THE FLAVOUR STRUCTURE OF THE LEFT

...and how to simplify running from *W* scale to *b* scale

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(based on WIP w/ S. Renner, D. Sutherland)

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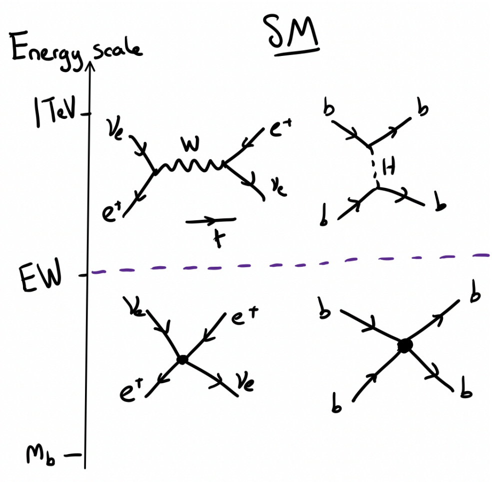
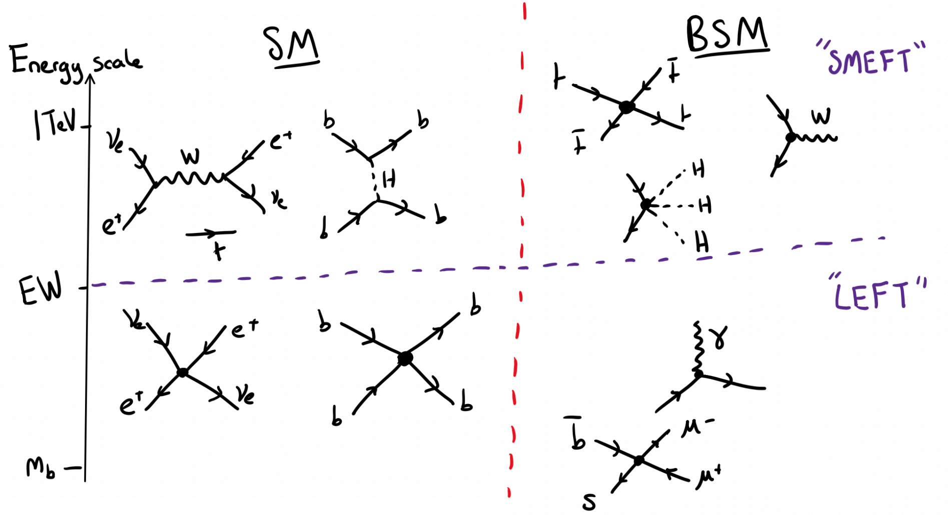
University of Glasgow


SM
Energy scale
1TeV
ν_e
W
e^+
e^+
ν_e
t
b
b
H
b
b
EW
ν_e
e^+
e^+
ν_e
b
b
b
b
m_b

EFFECTIVE FIELD THEORY



THE LOW-ENERGY EFFECTIVE FIELD THEORY

Effective field theory valid below the electroweak scale.

$$\mathcal{L}_{LEFT} = \mathcal{L}_{\mathrm{QCD+QED}} + \sum_{k,D>4} c_k^{(D)} \mathcal{O}_k^{(D)},$$

where $c_k^{(D)}$ have an implicit suppression of $\frac{1}{\Lambda_{EW}^{D-4}}$.

RUNNING AT ONE LOOP IN THE LEFT

Running first calculated at one-loop in (Jenkins, Manohar, and Stoffer 2018)

Focus on vectorial operators.

$$(\overline{\psi}\,\gamma_\mu P_{L/R}\,\psi)\,(\overline{\chi}\,\gamma^\mu P_{L/R}\,\chi) \qquad \psi,\chi \in \{d,e,u\}$$

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Focus on vectorial operators.

$$(\overline{\psi}\,\gamma_\mu P_{L/R}\,\psi)\,(\overline{\chi}\,\gamma^\mu P_{L/R}\,\chi) \qquad \psi, \chi \in \{d, e, u\}$$

