THE UNIVERSITY of EDINBURGH

Genealogical Constraints

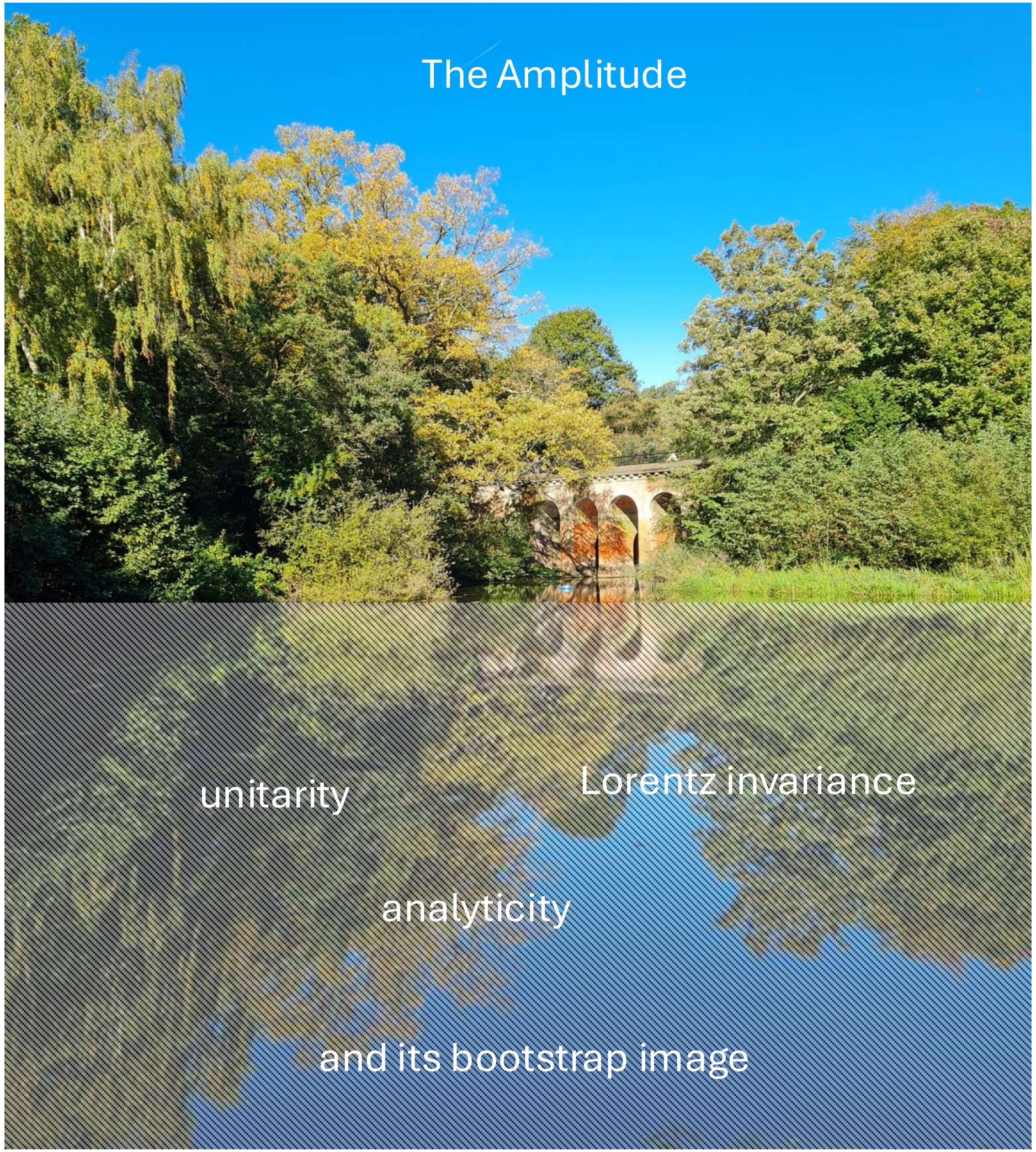
Maria Polackova

Based on arXiv:2406.05943

In collaboration with Hofie Hannesdottir, Luke Lippstreu, and Andrew McLeod

Feynman integrals are hard!

- Computing scattering amplitudes to higher loop orders is hard in general
- Computing them explicitly requires sophisticated methods (iterated integrals)
- Naturally, we seek for different methods using basic axioms


The Amplitude
unitarity
Lorentz invariance
analyticity
and its bootstrap image


The Amplitude
unitarity
Lorentz invariance
analyticity
and its bootstrap image
The Amplitude
Bootstrap ideal

What types of bootstrap do we know?

- S-matrix bootstrap:
  - Analyticity, unitarity, Lorentz invariance, locality, crossing symmetry
  - Choose EFT and fix coupling constants based on above principles, non-perturbative QFTs

What types of bootstrap do we know?

- S-matrix bootstrap:
  - Analyticity, unitarity, Lorentz invariance, locality, crossing symmetry
  - Choose EFT and fix coupling constants based on above principles, non-perturbative QFTs
- Conformal bootstrap:
  - Studies scale invariant critical points – phase transitions, non-perturbative QFTs, numerical bootstrap

What types of bootstrap do we know?

- S-matrix bootstrap:
  - Analyticity, unitarity, Lorentz invariance, locality, crossing symmetry
  - Choose EFT and fix coupling constants based on above principles, non-perturbative QFTs
- Conformal bootstrap:
  - Studies scale invariant critical points – phase transitions, non-perturbative QFTs, numerical bootstrap
- Landau bootstrap
  - Use analyticity to derive constraints on perturbative QFT

Feynman Integral

Momentum-space representation

$$I(p_i) = \frac{(-1)^E}{(i\pi)^{LD/2}} \int \frac{\mathrm{d}^D k_1 \cdots \mathrm{d}^D k_L}{(q_1^2 - m_1^2 - i\epsilon)\cdots(q_E^2 - m_E^2 - i\epsilon)},$$

Feynman Integral

Momentum-space representation

$$I(p_i) = \frac{(-1)^E}{(i\pi)^{LD/2}} \int \frac{\mathrm{d}^D k_1 \cdots \mathrm{d}^D k_L}{(q_1^2 - m_1^2 - i\epsilon) \cdots (q_E^2 - m_E^2 - i\epsilon)},$$

$$\frac{1}{\prod_{i=1}^{E} A_i} = \Gamma(d) \int_0^\infty \frac{1}{\mathrm{GL}(1)} \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{(\sum_{i=1}^{E} \alpha_i A_i)^E},$$

Feynman parametrisation representation

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

Feynman Integral

Momentum-space representation

$$I(p_i) = \frac{(-1)^E}{(i\pi)^{LD/2}} \int \frac{\mathrm{d}^D k_1 \cdots \mathrm{d}^D k_L}{(q_1^2 - m_1^2 - i\epsilon)\cdots(q_E^2 - m_E^2 - i\epsilon)},$$

$$\frac{1}{\prod_{i=1}^{E} A_i} = \Gamma(d) \int_0^\infty \frac{1}{\mathrm{GL}(1)} \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{(\sum_{i=1}^{E} \alpha_i A_i)^E},$$

Advantages of Feynman parametrisation:

a) Explicitly Lorentz invariant

b) Kinematic dependence in F polynomial

Feynman parametrisation representation

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

Choice of projection condition, e.g., $\delta(1 - \sum_i \alpha_i)$

Identifying singularities: Landau equations Landau (1959)

In Feynman parametrisation (FP) space we have two types of singularities:

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

a) Pinch singularity
b) End-point singularity

Identifying singularities: Landau equations Landau (1959)

In Feynman parametrisation (FP) space we have two types of singularities:

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{d\alpha_1 \cdots d\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

a) Pinch singularity
b) End-point singularity

$$\alpha_i = 0 \quad \text{or} \quad \frac{\partial F}{\partial \alpha_i} = 0 \quad \text{for } i = 1, \ldots, E$$

$$F = (\alpha_i - r_1)(\alpha_i - r_2) \ldots (\alpha_i - r_n)$$

Identifying singularities: Landau equations Landau (1959)

In Feynman parametrisation (FP) space we have two types of singularities:

a) Pinch singularity
b) End-point singularity

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

$$\alpha_i = 0 \text{ or } \frac{\partial F}{\partial \alpha_i} = 0 \quad \text{for } i = 1, \ldots, E$$

$$F = (\alpha_i - r_1)(\alpha_i - r_2)\ldots(\alpha_i - r_n)$$

in momentum space:

$$\alpha_i(q_i^2 - m_i^2) = 0,$$

$$\sum_{i \in a} \pm \alpha_i q_i^\mu = 0,$$

Identifying singularities: Landau equations Landau (1959)

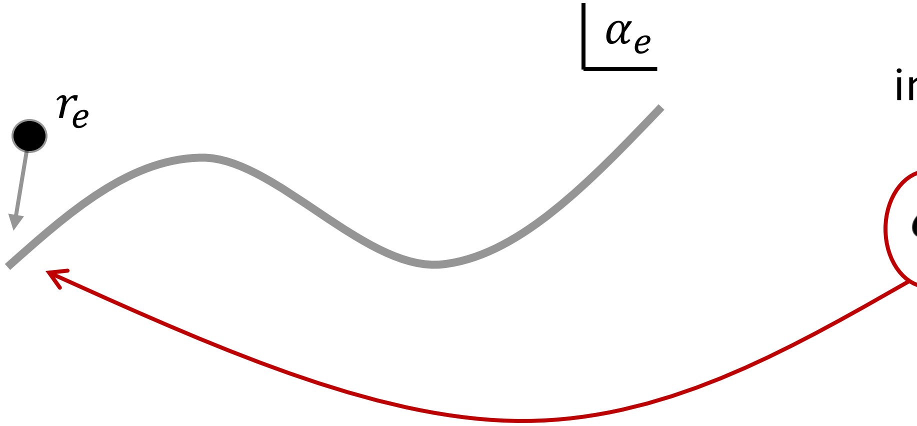
In Feynman parametrisation (FP) space we have two types of singularities:

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{d\alpha_1 \cdots d\alpha_E}{GL(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

a) Pinch singularity
b) End-point singularity

$$\alpha_i = 0 \text{ or } \frac{\partial F}{\partial \alpha_i} = 0 \quad \text{for } i = 1, \ldots, E$$

$$F = (\alpha_i - r_1)(\alpha_i - r_2)\ldots(\alpha_i - r_n)$$



in momentum space:

$$\alpha_i (q_i^2 - m_i^2) = 0,$$

$$\sum_{i \in a} \pm \alpha_i q_i^\mu = 0,$$

Identifying singularities: Landau equations Landau (1959)

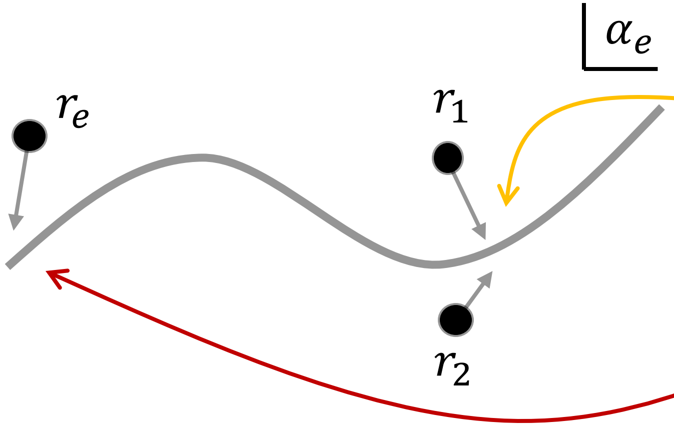
In Feynman parametrisation (FP) space we have two types of singularities:

a) Pinch singularity
b) End-point singularity

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d},$$

$$\alpha_i = 0 \text{ or } \frac{\partial F}{\partial \alpha_i} = 0 \quad \text{for } i = 1, \ldots, E$$

$$F = (\alpha_i - r_1)(\alpha_i - r_2)\ldots(\alpha_i - r_n)$$



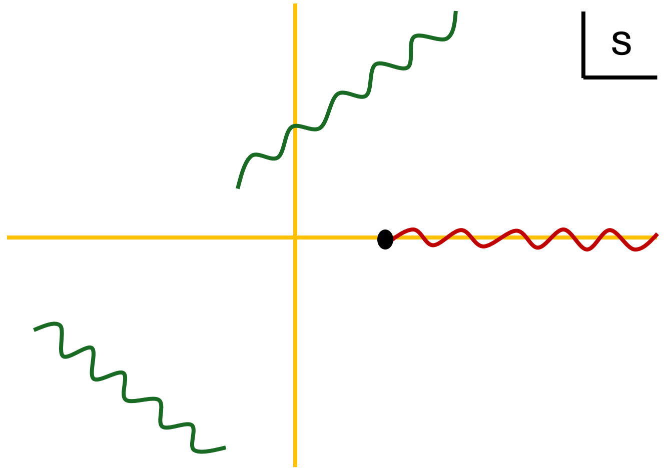
in momentum space:

$$\alpha_i (q_i^2 - m_i^2) = 0,$$

$$\sum_{i \in a} \pm \alpha_i q_i^\mu = 0,$$

How to extract these singularities?

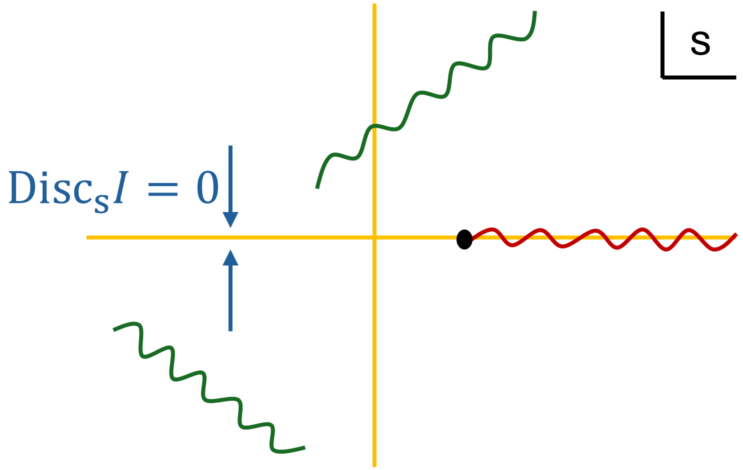
a) $\mathrm{Disc}_s I = ?$



Computing discontinuities –
as many as we can!

How to extract these singularities?

