Towards UV-completing supersymmetric extensions of the Standard Model

Gabriel Picanço[1,2]

in collaboration with: Andrew D. Bond[1], Gudrun Hiller[2], and Daniel F. Litim[1]

[1]University of Sussex - UK

[2]Technische Universität Dortmund - Germany

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Outline

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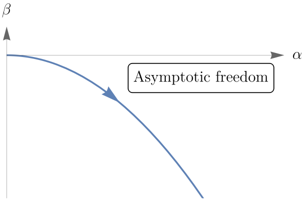
Asymptotic freedom (AF), Landau pole, and asymptotic safety (AS)



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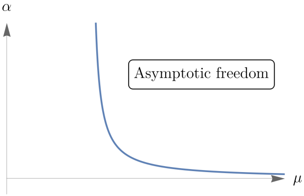

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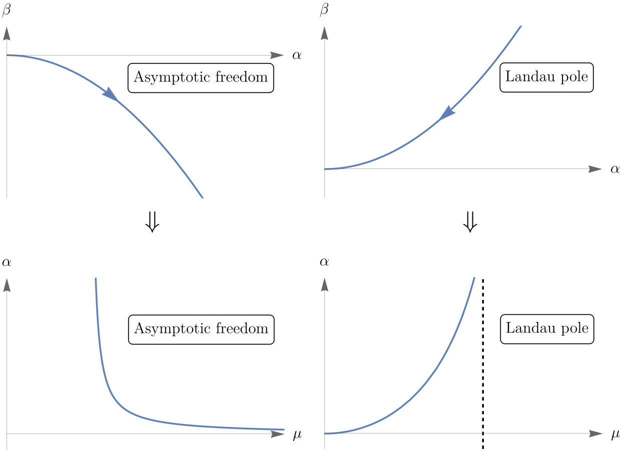

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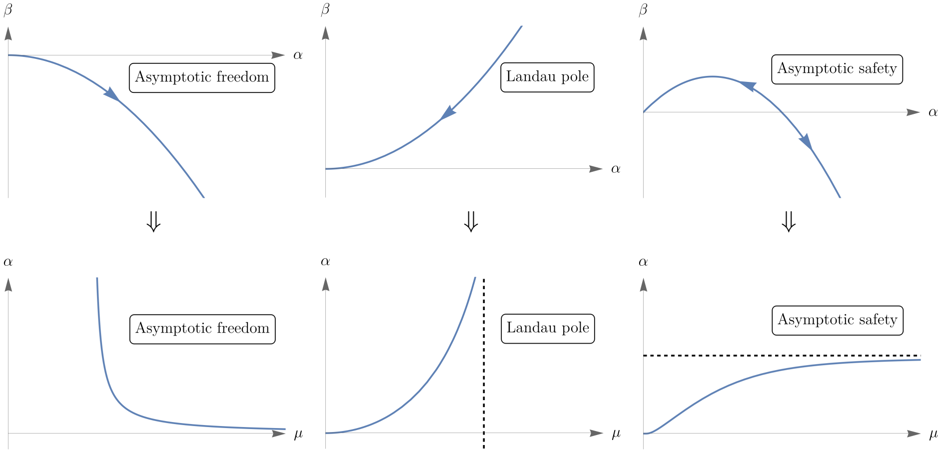

First instance of AS in 4D matter theories without gravity: [Litim and Sannino, 2014]. Various results followed:

Non-supersymmetric theories:

- General theorems on asymptotic safety of scalar-fermion-gauge theories, e.g. [Bond and Litim, 2017b].
- Two-loop analysis of FPs and phase diagrams [Bond and Litim, 2018].
- Valuable for model-building applications addressing BSM phenomenology[a].

