Rare *b* decays

An overview

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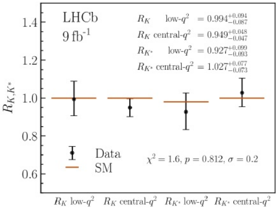
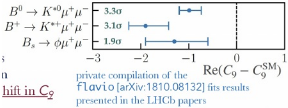
RIP the B anomalies ?

Latest progress and possible directions for rare $\not{B}b$ decays in 2025

- Rare b decays involve a FCNC loop/GIM-suppressed $b \to d/s$ transition.
- Propelling factor were anomalies seen in $b \to s$ transitions, one of only signs of BSM in 2014-2022 era. Deviations included BRs, angular observables and LU ratios.
- In $\sim$2022, anomaly in LU ratios disappeared
- Angular observables/BRs still not SM-like





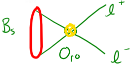
Classification of rare $b \to s$ decays

Radiative	Semi-leptonic $\ell^+\ell^- = \ell\ell/\nu\bar{\nu}$	Purely (radiative) leptonic
$B \to X_s\gamma$	$B \to X_s\ell\ell$	$B_s \to \ell^+\ell^-$
$B \to K^{(*)}\gamma$	$B \to K^{(*)}\ell\ell$	$B_s \to \gamma\ell^+\ell^-$
$B_s \to \phi\gamma$	$B_s \to \phi\ell\ell$	$B_s \to \ell^+\ell^-\ell'^+\ell'^-$
$\Lambda_b \to \Lambda\gamma$	$\Lambda_b \to \Lambda\ell\ell$	

Calculating exclusive rare decays

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}}\lambda \sum_i C_i O_i, \quad \mathcal{O}_7 = \frac{e}{g^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \quad \mathcal{O}_{9(10)} = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$$

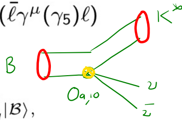
$B_s \to \ell^+\ell^-$



$$\begin{aligned}\langle \ell^+\ell^- |\mathcal{O}_i|\mathcal{B}\rangle &\sim \langle \ell^+\ell^- |j^i_\ell\, j_q|\mathcal{B}\rangle \\ &\sim \langle \ell^+\ell^- |j^i_\ell|0\rangle\ \langle 0|j_q|\mathcal{B}\rangle \\ &\sim f_{\mathcal{B}}\langle \ell^+\ell^- |j^i_\ell|0\rangle\end{aligned}$$

introducing B decay constant $f_{\mathcal{B}}$,

$B \to K^{(*)}\nu\bar{\nu}$



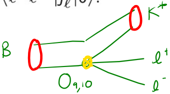
$$\begin{aligned}\langle \nu\bar{\nu}\mathcal{M}|\mathcal{O}_i|\mathcal{B}\rangle &\sim \langle \nu\bar{\nu}\mathcal{M}|j^i_\ell\, j^i_q|\mathcal{B}\rangle, \\ &\sim \langle \nu\bar{\nu}|j^i_\ell|0\rangle\ \langle \mathcal{M}|j^i_q|\mathcal{B}\rangle \\ &\sim F_i\langle \nu\bar{\nu}|j^i_\ell|0\rangle,\end{aligned}$$

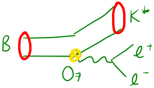
where F_i is the form factor

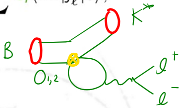
$$\langle \ell^+\ell^- |\mathcal{H}_{\rm eff}|\mathcal{B}\rangle \sim \frac{4G_F}{\sqrt{2}}\lambda f_{\mathcal{B}} \sum_i C_i\langle \ell^+\ell^- |j^i_\ell|0\rangle.$$

$$\langle \nu\bar{\nu}\mathcal{M}|\mathcal{H}_{\rm eff}|\mathcal{B}\rangle \sim \frac{4G_F}{\sqrt{2}}\lambda F_i \sum_i C_i\langle \nu\bar{\nu}|j^i_\ell|0\rangle,$$

