Multi-Metric Black Holes and the Gregory-Laflamme Instability

Kieran Wood[1], with Paul M. Saffin[1] & Anastasios Avgoustidis[1]

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YTF '24, IPPP, Durham, 19/11/24

Talk based on "Black Holes in Multimetric Gravity" Parts I and II

I. [2402.17835]

II. [2410.10976]

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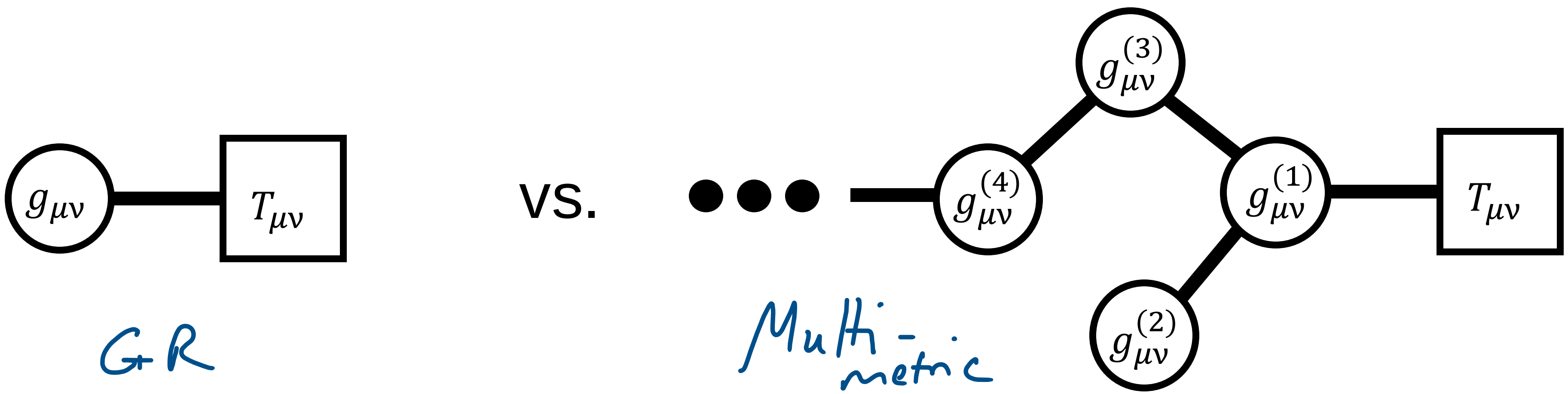
What is multi-metric gravity and why should we care?

As the name suggests, multi-metric gravity is a modified theory of gravity involving multiple interacting metric tensors rather than just one (as in standard GR)



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  - KW, Saffin, Avgoustidis, 2023 [2304.09205]

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- Can recover GR (for now) including viable cosmology so crucial to test!

See reviews by:
- de Rham [1401.4173]
- Hinterbichler [1105.3735]
- Schmidt-May, von Strauss [1512.00021]

See also:
- Hogas, Mortsell [2101.08794], [2101.08795]
- Akrami et al [1503.07521]

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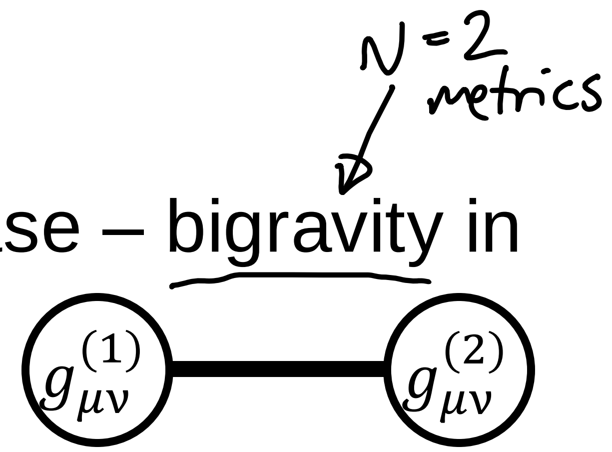

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- Additional class of BHs endowed with massive graviton hair found numerically

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Our work:

1. Why stop at 2 metrics and 4 dimensions?

2. Can we make more sense of the instability?

Short review of multi-metric theory

It's very tricky to couple multiple metrics without introducing ghosts, but it can be done! Need to work with dRGT framework

- de Rham, Gabadadze, 2010 [1007.0443]
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- $$S_K = \sum_{i=0}^{N-1} \frac{M_i^{D-2}}{2} \int \mathrm{d}^D x \sqrt{-\det g_{(i)}} R_{(i)}$$

← N copies of Einstein-Hilbert term

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- $S_K = \sum_{i=0}^{N-1} \frac{M_i^{D-2}}{2} \int \mathrm{d}^D x \sqrt{-\det g_{(i)}} R_{(i)}$ ← N copies of Einstein-Hilbert term
- $S_V = -\sum_{i,j} \int \mathrm{d}^D x \sqrt{-\det g_{(i)}} \sum_{m=0}^{D} \beta_m^{(i,j)} e_m(S_{i\to j})$ ← ghost free interaction terms.

Interaction structure

Let's examine the potential in more detail

$$S_V = -\sum_{i,j} \int \mathrm{d}^D x \sqrt{-\det g_{(i)}} \sum_{m=0}^{D} \beta_m^{(i,j)} e_m(S_{i\to j})$$

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- $\beta_m^{(i,j)} = \beta_m^{(j,i)}$ are arbitrary constants characterising the interactions
- $e_m(S)$ are elementary symmetric polynomials of the eigenvalues of S
- Interactions ghost free by virtue of special structure of $S_{i\to j}$

$$S_{i\to j} = \sqrt{g_{(i)}^{-1} g_{(j)}}$$

In the sense that $(S^2_{i\to j})^\mu{}_\nu = g^{(i)\mu\lambda} g^{(j)}_{\lambda\nu}$.

