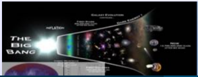
# Probing Spin-2 Ultralight Dark Matter with PTA and Gravitational Wave Detectors

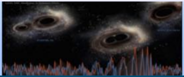
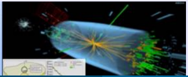
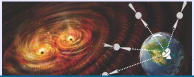
Speaker: Yun-Long Zhang (NAOC)

National Astronomical Observatories,
Chinese Academy of Sciences

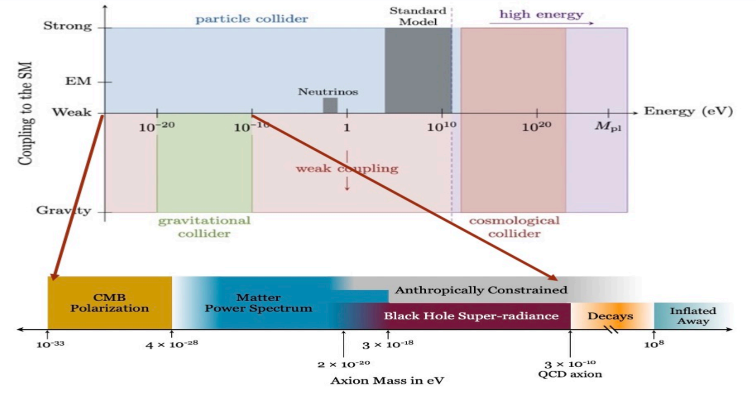
*arXiv:2501.11071, J.R.Zhang, J.Chen, H.S.Jiao, R.G.Cai & Y.L. Zhang*
*Phys.Rev.D 110 (2024) 044052, R.G. Cai, J.R. Zhang & Y.L. Zhang*
*Phys.Rev.D 106 (2022) 066006, S.C Sun, X.Y. Yang & Y.L. Zhang*

*2025, July 25@Durham[email: zhangyunlong@nao.cas.cn]*

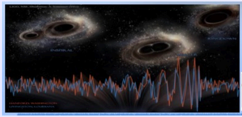
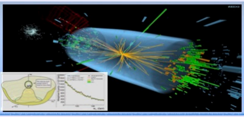
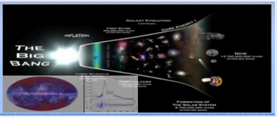


# Motivation: new physics in ultra-low energy

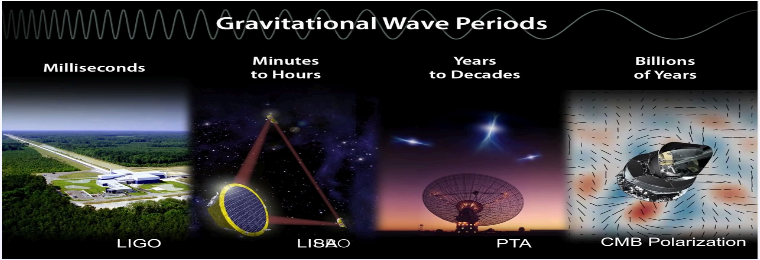


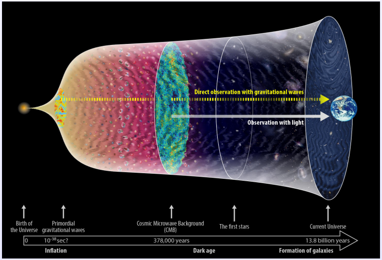

[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]