$B \to K^{(*)}\ell^+\ell^-$







$$\begin{aligned}\mathcal{A}_{\mathcal{B}\to\mathcal{M}\ell^+\ell^-} &= \langle \ell^+\ell^- \mathcal{M}|\mathcal{H}_{\rm eff}|\mathcal{B}\rangle \\ &= \frac{4G_F}{\sqrt{2}}\lambda \sum_i \langle \ell^+\ell^- \mathcal{M}|C_i\mathcal{O}_i|\mathcal{B}\rangle. \sim \mathcal{N}\left(C\mathcal{F}^V(q^2) \quad + \frac{1}{q^2}\left(C\mathcal{F}^T(q^2) - \mathcal{H}(q^2)\right)\right)\langle \ell^+\ell^- |j_\ell|0\rangle,\end{aligned}$$

for the Wilson coefficients C_i, form factors $\mathcal{F}^V_i(q^2)$ and $\mathcal{F}^T_i(q^2)$, and non-local MEs $\mathcal{H}(q^2)$.

$$\mathcal{H}^\mu(q^2) \equiv i\int d^4x e^{iqx}\langle \mathcal{M}(k)|T\{j^{\rm em}_\mu(x), (C_1\mathcal{O}_1 + C_1\mathcal{O}_2)(0)\}|\mathcal{B}(q+k)\rangle, \quad \text{where } j^{\rm em}_\mu = \sum_q Q_q\bar{q}\gamma_\mu q$$

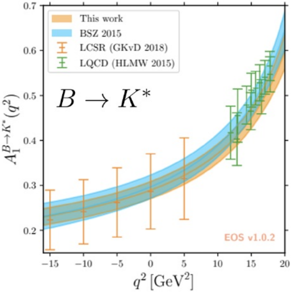
Form factors

Progress and future directions

- State of form factor calculations is advanced for $b \to s$ and $b \to d$ transitions in B, B_s and Λ_b decays
- Different methods are more suitable to different kinematic ranges and different decays, e.g. light cone sum rules (LCSR) better at lower q^2 and Lattice QCD better for higher q^2 and stable final-state mesons

Recent interesting results include:

- Dispersive analysis of $\Lambda_b \to \Lambda(1520)$ [Amhis++'22]
- Dispersive Analysis of $B \to K^*$ and $B_s \to \phi$ [Gubernari++'23]
- Light-Cone Sum Rules for S-wave $B \to K\pi$ [Descotes-Genon++'23]
- HPQCD analysis of $B_{(s)} \to \pi(K)$ and $D_{(s)} \to \pi(K)$ [Parrott++'22,Roberts++'25]
- $B_s \to \phi$ and $B \to K^*$ [Sanfillipo++ work in progress]

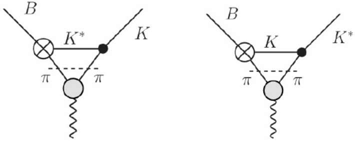


Improvements for Lattice $B \to K^*$, $B \to \pi K$, $B \to \rho$, $B \to \pi\pi$ desirable...

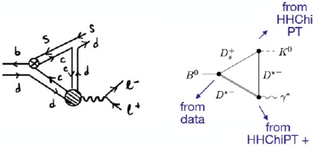
Theory progress in $B \to K^{(*)}\ell^+\ell^-$

Non-local contributions see A. Tinari and M. Hoferichter's talk at LHCb Implications workshop 2024

Contribution of anomalous thresholds in quark loop [Mutke++'24]



Estimate effect of rescattering of charm mesons [Isidori++'24]