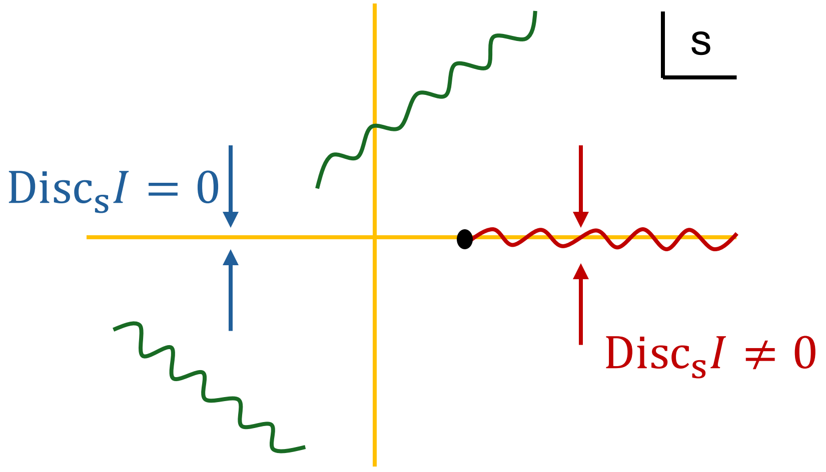
a) $\mathrm{Disc}_s I = ?$



Computing discontinuities – as many as we can!

How to extract these singularities?

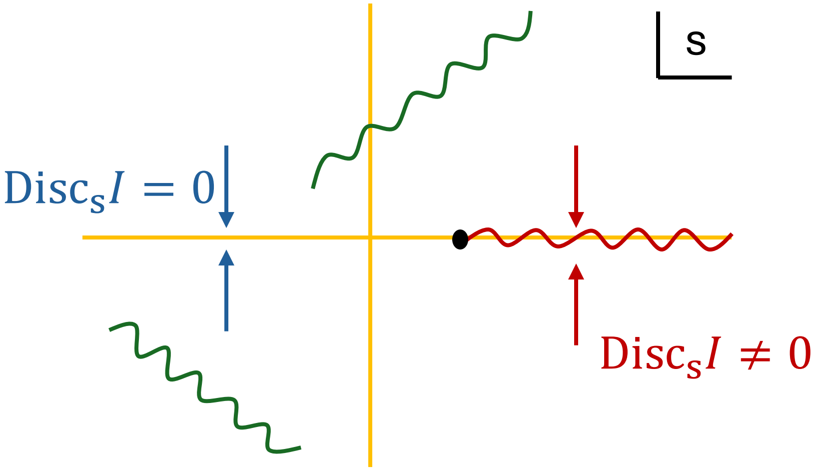
a) $\mathrm{Disc}_s I = ?$



Computing discontinuities –
as many as we can!

How to extract these singularities?

a) $\text{Disc}_s I = ?$



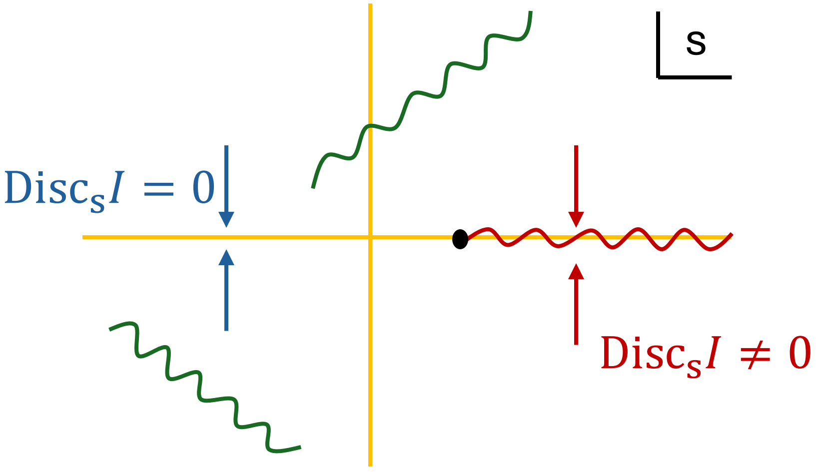
Computing discontinuities – as many as we can!

b) $\ldots \text{Disc}_{s_2} \text{Disc}_{s_1} I = ?$

Some discontinuities can be accessed only after taking first Disc_{s_1}, some of them are not allowed to be accessed after Disc_{s_1}

How to extract these singularities?

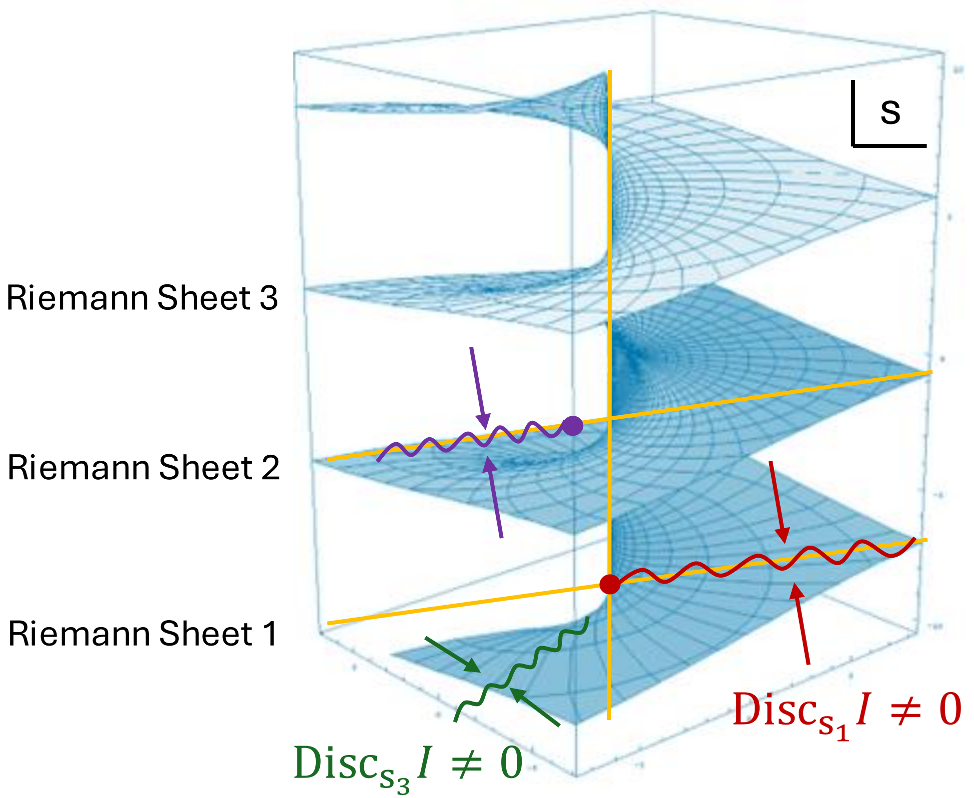
a) $\text{Disc}_s I = ?$



Computing discontinuities – as many as we can!

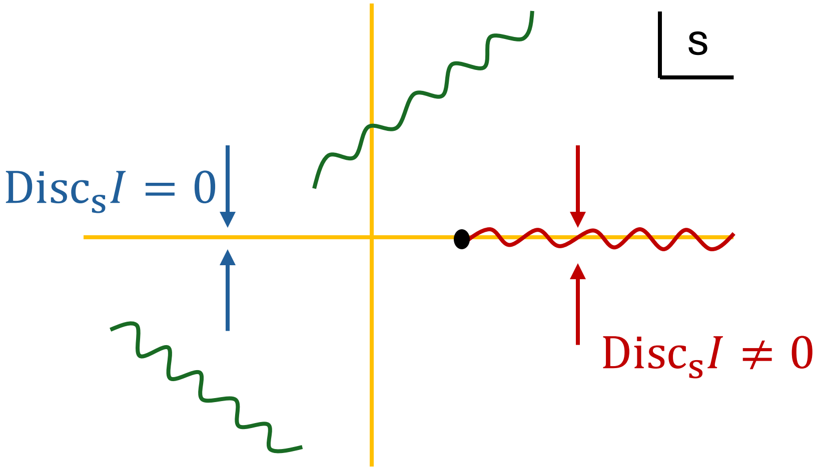
b) … $\text{Disc}_{s_2}\, \text{Disc}_{s_1} I = ?$

Some discontinuities can be accessed only after taking first Disc_{s_1}, some of them are not allowed to be accessed after Disc_{s_1}



How to extract these singularities?

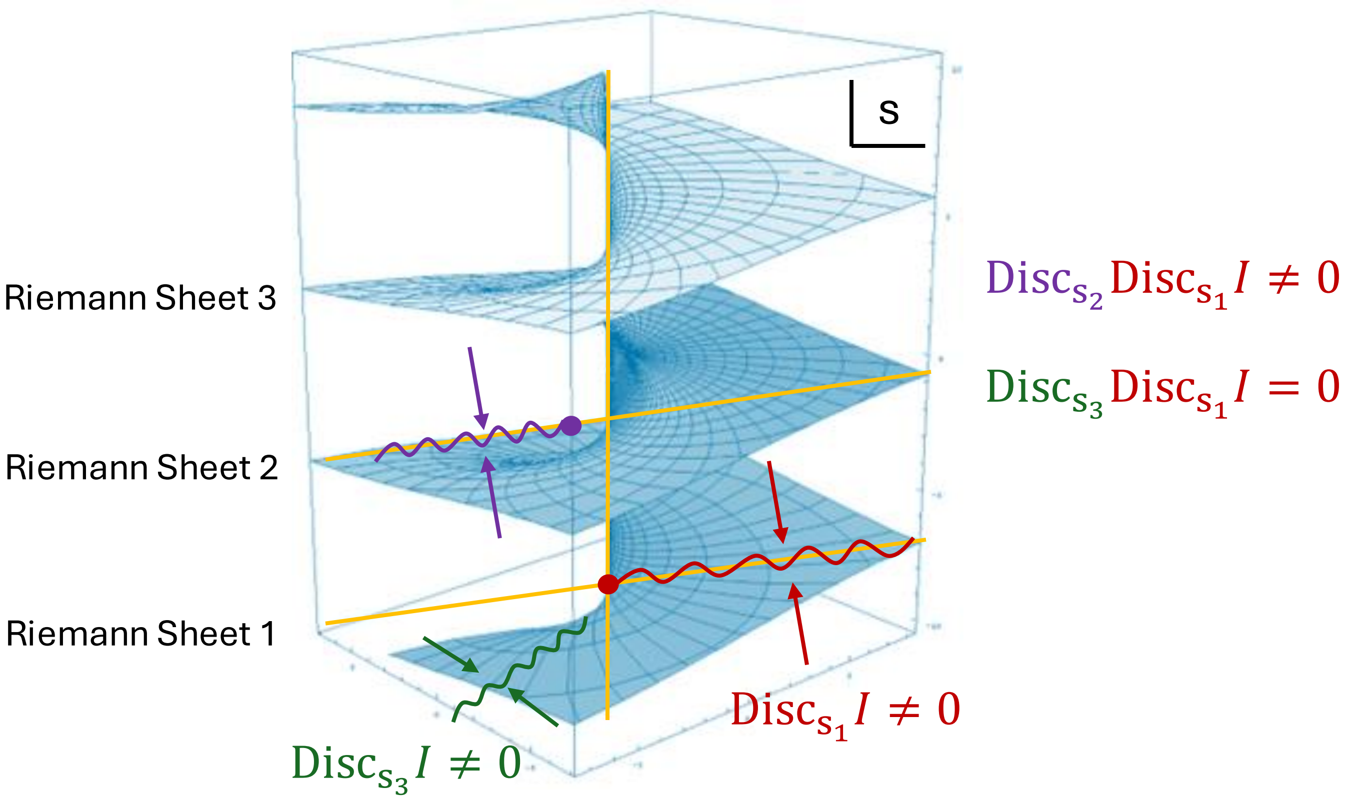
a) $\mathrm{Disc}_s I = ?$



Computing discontinuities – as many as we can!

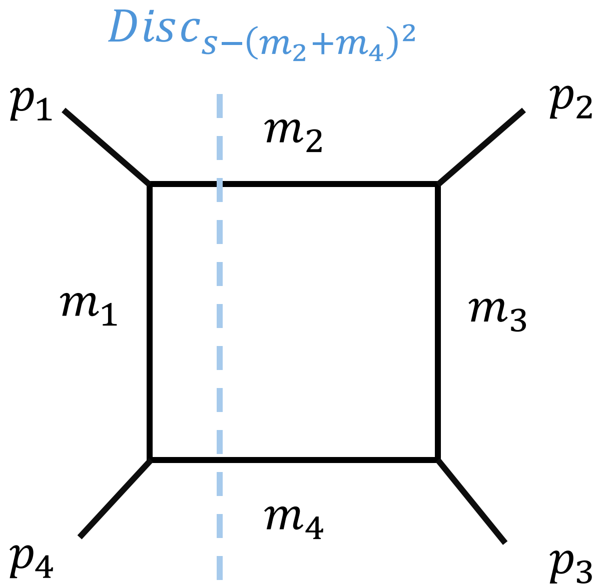
b) $\ldots \mathrm{Disc}_{s_2} \mathrm{Disc}_{s_1} I = ?$

Some discontinuities can be accessed only after taking first Disc_{s_1}, some of them are not allowed to be accessed after Disc_{s_1}



Disc rules: Hierarchical constraints

- Once we impose on-shell constraints for $\lambda = 0$, we cannot take them off-shell again



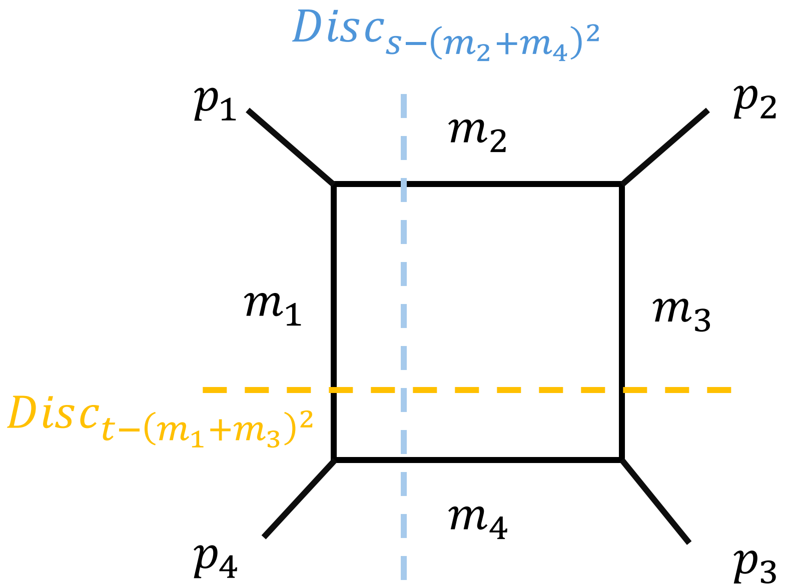
Cutkosky's rules:

Cutkosky (1960)

$$\mathrm{Disc}_{s-(m_2+m_4)^2} I(p_i) \propto \int_0^\infty \frac{\mathrm{d}^D k \, \delta(p_2^2 - m_2^2)\delta(p_4^2 - m_4^2)}{(p_1^2 - m_1^2)(p_3^2 - m_3^2)}$$

Disc rules: Hierarchical constraints

- Once we impose on-shell constraints for $\lambda = 0$, we cannot take them off-shell again



Cutkosky's rules:

Cutkosky (1960)

$$\mathrm{Disc}_{s-(m_2+m_4)^2} I(p_i) \propto \int_0^\infty \frac{\mathrm{d}^D k \; \delta(p_2^2 - m_2^2)\delta(p_4^2 - m_4^2)}{(p_1^2 - m_1^2)(p_3^2 - m_3^2)}$$

$$\mathrm{Disc}_{t-(m_1+m_3)^2} \mathrm{Disc}_{s-(m_2+m_4)^2} I(p_i) = 0$$

Hierarchical principle in practice

- Find all singularities: some endpoint singularities diverge faster than others and we require blow-ups to resolve them

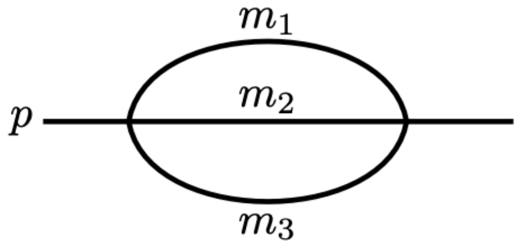
$$\alpha_i \to \epsilon^{w_i} \alpha_i, \quad \text{with} \quad \epsilon \to 0,$$

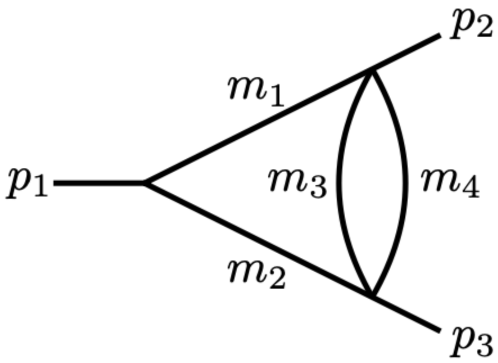
Hierarchical principle in practice

- Find all singularities: some endpoint singularities diverge faster than others and we require blow-ups to resolve them
- Find all solutions to Landau equations for all kinematic singularities - It is hard!

