Updates to the Dispersive Calculation of the Hadronic Vacuum Polarisation Contribution to Muon $g-2$

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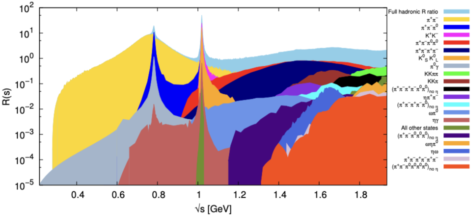

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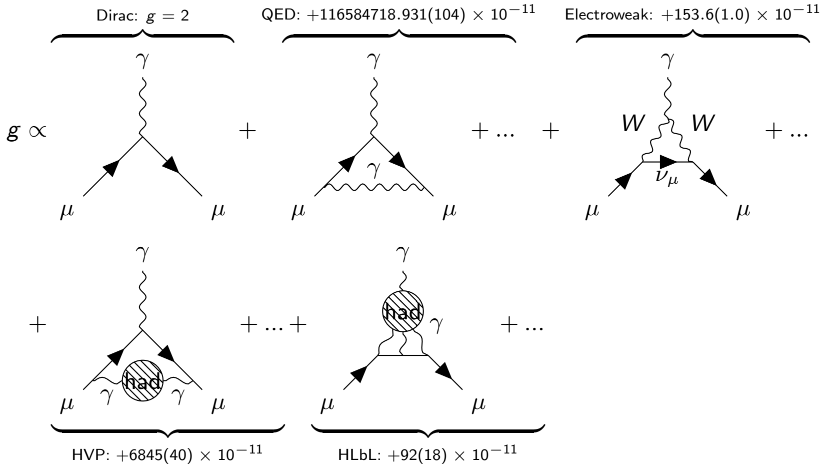
Spin Precession

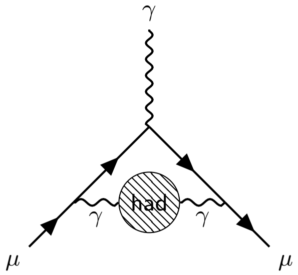
- The classical result for the magnetic moment of a particle with angular momentum $\boldsymbol{L}$ is
$$\boldsymbol{\mu}_{cl} = \frac{e}{2m}\boldsymbol{L}.$$
- The non-relativistic limit of the Dirac equation gives the Schrödinger equation with an interaction $-\boldsymbol{\mu} \cdot \boldsymbol{B}$ where
$$\boldsymbol{\mu} = g\frac{e}{2m}\boldsymbol{S}.$$
and $g = 2$.
- QFT corrections to the fermion-photon vertex produce deviations from the Dirac case $g = 2$.

HVP for $g-2$



- Hadronic terms are non-perturbative $\implies$ lattice or *dispersive* methods.

Dispersive Methods

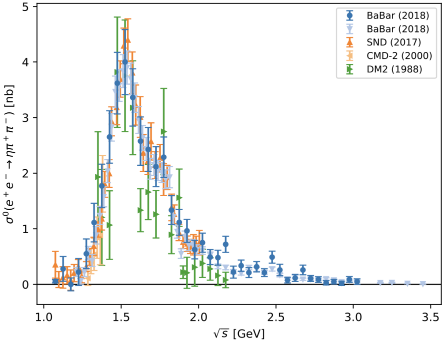


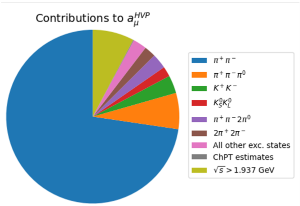
- HVP contribution a_μ^{HVP} (above) depends on HVP operator $\Pi\left(k^2\right)$.
- Analyticity and Cauchy's theorem imply $\Pi\left(k^2\right) = \frac{k^2}{\pi}\int_{s_{th}}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\left[\Pi\left(s\right)\right]}{s-k^2-i\varepsilon}$.
- Unitarity leads to the optical theorem $\implies \mathrm{Im}\left[\Pi\left(s\right)\right] = \frac{s}{4\pi\alpha}\sigma^0_{\mathrm{had}}\left(s\right)$.
- The matrix element of the above can be decomposed over form factors $F_1\left(q^2\right)$ and $F_2\left(q^2\right)$, and the anomalous magnetic moment $a_\mu = F_2\left(0\right)$.