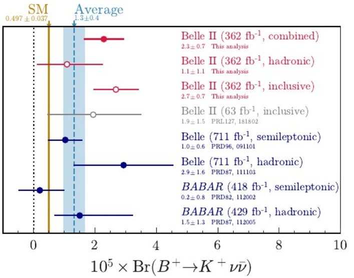


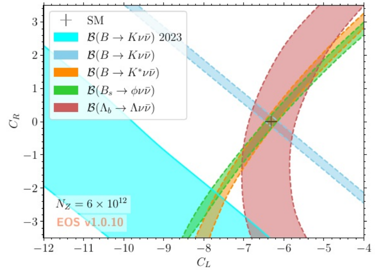
	MH, Kubis, Mutke 2024	Isidori, Polonsky, AT 2024
Framework	dispersion relations	loop integrals
	discontinuities $\simeq [B \to (P,V) X\bar{X}] \times [X\bar{X} \to \gamma^*]^*$ agree	
Key input	**couplings from *B*, *D*, ... decays, form factors**	
Key output	size of anomalous contributions by comparing normal and anomalous terms in dispersion relation for $B \to (P,V)\gamma^*$	size of $D\bar{D}$ rescattering via $D_s^*\bar{D}$, $D^*\bar{D}_s$ loop diagrams (including form factors) and coherent sum over other channels

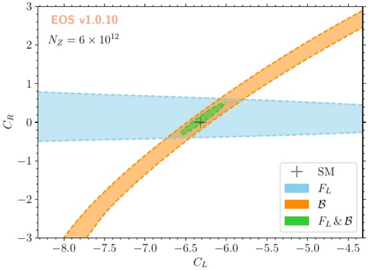
Maximum 10% effect in C_9, difficult to reconcile with 25% shift from data

Status of $b \to s\nu\bar{\nu}$

Access CKM elements, WCs, form-factors but experimentally challenging (missing energy, vertexing+model-dependent analyses. Combined measurement: 3.5σ evidence, 2.7σ 'tension' with SM prediction [Belle II'23].





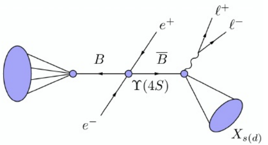


Analysis of FCCee+CEPC sensitivity for 4-body final states, see [Amhis++'23]

The inclusive alternative for $b \to s\ell^+\ell^-$

Semi-inclusive decays at LHCb see J Jenkins talk at LHCb Implications workshop 2024

B factories

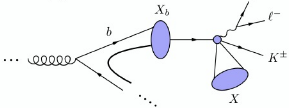


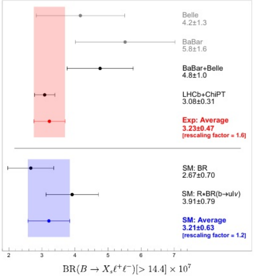
Sum over exclusive modes and isospin reweighting $B^{0,+} \to K^+(+n\pi^\pm)\mu^+\mu^-$ (avoid $\pi^0 s$) [Koppenburg'02]

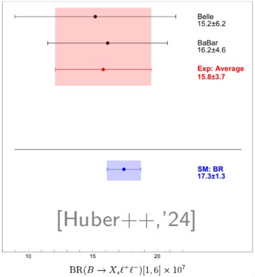
$X_b \to K^+\mu^+\mu^- X$ Isospin extrapolation from semi-inclusive (vertex 3 charged particles) [Amhis++'21]

Measure+subtract B_s and Λ_b contributions using extra K or p in final state.

LHCb (?)





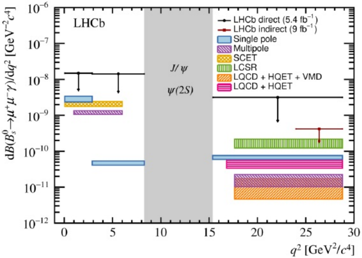


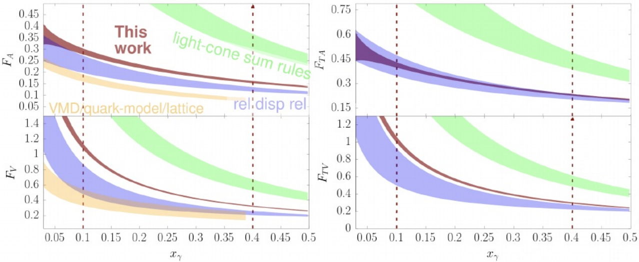
Theory advanced [Huber++,'24]: NNLO QCD, and to NLO in QED+power corrections $\mathcal{O}(1/m_b^3)$. Kruger-Sehgal approach to factorizable charmonium loops [Huber++'19].