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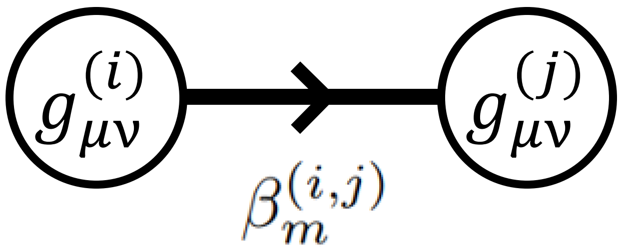
We can express these interactions diagrammatically



- β's live on interaction links
- Interactions are oriented in direction of arrows since $S_{i\to j} = S_{j\to i}^{-1}$

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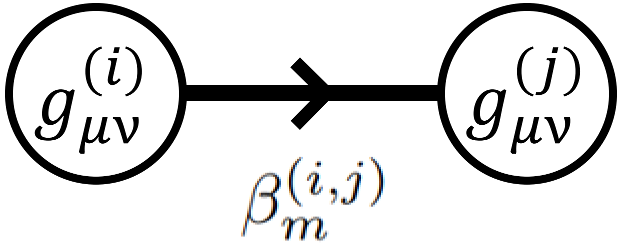

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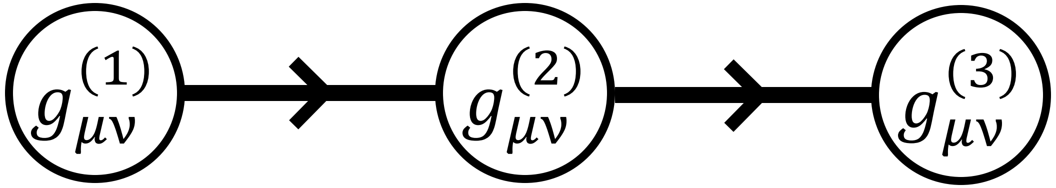
Example: 3 metric chain



$$S_V = -\int \mathrm{d}^D x \left[\sqrt{-g_{(1)}} \sum_{m=0}^{D} \beta_m^{(1,2)} e_m(S_{1\to 2}) + \sqrt{-g_{(2)}} \sum_{m=0}^{D} \beta_m^{(2,3)} e_m(S_{2\to 3}) \right]$$

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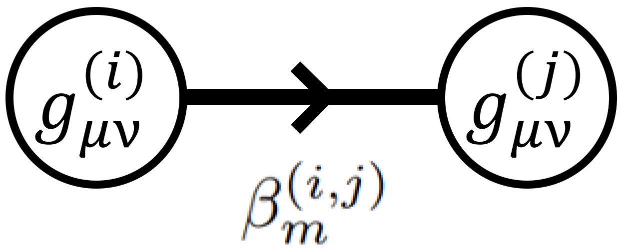

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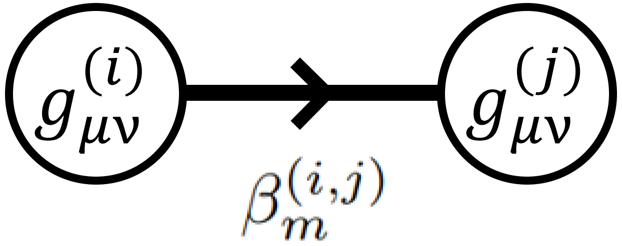

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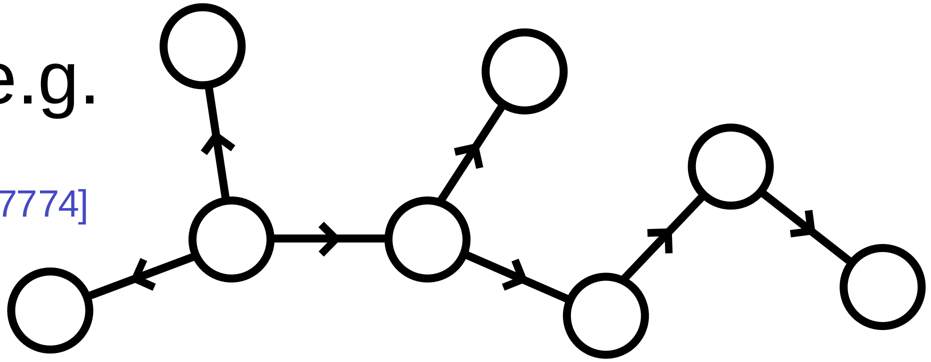

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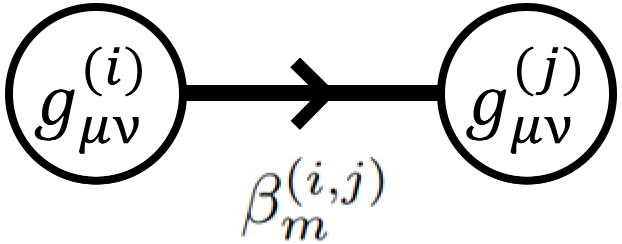

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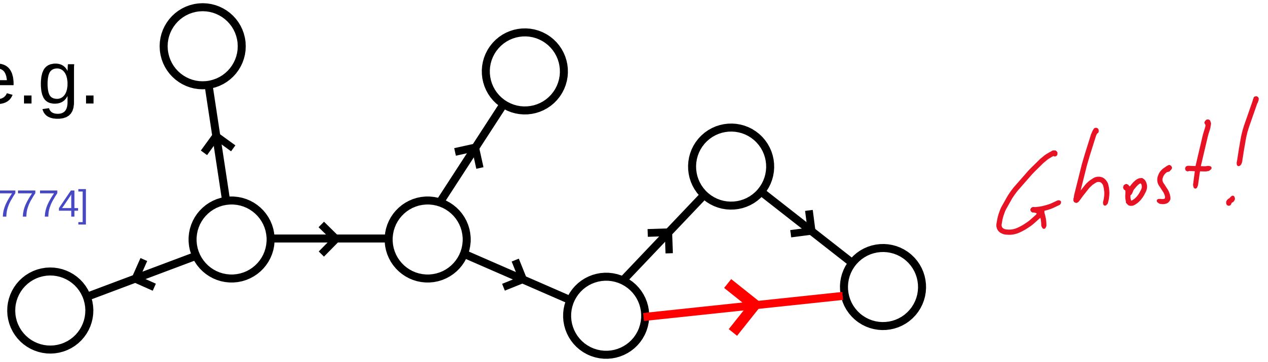