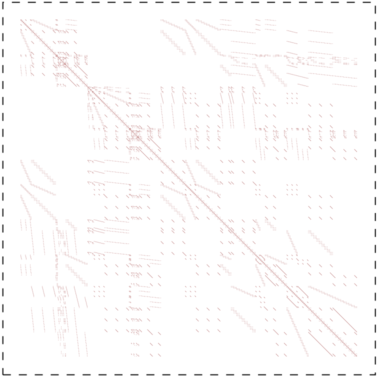
Consider effects of dipole operators.

$$(\overline{\psi}_L \sigma^{\mu\nu} \psi_R)\, F_{\mu\nu} \qquad \psi \in \{d, e, u\}$$

$$(\overline{\psi}_L \sigma^{\mu\nu} T^A \psi_R)\, G^A_{\mu\nu} \qquad \psi \in \{d, u\}$$

RUNNING AT ONE LOOP IN THE LEFT

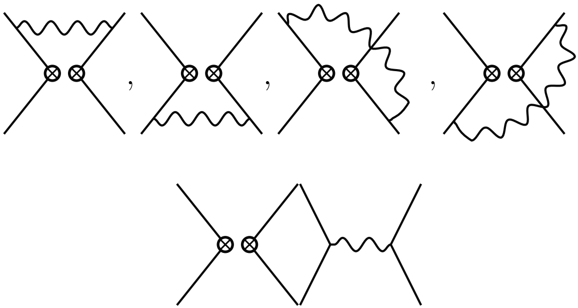
$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)} c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



DSixTools basis (Fuentes-Martin, Ruiz-Femenia, Vicente, and Virto 2021)

DIAGRAMMATIC INTERPRETATION

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)}\, c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



(+identical diagrams with gluons)

THE LEFT HAS A LARGE (BROKEN) FLAVOUR SYMMETRY

$$(U(3)_{d_L} \times U(3)_{d_R} \rtimes \mathbb{Z}_{2,d}) \times (U(3)_{e_L} \times U(3)_{e_R} \rtimes \mathbb{Z}_{2,e}) \\ \times (U(2)_{u_L} \times U(2)_{u_R} \rtimes \mathbb{Z}_{2,u}) \times U(3)_{\nu_L}$$

Kinetic terms invariant under $d_L^i \to U_{d_L}^{ij} d_L^j$, $d_R^i \to U_{d_R}^{ij} d_R^j$, $d_L \leftrightarrow d_R$, ...

Masses and other operators break this — their components are *charged* under the flavour group.

$$\mathcal{L} = i\overline{d}_L^i D d_L^i + i\overline{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ - [M^d]_{ij}\overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ + c_{ijkl}\left(\overline{d}_L^i \gamma d_L^j\right)\left(\overline{e}_L^k \gamma e_L^l\right) + [\text{other ops}]$$

(Neglecting purely gluonic operators)

WE USE A SMALLER LARGE (BROKEN) FLAVOUR SYMMETRY

$$SU(3)_d \times SU(3)_e \times SU(2)_u \times \mathbb{Z}_2$$

Kinetic terms invariant under $d_L^i \to U_d^{ij} d_L^j$, $d_R^i \to U_d^{ij} d_R^j$, ..., $(d_L, e_L, u_L) \leftrightarrow (d_R, e_R, u_R)$.

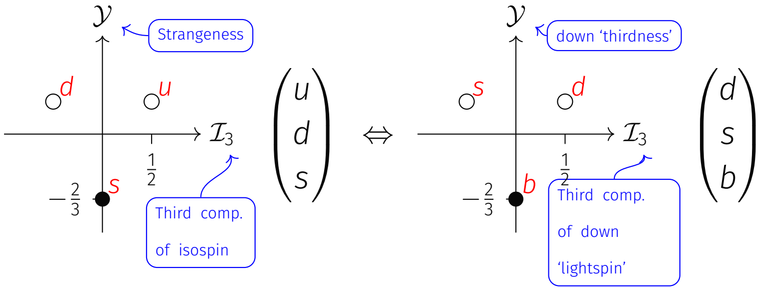
Masses and other operators break this — their components are *charged* under the flavour group.