Hierarchical principle in practice

- Find all singularities: some endpoint singularities diverge faster than others and we require blow-ups to resolve them
- Find all solutions to Landau equations for all these singularities
  - It is hard!
- Therefore, only few examples exist with fully computed hierarchical constraints





Landshoff, Olive, Polkinghorne (1965); Pham (1967); Berghoff, Panzer (2022)

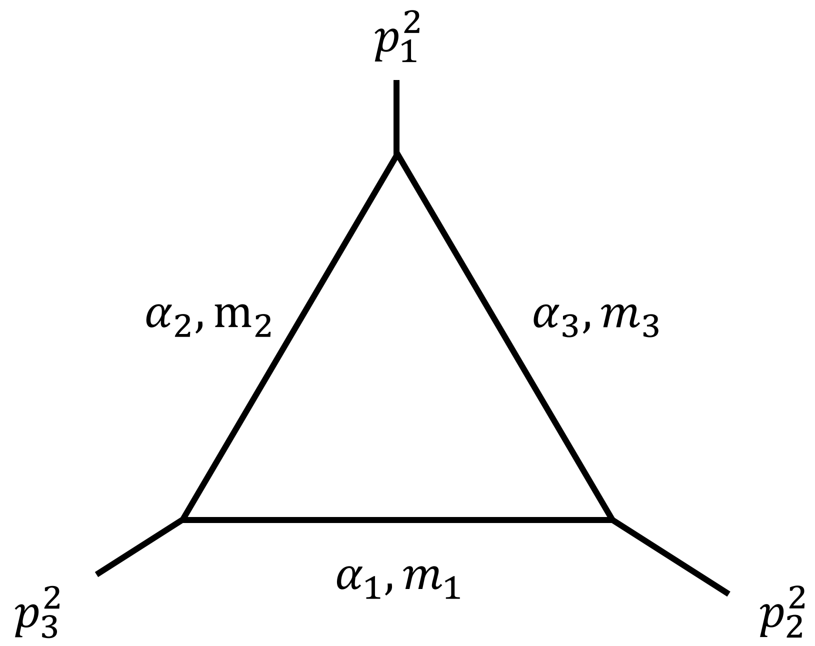
Instead: a) rethink on-shell conditions in FP space

$$I(p_i) = \Gamma(d) \lim_{\varepsilon \to 0^+} \int_0^\infty \frac{\mathrm{d}\alpha_1 \cdots \mathrm{d}\alpha_E}{\mathrm{GL}(1)} \frac{U^{d-D/2}}{(-F - i\varepsilon)^d} \,,$$

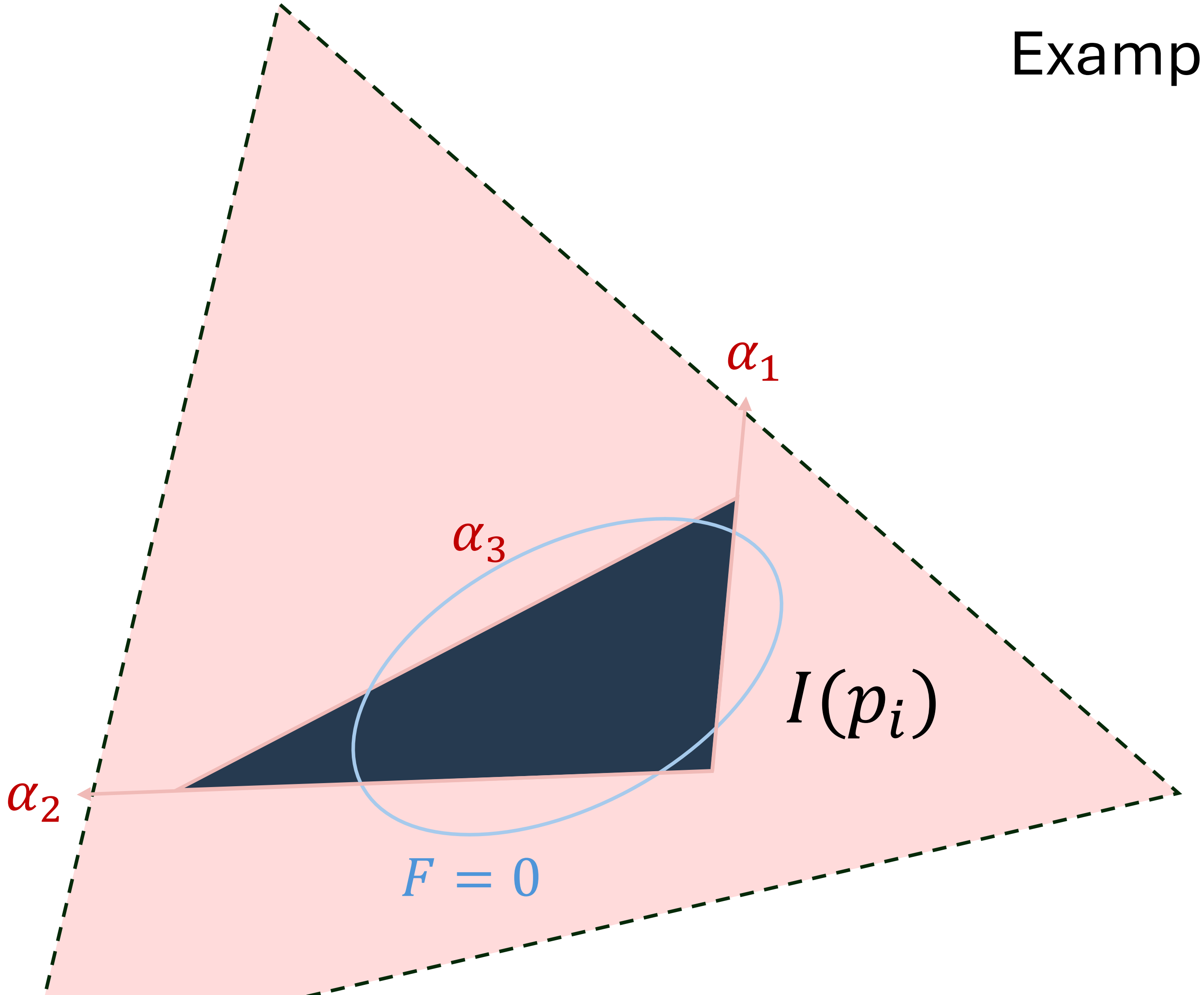
In FP space we can associate cutting edges to deleting boundaries of the α parameters

Britto (2023)

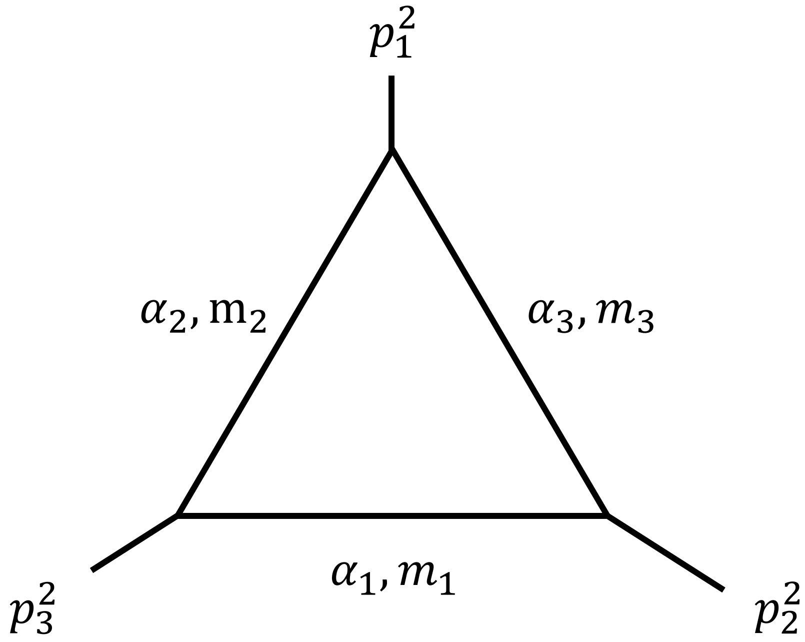
Example: triangle diagram



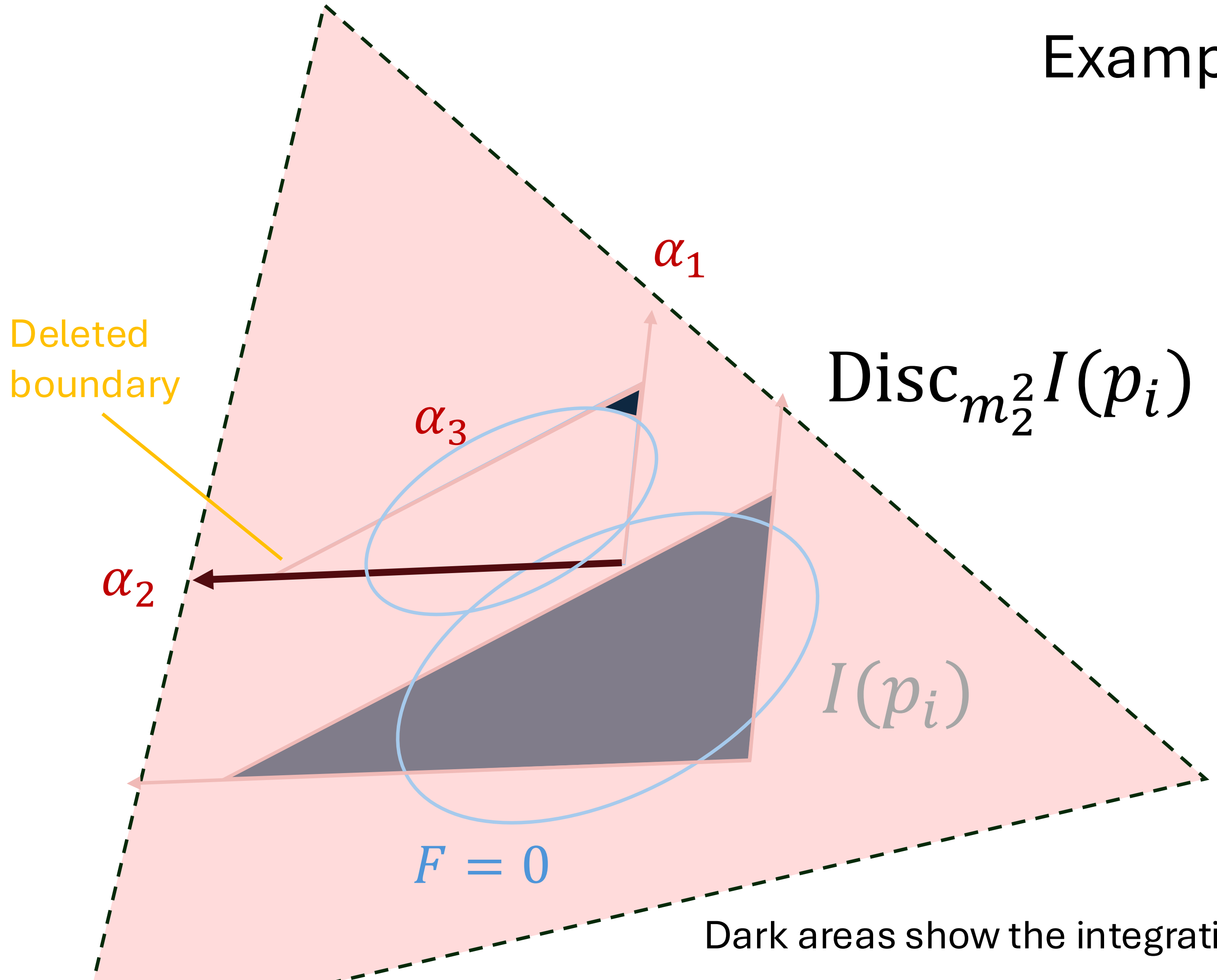
Instead: a) rethink on-shell conditions in FP space