Equations of motion

Einstein equations pick up an extra term which encodes the effect of the interactions

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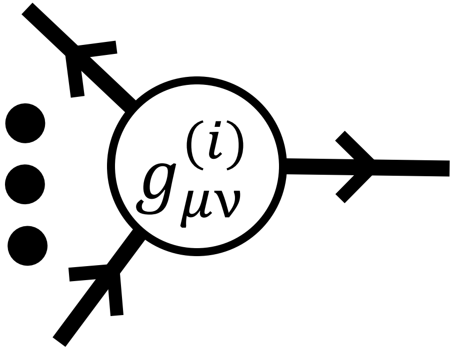

$$W^{(i)\mu}{}_{\nu} = \sum_j \sum_{m=0}^{D} (-1)^m \beta_m^{(i,j)} Y^{\mu}_{(m)\nu}(S_{i\to j}) + \sum_k \sum_{m=0}^{D} (-1)^m \beta_{D-m}^{(k,i)} Y^{\mu}_{(m)\nu}(S^{-1}_{k\to i})$$

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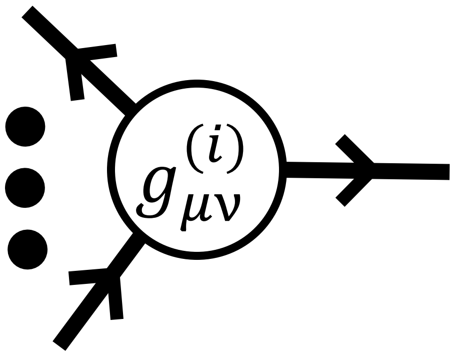

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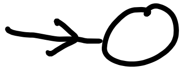

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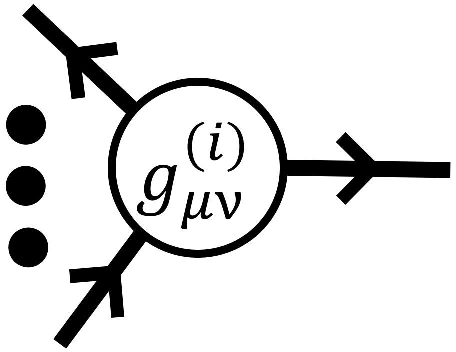

- Each interaction contributes a separate term to W-tensors of metrics it connects
- Inward and outward arrows contribute differently

Black hole solutions

Look for vacuum solutions describing rotating and asymptotically (A)dS BHs (can also add charge in $D = 4$, and hairy solutions exist but must be found numerically)

Ansatz based on Kerr-(A)dS metric of GR:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{r_s}{U(r)} l_\mu l_\nu$$

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$\bar{g}_{\mu\nu}$: (A)dS metric in D-dims

$l_\mu l_\nu$: Comps. of 1-form dual to null vector tangent to geodesic in (A)dS.

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$$g^{(i)}_{\mu\nu} = a_i^2 \left[\bar{g}_{\mu\nu} + \frac{r_{s,i}}{U(r)} l_\mu l_\nu \right]$$

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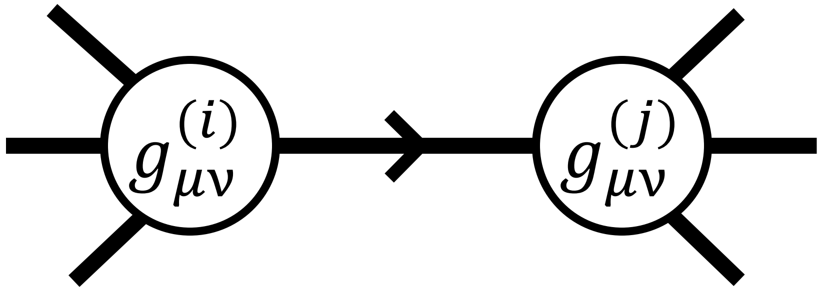

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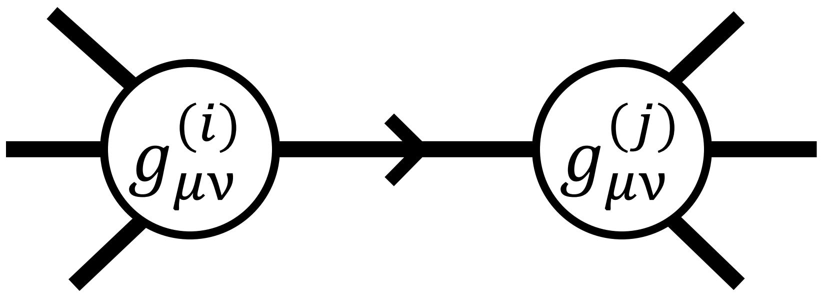

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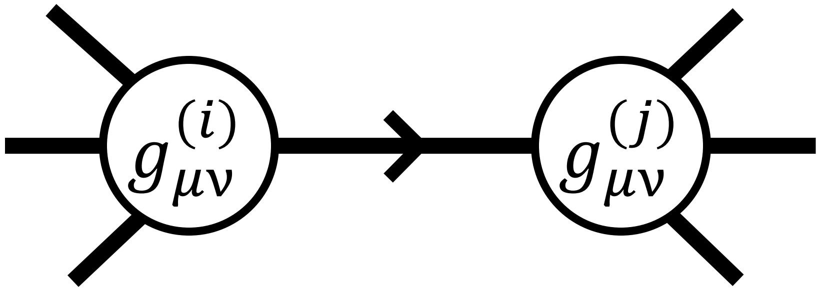

$$\sigma^{(+)}_{i,j} = \sum_{m=0}^{D} \beta^{(i,j)}_{m} \binom{D-2}{m-1} a_j^{m-1} a_i^{1-m} \,,$$

$$\sigma^{(-)}_{i,k} = \sum_{m=0}^{D} \beta^{(k,i)}_{D-m} \binom{D-2}{m-1} a_k^{m-1} a_i^{1-m} \,.$$