[a] *e.g.* [Bause et al., 2022; Bißmann et al., 2021; Hiller et al., 2019a,b, 2020; Kowalska et al., 2017]

Supersymmetric theories:

- First perturbative results ensuring AS: [Bond and Litim, 2017a].
- First exact conformal windows of interacting ultraviolet FP [Bond and Litim, 2022].
- Steps towards UV-completing extensions of the MSSM, e.g. [Hiller et al., 2022]

- New way of considering supersymmetry, from asymptotic safety:
  - physically motivated: UV completion beyond asymptotic freedom.
  - highly predictive and falsifiable.

- Theoretical framework advantages:[1]
  - Exact theorems constraining physical theories,
  - Infinite-order beta functions,
  - Uniqueness of fixed points, etc.

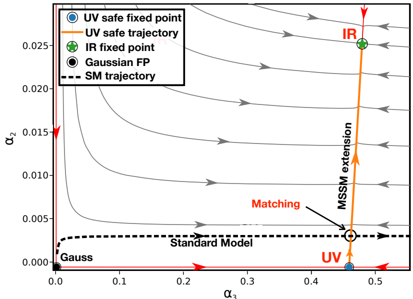

Fig. 1: Matching at LO of UV-complete supersymmetric theory with the SM. Extracted from [Hiller et al., 2022]

[1] [Intriligator and Wecht, 2003; Novikov et al., 1983; Seiberg, 1995] and more

Outline

Family of template models for asymptotic safety in particle physics

- Asymptotic safety of SYM theories requires semi-simple gauge groups and Yukawa couplings [Bond and Litim, 2017b; Hiller et al., 2022].

- Consider semi-simple ($\mathcal{N} = 1$) super-Yang–Mills theories with gauge group $SU(N_1) \times SU(N_2)$ with couplings α_1 and α_2.

- Matter content: chiral superfields (ψ, Ψ, χ, Q) with flavour multiplicities (N_F, N_Ψ, N_F, N_Q) charged according to

Chiral superfields	ψ_L	ψ_R	Ψ_L	Ψ_R	χ_L	χ_R	Q_L	Q_R
$\mathbf{SU(N_1)}$	$\overline{\square}$	$\square$	$\square$	$\overline{\square}$	1	1	1	1
$\mathbf{SU(N_2)}$	1	1	$\square$	$\overline{\square}$	$\overline{\square}$	$\square$	$\overline{\square}$	$\square$
multiplicity	N_F	N_F	N_Ψ	N_Ψ	N_F	N_F	N_Q	N_Q

Fig. 2: Chiral superfields and their gauge charges and flavour multiplicities.

- Interaction given by Yukawa superpotential $W[\psi, \Psi, \chi]$ such that $W = y\,\mathrm{Tr}[\psi_L \Psi_L \chi_L + \psi_R \Psi_R \chi_R]$.

Fixed-point (FP) structure of the theory

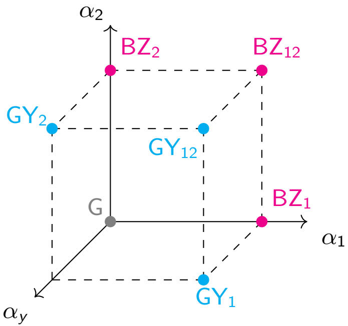

Fig. 3: Illustration of the 7 fixed points that may show up in such theories:

- the free Gaussian (G) FP, in gray,
- the 3 interacting Banks-Zaks (BZ) FPs, in magenta, and
- the 3 interacting gauge-Yukawa (GY) FPs, in cyan.

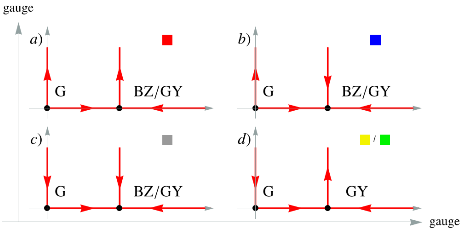

Fig. 4: Schematic flow diagrams for the gauge couplings in all scenarious:

- *a*) and *b*) represent asymptotically free theories,
- *c*) represents effective theories, and
- *d*) represents potentially asymptotically safe theories.