$$\mathcal{L} = i\overline{d}_L^i D d_L^i + i\overline{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L]$$
$$- [M^d]_{ij} \overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e]$$
$$+ c_{ijkl} \left(\overline{d}_L^i \gamma d_L^j\right) \left(\overline{e}_L^k \gamma e_L^l\right) + [\text{other ops}]$$

(Neglecting purely gluonic operators)

FLAVOUR DECOMPOSITION

Following (Machado, Renner, and Sutherland 2023) , Clebsch-Gordan decompose under $SU(3)_d \times SU(3)_e \times SU(2)_u$



There are 11 flavour quantum numbers in total

$$\{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_d \, , \{d, \mathcal{I}, \mathcal{I}_3, \mathcal{Y}\}_e \, , \{d, \mathcal{I}, \mathcal{I}_3\}_u$$

PARITY DECOMPOSITION (FOR VECTORIAL OPERATORS)

$$(\overline{\psi}\,\gamma_\mu P_{L/R}\,\psi)\,(\overline{\chi}\,\gamma^\mu P_{L/R}\,\chi) \qquad \psi, \chi \in \{d, e, u\}$$

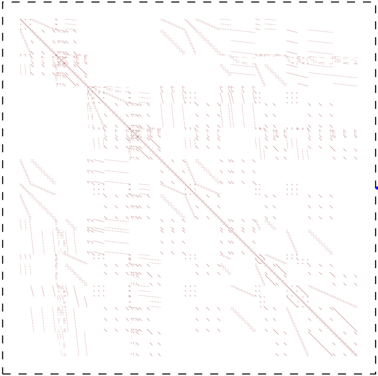
	$+$	$-$
'A' type	$LL + RR$	$LL - RR$
'B' type	$LR + RL$	$LR - RL$

There is 1 parity quantum number

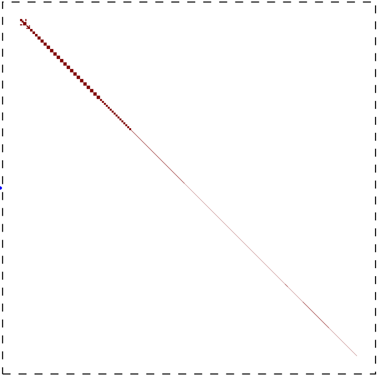
$$\pm$$

EFFECT ON ANOMALOUS DIMENSION MATRIX

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)}\, c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$

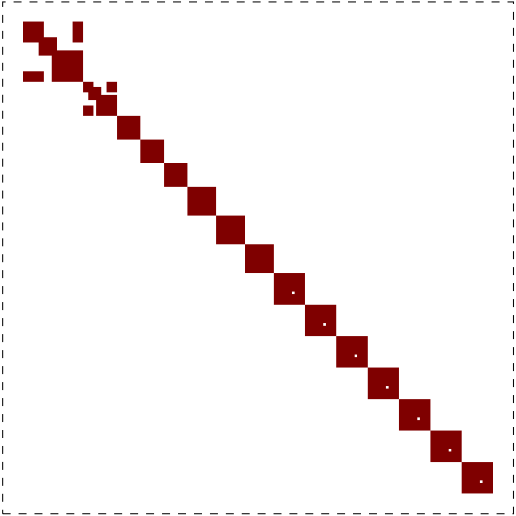


DSixTools/San Diego basis



Flavour & parity basis

FEW ZEROES WITHIN BLOCKS



We classify the blocks where source terms are present.

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \gamma(t) c_V(t) + \boxed{s_V(t)} \qquad (t \equiv \ln \mu)$$

Only present in RGEs for parity even blocks.