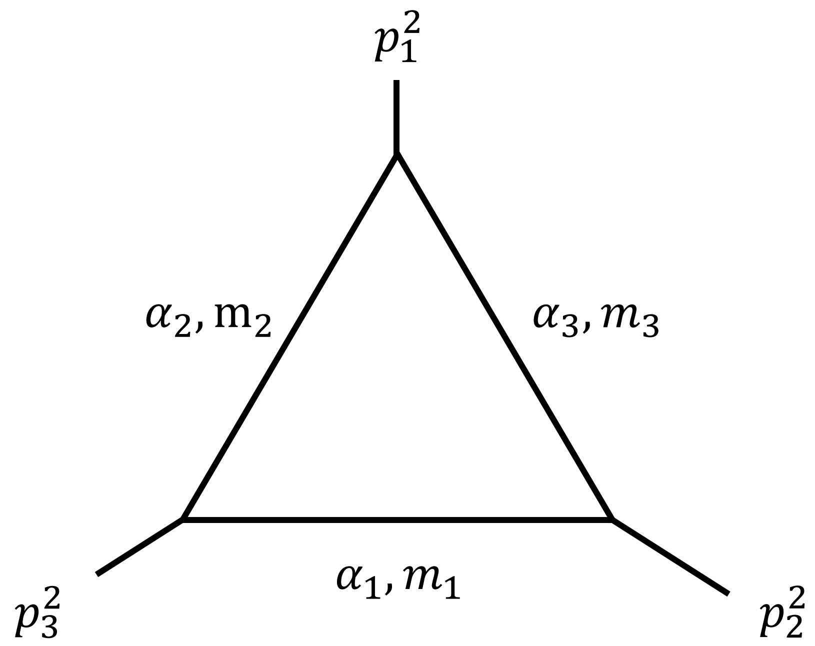
Example: triangle diagram



Instead: a) rethink on-shell conditions in FP space

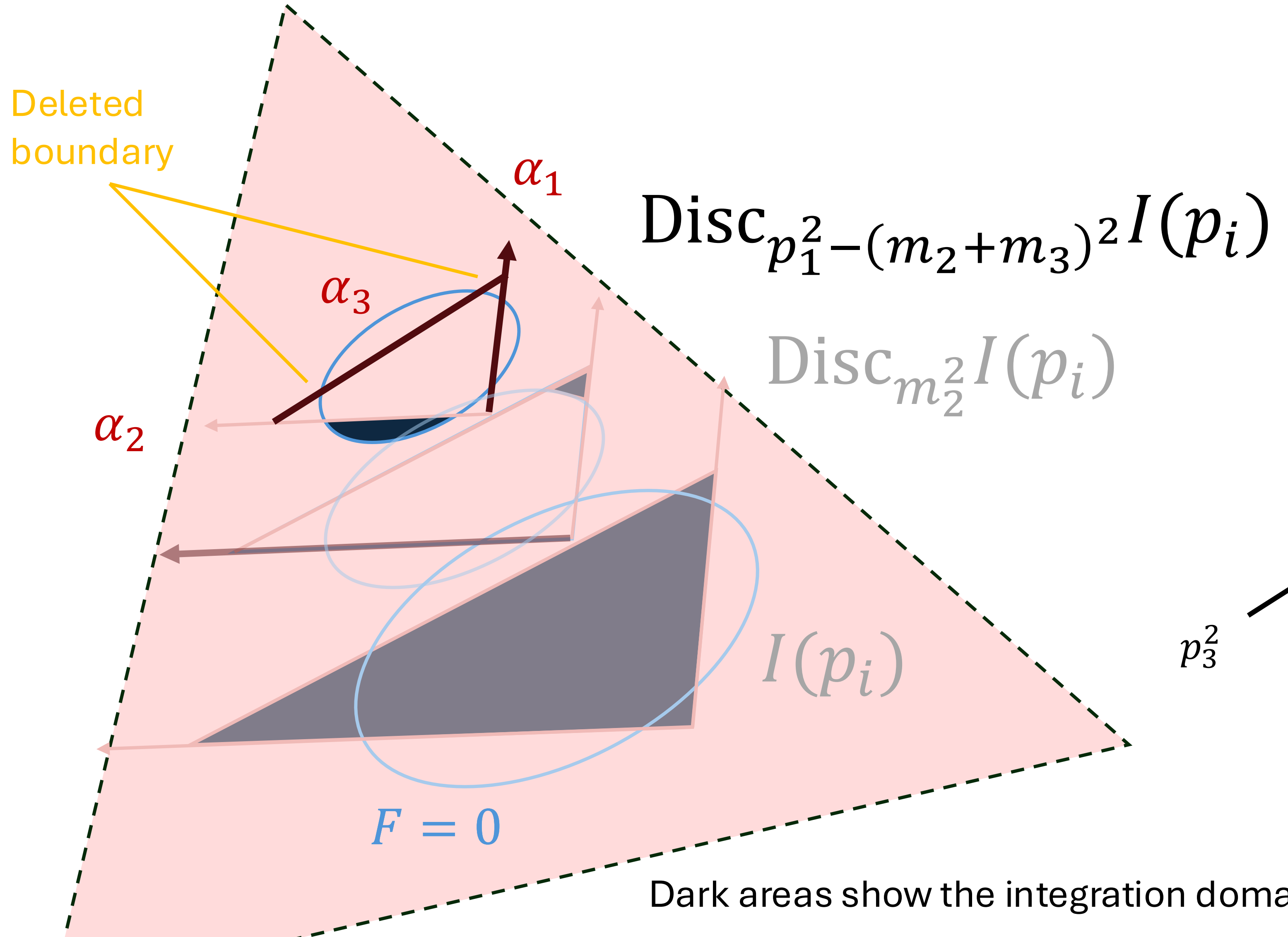


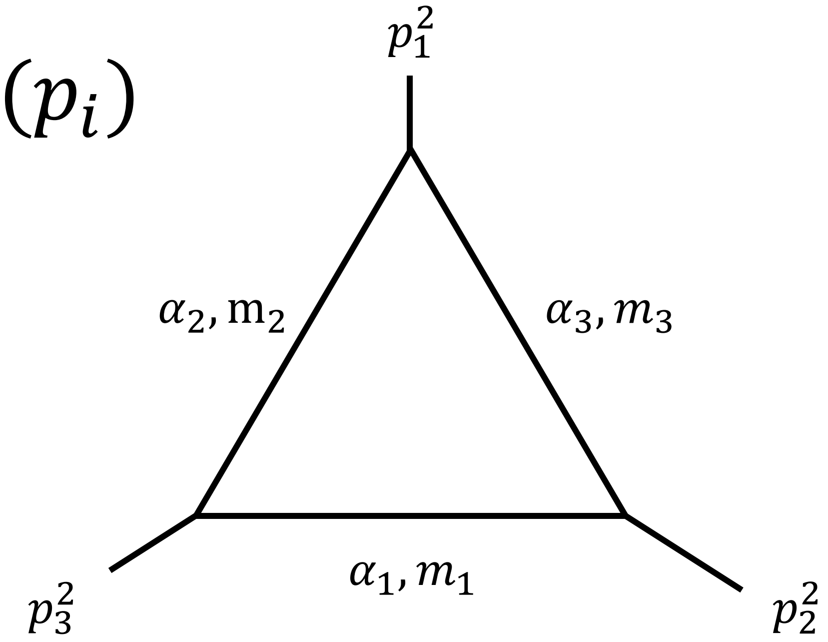
Example: triangle diagram



Dark areas show the integration domain when we cut out α axes in black

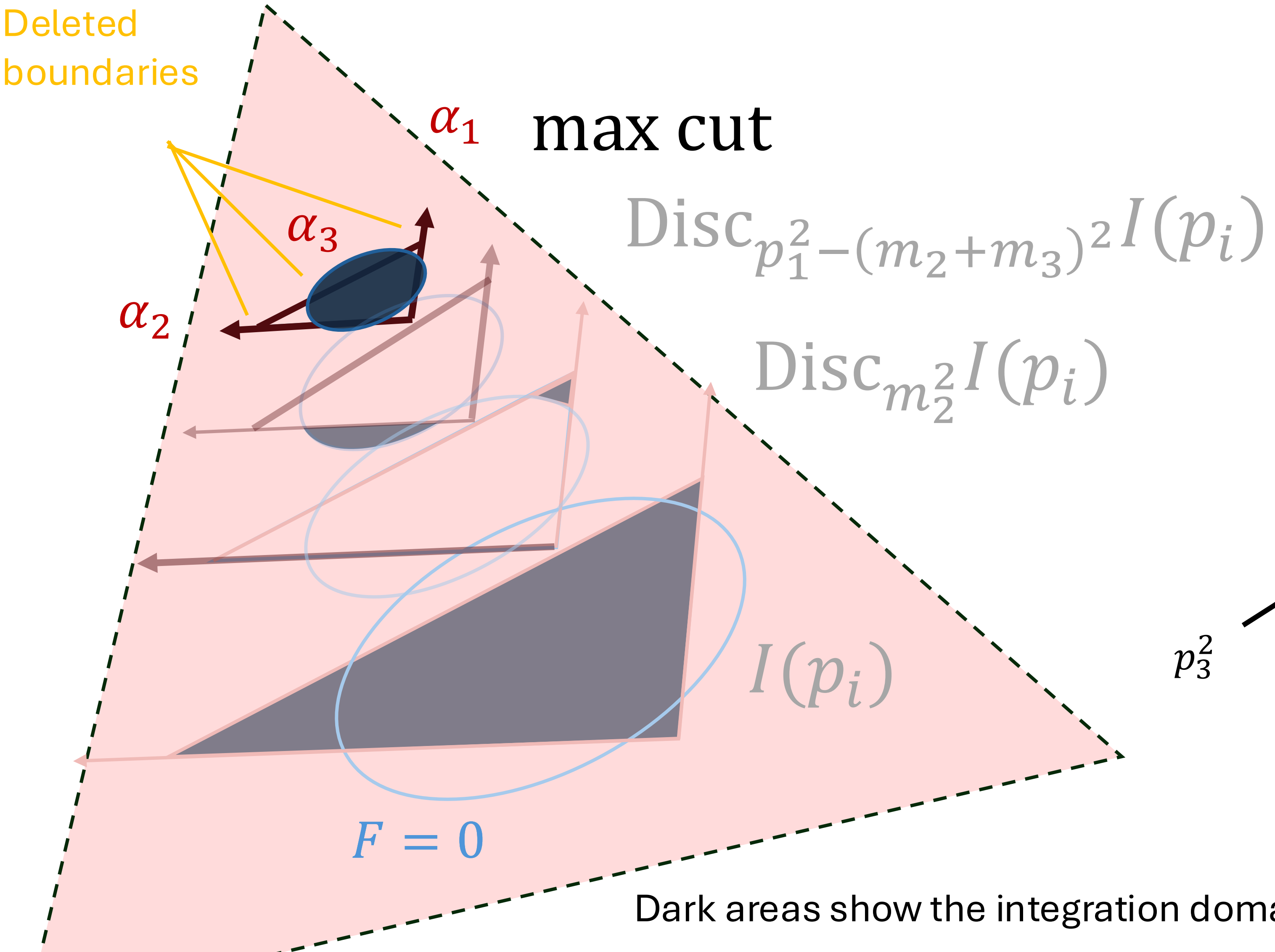
Instead: a) rethink on-shell conditions in FP space

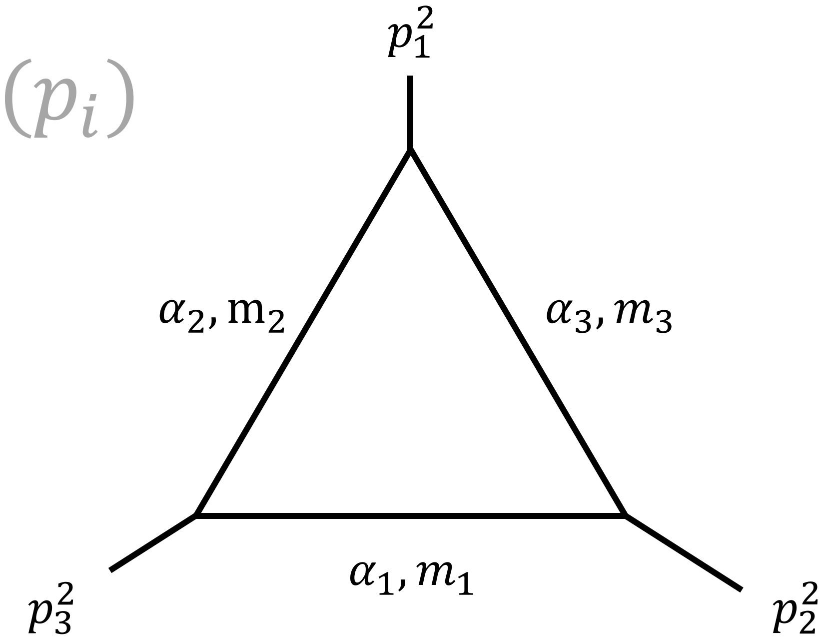




Dark areas show the integration domain when we cut out α axes in black

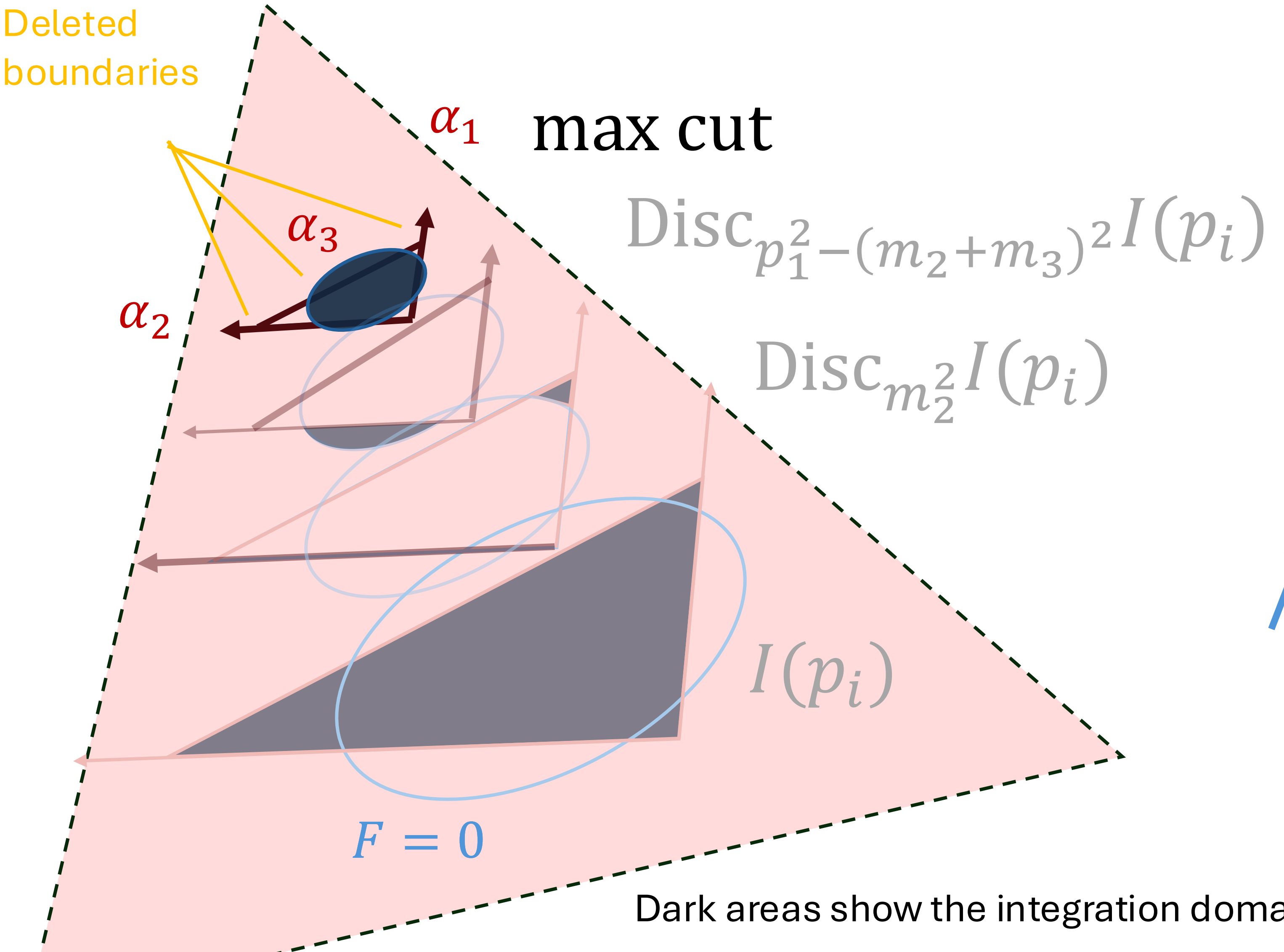
Instead: a) rethink on-shell conditions in FP space





