Asymptotically Safe Cosmology

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Cosmology and Conformal Symmetry

- The FRW metric is conformally flat:

$$ds^2 = a^2(\tau)(-d\tau^2 + dr^2 + r^2 d\Omega^2)$$

- The Big Bang singularity is special. All the Weyl-invariant curvature terms are finite. Other curvature terms are also meromorphic functions in the analytic continuation of spacetime.

Cosmology and Conformal Symmetry

- The CMB

$$\langle \delta\rho(x)\delta\rho(y)\rangle \sim \langle \delta\rho(kx)\delta\rho(ky)\rangle$$

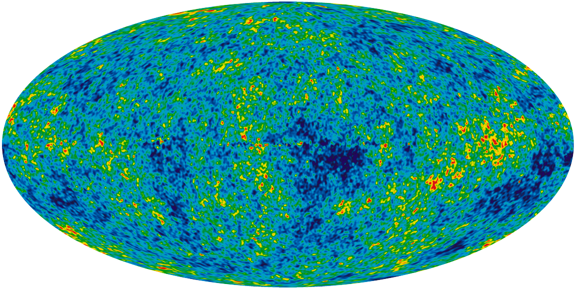

Figure: CMB. Image Source: Wikipedia

Cosmology and Conformal Symmetry

- Imposing classical Weyl symmetry is not sufficient. The effective action calculated from the path integral should also be Weyl invariant.
- The Weyl anomaly if that doesn't happen to be the case is given by:

$$\langle T^{\mu}_{\mu} \rangle = \frac{2}{\sqrt{g}} g_{\mu\nu} \frac{\delta \Gamma}{\delta g_{\mu\nu}}$$

- For conformal matter coupled to a gravitational background, the Weyl anomaly factors into three terms:

$$\langle T^{\mu}_{\mu} \rangle = aE + cC^2 + \Lambda$$

Cosmology and Conformal Symmetry

- The 'a' anomaly, the 'c' anomaly, and the vacuum energy cancel precisely if we have the following matter content (2110.06258 [hep-th]):

$$n_{1/2} = 4n_1; \quad n'_0 = 3n_1; \quad n_0 = 0$$

- This is true for the Standard Model which has 12 gauge bosons and three generations of 48 Weyl fermions.
- However, in this model, the Higgs field is absent. It should emerge after the conformal symmetry has been broken. We also need 36 dimension zero scalars with a fourth derivative Lagrangian.

Cosmology and Conformal Symmetry

- The dimension zero scalar action is given by:

$$\int d^4x\sqrt{g}\frac{1}{2}\phi'(\Delta_4)\phi'$$

- Where $\Delta_4 = \Box^2 + (2R^{\mu\nu} - \frac{2}{3}R\eta^{\mu\nu})\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$ is the fourth derivative operator that ensures local Weyl symmetry for the scalar ϕ' with conformal weight 0.
- These scalars have a trivial Heisenberg algebra and therefore they have no excited states (particles).
- They have scale-invariant correlation functions:

$$\langle\phi(t,x)\phi(t,x')\rangle = \int d^3k\frac{e^{ik.(x-x')}}{k^3} \quad (1)$$

Asymptotic Safety

- The Asymptotic Safety Scenario: The couplings reach a non-trivial fixed point in the UV.

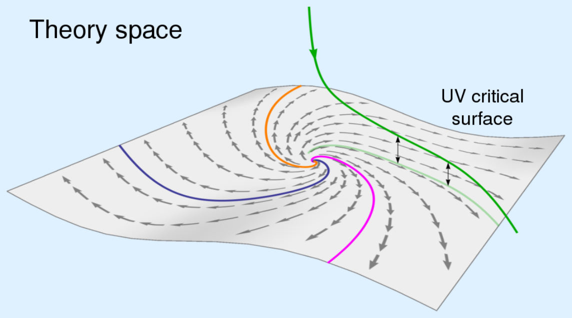

Figure: Asymptotic Safety. Image Source: Wikipedia

Methodology

- Consider the action:

$$e^{\Gamma_k[J]} = \int [D\phi] e^{-S[\phi]+J.\phi-\Delta_k[\phi]}$$

- Renormalization Group Equation:

$$\boxed{\frac{d}{dt}\Gamma_k[\Phi] = \frac{1}{2}\mathrm{Tr}\left(\left(\frac{\delta^2\Gamma_k[\Phi]}{\delta\Phi\delta\Phi} + R_k\right)^{-1}\partial_t R_k\right)}$$