SOLVING RUNNING

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \gamma(t) c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$

Solve running with integrating factor U

$$c_V(t_b) = U(t_b, t_W) c_V(t_W) + U(t_b, t_W) \int_{t_W}^{t_b} \mathrm{d}t\, U(t_W, t) s_V(t)\,.$$

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At first approximation

$$U(t_b, t_W) = e^{\frac{\langle\gamma\rangle}{16\pi^2}(t_b - t_W)} \, ,$$

where $\langle\gamma\rangle$ has SM couplings taking average values between M_W and m_b.

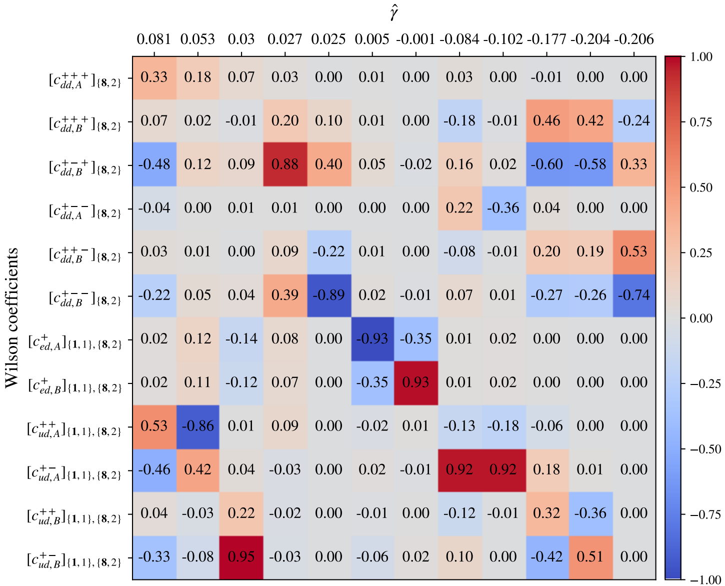
SOLVING RUNNING

Diagonalise U to understand RG flow basis-independently

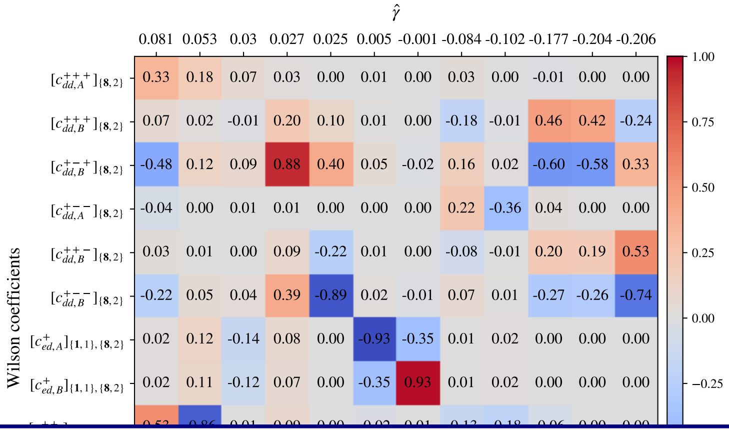
$$(S^{-1}US)_{ij} = \left(\frac{m_b}{m_W}\right)^{\frac{\hat{\gamma}_i}{(4\pi)^2}} \delta_{ij}$$

No mixing, directions with +ve $\hat{\gamma}$ shrink, -ve $\hat{\gamma}$ grow.

LEPTON UNIVERSAL OPERATORS MEDIATING $b \to s$



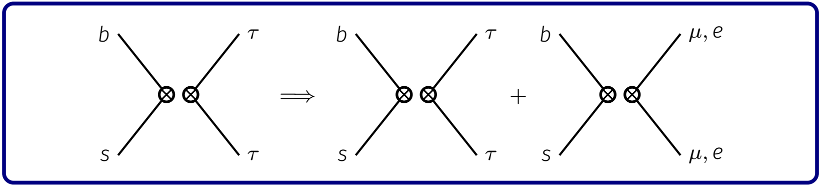
LEPTON UNIVERSAL OPERATORS MEDIATING $b \to s$