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Metrics proportional to each other, and to standard GR solution

Option 2: make $\sigma^{(+)}_{i,j} = 0$

Metrics not proportional, have their own horizon sizes

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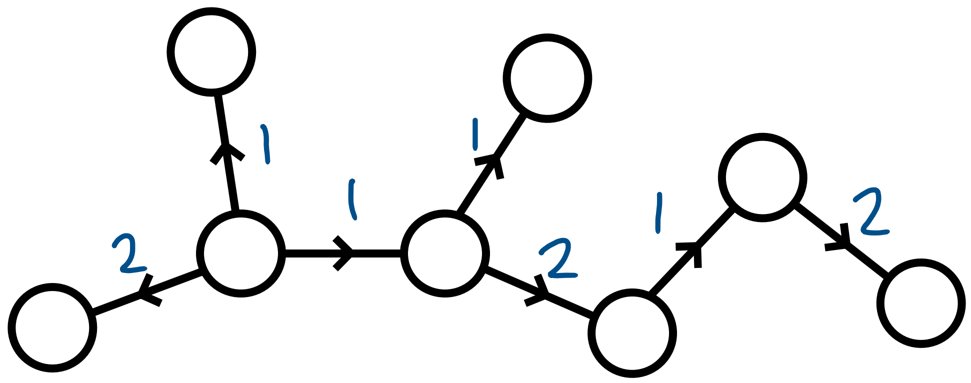
For a general multi-metric theory, need to choose either option 1 or option 2 for each interaction



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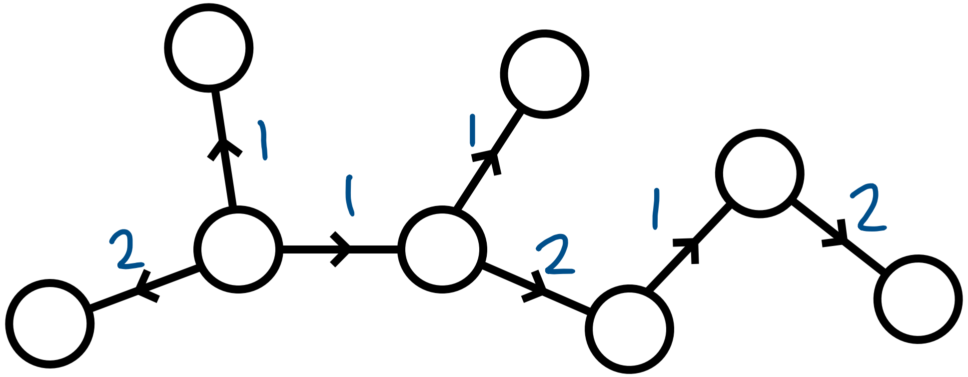

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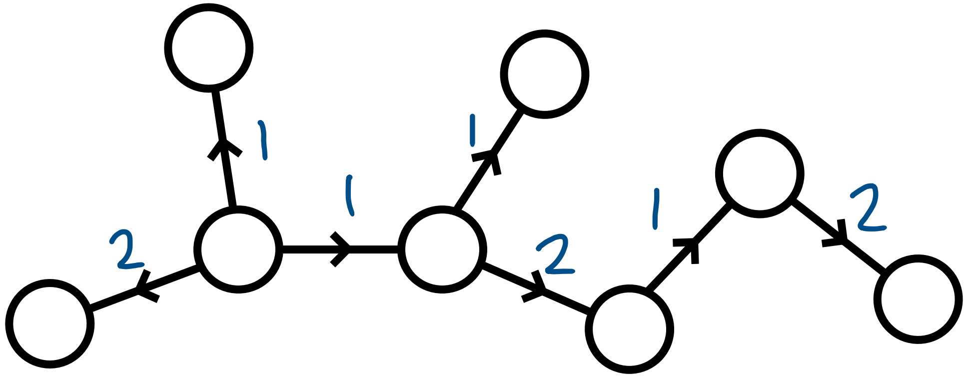

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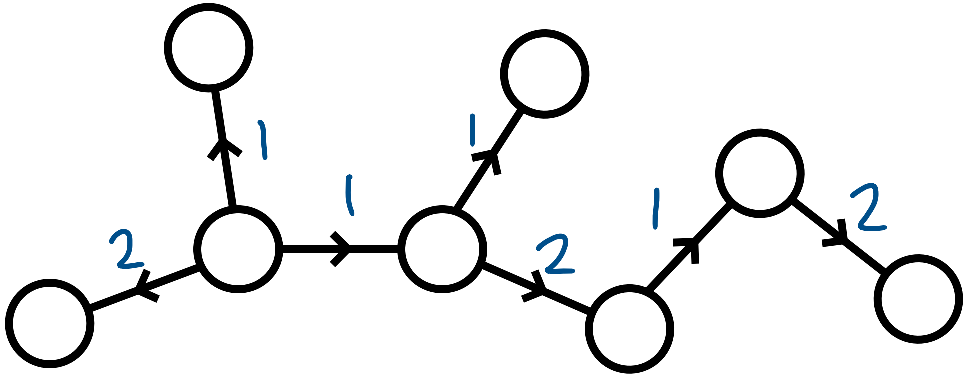

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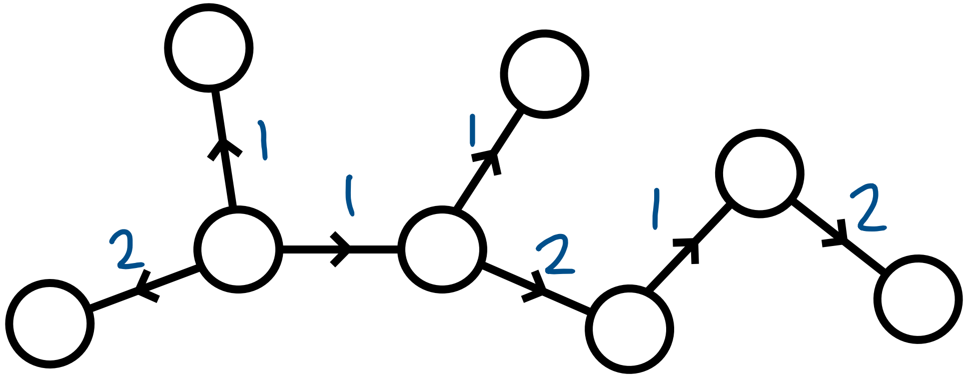