$$\implies a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds\left(\sigma^0_{\mathrm{had}}\left(s\right)K\left(s\right)\right)$$

Dispersive Methods

- This calculation requires low energy $e^+e^- \to$ *hadrons* cross sections.

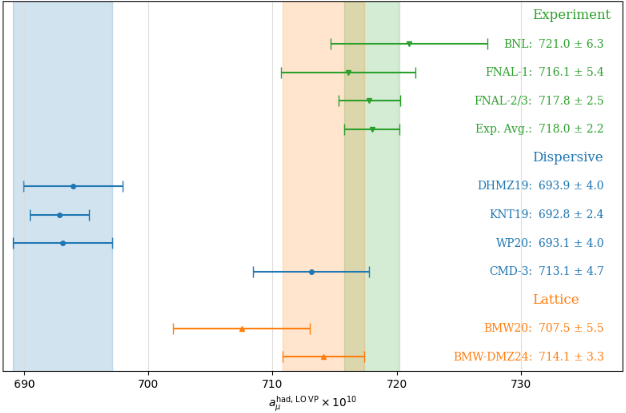




- Dependent on experimental data, especially the $\pi^+\pi^-$ channel (approximately $3/4$ of the total result.)
- Clear procedure: combine lattice and dispersive predictions for HVP contribution and compare to experiment?

"The Anomalous Anomaly"

- Theoretical picture for muon $g - 2$ is unclear.
- Lattice results are consistent with the $g - 2$ experiment results.
- Dispersive results were not, and are now internally inconsistent.
- Lots of work on the dispersive side before any conclusions can be safely drawn...



Relational Database

- Need to store cross section data (including energies and errors) and also steering flags at channel and dataset level.
- Data are stored in a relational SQL database used to build channel objects. (Replaces text files read in by FORTRAN.)

Channels → Datasets → Data → Covariances

	channel_id	channel_name	num_sets	use_fsr	use_vp	...
0	1	#pi^0#gamma	5	–	Y	...
1	2	#pi^+#pi^–	31	Y	Y	...
2	3	#pi^+#pi^–#pi^0	20	–	Y	...
3	4	#eta#gamma	11	–	Y	...
4	5	#pi^+#pi^–2#pi^0	9	–	Y	...
..	...	...	...	...	...	...
47	48	#eta#phi	6	–	Y	...
48	49	p#bar{p}	7	–	Y	...
49	50	n#bar{n}	4	–	Y	...
50	98	Inclusive data LOW	6	–	Y	...
51	99	Inclusive data HIGH	19	–	Y	...

	dataset_id	channel_id	use	num_data_points	experiment	year	...
0	199	1	y	11	SND	2018	...
1	200	1	y	62	SND	2016	...
2	201	1	y	46	CMD–2	2005	...
3	202	1	y	30	SND	2003	...
4	203	1	y	13	SND	2000	...
..	...	...	..	...	...	...	...
268	8	99	y	97	Crystal ball	1986	...
269	12	99	y	4	LENA	1982	...
270	14	99	y	1	DASP	1982	...
271	16	99	y	2	CUSB	1982	...
272	17	99	y	2	DHHM	1980	...