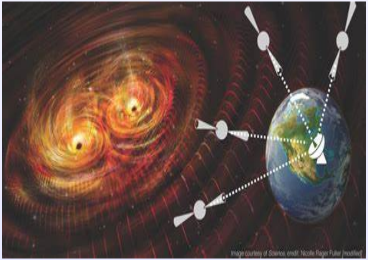




# Multi-band gravitational wave detection



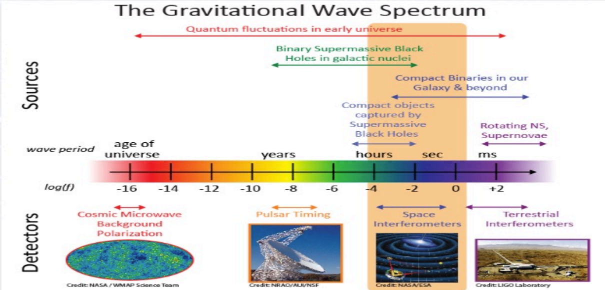


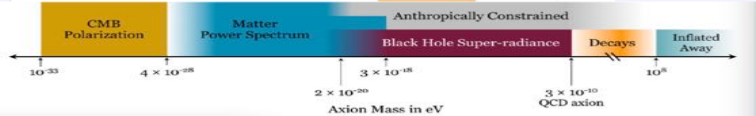



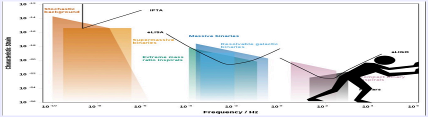



[cf. LISA/ Gravitational Waves Initiative]

# Spectrum of gravitational wave and axion mass



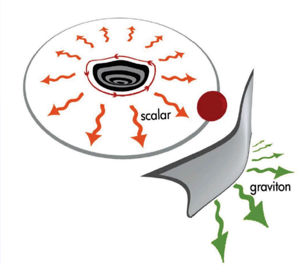



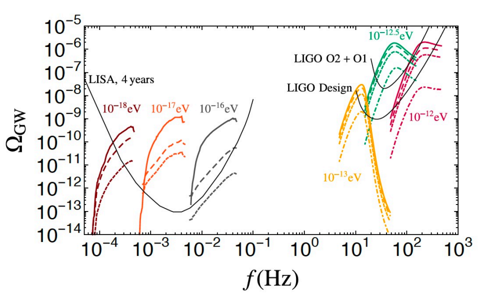




[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

# Axion annihilation and Stochastic GWs

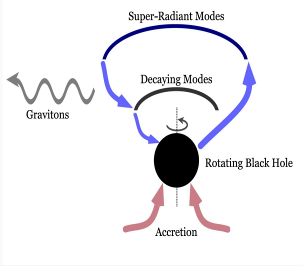


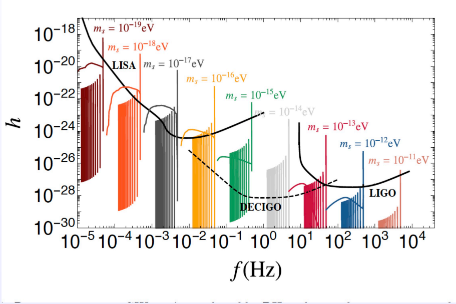



- Axion annihilation $\vartheta + \vartheta \to hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

# Energy level transition and Monochromic GW




- Energy transition $\vartheta^+ \to \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

# GW burst & high frequency detection

GW & ULDM

Ultralight DM

Axion-like DM

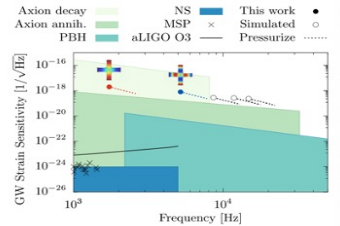
GW and DM

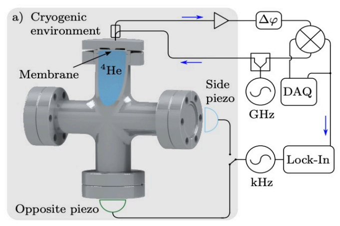
spin-0 DM

spin-2 DM

## Branch Ratio and GWs

- $\frac{\mathrm{Br}(\vartheta\to\mathbf{gg})}{\mathrm{Br}(\vartheta\to\gamma\gamma)} \simeq \frac{\alpha_{\mathbf{g}}^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42}$ergs/s).
- High frequency $h_{(\mathbf{g})} \sim 10^{-26}\left(\frac{1\mathrm{GHz}}{\nu}\right)\left(\frac{P_{(\mathbf{g})}}{P_{(\gamma)}}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$
- Low freq. $h_{(\mathbf{g})} \sim 10^{-21}\left(\frac{10^{-2}\mathrm{Hz}}{\nu}\right)^{1/2}\left(\frac{M_{\mathrm{BH}}}{10^7 M_\odot}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$




cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.
L.H. Du's group, Evidence for chiral graviton modes in fractional quantum Hall liquids