$$\delta E_n = \left\langle n \middle| \hat{H}_1 \middle| n \right\rangle$$

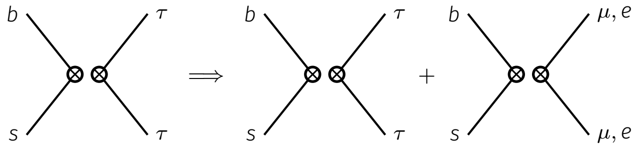
$$\delta \left| n \right\rangle = \sum_{k \neq n} \frac{\left\langle k \middle| \hat{H}_1 \middle| n \right\rangle}{E_n - E_k} \left| k \right\rangle \sim \sum_{k \neq n} \frac{O(10^{-4})}{\hat{\gamma}_n - \hat{\gamma}_k} \left| k \right\rangle \implies$$

higher loop corrections only a large effect for nearly degenerate eigenvectors



τ only at M_W scale $\implies$ τ, and some e and μ, at m_b scale

PHENO EXAMPLE

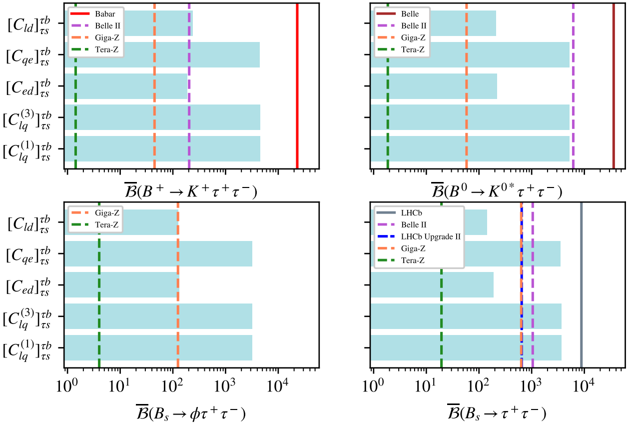


τ only at M_W scale $\implies$ τ, and some e and μ, at m_b scale

$$3\bar{b}\gamma s\,\bar{\tau}\gamma\tau = \bar{b}\gamma s\,\underbrace{(\bar{e}\gamma e + \bar{\mu}\gamma\mu + \bar{\tau}\gamma\tau)}_{\text{LFU}} - \bar{b}\gamma s\,\underbrace{(\bar{e}\gamma e + \bar{\mu}\gamma\mu - 2\bar{\tau}\gamma\tau)}_{\text{LFNU}}$$

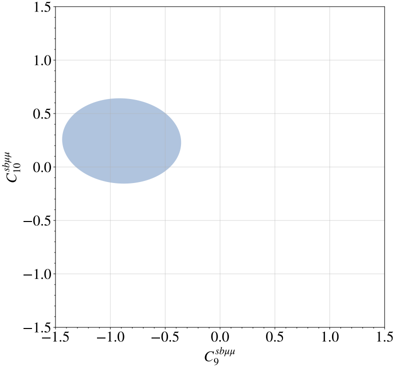
PHENO EXAMPLE

$bs\mu\mu$ (teal bars) can be better than current/projected $bs\tau\tau$ (solid/dashed lines)

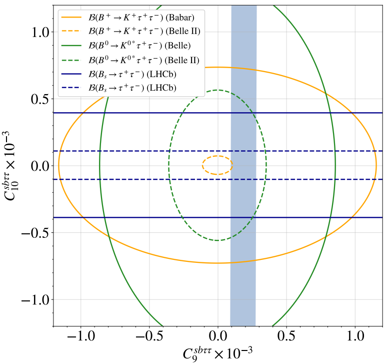


Also (Cornella, Faroughy, Fuentes-Martin, Isidori, and Neubert 2021)

PHENO EXAMPLE



$bs\mu\mu$



$bs\tau\tau$

SUMMARY

- Flavour and parity simplify running in the LEFT
- γ can be block diagonalised to all orders
- The map $M_W \to m_b$ is fully understandable in terms of eigenvalues and eigenvectors
- Many possible pheno applications!