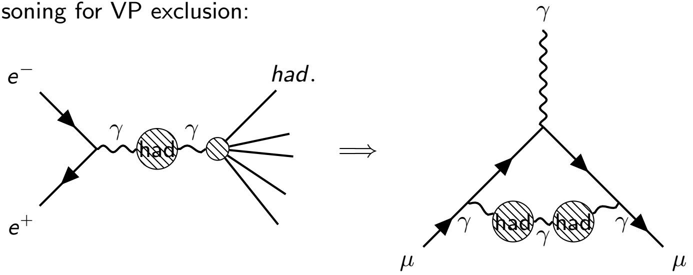
	data_id	channel_id	dataset_id	energy_min	energy_max	cross_section	stat_error	syst_error
0	7403	1	199	1.075	1.075	0.093	0.045	0.0040
1	7404	1	199	1.119	1.119	0.048	0.029	0.0030
2	7405	1	199	1.200	1.200	0.060	0.026	0.0020
3	7406	1	199	1.284	1.284	0.050	0.021	0.0020
4	7407	1	199	1.353	1.353	0.023	0.019	0.0010
...	...	...	...	...	...	...	...	...
9987	472	99	14	9.510	9.510	3.730	0.160	0.2800
9988	474	99	16	10.340	10.520	3.540	0.050	0.5310
9989	475	99	16	10.580	11.600	3.850	0.050	0.5775
9990	476	99	17	9.400	9.500	3.800	0.270	0.4200
9991	477	99	17	9.980	10.100	3.600	0.360	0.5400

	data_id1	data_id2	cmat_type	value
0	14	14	stat	0.019321
1	14	15	stat	0.000000
2	14	16	stat	0.000000
3	14	17	stat	0.000000
4	14	18	stat	0.000000
...	...	...	...	...
1174443	10742	10743	syst	0.000060
1174444	10742	10744	syst	0.000067
1174445	10743	10743	syst	0.000047
1174446	10743	10744	syst	0.000053
1174447	10744	10744	syst	0.000059

- Data are then processed according to channel and dataset flags.
- Future Aim: This data and the results of the following routines will be publicly accessible via a to-be-built interface.

Bare Cross Sections

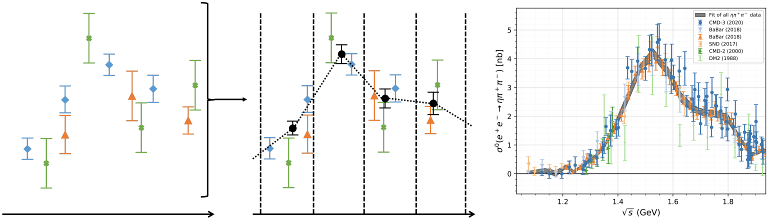
- Cross sections must be *bare*: *inclusive* of final state radiation (FSR) and *exclusive* of vacuum polarisation (VP).
- Reasoning for VP exclusion:



- VP corrections are included in all channels for required datasets. Some datasets require redressing.
- FSR indistinguishable from hadronic insertion so must be included.
- Future Aim: FSR beyond $\pi^+\pi^-$ channel. Refinements to VP routine.

Clustering and Fitting

- Different experiments have different binnings and measure cross sections over different energy ranges.
- Dynamic clustering routine: effectively rebin the data with fitted bin width parameter.

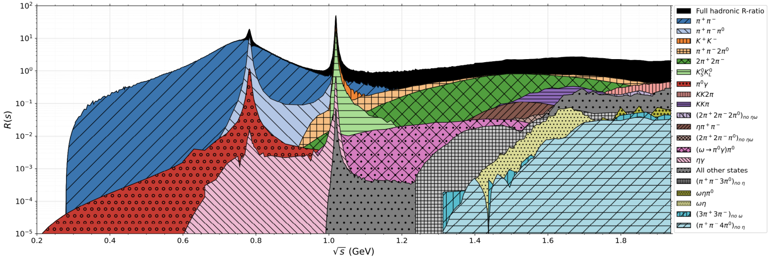


- Typical weighted average suffers from d'Agostini bias (due to correlations) $\Longrightarrow$ iterative fit procedure.
- Future Aim: Spline interpolation of data to potentially avoid clustering/fitting and better describe resonance lineshapes.