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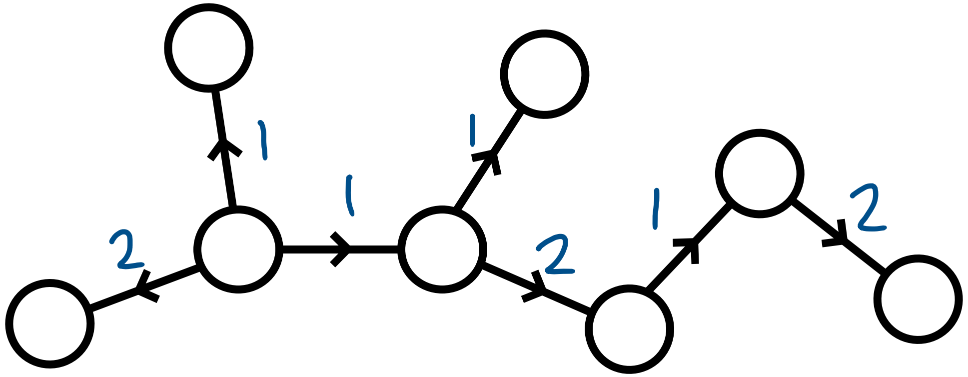

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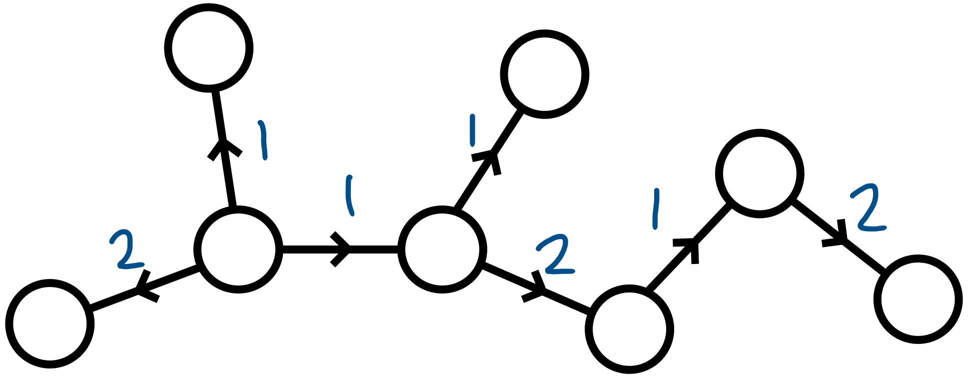

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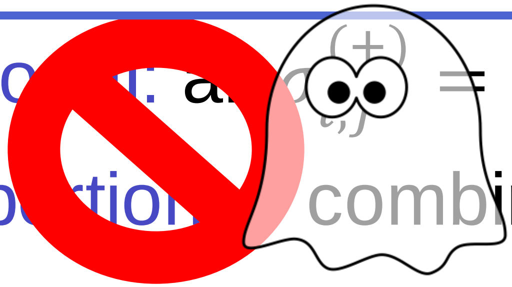

Instability of proportional solutions

Around proportional solutions, perturbations take the form:

i-th mass eigenstate

$$\bar{\Box} H^{(i)}_{\mu\nu} + 2\bar{R}^{\alpha\ \beta}_{\ \mu\ \nu} H^{(i)}_{\alpha\beta} = m_i^2 H^{(i)}_{\mu\nu}$$

i-th graviton mass

(1 massless, N-1 massive).

The structure of the graviton mass modes is determined entirely from the interaction coefficients.

Instability of proportional solutions

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Precisely those eqs studied in the context of the GL instability plaguing black strings in 5D!

- Gregory, Laflamme, 1993 & 1994 [hep-th/9301052] & [hep-th/9404071]

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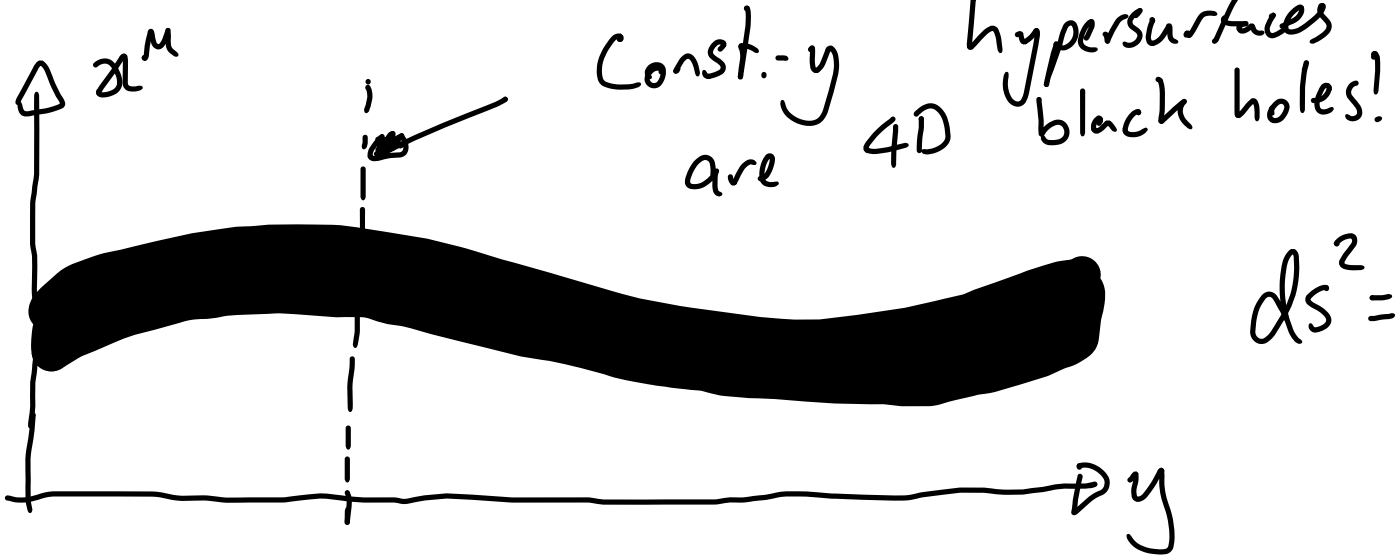