Parameters defining the family of theories

- Theory characterized by field multiplicities $(N_1, N_2, N_F, N_Q, N_\Psi)$.
- β_i and γ_i can be rewritten in terms of (R, ϵ, P, N_Ψ), where

$$R := \frac{N_2}{N_1}\,, \quad \epsilon := \text{one-loop coeff. of } \beta_1\,, \quad \text{and} \quad P := \frac{\text{one-loop coeff. of } \beta_2}{\text{one-loop coeff. of } \beta_1}\,. \qquad (1)$$

- Notice: asymptotic safety requires $P < 0$!

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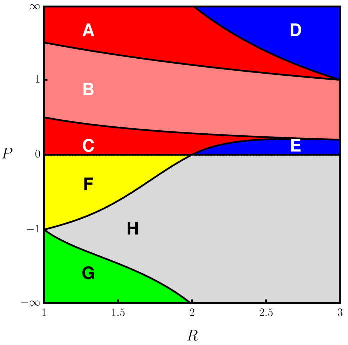
- Notice: asymptotic safety requires $P < 0$!

- Positivity of (N_1, N_2, N_F, N_Q) impose $0 \lesssim R \lesssim 3$ (depending on ϵ and P).

- Large-N Veneziano limit: (R, ϵ, P) become continuous. In the strictly perturbative $|\epsilon|, |P\epsilon| \ll 1$ limit, constraints reduce to

$$1 < R < \tfrac{3}{N_\Psi} \qquad (\implies N_\Psi = 1 \text{ or } 2!) \tag{2}$$



case	region	eps	G	BZ1	BZ2	BZ12	GY1	GY2	GY12	UV		IR 0	IR y
			complete asymptotic freedom (eps < 0, P > 0)										
1	**A**	–	1---	2--+	3-++		5-++	6-++	7+++	**1**	**AF**	**3**	**7**
2	**B**	–	1---	2--+	3--+	4-++	5-++	6-++	7+++	**1**	**AF**	**4**	**7**
3	**C**	–	1---	2-++	3--+		5-++	6-++	7+++	**1**	**AF**	**2**	**7**
4	**D**	–	1---	2--+	3-++		5-++	6+++		**1**	**AF**	**3**	**6**
5	**E**	–	1---	2-++	3--+		5+++	6-++		**1**	**AF**	**2**	**5**
			asymptotic safety or effective theories (P < 0)										
6	**F**	–	1--+	2-++			5-++		7+++	**5**	**AS**	**2**	**7**
7	**G**	–	1--+	2-++			5+++			eff.		**2**	**5**
8	**H**	–	1--+	2-++			5+++			eff.		**2**	**5**
9	**F**	+	1--+		3-++			6+++		eff.		**3**	**6**
10	**G**	+	1--+		3-++			6-++	7+++	**6**	**AS**	**3**	**7**
11	**H**	+	1--+		3-++			6+++		eff.		**3**	**6**

Fig. 5: Parametric conformal windows and relevancy of all FPs in LO.

Effect of finite ϵ on conformal windows at NLO: GY_1

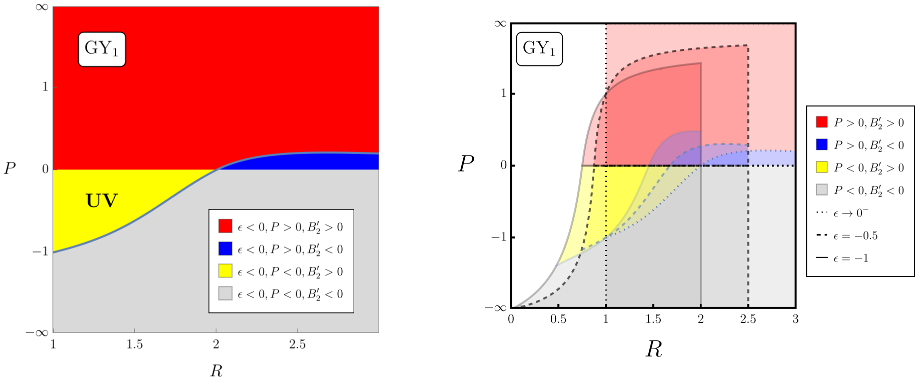


Fig. 6: Parametric conformal windows of the GY_1 fixed point at LO and NLO.