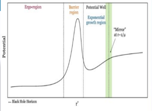
# Ultralight dark matter and gravitational wave

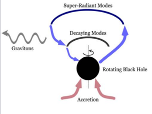
## GW & EM signals from axion like DM

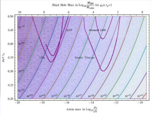
- Axion annihilation $\vartheta + \vartheta \rightarrow h$(Stochastic GW)
  Energy transition $\vartheta^{+} \rightarrow \vartheta^{-} + h$(Monochromic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left(\frac{M_{BH}}{M_{\odot}}\right)\left(\frac{m_{\vartheta}}{10^{-10}\text{eV}}\right)$
- Fast Radio Burst from Axion $\sim \vartheta F\tilde{F}$ $(\vartheta \rightarrow \gamma\gamma)$
- GW burst from Axion $\sim \vartheta R\tilde{R}$ $(\vartheta \rightarrow hh)$

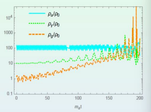
## GW detection and Ultra-light DM

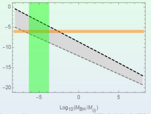
- Tabletop exp: QCD axion & GW burst ($\sim$ GHz)
- LISA & LVK: BH Superradiance ( $\sim$ mHz - kHz)
- PTA & SKA : Ultra-light DM ($\sim$ nHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)



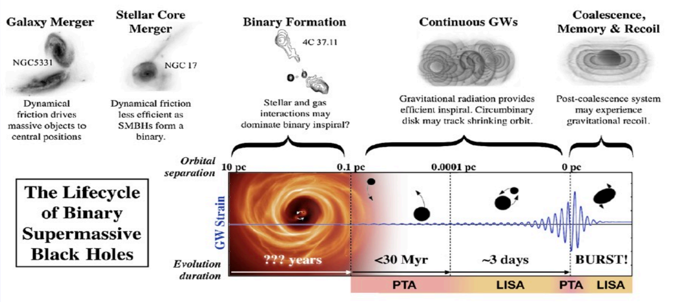







# Pulsar timing residual and cold dark matter



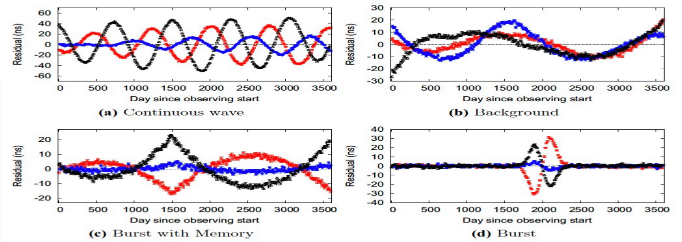


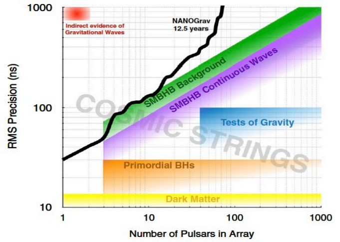

[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves" ]

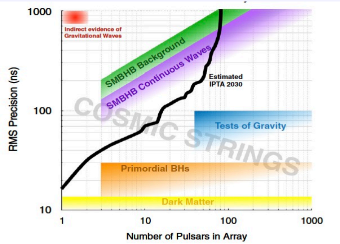
## Cold Dark Matter induced Pulsar timing residual

- Shapiro delay: Propagation through a distribution of dark-matter sub-structure (integrated Sachs–Wolfe effect)
- Doppler delay: Acceleration of the Earth or pulsar caused by the nearby passage of dark matter clump.