Dark areas show the integration domain when we cut out α axes in black

Instead: a) rethink on-shell conditions in FP space



$$\alpha_i(q_i^2 - m_i^2) = 0,$$

Dark areas show the integration domain when we cut out α axes in black

Instead: b) Euler characteristics test

- Space *Y* of the integration contour degenerates when $\lambda_i = 0$ occurs

Pham (1967)

- This means we can remove some $\alpha_i \dots \alpha_j$ boundaries and then ask if the $\lambda_i = 0$ condition changed the topology of the space $Y_{i\dots j}$

Instead: b) Euler characteristics test

- Space Y of the integration contour degenerates when $\lambda_i = 0$ occurs

Pham (1967)

- This means we can remove some $\alpha_i \dots \alpha_j$ boundaries and then ask if the $\lambda_i = 0$ condition changed the topology of the space $Y_{i \dots j}$

Space with removed singular loci:

$$Y = \mathbb{C}^{E-1} \setminus (F = 0 \cup U = 0 \cup_{e=1}^{E} \alpha_e = 0)$$

Instead: b) Euler characteristics test

- Space Y of the integration contour degenerates when $\lambda_i = 0$ occurs

Pham (1967)

- This means we can remove some $\alpha_i \dots \alpha_j$ boundaries and then ask if the $\lambda_i = 0$ condition changed the topology of the space $Y_{i \dots j}$

Space with removed singular loci: $Y = \mathbb{C}^{E-1} \setminus (F = 0 \cup U = 0 \cup_{e=1}^{E} \alpha_e = 0)$

Remove α_e boundaries from the singular loci: $Y_{i \dots j} = \mathbb{C}^{E-1} \setminus \left(F = 0 \cup U = 0 \cup_{e \notin \{i \dots j\}} \alpha_e = 0\right)$

Instead: b) Euler characteristics test

- Space Y of the integration contour degenerates when $\lambda_i = 0$ occurs

Pham (1967)

- This means we can remove some $\alpha_i \ldots \alpha_j$ boundaries and then ask if the $\lambda_i = 0$ condition changed the topology of the space $Y_{i\ldots j}$

Space with removed singular loci: $Y = \mathbb{C}^{E-1} \setminus (F = 0 \cup U = 0 \cup_{e=1}^{E} \alpha_e = 0)$

Remove α_e boundaries from the singular loci: $Y_{i\ldots j} = \mathbb{C}^{E-1} \setminus \left(F = 0 \cup U = 0 \cup_{e \notin \{i\ldots j\}} \alpha_e = 0\right)$

Has $Y_{i\ldots j}|_{\lambda_i=0}$ changed from $Y_{i\ldots j}$?

Instead: b) Euler characteristics test

- This change in the topology is captured by Euler characteristic $\chi(Y)$
- Euler characteristic corresponds to:
  a) Number of solutions to the equation:

$$\frac{\mu_1}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial \alpha_e} + \frac{\mu_2}{\mathcal{U}} \frac{\partial \mathcal{U}}{\partial \alpha_e} + \frac{\nu_e}{\alpha_e} = 0 \quad \text{for } e \in \{1, 2, \ldots, E\}$$

Huh (2013)

Instead: b) Euler characteristics test

- This change in the topology is captured by Euler characteristic $\chi(Y)$
- Euler characteristic corresponds to:
  a) Number of solutions to the equation:

$$\frac{\mu_1}{\mathcal{F}}\frac{\partial \mathcal{F}}{\partial \alpha_e} + \frac{\mu_2}{\mathcal{U}}\frac{\partial \mathcal{U}}{\partial \alpha_e} + \frac{\nu_e}{\alpha_e} = 0 \quad \text{for } e \in \{1, 2, \ldots, E\}$$

Huh (2013)

  b) Number of master integrals of a given Feynman diagram

Bitoun, Bogner, Klausen, Panzer (2018)

Instead: b) Euler characteristics test

Remove α_e boundaries from the singular loci:

$$Y_{i\dots j} = \mathbb{C}^{E-1} \setminus \left(F = 0 \,\cup\, U = 0 \,\cup_{e \notin \{i\dots j\}} \alpha_e = 0\right)$$

$$\left|\chi\left(Y_{i\dots j}\,\Big|_{\lambda_i=0}\right)\right| \overset{?}{<} \left|\chi\left(Y_{i\dots j}\right)\right|$$

Fevola, Mizera, Telen (2023)

Instead: b) Euler characteristics test

Remove α_e boundaries from the singular loci:

$$Y_{i\ldots j} = \mathbb{C}^{E-1} \setminus \left(F = 0 \cup U = 0 \cup_{e \notin \{i\ldots j\}} \alpha_e = 0\right)$$

$$\left|\chi\left(Y_{i\ldots j}\Big|_{\lambda_i=0}\right)\right| \overset{?}{<} \left|\chi\left(Y_{i\ldots j}\right)\right|$$

Fevola, Mizera, Telen (2023)

Yes, the space Y degenerates and discontinuity w.r.t. λ_i can be non-zero

No, the space Y does not degenerate and discontinuity w.r.t. λ_i is zero

What we have learnt so far:

- We established cutting edges in Feynman parameter space
- Use Euler characteristics test for a space degeneracy, i.e., “is the discontinuity in λ_i possible?”

What we have learnt so far:

- We established cutting edges in Feynman parameter space
- Use Euler characteristics test for a space degeneracy, i.e., “is the discontinuity in λ_i possible?”
- How do we identify which α_e boundaries remove?

Instead of solving Landau equations,
we can use minimal cuts!

$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts

$s_{ij} = (p_i + p_j)^2$

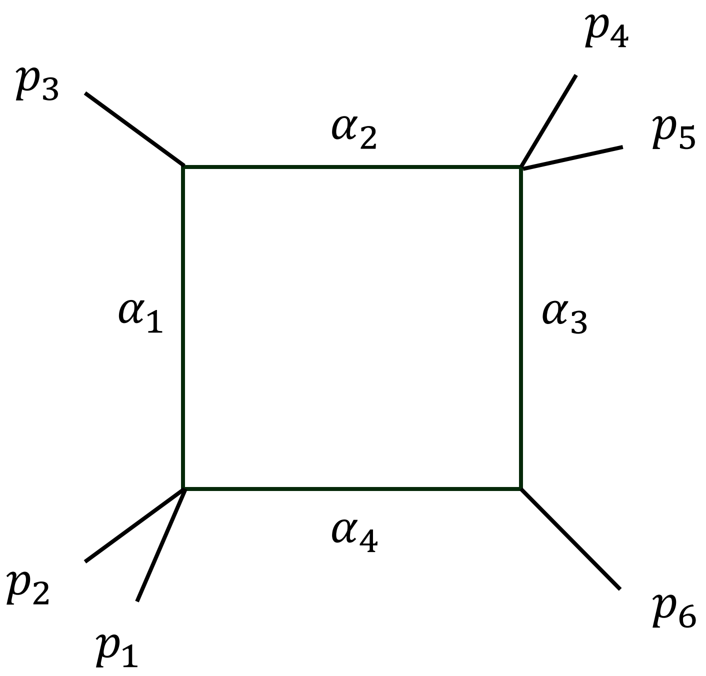
Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts

$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts

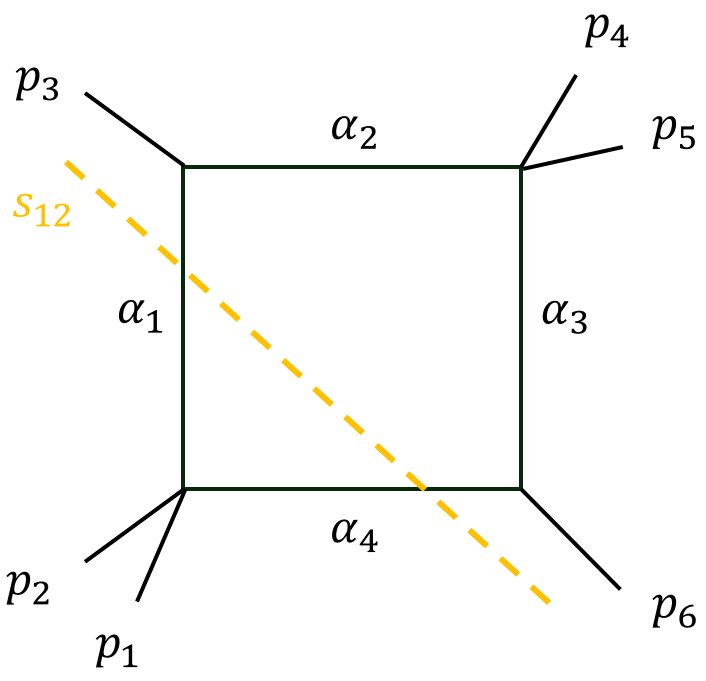


$\lambda = s_{12}$

$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts

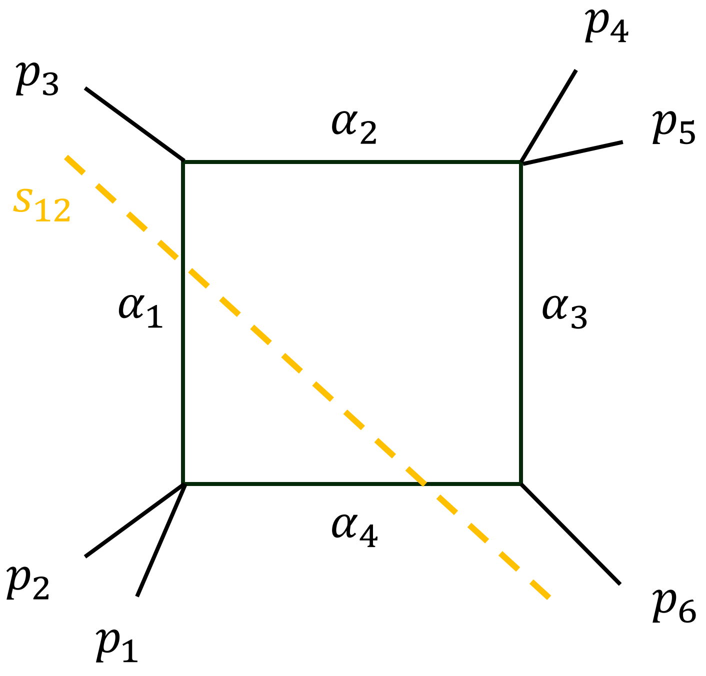


$\lambda = s_{12}$

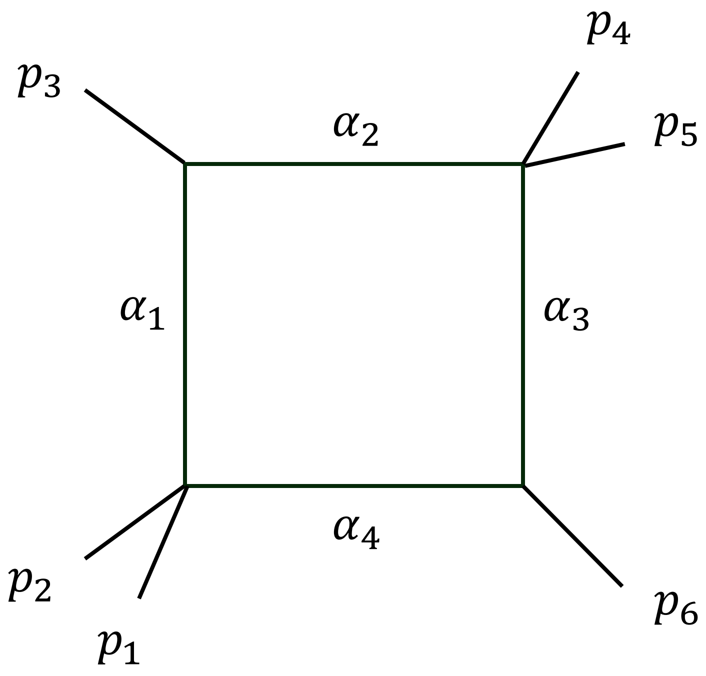
$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts



$\lambda = s_{12}$

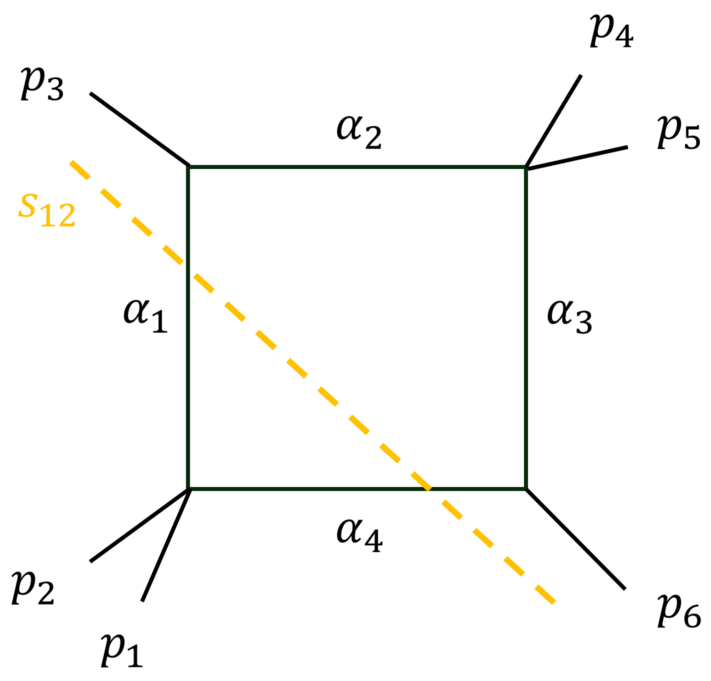


$\lambda = s_{345}$

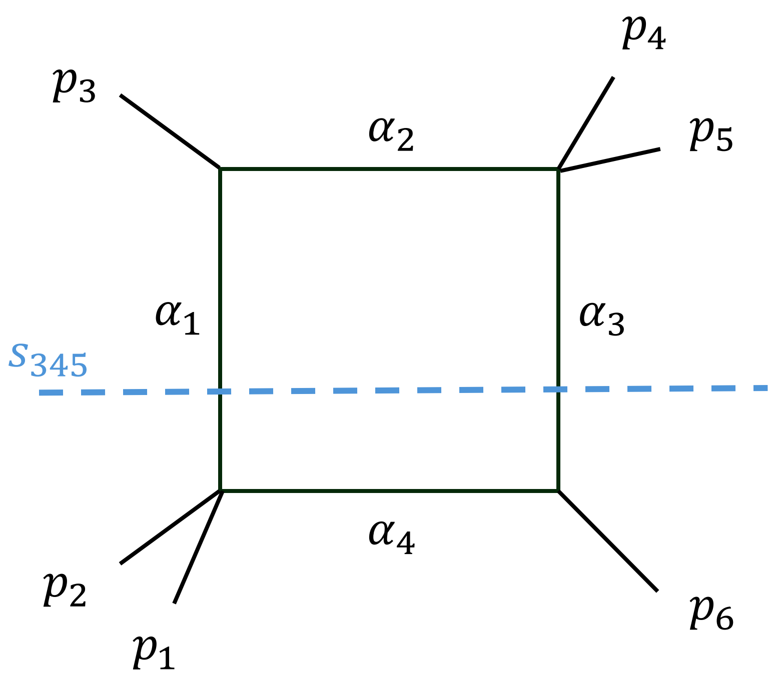
$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts



$\lambda = s_{12}$

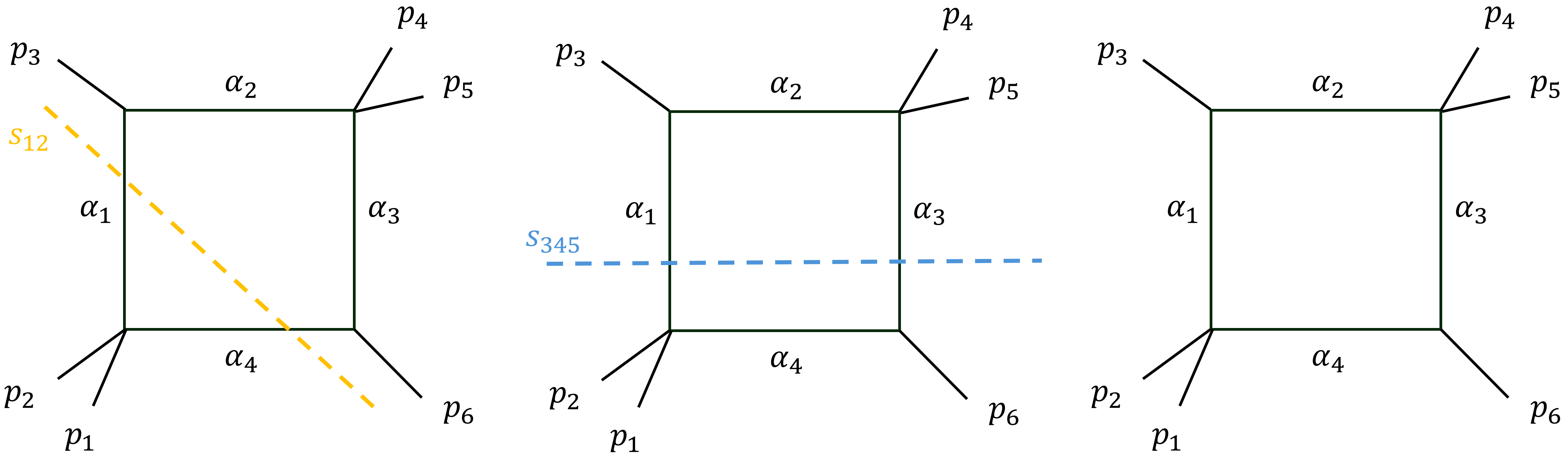


$\lambda = s_{345}$

$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts

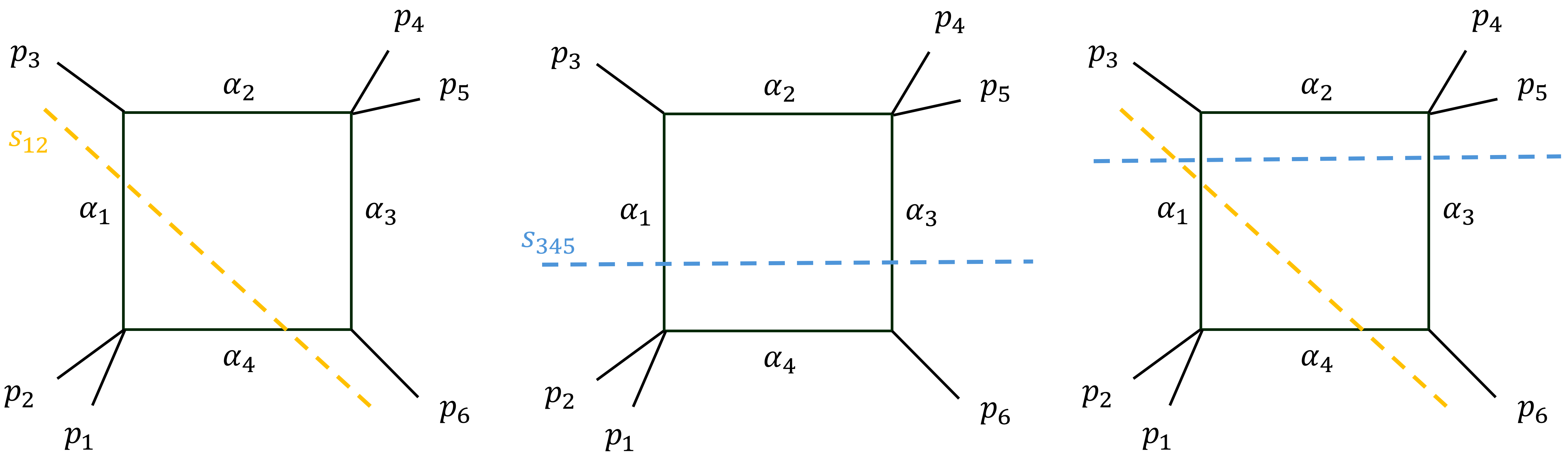


$\lambda = s_{12}$ $\lambda = s_{345}$ $\lambda = s_{12} - s_{345}$

$s_{ij} = (p_i + p_j)^2$

Minimal cuts

- Conservative choice of the Landau equations solutions
- Cut the diagram such that kinematic variables in $\lambda_i = 0$ are resolved by the cuts



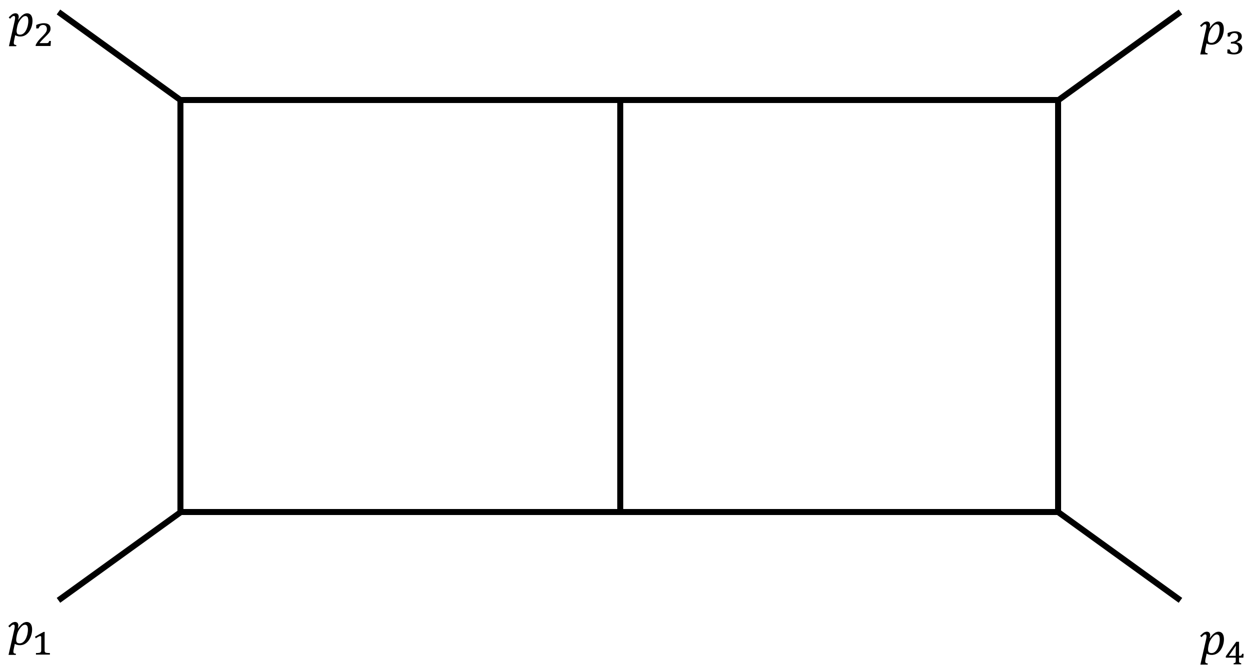
$\lambda = s_{12}$ $\lambda = s_{345}$ $\lambda = s_{12} - s_{345}$

Minimal cuts

- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries

Minimal cuts

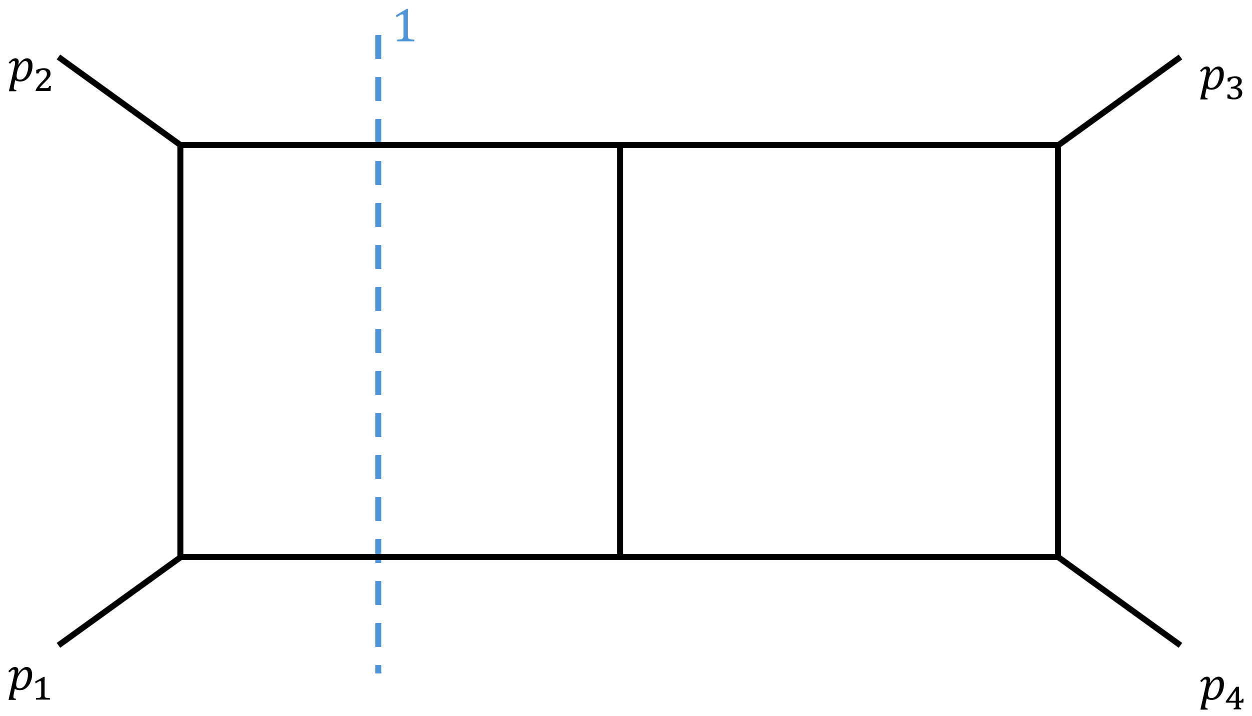
- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



$\lambda = s_{12}$

Minimal cuts

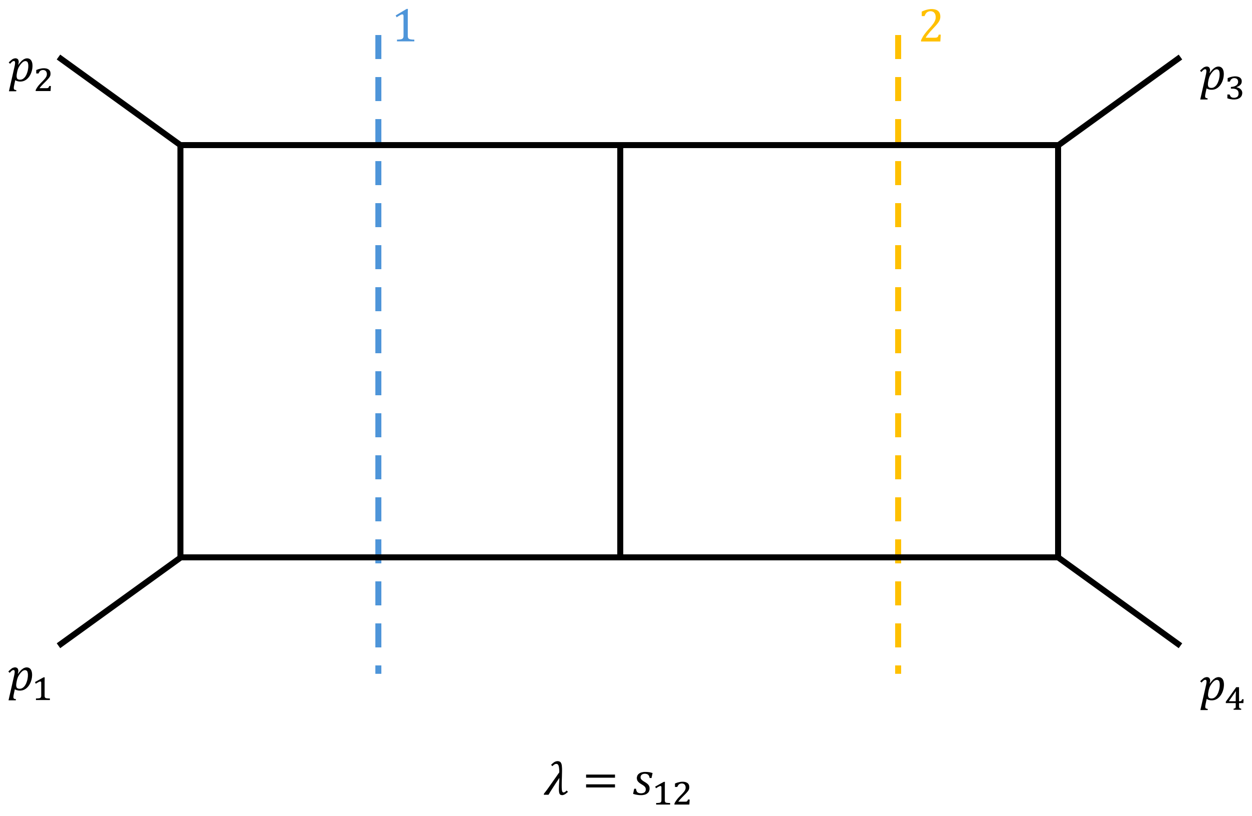
- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