LO vs NLO vs infinite-order conformal windows

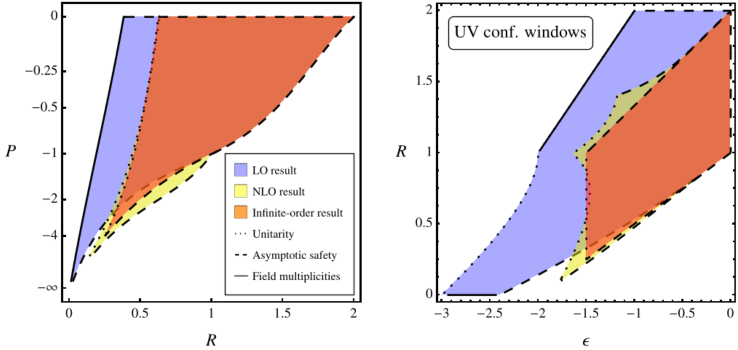


Fig. 7: Comparison of LO, NLO, and infinite-order GY_1 conformal windows projected at the (R, P) and (ϵ, R) planes. Constraints resulting in each boundary are identified.

Application: model building for BSM physics

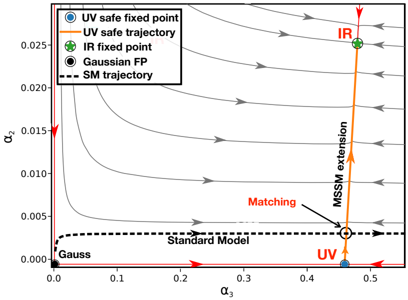


Fig. 1, again: Matching at LO of UV-complete supersymmetric theory with the SM.

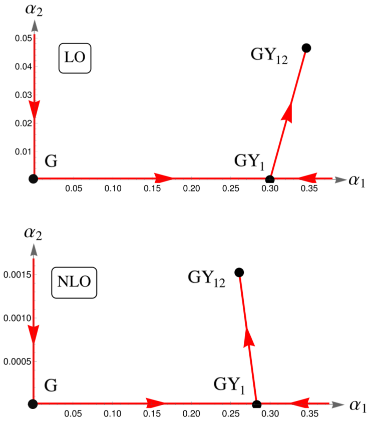


Fig. 8: Higher-order effects raising the matching scale μ_{SM}.

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Outlook and future directions

- At LO and NLO: full classification of FPs, completing the phase diagram.
- Rich variety of behaviours for non-AS theories.
- At infinite-order: qualitative NLO conformal306 windows confirmed.
- For BSM physics: mechanisms of compatibilisation of AS MSSM extensions found.

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- Rich variety of behaviours for non-AS theories.
- At infinite-order: qualitative NLO conformal306 windows confirmed.
- For BSM physics: mechanisms of compatibilisation of AS MSSM extensions found.
- Some future directions:
  - Couplings and critical exponents from NSVZ.
  - Abelian sector.
  - Formal aspects of QFTs with exact results.
  - Interacting UV FPs in MSSM extensions including new quarks or leptons.
  - Phenomenological signatures of MSSM extensions with new dark gauge sectors.

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Thanks for your attention!

General two-loop gauge beta functions

- Gauge beta functions w/o Yukawas in general QFTs, gauge group $G = \prod_i G_i$:

$$\beta_i = \alpha_i^2(-B_i + C_{ij}\alpha_j) + \mathcal{O}(\alpha^4). \quad (3)$$

– Necessary for UV interacting FP: at least one $B_i < 0$.