Final Combination

- Not all final states measured - supplement with isospin class estimation.
- Some overlap between final states - avoid double counting.
- Sum of non-overlapping clustered and fitted bare cross section data:

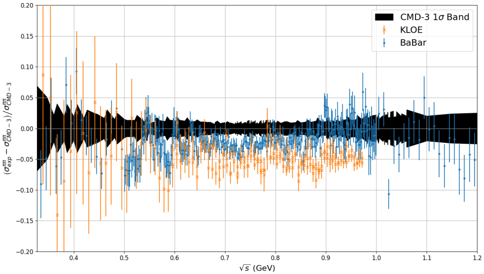
$$a_\mu^{HVP}[KNT19] = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \left(\sigma_{\text{had}}^0(s)\, K(s) \right) = (692.78 \pm 2.42) \times 10^{-10}$$

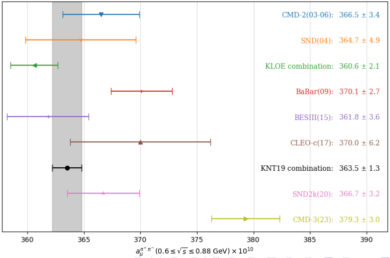


- Future Aim: An accurate and precise dispersive calculation, taking account of all available data ($a_\mu^{HVP}[KNTW25?]$).

Tensions

- Particularly in the all-important $\pi^+\pi^-$ channel, there are tensions between datasets.
- Present KLOE-BaBar method of including the difference as an additional systematic insufficient when CMD-3 must also be considered - precision becomes unworkably large.
- Massive community work to understand $\pi^+\pi^-$ channel tensions and dispersive-lattice tensions.





Investigation

- Massive community work to understand $\pi^+\pi^-$ channel tensions and dispersive-lattice tensions.

Suggestion	Response
There is something wrong with the CMD-3 data.	The cross section data were interrogated for nearly two years between preprint and publication. No faults were identified.
The old data are incorrect since they disagree with CMD-3/lattice results.	There is presently no evidence to support this. Despite other (smaller) tensions, results were stable for > 20 years.
The KLOE (and BES-III) radiative corrections rely on a faulty Monte Carlo.	Based on a BaBar study of additional radiation. Comparison to other generators reveals no significant differences.
Data from hadronic tau decays should be re-included.	Data were not included previously due to limited understanding of isospin-breaking corrections. This has not *(yet)* changed.
"Consistent" low energy dispersive data can supplement the lattice long distance.	No substantial tensions at low energies. Overlooks the existing tensions in the dispersive data and between methods.

- Outlook: We do not know why there are tensions. This is a challenge that needs meeting...

Blinding

- The new KNTW analysis will be *blinded* - c.f. Muon $g-2$: blinding for data-driven hadronic vacuum polarization; A. Keshavarzi, D. Nomura, T. Teubner and A. Wright (arXiv:2409.02827v1).
- Important to blind as all of the above will be re-examined and (hopefully) improved upon and we do not want to bias our results.
- Blinding will occur using the modified integral:

$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds \left(\sigma^0_{\text{had}}(s)\, K(s)\, B(s)\right)$$

$$B(s) = a \cdot b \cdot (s+s_0)^c$$

  where the sign a, scale b, offset s_0 and power c are random variables KNTW do not hold the blinding seeds for.
- Two stage blinding:
  - Full Blinding: Channel numbers blinded. Each channel has a different a, b, s_0 and c.
  - Relative Unblinding: Channel numbers known. Universal a, b, s_0 and c.

General Outlook

- Expect the final results of Fermilab $g-2$ spring 2025, but other experiments set to start in the next few years.
- New $\pi^+\pi^-$ measurements expected from the major collaborations 2025-26.
- Should help improve our understanding of the dispersive calculation, with the aim of understanding the discrepancies.

Summary

- The HVP contribution to muon $g-2$ can be calculated using $e^+e^- \to \textit{hadrons}$ data.
- Non-trivial processing is required.
- Tensions exist that must be resolved to properly understand the muon $g-2$ anomaly, and potentially search for new physics.