Massive spinning fields in 3d quantum gravity

Robert Bourne

DAMTP, University of Cambridge

YTF 2024

Based on 2407.09608 with Alejandra Castro, Jackson Fliss

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!
Solution: Let's look at a toy model.

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!
Solution: Let's look at a toy model.

In 2+1 dimensions general relativity is

- topological

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!
Solution: Let's look at a toy model.

In 2+1 dimensions general relativity is

- topological
- renormalizeable

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!
Solution: Let's look at a toy model.

In 2+1 dimensions general relativity is

- topological
- renormalizeable
- contains interesting solutions such as the BTZ black hole.

Gravity in 2+1 dimensions

Problem: Quantum gravity is hard!
Solution: Let's look at a toy model.

In 2+1 dimensions general relativity is

- topological
- renormalizeable
- contains interesting solutions such as the BTZ black hole.

I will mostly discuss $\Lambda > 0$.

Gravity and Chern-Simons theory

We can write 3d gravity in terms of the dreibein and spin connection

$$e^a, \qquad \omega^a \ .$$

Gravity and Chern-Simons theory

We can write 3d gravity in terms of the dreibein and spin connection

$$e^a, \qquad \omega^a \ .$$

- Transform under isometry group $SO(4) = SU(2)_L \times SU(2)_R/\mathbb{Z}_2$

Gravity and Chern-Simons theory

We can write 3d gravity in terms of the dreibein and spin connection

$$e^a, \qquad \omega^a \ .$$

- Transform under isometry group $SO(4) = SU(2)_L \times SU(2)_R/\mathbb{Z}_2$
- Dreibein must give an invertible metric

Gravity and Chern-Simons theory

We can write 3d gravity in terms of the dreibein and spin connection

$$e^a, \qquad \omega^a \ .$$

- Transform under isometry group $SO(4) = SU(2)_L \times SU(2)_R/\mathbb{Z}_2$
- Dreibein must give an invertible metric

Rewrite these as a pair of $\mathfrak{su}(2)$ connections

$$A_L = i\left(\omega^a + \frac{e^a}{\ell_{dS}}\right) L_a \ , \qquad A_R = i\left(\omega^a - \frac{e^a}{\ell_{dS}}\right) \bar{L}_a \ .$$

Gravity and Chern-Simons theory

At the level of the action and equations of motion, 3d gravity looks like two Chern-Simons theories

$$S_{EH} = kS_{CS}[A_L] - kS_{CS}[A_R] \, , \qquad k = \frac{\ell_{dS}}{4G_N}$$

$$G_{\mu\nu} + \frac{1}{\ell_{dS}^2} g_{\mu\nu} = 0 \Leftrightarrow \begin{cases} F_L = \mathrm{d}A_L + A_L \wedge A_L = 0 \\ F_R = \mathrm{d}A_R + A_R \wedge A_R = 0 \end{cases} \qquad .$$

Gravity and Chern-Simons theory

At the level of the action and equations of motion, 3d gravity looks like two Chern-Simons theories

$$S_{EH} = kS_{CS}[A_L] - kS_{CS}[A_R]\ , \qquad k = \frac{\ell_{dS}}{4G_N}$$

$$G_{\mu\nu} + \frac{1}{\ell_{dS}^2} g_{\mu\nu} = 0 \Leftrightarrow \begin{cases} F_L = \mathrm{d}A_L + A_L \wedge A_L = 0 \\ F_R = \mathrm{d}A_R + A_R \wedge A_R = 0 \end{cases} .$$

But...

Gravity and Chern-Simons theory

At the level of the action and equations of motion, 3d gravity looks like two Chern-Simons theories

$$S_{EH} = kS_{CS}[A_L] - kS_{CS}[A_R]\ , \qquad k = \frac{\ell_{dS}}{4G_N}$$

$$G_{\mu\nu} + \frac{1}{\ell_{dS}^2} g_{\mu\nu} = 0 \Leftrightarrow \begin{cases} F_L = \mathrm{d}A_L + A_L \wedge A_L = 0 \\ F_R = \mathrm{d}A_R + A_R \wedge A_R = 0 \end{cases} .$$

But...

- Chern-Simons theory has a larger phase space. It does not constraint the dreibein to be invertible.

Gravity and Chern-Simons theory

At the level of the action and equations of motion, 3d gravity looks like two Chern-Simons theories

$$S_{EH} = kS_{CS}[A_L] - kS_{CS}[A_R]\ , \qquad k = \frac{\ell_{dS}}{4G_N}$$

$$G_{\mu\nu} + \frac{1}{\ell_{dS}^2} g_{\mu\nu} = 0 \Leftrightarrow \begin{cases} F_L = \mathrm{d}A_L + A_L \wedge A_L = 0 \\ F_R = \mathrm{d}A_R + A_R \wedge A_R = 0 \end{cases} .$$

But...

- Chern-Simons theory has a larger phase space. It does not constraint the dreibein to be invertible.
- Chern-Simons theory exists on a fixed topology, (quantum) gravity should not.