- Example (scalar field):

$$\begin{aligned}
\Gamma_k[\Phi] &= \int d^d x \left(\frac{1}{2}(\partial_\mu \Phi)^2 + V_k(\Phi^2)\right) \\
V_k(\Phi^2) &= \Sigma_n \lambda_{2n} \Phi^{2n} \\
\frac{d}{dt}\Gamma_k[\Phi] &= \int d^d x \partial_t V_k(\Phi^2) = \int d^d x \beta_{2n} \Phi^{2n}
\end{aligned}$$

Methodology

- We consider the gravitational action:

$$\int d^4x\sqrt{g}\left[\frac{1}{16\pi G}(R+2\Lambda)+g_1C^2+g_2E+g_3R^2+g_4\Box R\right]$$

- And the conformal matter action

$$S_{\text{matter}}=\int d^4x\sqrt{g}\left[\frac{-1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\phi(-\Box+\frac{R}{6})\phi+\bar{\psi}e^{\mu}_{a}\gamma^{a}\partial_{\mu}\psi\right.$$
$$\left.+\frac{1}{2}\phi'(\Delta_4)\phi'\right]$$

Methodology

- Note that we do not have any interaction in the matter sector. Therefore, when we truncate the effective action and take its derivative w.r.t. the renormalization time we get the beta functions for gravitational couplings.
- To evaluate the right-hand side of the ERGE, we need to calculate the trace of the effective propagator for vector bosons, scalars, Dirac fermions, and dimension zero scalars.
- We do not path integrate over the graviton.

Results

$$
\begin{aligned}
\beta^{(0)} &= \frac{k^4}{32\pi^2}\left(n_0 - 2n_{1/2} + 2n_1 + 2n_0'\right)\\
\beta^{(1)} &= \frac{k^2}{96\pi^2}\left(n_{1/2} - 4n_1 + \sqrt{\pi}n_0'\right)\\
\beta_1^{(2)} &= \frac{1}{(4\pi)^2 180}(3n_0/2 + 9n_{1/2}/2 + 18n_1 - 12n_0')\\
\beta_2^{(2)} &= \frac{1}{(4\pi)^2 180}(-n_0/2 - 11n_{1/2}/4 - 31n_1 + 14n_0')\\
\beta_3^{(2)} &= 0\\
\beta_4^{(2)} &= \frac{6}{(4\pi)^2 180}(n_0 + n_{1/2}/2 - 3n_1 + 2n_0')\,.
\end{aligned}
$$

Results

- Since G and Λ are dimensionful couplings, we need to rescale them to get dimensionless couplings:

$$\tilde{G} = k^2 G; \quad \tilde{\Lambda} = k^{-2} \Lambda .$$

- The fixed points for these couplings are given when the beta functions $\tilde{\beta}^{(0)}$ and $\tilde{\beta}^{(1)}$ vanish:

$$\begin{aligned} \tilde{G}_* &= \frac{12\pi}{(n_{1/2} - 4n_1 + \sqrt{\pi} n_0')} \\ \tilde{\Lambda}_* &= \frac{3(n_0 - 2n_{1/2} + 2n_1 + 2n_0')}{4(n_{1/2} - 4n_1 + \sqrt{\pi} n_0')} . \end{aligned}$$

Results

- The condition for the beta functions of couplings of C^2, E to vanish and the fixed point for the cosmological constant to be zero is:

$$n_{1/2} = 4n_1; \quad n'_0 = 3n_1; \quad n_0 = 0 \tag{2}$$

- This is the same matter content needed to cancel the Weyl anomaly!
- All the beta functions vanish except for the coupling of $\Box R$ term. But this term will not contribute if the manifold is compact in Euclidean signature or in the context of Lorentzian cosmology has reflecting boundary conditions from the Big Bang.

Thank You!