$B_q \to \gamma\ell^+\ell^-$, an unlikely newcomer

LHCb or Belle II?

- Sensitive to a larger set of Wilson coefficients than $B_q \to \ell^+\ell^-$
- Suppressed by $\alpha_{\rm em}$ compared to $B_q \to \ell^+\ell^-$ but not helicity suppressed
- Cleaner access to e.g. $C_{9,10}^{(\prime)}$ as penguin contribution suppressed





See talk by F. Sanfilippo @Lattice'24

- LCSR [Janowski++'21]
- Rel. dispersion relations [Kozachuk++'18]
- VMD quark model/Lattice [Guadagnoli++'23]
- This work [ETM'24]

Thoughts

Large effects in lepton universality ratios gone, but $b \to s$ and $b \to d$ transitions can still provide meaningful constraints on BSM physics

- Try to understand tension in angular observables and branching ratios in terms of hadronic effects:
  1. Need LQCD progress in form factors for unstable, interesting work in progress by the ETM collaboration
  2. Anomalous thresholds in quark loop could account for some of discrepancy, but current calculations predict $< 10\%$ effect. Further work needed, as there is a lack of data to make accurate predictions.
- Continue to probe $C_9^{(\prime)}$, $C_{10}^{(\prime)}$ by considering complementary channels:
  1. Consider theoretically clean channels with neutrinos in final state, experimentally challenging: while at Belle II a $\sim 10\%$ uncertainty is expected, a recent sensitivity study showed that at the FCC-ee a 0.53% uncertainty for $B \to K^* \nu \bar{\nu}$ would be achievable
  2. Recent idea to use semi-inclusive decays at LHCb, theory at high q^2 in advanced state and agrees with experiment
  3. $B_s \to \gamma \ell^+ \ell^-$ provide another opportunity for LHCb and Belle II to access the Wilson coefficients C_9 and C_{10}

Form factors for $B \to V$

$$
\begin{aligned}
\langle V(k)|\bar{s}\gamma^{\mu}(1 \mp \gamma_5)b|\bar{M}(p)\rangle &= P_1^{\mu}\,\mathcal{V}_1(q^2) \pm \sum_{i=2,3,P} P_i^{\mu}\,\mathcal{V}_i(q^2)\ , \\
\langle V(k)|\bar{s}iq_{\nu}\sigma^{\mu\nu}(1 \pm \gamma_5)b|\bar{M}(p)\rangle &= P_1^{\mu}\,T_1(q^2) \pm P_2^{\mu}\,T_2(q^2) \pm P_3^{\mu}\,T_3(q^2)\ , \qquad (1)
\end{aligned}
$$

where the Lorentz structures P_i^{μ} are defined as in [?] as

$$
\begin{aligned}
P_1^{\mu} &= 2\epsilon^{\mu}{}_{\alpha\beta\gamma}\eta^{*\alpha}k^{\beta}q^{\gamma}\ , \quad P_2^{\mu} = i\{(m_M^2 - m_V^2)\eta^{*\mu} - (\eta^* \cdot q)(p+k)^{\mu}\}\ , \\
P_3^{\mu} &= i(\eta^* \cdot q)\{q^{\mu} - \frac{q^2}{m_M^2 - m_V^2}(p+k)^{\mu}\}\ , \quad P_P^{\mu} = i(\eta^* \cdot q)q^{\mu}\ , \qquad (2)
\end{aligned}
$$

