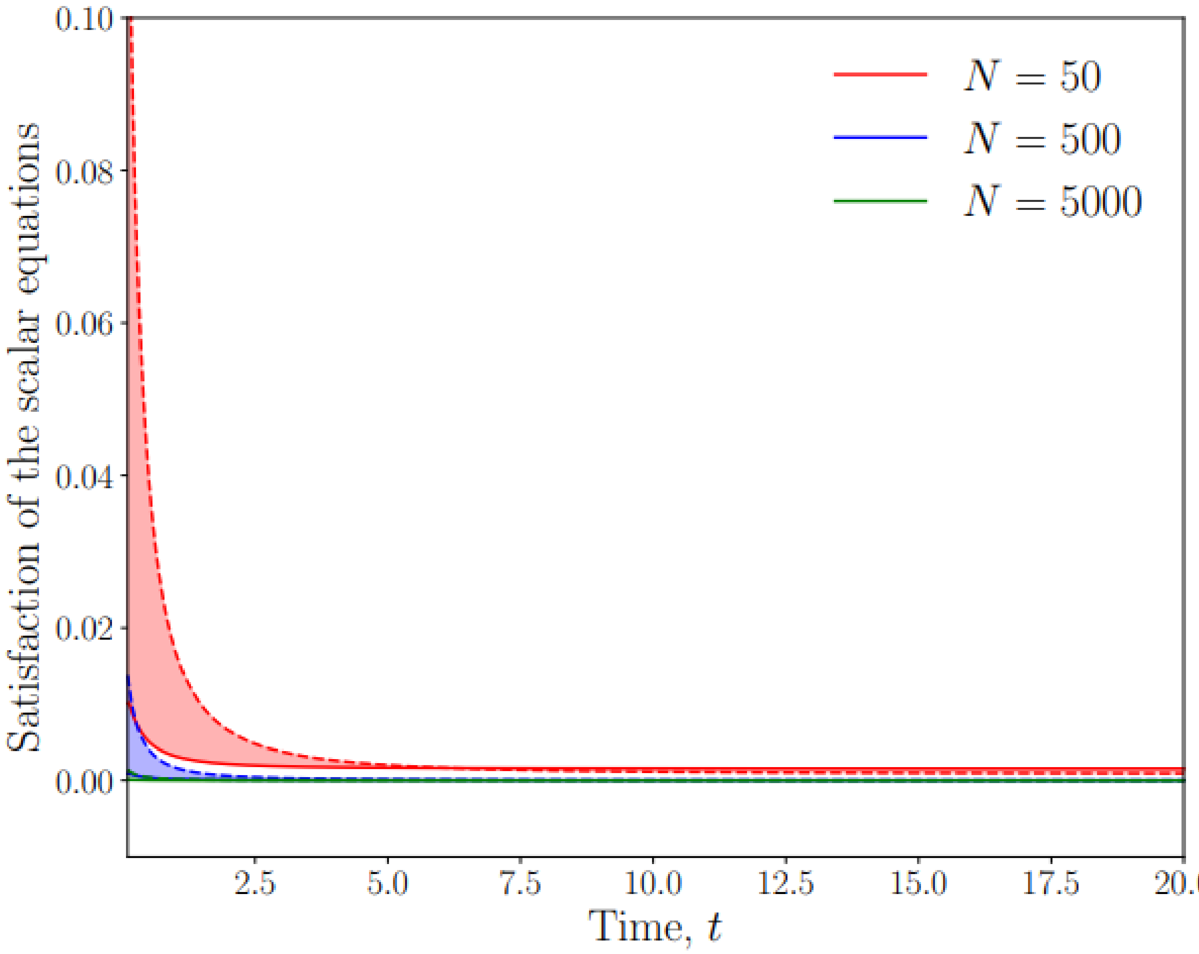


Dynamical eqs (RS Friedmann eqs)



Scalar eqs (Hamiltonian constraint)

Scalar-tensor multi-gravity

Readily generalises to arbitrary STMG theories, which are all ghost free, perfectly viable modified gravity EFTs in their own right

$$I_K = \sum_{i=0}^{N-1} \int \mathrm{d}^D x \sqrt{-\det g_{(i)}} \left[\frac{F_i(\phi_i)}{2} R_{(i)} - \frac{G_i(\phi_i)}{2} g^{(i)\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - U_i(\phi_i) \right]$$

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Potentially interesting phenomenology e.g. screening mechanisms:

$$\tilde{M}_i^{D-2} \tilde{G}_{\mu\nu}^{(i)} + \tilde{W}_{\mu\nu}^{(i)}(\tilde{\boldsymbol{\chi}}) = A_i^2(\tilde{\chi}_i) T_{\mu\nu}^{(i,m)} + \tilde{T}_{\mu\nu}^{(i,\tilde{\chi}_i)}$$

$$\tilde{\Box}^{(i)} \tilde{\chi}_i = \frac{\partial \tilde{V}_i}{\partial \tilde{\chi}_i} + \tilde{\mathbb{X}}_i(\tilde{\boldsymbol{\chi}}) - A_i^3(\tilde{\chi}_i) \frac{\partial A_i}{\partial \tilde{\chi}_i} T_{(i,m)}$$

(χ and Tilde quantities in Einstein frame)

Summary and future directions

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- Main takeaway: can’t get standard massive/multi-gravity from an extra dimension consistently, but can get multi-gravity + scalar fields

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- Main takeaway: can’t get standard massive/multi-gravity from an extra dimension consistently, but can get multi-gravity + scalar fields
- New theory STMG is a perfectly viable ghost free modified theory of gravity away from continuum limit
- Lots of potentially interesting phenomenology for the new scalars (e.g. screening mechanisms)
- Can potentially use it to ask questions about higher dimensional physics through a lower dimensional lens (e.g. Gregory-Laflamme instability exists in both 5D GR and 4D multi-gravity)

Backup slides

Interaction structure

Let's examine the potential in more detail:

$$I_V = -\sum_{i_1 \ldots i_D = 0}^{N-1} \int_{\mathcal{M}_D} \varepsilon_{a_1 \ldots a_D} T_{i_1 \ldots i_D} e^{(i_1) a_1} \wedge \ldots \wedge e^{(i_D) a_D}$$

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$e^{(i)a} = e_\mu^{(i)a} dx^\mu$

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- Constructed from the vielbeins of each metric: $g_{\mu\nu}^{(i)} = e_\mu^{(i)a} e_\nu^{(i)b} \eta_{ab}$

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↪ only permit terms like $T_{iii\dots}$, $T_{iij\dots}$, $T_{ijj\dots}$ $\Rightarrow$ $e^{(i)\mu}{}_a e^{(j)}{}_\nu{}^a$.

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- Does the number of d.o.f. check out?
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