# The timing residuals of nHz gravitational waves



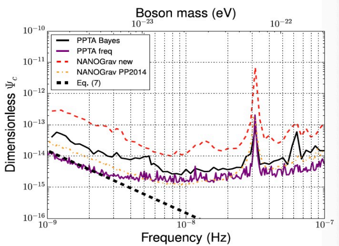



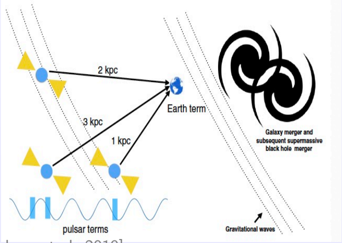

[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

# Timing residuals & Ultralight dark matter

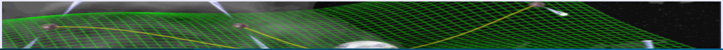
## PTA & Ultra light dark matter

- Fuzzy like DM $f_c = \frac{m}{\pi} \simeq 4.8\,\text{nHz}\left(\frac{m}{10^{-23}\text{eV}}\right)$
- Coherence length: $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4\text{kpc}\left(\frac{10^{-23}\text{eV}}{m}\right)\left(\frac{10^{-3}}{v}\right)$
- Pulsar timing residuals: $R_c(f) = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$




[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



# Oscillation of fuzzy dark matter

## DM oscillation induced time residual

- Metric: $ds^2 = -(1+2\Phi)\,dt^2 + [(1-2\Psi)\,\delta_{ij} + h_{ij}]\,dx^i dx^j$.
- e.g. the scalar field $\phi(x,t) = \phi(x)\cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t')dt'$
- **Strain** $h_\phi = 2\sqrt{3}\,\Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2}\frac{\rho_\phi}{m^2} \simeq 5.2\times 10^{-17}\alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- **GW Timing residual** $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014) ]

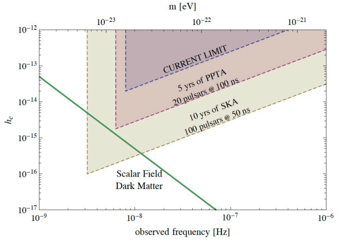
# Pulsar timing constraints on ULDM

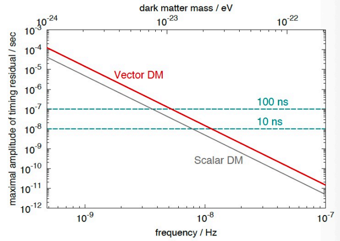


## DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014) ]



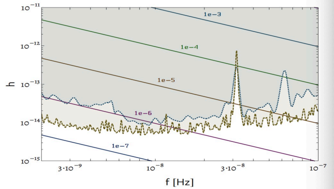



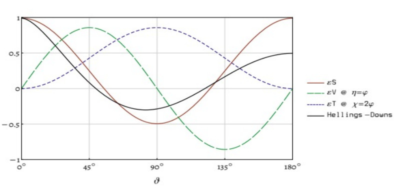

[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020) ]

# Pulsar timing constraints on spin-2 ULDM

## spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,
$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 \left(M_{\mu\nu}M^{\mu\nu} - M^2\right)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos\left[mt + \theta_2(x)\right]\varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t)-\omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



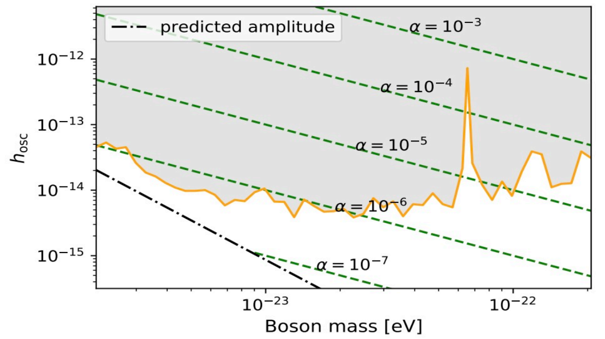




[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)
Gromov-Son, "Bimetric Theory of Fractional Quantum Hall States" PRX(2017) ]

# Gravitational effects & coupling effects

$$S = \frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\left[\sqrt{|g|}R(g) + \alpha^2\sqrt{|f|}R(f)\right.$$
$$\left. -2\frac{\alpha^2 M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\right] + \int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}\left[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G}) + \mathcal{L}^{(2)}_{\rm FP}(M) + \right.$$
$$\left. -\frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\right],$$