$$ds^2 = g^{(BH)}_{\mu\nu} dx^\mu dx^\nu + dy^2 .$$

Instability of proportional solutions

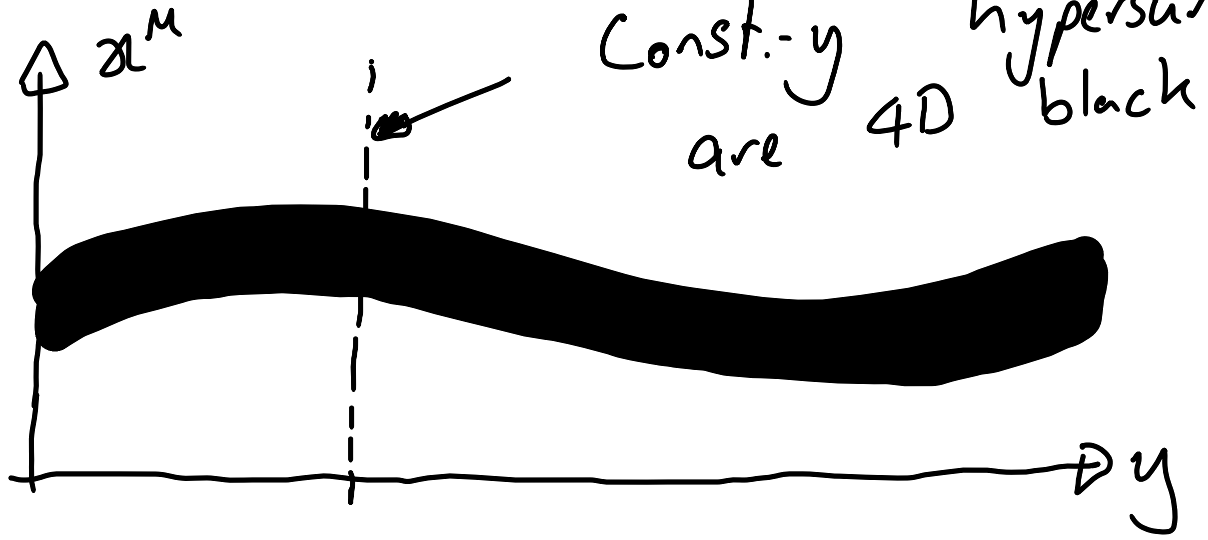
$$\bar{\Box} H^{(i)}_{\mu\nu} + 2\bar{R}^{\alpha\ \beta}_{\ \mu\ \nu} H^{(i)}_{\alpha\beta} = m_i^2 H^{(i)}_{\mu\nu}$$

In the GL case:

- $H^{(i)}_{\mu\nu}$ are Fourier modes
- $m_i \to k_i$ is the Fourier momentum.

Precisely those eqs studied in the context of the GL instability plaguing black strings in 5D!

- Gregory, Laflamme, 1993 & 1994 [hep-th/9301052] & [hep-th/9404071]



Const.-y hypersurfaces are 4D black holes!

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Solution unstable whenever $m_i r_s \lesssim 876$. Only stable iff lightest graviton evades bound

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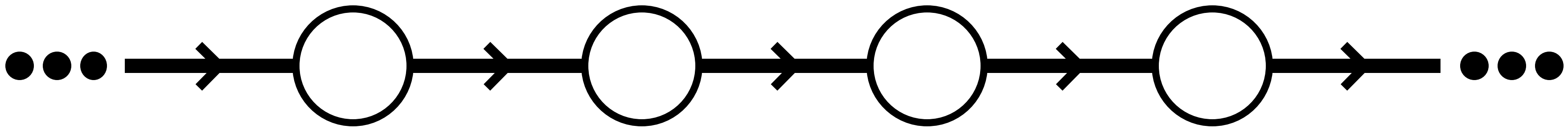
2. Kerr $\bar{g}_{\mu\nu}$

Additional superradiant instability in same range $m_i r_s \sim \mathcal{O}(1)$ but subdominant to GL instability

- East, Siemonsen, 2023 [2309.05096]