with the $\epsilon_{0123} = +1$ convention for the Levi-Civita tensor and with η the polarization of the vector meson. Note that $T_1(0) = T_2(0)$ holds algebraically. This parameterisation is chosen as it simplifies results for decay rates, and makes the relation between the tensor and vector form factors clear. The $\mathcal{V}_{P,1,2,3}$ can be related to the more traditional $A_{0,1,2,3}$ and V via

$$
\begin{aligned}
\mathcal{V}_P(q^2) &= \frac{-2m_V}{q^2}A_0(q^2)\ , \quad \mathcal{V}_1(q^2) = \frac{-V(q^2)}{m_M + m_V}\ , \quad \mathcal{V}_2(q^2) = \frac{-A_1(q^2)}{m_M - m_V}\ , \\
\mathcal{V}_3(q^2) &= \left(\frac{m_M + m_V}{q^2}A_1(q^2) - \frac{m_M - m_V}{q^2}A_2(q^2)\right) \equiv \frac{2m_V}{q^2}A_3(q^2)\ . \qquad (3)
\end{aligned}
$$

where $A_3(0) = A_0(0)$ insures that the matrix elements are finite at $q^2 = 0$.

Definition of operators for $b \to s$

The relevant effective Hamiltonian for decays involving $b \to s\gamma$ and $b \to s\ell^+\ell^-$ can be expressed by [Bobeth++'99,'01]

$$\mathcal{H}_{\text{eff}} = -\frac{4\,G_F}{\sqrt{2}} \left(\lambda_t \mathcal{H}_{\text{eff}}^{(t)} + \lambda_u \mathcal{H}_{\text{eff}}^{(u)}\right) \quad (4)$$

where G_F is the Fermi constant, $\lambda_q = V_{qb}V_{qs}^*$ and

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{(t)} &= C_1\mathcal{O}_1^c + C_2\mathcal{O}_2^c + \sum_{i=3}^{6} C_i\mathcal{O}_i + \sum_{i=7,8,9,10,P,S} (C_i\mathcal{O}_i + C_i'\mathcal{O}_i')\,, \\ \mathcal{H}_{\text{eff}}^{(u)} &= C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2(\mathcal{O}_2^c - \mathcal{O}_2^u)\,. \end{aligned}$$

The operators $\mathcal{O}_{i\leq 6}$ are given by the P_i given in [Bobeth++'99], while the remaining ones are given by

$$\mathcal{O}_7 = \frac{e}{g^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \qquad \mathcal{O}_7' = \frac{e}{g^2} m_b(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu}, \quad (5)$$

$$\mathcal{O}_8 = \frac{1}{g_s} m_b(\bar{s}\sigma_{\mu\nu}T^a P_R b)G^{\mu\nu\,a}, \qquad \mathcal{O}_8' = \frac{1}{g_s} m_b(\bar{s}\sigma_{\mu\nu}T^a P_L b)G^{\mu\nu\,a}, \quad (6)$$

$$\mathcal{O}_9 = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu\mu), \qquad \mathcal{O}_9' = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu\mu), \quad (7)$$

$$\mathcal{O}_{10} = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu\gamma_5\mu), \qquad \mathcal{O}_{10}' = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu\gamma_5\mu), \quad (8)$$

$$\mathcal{O}_S = \frac{e^2}{16\pi^2} m_b(\bar{s}P_R b)(\bar{\mu}\mu), \qquad \mathcal{O}_S' = \frac{e^2}{16\pi^2} m_b(\bar{s}P_L b)(\bar{\mu}\mu), \quad (9)$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2} m_b(\bar{s}P_R b)(\bar{\mu}\gamma_5\mu), \qquad \mathcal{O}_P' = \frac{e^2}{16\pi^2} m_b(\bar{s}P_L b)(\bar{\mu}\gamma_5\mu), \quad (10)$$