$\lambda = s_{12}$

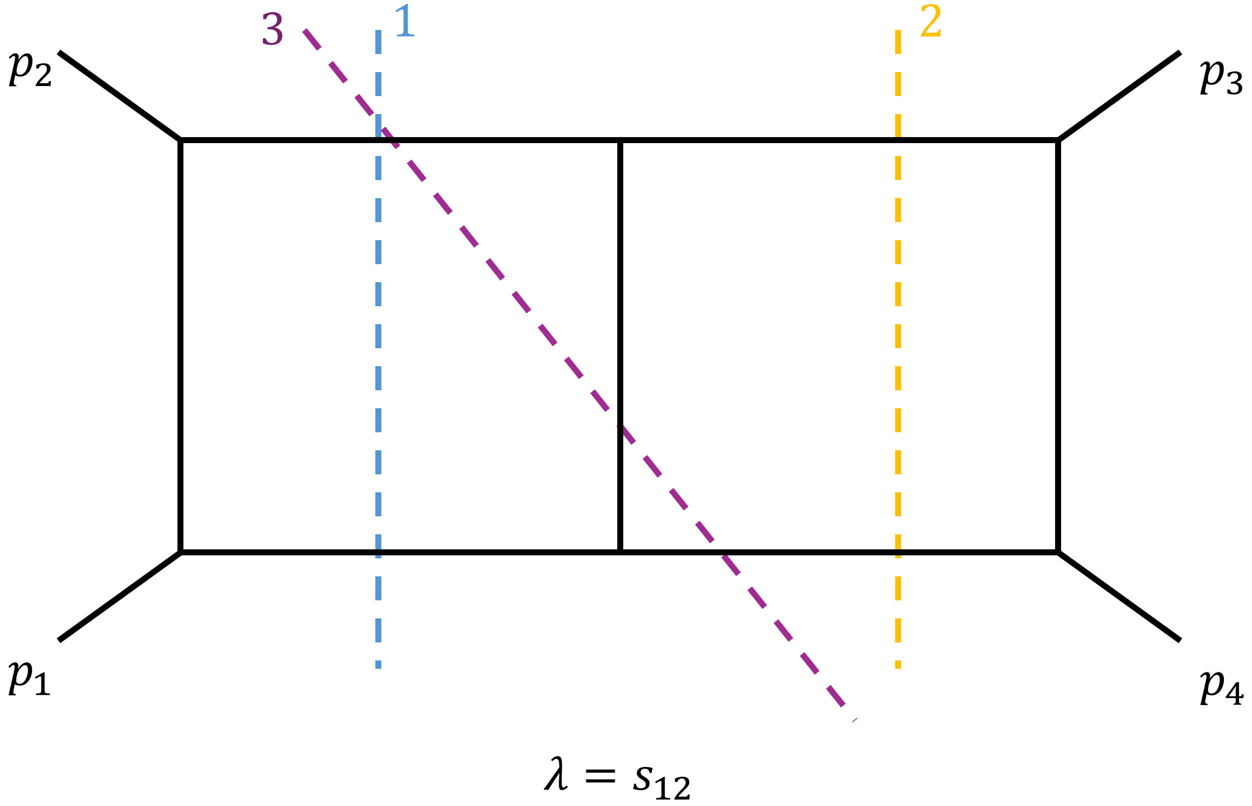
Minimal cuts

- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



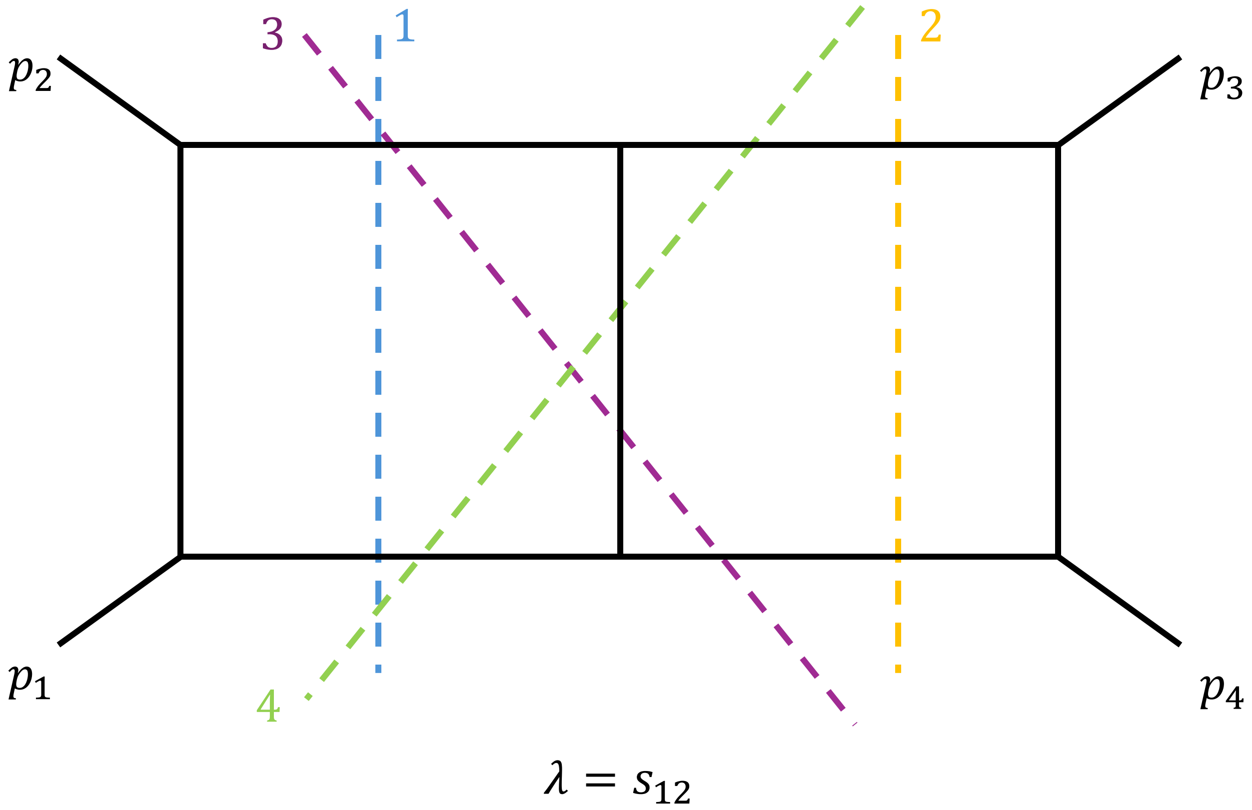
Minimal cuts

- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



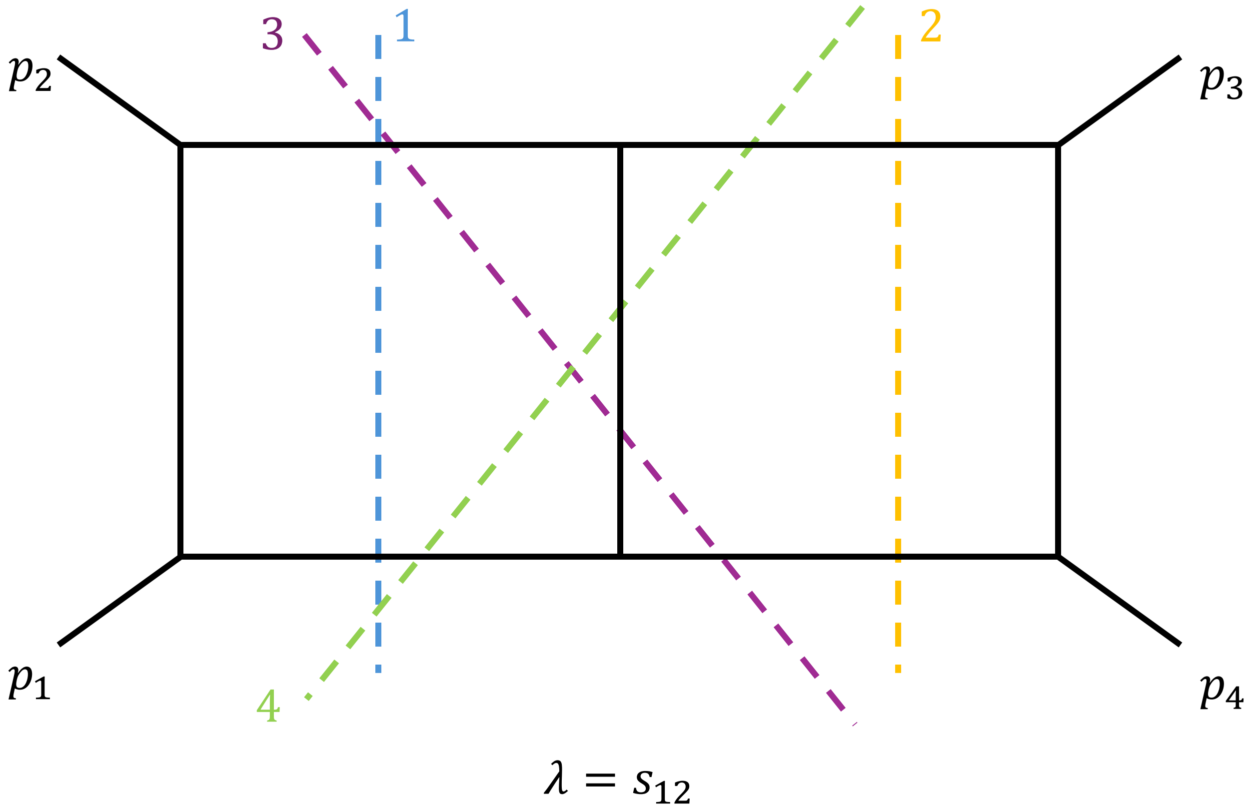
Minimal cuts

- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



Minimal cuts

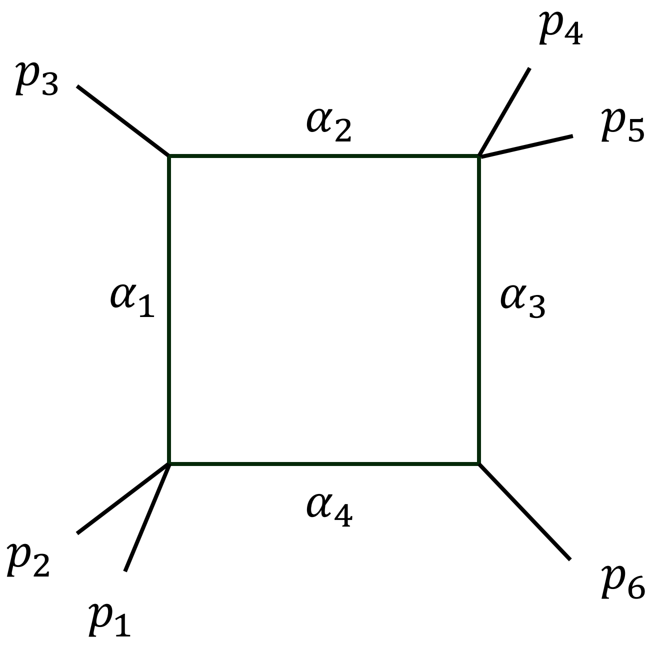
- Minimal cuts are conservative choice, i.e. more propagators can be put on-shell and we could drop more α_i boundaries



We choose to under-constrain the space of the integration contour for our method to be easily implemented

Hierarchical constraints which follow from minimal cuts: Genealogical constraints

