Updates to the Dispersive Calculation of the Hadronic Vacuum Polarisation Contribution to Muon g - 2

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Spin Precession

• The classical result for the magnetic moment of a particle with angular momentum *L* is

$$\boldsymbol{\mu_{cl}} = \frac{e}{2m}\boldsymbol{L}.$$

• The non-relativistic limit of the Dirac equation gives the Schrödinger equation with an interaction $-\mu \cdot B$ where

$$\mu = g \frac{e}{2m} \boldsymbol{S}.$$

and g = 2.

• QFT corrections to the fermion-photon vertex produce deviations from the Dirac case g = 2.

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HVP for g - 2



• Hadronic terms are non-perturbative \implies lattice or *dispersive* methods.

Dispersive Methods



- HVP contribution a_{μ}^{HVP} (above) depends on HVP operator $\Pi(k^2)$.
- Analyticity and Cauchy's theorem imply $\Pi(k^2) = \frac{k^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds}{s} \frac{\text{Im}[\Pi(s)]}{s k^2 i\varepsilon}$.
- Unitarity leads to the optical theorem \implies Im $[\Pi(s)] = \frac{s}{4\pi\alpha} \sigma_{had}^0(s)$.
- The matrix element of the above can be decomposed over form factors $F_1(q^2)$ and $F_2(q^2)$, and the anomalous magnetic moment $a_{\mu} = F_2(0)$.

$$\implies a_{\mu}^{HVP} = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \left(\sigma_{had}^0 \left(s \right) K \left(s \right) \right)$$

Dispersive Methods

• This calculation requires low energy $e^+e^- \rightarrow hadrons$ cross sections.



- Dependent on experimental data, especially the $\pi^+\pi^-$ channel (approximately 3/4 of the total result.)
- Clear procedure: combine lattice and dispersive predictions for HVP contribution and compare to experiment?

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Motivation

KNTW Procedure Situation and Outlook

"The Anomalous Anomaly"

- Theoretical picture for muon g 2 is unclear.
- Lattice results are consistent with the g 2 experiment results.
- Dispersive results were not, and are now internally inconsistent.
- Lots of work on the dispersive side before any conclusions can be safely drawn...



Relational Database

- Need to store cross section data (including energies and errors) and also steering flags at channel and dataset level.
- Data are stored in a relational SQL database used to build channel objects. (Replaces text files read in by FORTRAN.)

| | channel id | cha | nnel name | num sets use | far use vp | | | dataset | _id | channel_id | use | num_data_points | experime | nt year | |
|---|---|--|---|---|--|--|--|---|-----|------------|---|--|--|--|--|
| 0 | - 1 | #1 | i^0¥gamma | - 5 | Y | | | 8 | 199 | 1 | y. | 11 | S | ND 2018 | |
| 1 | 2 | - | pi^+#pi^- | 31 | Y Y | | | 1 | 200 | 1 | ÿ. | 62 | s | ND 2016 | |
| 2 | 3 | #pi^+# | pi^-#pi^0 | 20 | - Y | | | 2 | 201 | 1 | y. | 46 | CMD | -2 2005 | |
| 3 | 4 | | eta#gamma | 11 | - Y | | | 3 | 202 | 1 | Y | 30 | s | ND 2003 | |
| 4 | 5 | #pi^+#p | i^-2#pi^0 | 9 | - Y | | | 4 | 203 | 1 | ý. | 13 | S | ND 2808 | |
| | | | | | | | | | | | | | | | · · · · |
| 47 | 48 | | #eta#phi | 6 | - Y | | | 268 | 8 | 99 | У | 97 | Crystal ba | 11 1986 | |
| 48 | 49 | | p#bar(p) | 7 | - Y | | | 269 | 12 | 99 | y | 4 | LE | NA 1982 | |
| 49 | 50 | | n#bar{n} | 4 | - Y | | | 270 | 14 | 99 | y . | 1 | DA | SP 1982 | |
| 50 | 98 | Inclusive | data LOW | 6 | - Y | | | 271 | 16 | 99 | У | 2 | CU | SB 1982 | |
| 51 | 99 | Inclusive | data HIGH | 19 | - Y | | | 272 | 17 | 99 | ý. | 2 | DH | HM 1988 | |
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| | data_id | channel_id | dataset_id | energy_min | energy_max | cross_section | stat_error | syst_error | 1 | | | data_id1 | data_id2 c | mat_type | value |
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| 0 | data_id 7403 7404 | channel_id 1 | dataset_id 199 199 | energy_mi 1.07 1.11 | energy_max 1.075 1.119 | cross_section 0.893 0.848 | stat_error 8.045 8.029 | syst_error 0.0040 0.0030 |] | | 0 | data_id1 14 14 | data_id2 cm 14 15 | mat_type stat stat | value 0.019321 0.080080 |
| 0 1 2 | data_id 7403 7404 7405 | channel_id 1 1 | dataset_id 199 199 | energy_min 1.07 1.11 1.28 | energy_max 1.075 1.119 1.208 | cross_section 0.893 0.848 0.868 | stat_error 0.045 0.029 0.026 | syst_error 0.0040 0.0030 0.0020 |] | | 0 1 2 | data_id1 14 14 | data_id2 c 14 15 16 | mat_type stat stat stat | value 0.019321 0.000000 0.000000 |
| 0 1 2 3 | data_id 7403 7404 7405 7406 | channel_id 1 1 1 | dataset_id 199 199 199 199 | energy_min 1.07 1.11 1.20 1.28 | energy_max 1.075 1.119 1.200 1.284 | cross_section 0.093 0.848 0.868 0.868 | stat_error 8.845 8.829 8.826 8.826 8.821 | syst_error 0.8840 0.8830 0.8020 0.8020 |] | | 0 1 2 3 | data_id1 14 14 14 | data_id2 c 14 15 16 17 | mat_type stat stat stat stat | value 0.019321 0.000000 0.000000 0.000000 |
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 $\mathsf{Channels} \to \mathsf{Datasets} \to \mathsf{Data} \to \mathsf{Covariances}$