[cf. Y. M. Wu, Z. C. Chen, Q. G. Huang, JCAP **09**, 021 (2023)

"Pulsar timing residual induced by ultralight tensor dark matter," ]

# Angular correlation & Hellings-Downs curves



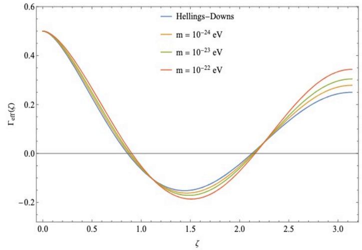



FIG. 2: Effective cross-correlation curves with $\alpha = 10^{-6}$ and mass ranging from $10^{-24}$ to $10^{-22}$. It can be seen that in this range, the deformation of spin-2 dark matter on the Hellings-Downs curve is relatively small.

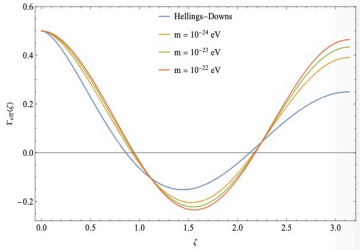



FIG. 3: Effective cross-correlation curves with $\alpha = 10^{-5.5}$. The deformation is very strong in this range, suggesting that if the coupling constant $\alpha$ is above this magnitude, existing ultralight mass spin-2 dark matter would have considerable effects on the deformation of the Hellings-Downs curve at corresponding frequency.

$$S = \frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\left[\sqrt{|g|}R(g) + \alpha^2\sqrt{|f|}R(f) - 2\frac{\alpha^2 M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\right] + \int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}\left[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G}) + \mathcal{L}^{(2)}_{\rm FP}(M) + - \frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\right] ,$$

[cf. R.G.Cai, J.R.Zhang, Y.L. Zhang, arXiv: 2402.03984,

Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter]

# Multi fields: Marcenko-Pastur distribution

GW & ULDM

Ultralight DM

Axion-like DM

GW and DM

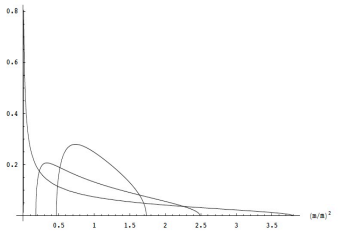
spin-0 DM

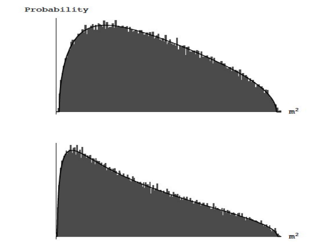
spin-2 DM

## Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m)$, $\int dm P(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices," (1967) ]



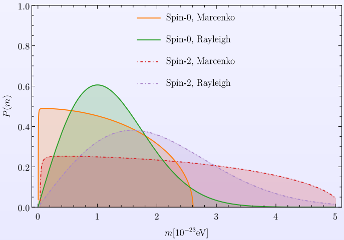



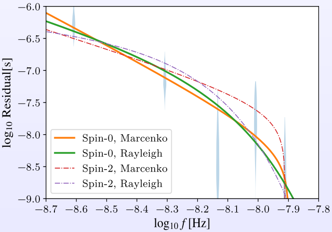

[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)
Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009) ]

# Phenomenological fitting results

## Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.

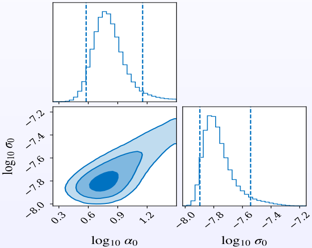


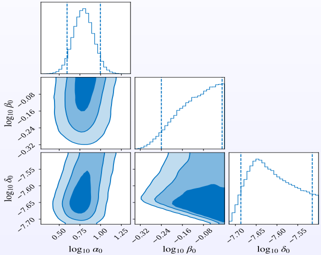



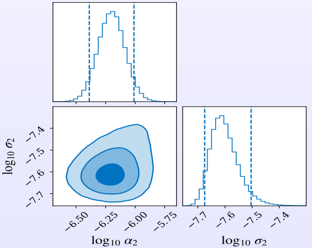

[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter" ]