– From [Bond and Litim, 2017b]: $B_i \leq 0 \implies C_{ij} \geq 0$.

– Without Yukawa couplings, interacting FPs (Banks–Zaks-type) can only be IR!

- Yukawa sector comes to rescue:

$$\beta_i \to \beta_i = \alpha_i^2(-B_i + C_{ij}\alpha_j - 2D_{ij}\alpha_j) + \mathcal{O}(\alpha^4)\,, \quad (4)$$

with $D_{ij} > 0 \implies$ interacting UV fixed points possible!

- Theory characterized by field multiplicities $(N_1, N_2, N_F, N_Q, N_\Psi)$, with couplings

$$\alpha_{1,2} = N_{1,2}\left(\frac{g_{1,2}}{4\pi}\right)^2 \quad \text{and} \quad \alpha_y = N_1\left(\frac{y}{4\pi}\right)^2 , \tag{5}$$

where $g_{1,2}$ are the usual gauge couplings.

- Physical quantities will be functions of (R, ϵ, P, N_Ψ), with

$$R := \frac{N_2}{N_1} , \quad \epsilon := \frac{N_F + N_\Psi N_2 - 3N_1}{N_1} , \quad \text{and} \quad P := \frac{N_1}{N_2}\frac{N_F + N_Q + N_\Psi N_1 - 3N_2}{N_F + N_\Psi N_2 - 3N_1} , \tag{6}$$

where (R, ϵ, P) become continuous in the large-N Veneziano limit.

- Positivity of (N_1, N_2, N_F, N_Q) impose

$$R > 0 , \quad R < \frac{3+\epsilon}{N_\Psi} , \quad \text{and} \quad R > 1 + \left(\frac{1 - RP}{3 + N_\Psi}\right)\epsilon \tag{7}$$

which, in the strictly perturbative limit $|\epsilon|, |P\epsilon| \ll 1$, reduce to

$$P = \text{finite} \quad \text{and} \quad 1 < R < \frac{3}{N_\Psi} \qquad (\implies N_\Psi = 1 \text{ or } 2!) \tag{8}$$

- Leading order in perturbation theory: complete $\mathcal{O}(\epsilon^1)$ expressions. Requires 2-loop in gauge sectors and 1-loop in the Yukawa sector.
- Let $\beta_i := \mathrm{d}\alpha_i/\mathrm{d}\log\mu$, with μ being the RG scale. At LO:

$$\begin{aligned}
\beta_1^{(1+2)} &= \underbrace{2\alpha_1^2\epsilon}_{1\text{ loop}} + \underbrace{2\alpha_1^2\big[(6+4\epsilon)\alpha_1 + 2R\alpha_2 - 4R(3+\epsilon-R)\alpha_y\big]}_{2\text{ loop}},\\
\beta_2^{(1+2)} &= \underbrace{2\alpha_2^2 P\epsilon}_{1\text{ loop}} + \underbrace{2\alpha_1^2\big[(6+4P\epsilon)\alpha_2 + (2/R)\alpha_1 - (4/R)(3+\epsilon-R)\alpha_y\big]}_{2\text{ loop}},\\
\beta_y^{(1)} &= 2\alpha_y\big[-2\alpha_1 - 2\alpha_2 + (4+\epsilon)\alpha_y\big].
\end{aligned} \tag{9}$$

- The FPs are:

	BZ_1	BZ_2	GY_1	GY_2	BZ_{12}	GY_{12}
α_1^*	$-\frac{\epsilon}{6}$	0	$\frac{-\epsilon}{2(R^2-3R+3)}$	0	$\frac{RP-3}{16}\epsilon$	$\frac{R^2(R-2)P-4R+3}{(R-1)(3R^2-8R+9)}\frac{\epsilon}{2}$
α_2^*	0	$-\frac{\epsilon P}{6}$	0	$\frac{-\epsilon PR}{2(4R-3)}$	$\frac{1-3RP}{16R}\epsilon$	$\frac{R(R^2-3R+3)P-R+2}{(R-1)(3R^2-8R+9)}\frac{\epsilon}{2}$
α_y^*	0	0	$\frac{1}{2}\alpha_1^*$	$\frac{1}{2}\alpha_2^*$	0	$\frac{1}{2}(\alpha_1^*+\alpha_2^*)$