Instability of proportional solutions

Does this make sense? Well, let's consider the 'chain' theory



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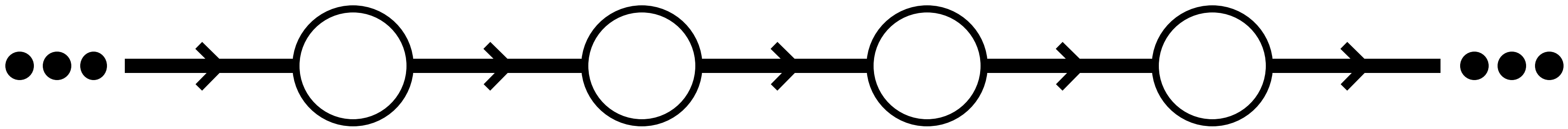
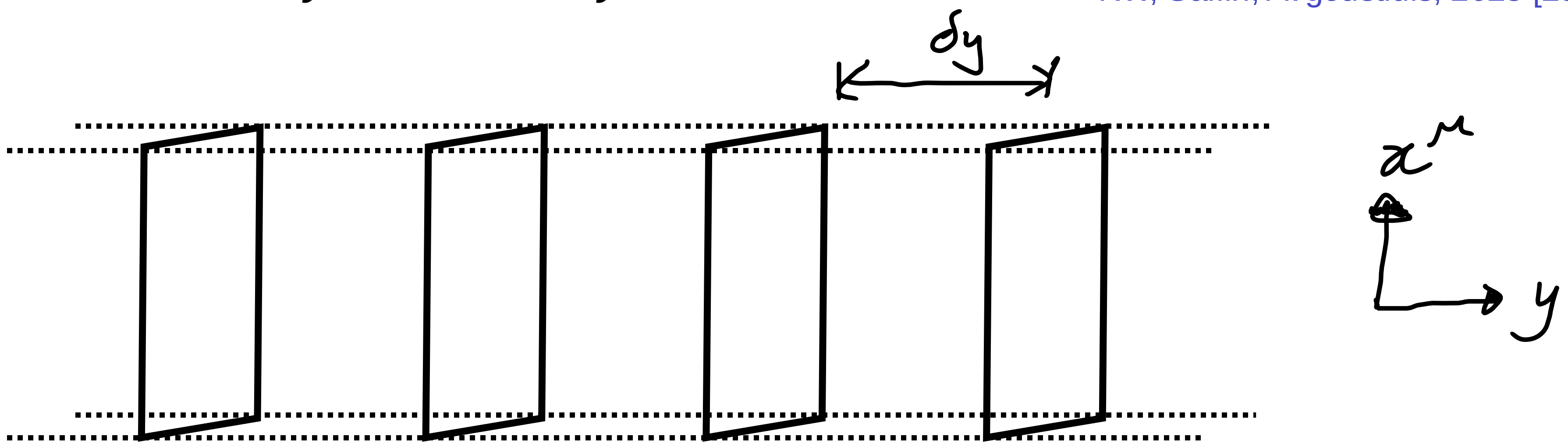


Think of each metric as defining a hypersurface in a compact extra dimension, separated by distance δy, construct extra dimension by taking limit $N \to \infty,\ \delta y \to 0,\ N\delta y = L$.

- de Rham, Matas, Tolley, 2013 [1308.4136]
- KW, Saffin, Avgoustidis, 2023 [2304.09205]



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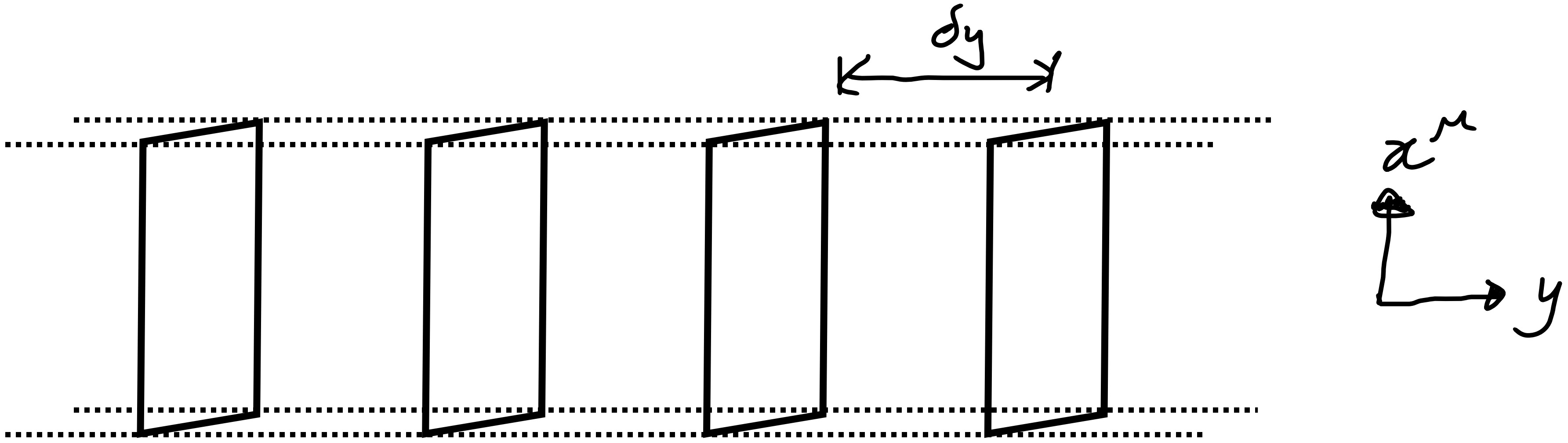

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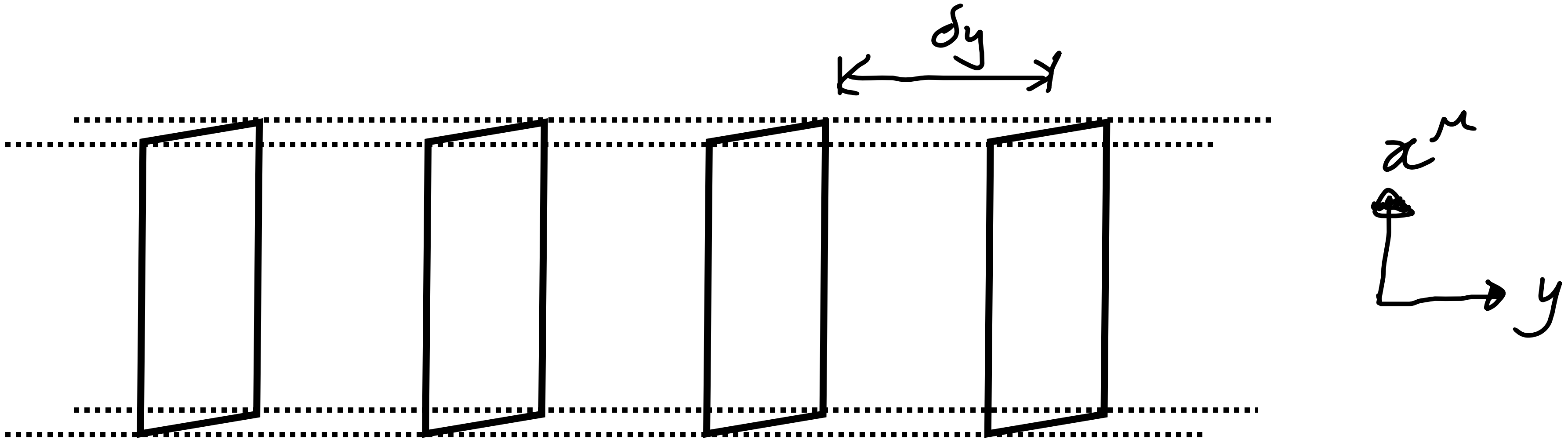

GL instability exists in 5D context, where each hypersurface is Schwarzchild. Obvious, then, that same behaviour should show in deconstructed theory.

