The Nature of Dark Energy

Laura Mersini-Houghton
UNC - Chapel Hill
Durham University 2025

The Phantom Dark Energy Model in an FRW background:

Non canonical Lagrangian:

$L = f(\phi)g(X) - V(\phi)$, where: $X = g\mu\nu\ \nabla\mu\phi\nabla\nu\phi/2$.

Assume the background field is homogeneous. Fluctuations around it are functions of space and time.

The multivalued Hamiltonian obtained from L is:

$H + L = \Pi\ \phi =$

$H = f(\phi)(2Xg'(X) - g(X)) + V(\phi)$

where the conjugate momentum is :

$\Pi = \partial L/\partial\dot{}\phi = f(\phi)g'(X)\ \phi.$

Require three conditions:

- *Equation of state is $w<-1$, (Phantom).*
- *Energy density of the field is nonnegative*
- *The speed of sound squared is positive, (stability).*

Bianchi Identity:

$$[\dot{\Pi} + 3H\Pi + \frac{\partial V(\phi)}{\partial \phi} - g(X)\frac{\partial f(\phi)}{\partial \phi}] = 0$$

In phantom

...nergy universe, the Hubble parameter ... time as $H \simeq a(t)^{-\frac{3}{2}(1+w_\phi)}$, where $(1+w_\phi) \leq 0$ ... $a(t) = a(t_0)[-w_\phi + (1+w_\phi)t/t_0]^{\frac{2}{3(1+w_\phi)}}$ is the scale factor, with t_0) the present time. Clearly, the scale factor diverges within a finite time $t \simeq t_0 w_\phi/(1+w_\phi)$ from present day t_0.The Hubble friction term quickly dominates over the potential energy term $\frac{\frac{\partial V(\phi)}{\partial \phi}}{3H} \simeq \frac{1/a(t)^2}{3a(t)^{-3(1+w)/2}} \to 0$ ...onable potential rendering the potential term

insignificant

Stability of perturbations:

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{C_s^2 k2}{a^2}\delta\phi + \frac{V_{\phi\phi}}{g' + 2Xg''}\delta\phi \simeq 0$$

$$\delta^2 E = \frac{\rho_X}{2}\left[\delta\dot{\phi}^2 + \frac{C_s^2 k^2}{a^2}\delta\phi^2 + \frac{V_{\phi\phi}}{\rho_X}\delta\phi^2\right]$$

$$d(\delta^2 E)/dt \simeq -6H\delta\dot{\phi}^2 < 0$$