Higgs Mechanism in Plenbanski Gravity

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From Einstein-Cartan to Plebanski

EC action

$$S = \frac{1}{4\ell_P^2} \int \epsilon_{IJKL} \left[e^I e^J R^{KL} - \frac{\Lambda}{6} e^I e^J e^K e^L \right] \tag{1}$$

Projector operator to SD space

$$P_{+\ \ KL}^{IJ} = \frac{1}{2} \left(\delta_K^{[I} \delta_L^{J]} - \frac{i}{2} \epsilon^{IJ}{}_{KL} \right) \tag{2}$$

SD area form

$$\Sigma^i \equiv 2i P_{+\ \ kl}^{0i} \Sigma^{kl} \tag{3}$$

Chiral Plebanski action

$$S = \frac{i}{\ell_P^2} \int \Sigma^i F_i - \frac{1}{2} M_{ij} \Sigma^i \Sigma^j + \frac{1}{2} \omega (\mathrm{tr}\, M - \Lambda) \tag{4}$$

From Einstein-Cartan to Plebanski

SD Curvature

$$F^i = dA^i + \frac{1}{2}\epsilon^i{}_{jk}A^j A^k \tag{5}$$

Eqs of Motion

$$D_A\Sigma^i = d\Sigma^i + \epsilon^i{}_{jk}A^j\Sigma^k = 0 \tag{6}$$

$$F^i = M^{ij}\Sigma_j, \quad \Sigma^i\Sigma^j = \delta^{ij}\omega \tag{7}$$

Reality conditions

$$\mathrm{Re}(\Sigma^i\Sigma_i) = \Sigma^i\bar{\Sigma}^j = 0 \tag{8}$$

$$\Sigma^i = ie^0e^i - \frac{1}{2}\epsilon^i{}_{jk}e^j e^k \tag{9}$$

Urbantke metric

$$g_{\mu\nu}\,\epsilon_\Sigma = -\frac{i}{6}\epsilon_{ijk}\, i_\mu\Sigma^i\, i_\nu\Sigma^j\,\Sigma^k \quad \epsilon_\Sigma = \frac{i}{6}\Sigma^i\Sigma_i = \sqrt{-g}\,d^4x \tag{10}$$

Gravity with a Higgs Field

Coupling Higgs field

$$S = \frac{i}{\ell_P^2}\int \Sigma^i F_i - \frac{1}{2}M_{ij}\Sigma^i\Sigma^j + \frac{1}{2}\omega[\mathrm{tr}\, M - \ell_P^2 V(\phi)] + + \int \frac{1}{2}D_A\phi^i * D_A\phi_i \qquad (11)$$

$$V(\phi) = \frac{\lambda}{4}(\phi_i^2 - v^2)^2 \qquad (12)$$

Eqs of motion

$$D_A\Sigma^i = i\ell_P^2\epsilon^i{}_{jk}\phi^j * D_A\phi^k \qquad (13)$$

$$F^i = M^{ij}\Sigma^j - \ell_P^2\tau^i \qquad (14)$$

$$\mathrm{tr}\, M = \ell_P^2 V(\phi) \qquad (15)$$

$$D_A * D_A\phi_i = -\frac{iV'}{6\ell_P^2}\Sigma^j\Sigma_j, \qquad (16)$$

$$\tau_i = \frac{\delta S_{D_A\phi}}{i\delta\Sigma^i} \qquad (17)$$

Gravity with a Higgs Field

Expansion around vev

$$\phi^i = \phi_0^i + \varphi^i = v\delta_3^i + \varphi^i \tag{18}$$

$$\begin{aligned} S = \; & \frac{i}{\ell_P^2} \int \Sigma^i F_i - \frac{1}{2} M_{ij} \Sigma^i \Sigma^j + \frac{1}{2}\omega[\mathrm{tr}\, M - \ell_P^2 V(\varphi_3)] + \\ & + \int \frac{1}{2} d\varphi_3 * d\varphi_3 + \frac{1}{2} \int v^2 (A^1 * A^1 + A^2 * A^2). \end{aligned} \tag{19}$$

$V(\varphi_3) = v^2 \lambda \varphi_3^2$

$$D_A \Sigma^i = i\ell_P^2 v^2 (*A^1 \delta^{i1} + *A^2 \delta^{i2}) \tag{20}$$

$$d * d\varphi_3 = -\frac{i}{6} \Sigma^i \Sigma_i V' \tag{21}$$

$$\Rightarrow \Box \varphi_3 = V' \tag{22}$$

Modified Bianchi I

Metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t)dz^2, \tag{23}$$

SD 2-form

$$\Sigma^1 = iadtdx^1 - abdx^2dx^3 \tag{24}$$
$$\Sigma^2 = iadtdx^2 - abdx^3dx^1 \tag{25}$$
$$\Sigma^3 = ibdtdx^3 - a^2dx^1dx^2. \tag{26}$$