- Data are then processed according to channel and dataset flags.
- Future Aim: This data and the results of the following routines will be publicly accessible via a to-be-built interface.

Bare Cross Sections

• Cross sections must be *bare*: *inclusive* of final state radiation (FSR) and *exclusive* of vacuum polarisation (VP).



- VP corrections are included in all channels for required datasets. Some datasets require redressing.
- FSR indistinguishable from hadronic insertion so must be included.
- Future Aim: FSR beyond $\pi^+\pi^-$ channel. Refinements to VP routine.

Clustering and Fitting

- Different experiments have different binnings and measure cross sections over different energy ranges.
- Dynamic clustering routine: effectively rebin the data with fitted bin width parameter.



- Typical weighted average suffers from d'Agostini bias (due to correlations) iterative fit procedure.
- Future Aim: Spline interpolation of data to potentially avoid clustering/fitting and better describe resonance lineshapes.

Final Combination

- Not all final states measured supplement with isospin class estimation.
- Some overlap between final states avoid double counting.
- Sum of non-overlapping clustered and fitted bare cross section data:

$$a_{\mu}^{HVP}[KNT19] = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \left(\sigma_{had}^0\left(s\right) K\left(s\right)\right) = (692.78 \pm 2.42) \times 10^{-10}$$



 Future Aim: An accurate and precise dispersive calculation, taking account of all available data (a^{HVP}_μ[KNTW25?]).

Motivation Situation and Outlook

Tensions

- Particularly in the all-important $\pi^+\pi^$ channel, there are tensions between datasets
- Present KLOE-BaBar method of including the difference as an additional systematic insufficient when CMD-3 must also be considered - precision becomes unworkably large.
- Massive community work to understand $\pi^+\pi^$ channel tensions and dispersive-lattice tensions.



Investigation

• Massive community work to understand $\pi^+\pi^-$ channel tensions and dispersive-lattice tensions.

| Suggestion | Response |
|---|---|
| There is something wrong with the CMD-3 data. | The cross section data were interrogated for nearly two years between preprint and publication. No faults were identified. |
| The old data are incorrect since they disagree with CMD-3/lattice results. | There is presently no evidence to support this. Despite other (smaller) tensions, results were stable for > 20 years. |
| The KLOE (and BES-III) radiative corrections rely on a faulty Monte Carlo. | Based on a BaBar study of additional radiation. Comparison to other generators reveals no significant differences. |
| Data from hadronic tau decays should be re-included. | Data were not included previously due to limited understanding of isospin-breaking corrections. This has not (yet) changed. |
| "Consistent" low energy dispersive data can supplement the lattice long distance. | No substantial tensions at low energies. Overlooks the existing tensions in the dispersive data and between methods. |

• Outlook: We do not know why there are tensions. This is a challenge that needs meeting...

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Blinding

- The new KNTW analysis will be *blinded* c.f. Muon g 2: blinding for data-driven hadronic vacuum polarization; A. Keshavarzi, D. Nomura, T. Teubner and A. Wright (arXiv:2409.02827v1).
- Important to blind as all of the above will be re-examined and (hopefully) improved upon and we do not want to bias our results.
- Blinding will occur using the modified integral:

$$\begin{aligned} a_{\mu}^{HVP} &= \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \left(\sigma_{had}^0 \left(s \right) K \left(s \right) B \left(s \right) \right) \\ B \left(s \right) &= a \cdot b \cdot \left(s + s_0 \right)^c \end{aligned}$$

where the sign a, scale b, offset s_0 and power c are random variables KNTW do not hold the blinding seeds for.

- Two stage blinding:
 - Full Blinding: Channel numbers blinded. Each channel has a different a, b, s_0 and c.
 - Relative Unblinding: Channel numbers known. Universal a, b, s_0 and c.

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General Outlook

- Expect the final results of Fermilab g 2 spring 2025, but other experiments set to start in the next few years.
- New $\pi^+\pi^-$ measurements expected from the major collaborations 2025-26.
- Should help improve our understanding of the dispersive calculation, with the aim of understanding the discrepancies.

Summary

- The HVP contribution to muon g-2 can be calculated using $e^+e^- \rightarrow hadrons$ data.
- Non-trivial processing is required.
- Tensions exist that must be resolved to properly understand the muon g-2 anomaly, and potentially search for new physics.

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