Gravity and Chern-Simons theory

At the level of the action and equations of motion, 3d gravity looks like two Chern-Simons theories

$$S_{EH} = kS_{CS}[A_L] - kS_{CS}[A_R] \,, \qquad k = \frac{\ell_{dS}}{4G_N}$$

$$G_{\mu\nu} + \frac{1}{\ell_{dS}^2} g_{\mu\nu} = 0 \Leftrightarrow \begin{cases} F_L = \mathrm{d}A_L + A_L \wedge A_L = 0 \\ F_R = \mathrm{d}A_R + A_R \wedge A_R = 0 \end{cases} .$$

But...

- Chern-Simons theory has a larger phase space. It does not constraint the dreibein to be invertible.
- Chern-Simons theory exists on a fixed topology, (quantum) gravity should not.
- Chern-Simons theory does *not* include the large diffeomorphisms of gravity.

Adding matter

It is possible to address these issues precisely for pure $\Lambda < 0$ gravity [Collier, Eberhardt, Zhang '23].

Adding matter

It is possible to address these issues precisely for pure $\Lambda < 0$ gravity [Collier, Eberhardt, Zhang '23].

To add matter we shall instead work perturbatively around each saddle point of the action.

Adding matter

It is possible to address these issues precisely for pure $\Lambda < 0$ gravity [Collier, Eberhardt, Zhang '23].

To add matter we shall instead work perturbatively around each saddle point of the action.

Schematically

$$Z \sim \int \mathcal{D}g \;\; e^{-S_{EH}[g]} Z_{\text{matter}}[g] \;.$$

Adding matter

It is possible to address these issues precisely for pure $\Lambda < 0$ gravity [Collier, Eberhardt, Zhang '23].

To add matter we shall instead work perturbatively around each saddle point of the action.

Schematically

$$Z \sim \int \mathcal{D}g \; e^{-S_{EH}[g]} Z_{\text{matter}}[g] \; .$$

We want to find a Chern-Simons version of this

$$Z \sim \int \mathcal{D}A_L \mathcal{D}A_R \; e^{-kS_{CS}[A_L]+kS_{CS}[A_R]} Z_{\text{matter}}[A_L, A_R] \; .$$

Adding matter

What should $Z_{\text{matter}}[A_L, A_R]$ look like?
Some hints:

Adding matter

What should $Z_{\text{matter}}[A_L, A_R]$ look like?
Some hints:

- Wilson loops in suitable representations correspond to particle worldline quantum mechanics [Castro, Iqbal, Llabrés '20]

Adding matter

What should $Z_{\text{matter}}[A_L, A_R]$ look like?
Some hints:

- Wilson loops in suitable representations correspond to particle worldline quantum mechanics [Castro, Iqbal, Llabrés '20]
- On the classical background our expression should reproduce the known one-loop determinant for a given field [Anninos, Denef, Law, Sun '22]

Adding matter

What should $Z_{\text{matter}}[A_L, A_R]$ look like?
Some hints:

- Wilson loops in suitable representations correspond to particle worldline quantum mechanics [Castro, Iqbal, Llabrés '20]
- On the classical background our expression should reproduce the known one-loop determinant for a given field [Anninos, Denef, Law, Sun '22]

An on-shell derivation can be performed from a heat kernel or quasi-normal mode analysis.

Adding matter

Our final result on a simple background such as S^3 takes the form of a contour integral over Wilson loops. For a minimally coupled spinning field

$$\log Z_{\text{matter}}^{\Delta,\text{s}} = \frac{i}{8}\int_{\mathcal{C}} \frac{\mathrm{d}\alpha}{\alpha}\frac{\cos\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}\left(1+2\text{s}^2\sin^2\left(\frac{\alpha}{2}\right)\right)\times$$
$$\sum_{\mathcal{R}_{\Delta,\text{s}}} \text{Tr}_{\text{R}_L}\left(\mathcal{P}e^{\frac{\alpha}{2\pi}\oint_\gamma A_L}\right)\text{Tr}_{\text{R}_R}\left(\mathcal{P}e^{-\frac{\alpha}{2\pi}\oint_\gamma A_R}\right).$$

Conclusions and future directions

- On a number of backgrounds we can write down a one-loop determinant for massive matter fields in the language of Chern-Simons theory.

Conclusions and future directions

- On a number of backgrounds we can write down a one-loop determinant for massive matter fields in the language of Chern-Simons theory.
- When $\Lambda > 0$ we understand how to evaluate such determinants order by order in Chern-Simons theory very efficiently.

Conclusions and future directions

- On a number of backgrounds we can write down a one-loop determinant for massive matter fields in the language of Chern-Simons theory.
- When $\Lambda > 0$ we understand how to evaluate such determinants order by order in Chern-Simons theory very efficiently.
- We are working to extend this approach to backgrounds with more complicated topologies.

Conclusions and future directions

- On a number of backgrounds we can write down a one-loop determinant for massive matter fields in the language of Chern-Simons theory.
- When $\Lambda > 0$ we understand how to evaluate such determinants order by order in Chern-Simons theory very efficiently.
- We are working to extend this approach to backgrounds with more complicated topologies.
- It would be interesting to link this approach more closely with worldline quantum mechanics calculations.