SD connection

$$A^1 = i\dot{a}dx^1 \tag{27}$$
$$A^2 = i\dot{a}dx^2 \tag{28}$$
$$A^3 = i(\dot{b} - \ell_P^2 v^2 b\dot{a}/a)dx^3 \tag{29}$$

SD curvature

$$F^i = \left[\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} - \ell_P^2 v^2 \frac{\dot{a}^2}{a^2}\right]\frac{\Sigma^i}{2} + \tag{30}$$

$$+ \left[\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab} + \ell_P^2 v^2 \frac{\dot{a}^2}{a^2}\right]\frac{\bar{\Sigma}^i}{2} \tag{31}$$

$$F^3 = \left[\frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} - \ell_P^2 v^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}^2}{a^2}\right)\right]\frac{\Sigma^3}{2} + \tag{32}$$

$$+ \left[\frac{\ddot{b}}{b} - \frac{\dot{a}^2}{a^2} - \ell_P^2 v^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}^2}{a^2}\right)\right]\frac{\bar{\Sigma}^3}{2} \tag{33}$$

Modified Bianchi I

2-form stress tensor

$$\tau^i = \frac{T}{12}\Sigma^i - \frac{1}{2}\Sigma^i{}_\mu{}^\rho \tilde{T}_{\rho\nu} dx^\mu dx^\nu \quad (34)$$

Stress-energy tensor

$$T_{\mu\nu} = \quad v^2(A^1_\mu A^1_\nu + A^2_\mu A^2_\nu) - g_{\mu\nu}\frac{v^2}{2}(A^1_\alpha A^{1\alpha} + A^2_\alpha A^{2\alpha}) + \quad (35)$$

$$+\partial_\mu\varphi_3\partial_\nu\varphi_3 - \frac{1}{2}g_{\mu\nu}(\partial\varphi_3)^2. \quad (36)$$

Friedmann equations

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}^2}{a^2}(1 - \ell_P^2 v^2) = \ell_P^2\left(\frac{\dot{\varphi}_3^2}{2} + V\right) \quad (37)$$

$$2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} - \ell_P^2 v^2\left(\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{a}^2}{a^2}\right) = \ell_P^2\left(-\dot{\varphi}_3^2 + V\right) \quad (38)$$

Proca Term

$$S = \frac{i}{\ell_P^2}\int \Sigma^i F_i - \frac{1}{2}M_{ij}\Sigma^i\Sigma^j + \frac{1}{2}\omega[\mathrm{tr}\, M - \Lambda] + \\ + \frac{1}{2}\int m^2 A^i * A^i \quad (39)$$

Equation of motion

$$D_A\Sigma^i = i\ell_P^2 m^2 * A^i \quad (40)$$

SD connection

$$A^i = \frac{2i\dot{a}}{2+\ell_P^2 m^2}dx^i \quad (41)$$

Friedmann equation

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda_{\mathrm{eff}}}{3} \quad (42)$$

$$\Lambda_{\mathrm{eff}} = \frac{2+m^2\ell_P^2}{2}\Lambda \quad (43)$$

Gravitational Waves in Plebański Gravity

Tensorial perturbations

$$\Sigma^i = \Sigma^i_0 + \delta\Sigma^i = \Sigma^i_0 + h^{ij}\bar{\Sigma}^j_0 \tag{44}$$

$$\Sigma^i_0 = idtdx^i - \frac{1}{2}\epsilon^i{}_{jk}dx^j dx^k. \tag{45}$$

SD connection

$$A^i = A^i_0 + \delta A^i = \delta A^i, \tag{46}$$

$$d\delta\Sigma^i + \epsilon^i{}_{jk}\delta A^j \Sigma^k_0 = i\ell_P^2 m^2 * \delta A^i \tag{47}$$

$$\delta A^i = \frac{1}{1+\ell_P^2 m^2}(i\dot{h}^{ij} - h^{ik}{}_{,l}\epsilon^j{}_{kl})dx^j \tag{48}$$

Gravitational Waves in Plebański Gravity

Curvature

$$F^i = F_0^i + \delta F^i = d\delta A^i \tag{49}$$
$$\delta F^i = \psi^{ij}\Sigma^j + \bar{\psi}^{ij}\bar{\Sigma}^j \tag{50}$$

Wave equation

$$\Box h^{ij} = 0 \tag{51}$$

Weyl tensor

$$\psi^{ij} = \frac{1}{1+\ell_P^2 m^2}\left(\frac{1}{2}\ddot{h}^{ij} + i\epsilon^{jkl}\dot{h}^{ik}{}_{,l} + \frac{1}{2}\nabla^2 h^{ij}\right) \tag{52}$$

Outlook

- Consider more complex cosmological models
- Numerical solutions
- Gravitational waves and Black Holes
- Computing τ directly from action variation