- Gauge NSVZ infinite-order β-functions are given by [Novikov et al., 1983]:

$$\beta_1 = \frac{2\alpha_1^2}{F(\alpha_1)}\left[N_F(1-2\gamma_\psi) + N_2 N_\Psi(1-2\gamma_\Psi) - 3N_1\right] \,,$$

$$\beta_2 = \frac{2\alpha_2^2}{F(\alpha_2)}\left[N_F(1-2\gamma_\chi) + N_1 N_\Psi(1-2\gamma_\Psi) + N_Q(1-2\gamma_Q) - 3N_2\right] \,,$$

where $F(\alpha) = 1 - 2C_2^G\alpha$ and C_2^G is the quadratic Casimir in the adjoint.

- Non-renormalization theorem for Yukawa superpotential: non-perturbative Yukawa β-function simply reads

$$\beta_y = 2\alpha_y\left[\gamma_\psi + \gamma_\Psi + \gamma_\chi\right] \,.$$

- Infinite-order expressions for γ_is were obtained using *a*-maximisation [Intriligator and Wecht, 2003] and conformal windows were extracted from unitarity.
- Qualitative NLO conclusions confirmed non-perturbatively, *e.g.* that the GY_1 can be interacting UV FP with or without IR conformality.
- Exact treatment is more limited. More work is needed.

Infinite-order UV GY_1 vs IR GY_{12}

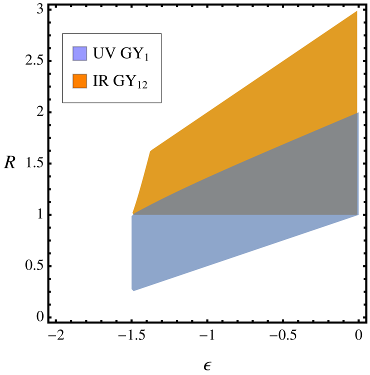

Fig. 8: Projection at the (ϵ, R)-plane of the infinite-order conformal windows of the UV GY_1 and IR GY_{12}.

Effect of finite ϵ on conformal windows at NLO: GY_2

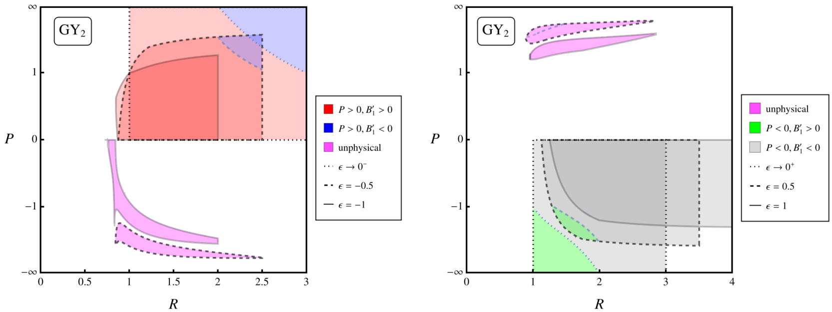

Fig. 9: Parametric conformal windows of the GY_2 fixed point at NLO for various $\epsilon > 0$.

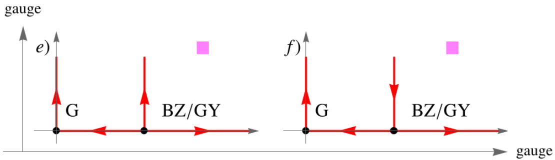

Fig. 10: Unphysical flow diagrams.