What *is* interesting is that instability exists even away from continuum limit, and away from the chain theory! Extra dimensions unnecessary! GL instability says something fundamental about spin-2 interactions?

More on the GL instability

Graviton masses heavily constrained by (e.g.) Solar System tests of gravity. 2 regimes are still allowed:

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Graviton masses heavily constrained by (e.g.) Solar System tests of gravity. 2 regimes are still allowed:

1. Very light masses

Vainshtein mechanism ensures GR recovered. Most BHs unstable, but timescale of order age of Universe – not physically relevant!

2. Very heavy masses

Tiny Compton wavelength ensures GR recovered. Only lightest BHs unstable, but instability is rapid, end state unclear!

How does the instability saturate?

The proportional solutions are unstable, but this instability must lead somewhere that (presumably) *is* stable. So, look for generic spherically symmetric solutions to field equations:

$$\mathrm{d}s^2_{(i)} = -p_i^2(r)\mathrm{d}t^2 + \frac{U_i'^2(r)}{Y_i^2(r)}\mathrm{d}r^2 + U_i^2(r)\mathrm{d}\Omega_2^2$$

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But, very complicated! (already the equations for just 3 metrics take up a 27000-line text file)

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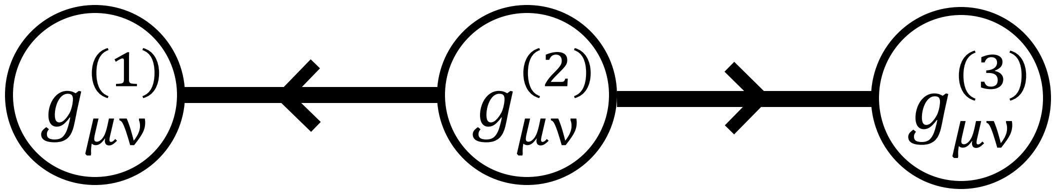

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Use gauge freedom to fix $U_2(r) = r$, rest of the functions must remain free. Can reduce the system down to:

$$\begin{aligned}
Y_1' &= \mathcal{F}_1(r, U_1, U_3, Y_1, Y_2, Y_3) \\
Y_2' &= \mathcal{F}_2(r, U_1, U_3, Y_1, Y_2, Y_3) \\
Y_3' &= \mathcal{F}_3(r, U_1, U_3, Y_1, Y_2, Y_3) \\
U_1' &= \mathcal{F}_4(r, U_1, U_3, Y_1, Y_2, Y_3) \\
U_3' &= \mathcal{F}_5(r, U_1, U_3, Y_1, Y_2, Y_3)
\end{aligned}$$

$$\begin{aligned}
p_2' &= p_2 F_2(r, U_1, U_3, Y_1, Y_2, Y_3) \\
p_1 &= p_2 G_1(r, U_1, U_3, Y_1, Y_2, Y_3) \\
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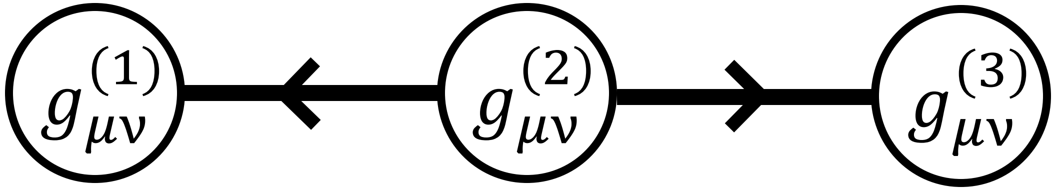

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Far away from the black hole, as $r \to \infty$, we can write:

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$$\delta Y_1 = -\frac{A}{2r} + \frac{B}{r}(1+mr)\,e^{-mr} + \frac{C}{r}\left(1+\sqrt{3}mr\right)e^{-\sqrt{3}mr}\,,$$
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Newtonian piece

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Newtonian piece

Yukawa, mass m

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Newtonian piece

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Yukawa, mass $\sqrt{3}m$.

How does the instability saturate?

Result is a superposition of a Newtonian piece from the massless graviton and Yukawa pieces from each of the two massive gravitons.

This is interpreted as the black hole (probably) gaining massive graviton hair once it crosses the GL instability bound - full numerical calculation of whole solution still to come in separate paper.

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One can't say with *certainty* that this is how the instability saturates – need numerical relativity simulations. However, multi-metric gravity has no well-posed dynamical formulation suited to doing this yet.

Summary

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- Black holes in the theory come in a few different guises. Proportional solutions suffer GL instability, non-proportional solutions linearly stable but likely nonlinearly unstable
- GL instability existing in these systems maybe says something fundamental about massive spin-2 interactions
- Likely end state is black hole endowed with massive graviton hair; potential observational consequences in (e.g.) GWs to be worked out down the line

Thanks for listening!

Email: kieran.wood@nottingham.ac.uk

Backup slides

Black hole solutions

With this ansatz, the Einstein tensor components are:

$$G^{(i)\mu}{}_{\nu} = -\frac{\Lambda}{a_i^2}\delta^{\mu}_{\nu}$$

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$$\sigma_{i,j}^{(+)} \propto \sigma_{j,i}^{(-)} \,.$$

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- Diagonal (δ^{μ}_{ν}) part of W-tensor gives N simultaneous equations for the cosmological constant and $N-1$ conformal factors (can always fix one by coordinate rescaling)
- Off-diagonal ($l^\mu l_\nu$) part must vanish!