BACKUP SLIDES

RUNNING AT ONE LOOP IN THE LEFT

Schematically (Jenkins, Manohar, and Stoffer 2018)

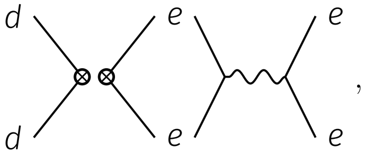
$$
\begin{aligned}
\text{masses} \to (4\pi)^2\,\dot{M} &= (e^2+g^2)M + (e+g)dM^2 + c_S M^3 + c_V M^3 + d^2 M^3\,, \\
\text{QED} \to (4\pi)^2\,\dot{e} &= e^3 + e^2 dM + d^2 M^2\,, \\
\text{QCD} \to (4\pi)^2\,\dot{g} &= g^3 + g^2 dM + d^2 M^2\,, \\
\text{dipoles} \to (4\pi)^2\,\dot{d} &= (e^2+g^2+eg)d + eM(c_S+c_T) + (e+g)d^2 M\,, \\
\text{4f scalar} \to (4\pi)^2\,\dot{c}_S &= (e^2+g^2)(c_S+c_T) + (e^2+g^2+eg)d^2\,, \\
\text{4f tensor} \to (4\pi)^2\,\dot{c}_T &= (e^2+g^2)(c_S+c_T)\,, \\
\text{4f vector} \to (4\pi)^2\,\dot{c}_V &= (e^2+g^2)c_V + (e^2+g^2+eg)d^2\,,
\end{aligned}
$$

Neglect operators in grey at $O\,(0.1\%)$ accuracy.

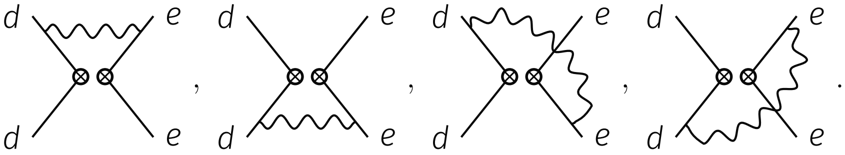
$\{c_S, c_T\}$ and c_V do not mix due to helicity selection rules. (Cheung and Shen 2015)

ACCIDENTAL ZERO

For each parity structure, zero due to contributions from



cancelling with contributions from



(Only occurs for this combination of charges.)

SOLVING RUNNING

$$\gamma(t) = e^2(t)\hat{\gamma}_e + g^2(t)\hat{\gamma}_g.$$

$$\ln U(t_b, t_W) = \frac{1}{(4\pi)^2}\int \gamma(t_1) + \frac{1}{2\,(4\pi)^4}\int_{t_1>t_2}[\gamma(t_1), \gamma(t_2)]$$
$$+ \frac{1}{6\,(4\pi)^6}\int_{t_1>t_2>t_3}([\gamma(t_1), [\gamma(t_2), \gamma(t_3)]] + [\gamma(t_3), [\gamma(t_2), \gamma(t_1)]]) + \ldots\,,$$
$$= -\,\hat{\gamma}_e \times 1.803 \times 10^{-3} - \hat{\gamma}_g \times 3.783 \times 10^{-2} - \frac{1}{2}[\hat{\gamma}_e, \hat{\gamma}_g] \times 7.379 \times 10^{-6} + \ldots$$

Neglecting higher order terms matches fully numerical solution to $O(0.0001\%)$ accuracy for lepton universal $b \to s$ block.

TWO-LOOP ESTIMATION

$$\delta |n\rangle = \sum_{k\neq n} \frac{\left\langle k \middle| \hat{H}_1 \middle| n \right\rangle}{E_n - E_k} |k\rangle \sim \sum_{k\neq n} \frac{\left(\frac{g}{4\pi}\right)^4 \times 10}{E_n - E_k} |k\rangle$$

$$\sim \sum_{k\neq n} \frac{\left(\frac{1}{4\pi}\right)^4 \times 10}{E_n - E_k} |k\rangle \sim \sum_{k\neq n} \frac{1 \times 10^{-4}}{E_n - E_k} |k\rangle$$

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