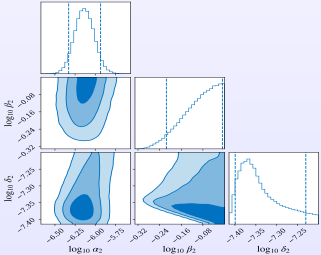
# Corner Figures of Bayesian Fitting





# Phenomenological fitting results

## The effective strain

- $h_c^{\phi}(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^{M}(f) = \frac{\alpha_2}{M_{pl}} \frac{m\mathcal{M}P(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

| | Parameters | spin-0 | spin-1 | spin-2 |
|---|---|---|---|---|
| Marcenko | $\alpha_i$ | $5.9^{+1.9}_{-1.3}$ | $\sim 3\alpha_0$ | $7.6^{+2.2}_{-1.7} \times 10^{-7}$ |
| | $m_-^i/(10^{-23}\text{eV})$ | $2.9^{+3.6}_{-0.3} \times 10^{-3}$ | $\sim \delta_0(1-\sqrt{\beta_0})$ | $6.3^{+6.0}_{-1.7} \times 10^{-3}$ |
| | $m_+^i/(10^{-23}\text{eV})$ | $2.61^{+0.21}_{-0.01}$ | $\sim \delta_0(1+\sqrt{\beta_0})$ | $5.08^{+0.02}_{-0.01}$ |
| Rayleigh | $\alpha_i$ | $5.6^{+3.8}_{-1.0}$ | $\sim 3\alpha_0$ | $6.1^{+2.1}_{-1.3} \times 10^{-7}$ |
| | $\sigma_i/(10^{-23}\text{eV})$ | $1.0^{+0.4}_{-0.1}$ | $\sim \sigma_0$ | $1.6^{+0.3}_{-0.1}$ |

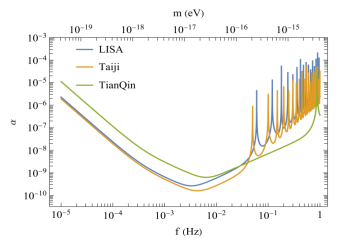
[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter" ]

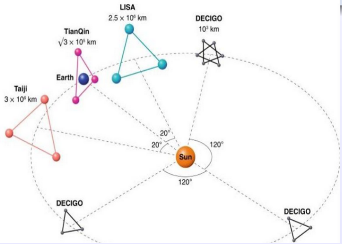
# Probing Spin-2 ULDM with GW detectors

## Frequency shift of the Laser photon

$$z_{rs}(t) = \frac{\Delta\nu}{\nu_o} = -\frac{\alpha}{2M_{\rm Pl}(1+\vec{v}\cdot\hat{n}_{rs})}\hat{n}^i_{rs}\hat{n}^j_{rs} \times \left[M_{ij}(t_r,\vec{x}_r) - M_{ij}(t_s,\vec{x}_s)\right],$$

$$M_{ij}(t,\vec{x}) = \frac{\sqrt{2\rho_{\rm dm}}}{\sqrt{5}m}\sum_A \varepsilon^A_{ij} e^{i(\omega t - m\vec{v}\cdot\vec{x})}$$




[Zhang et al, "Probing Spin-2 Ultralight Dark Matter with Space-based Gravitational Wave Detectors in Millihertz", arXiv:2501.11071, Gong, Luo & Wang, Nat Astron 5, 881–889 (2021)]

# ULDM & GWs: Multi-bands detection

## ULDM with spin 0,1,2 & GW effects

- Spin-2 ultralight dark matter & Bi-metric gravity
- **Wideband mass spectrum** extension of ULDM
- Search/constraint for ULDM with GW detection
- Angular correlation & **Deformed Hellings-Downs**

## GW detection & Ultra-light DM with multi band

- Tabletop exp: GW burst & axion ($\sim$ kHz - GHz)
- LVK & LISA: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Fuzzy dark matter ($\sim$ nHz- uHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)