Example: 2-mass easy box

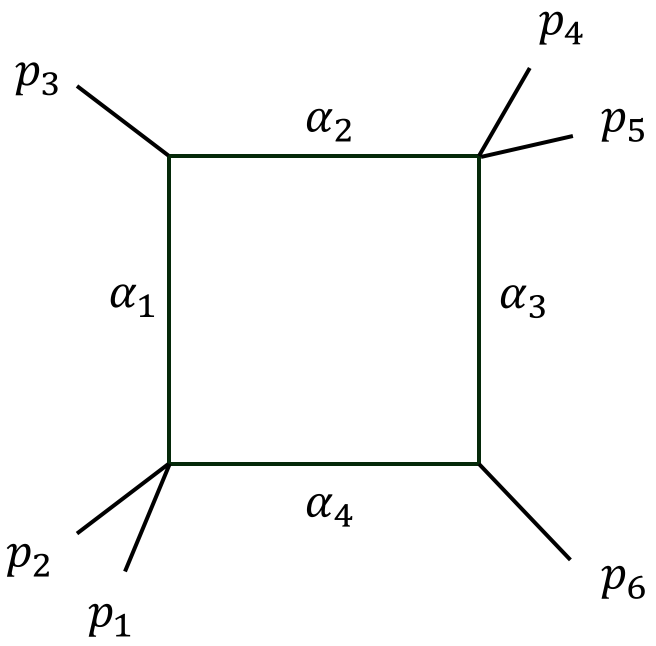


Minimal cuts

→

Euler characteristic test

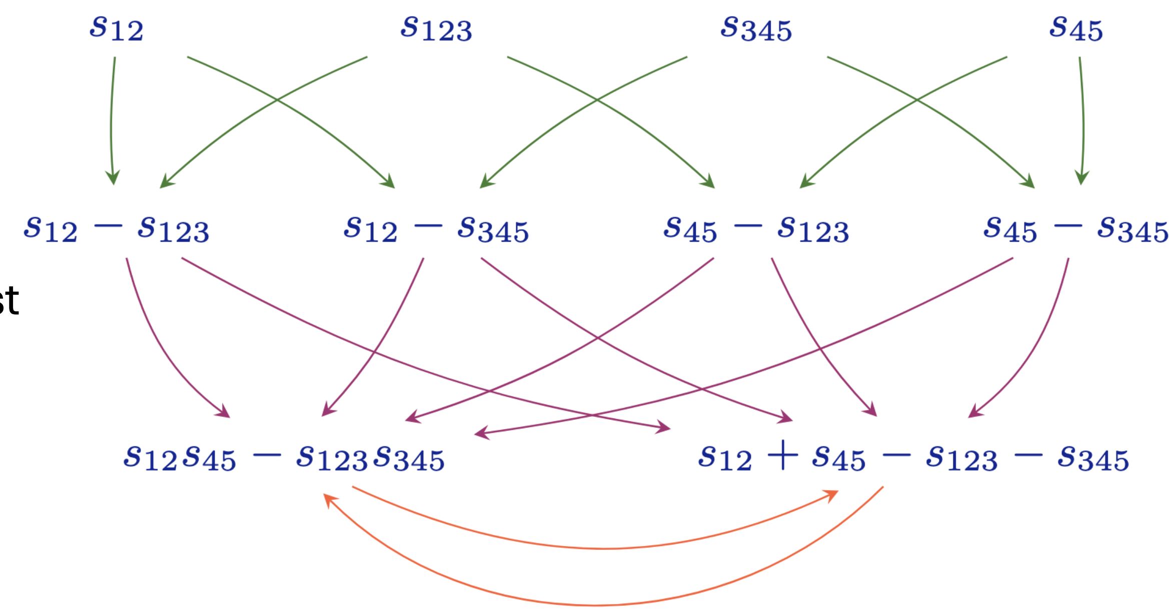
Example: 2-mass easy box



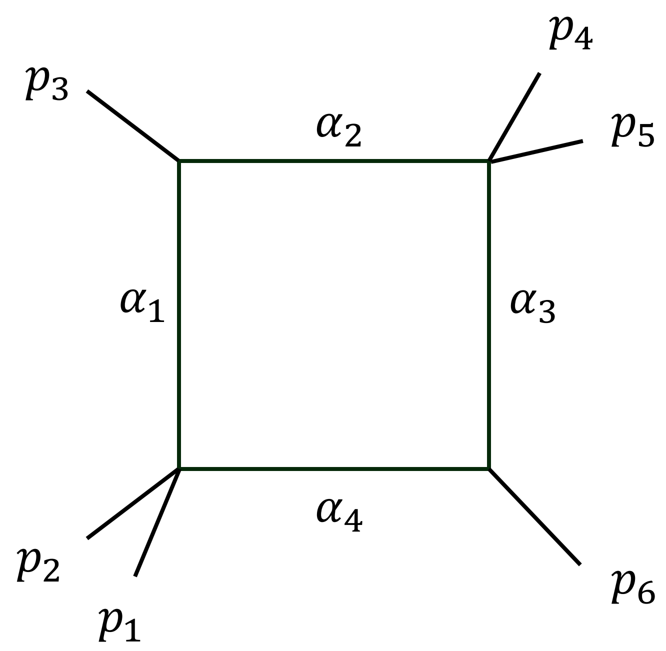
Minimal cuts

Euler characteristic test

Allowed discontinuities by genealogical constraints



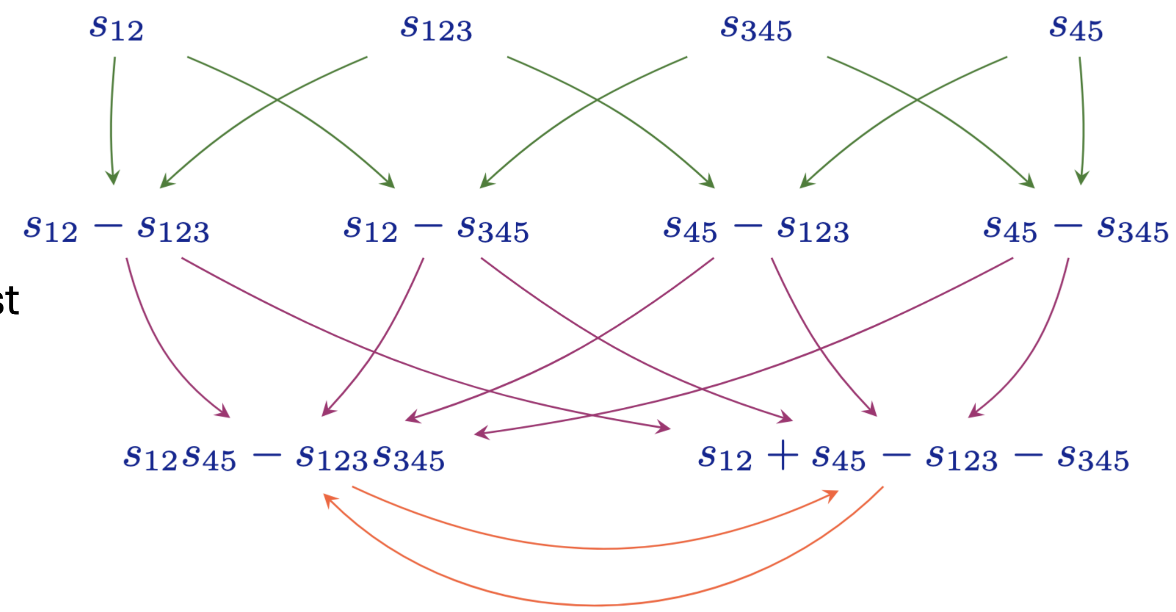
Example: 2-mass easy box



Minimal cuts

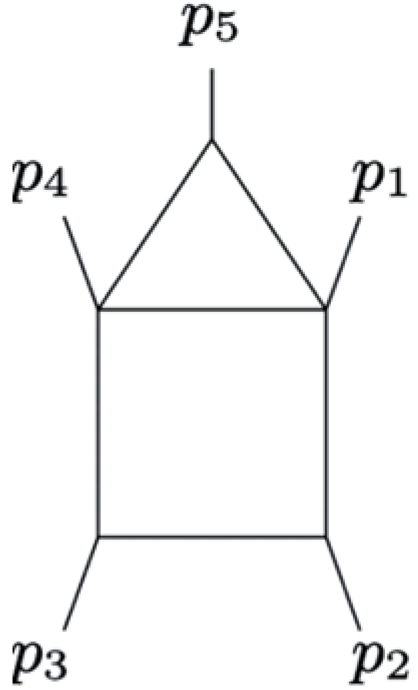
Euler characteristic test

Allowed discontinuities by genealogical constraints



All 64 hierarchical constraints of the type $\ldots \mathrm{Disc}_{\lambda'} \ldots \mathrm{Disc}_{\lambda} \ldots I(p_i) = 0$

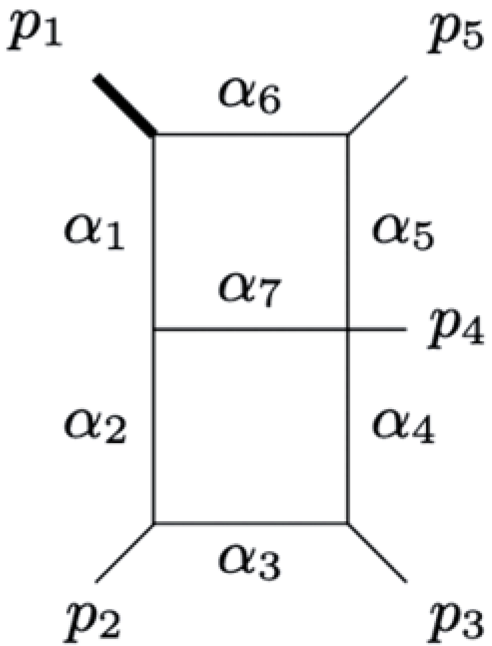
Two-loop examples



156 genealogical constraints

miss only 31 constraints

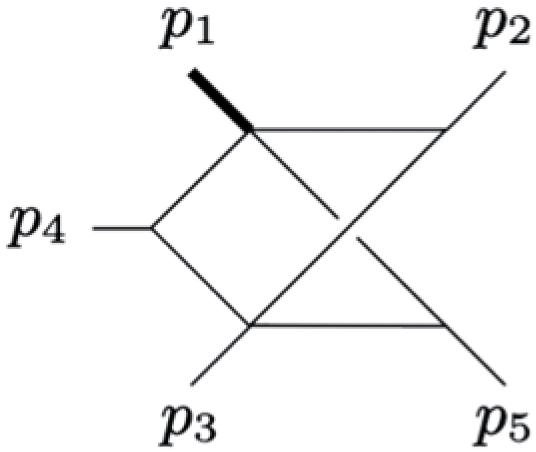
Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser (2019)



620 genealogical constraints

miss only 25 constraints

Abreu, Ita, Moriello, Page, Tschernow, Zeng (2020)



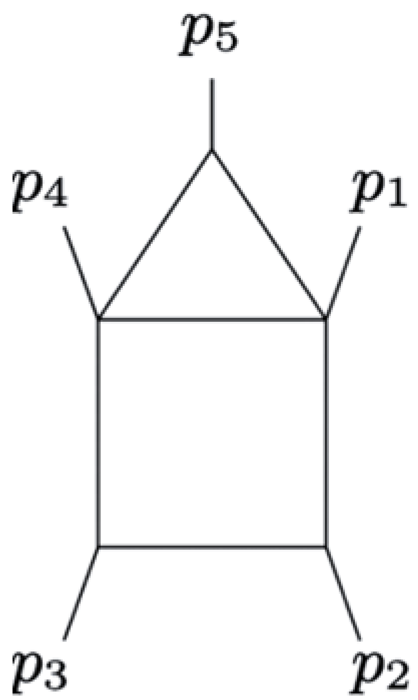
540 genealogical constraints

miss only 9 constraints

Abreu, Ita, Page, Tschernow (2022)

Two-loop examples

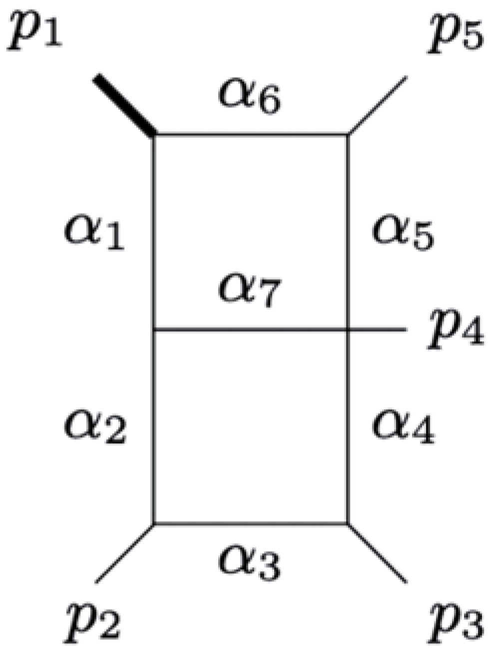
Compared to Steinman relations, for double-box diagram in the middle, we get 305 more constraints on the symbol.



156 genealogical constraints

miss only 31 constraints

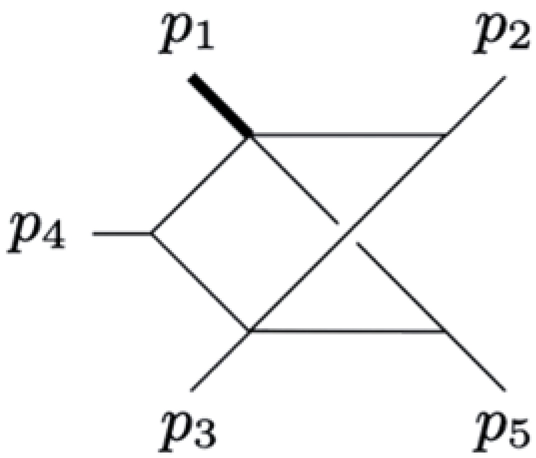
Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser (2019)



620 genealogical constraints

miss only 25 constraints

Abreu, Ita, Moriello, Page, Tschernow, Zeng (2020)



540 genealogical constraints

miss only 9 constraints

Abreu, Ita, Page, Tschernow (2022)

Missing constraints

Missing constraints

- The Euler characteristic test cannot tell if sequences of type

$$\ldots \mathrm{Disc}_\lambda \ldots \mathrm{Disc}_\lambda \ldots I(p_i) = 0$$

  can happen since no change in topology happens

Missing constraints

- The Euler characteristic test cannot tell if sequences of type

$$\ldots \mathrm{Disc}_{\lambda} \ldots \mathrm{Disc}_{\lambda} \ldots I(p_i) = 0$$

  can happen since no change in topology happens

- In Steinmann relations

$$\ldots \mathrm{Disc}_{s_{ij}} \mathrm{Disc}_{s_{jk}} I(p_i) = 0$$

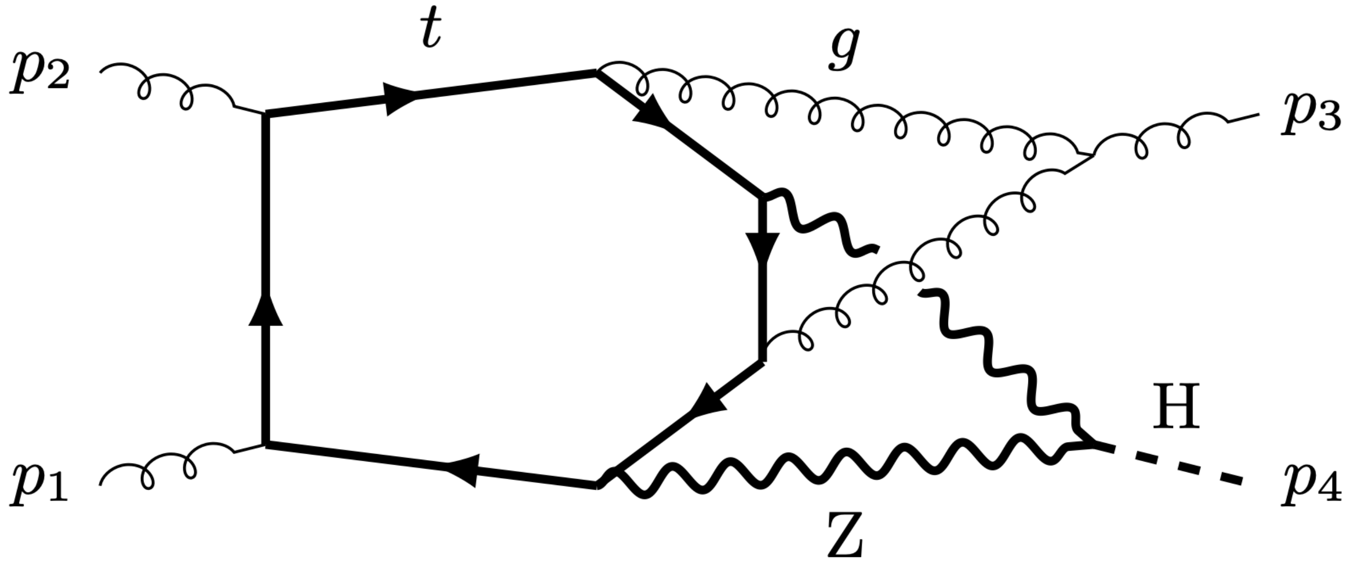
  these sequences are not allowed as first two discontinuities but could happen further in the sequence of discontinuities.
  Euler Characteristic test does not distinguish between these two scenarios

Three-loop example

- Even though we do not know the complete set of kinematic singularities of more complicated diagrams, we can derive some genealogical constraints nevertheless

Three-loop example

- Even though we do not know the complete set of kinematic singularities of more complicated diagrams, we can derive some genealogical constraints nevertheless



$$\Delta_{s_{12}+s_{23}-m_H^2+m_Z^2} \cdots \Delta_{m_H^2-4m_Z^2} \cdots \mathcal{I} = 0$$

Summary

- Genealogical constraints find a rich number of hierarchical constraints on the analytical structure
- Genealogical constraints hold for all orders in dimensional regularisation
- Can be easily derived for any type of massive or massless kinematic configurations
- Further analysis can be conducted focusing on higher power propagators and integrals with numerators

Thank you!