

CP violation, B mixing and lifetimes

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Outline

- Flavour in a nutshell
- $B-\bar{B}$ mixing: β and β_s
- $B-\bar{B}$ mixing: width difference
- Summary and outlook

Flavour in a nutshell

Flavour physics

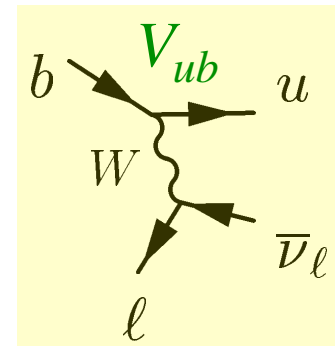
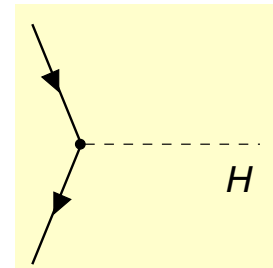
- studies transitions between different fermions.
- Standard Model: from **Higgs Yukawa** interaction
- encoded in Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.96 & 0.23 & 0.004e^{-i\gamma} \\ -0.23 & 0.96 & 0.040 \\ 0.009e^{-i\beta} & -0.04e^{i\beta_s} & 1.0 \end{pmatrix}$$

- Charge-parity (CP) violating quantities involve

$$\gamma \approx 66^\circ, \quad \beta \approx 23^\circ, \quad \beta_s \approx 1.1^\circ.$$

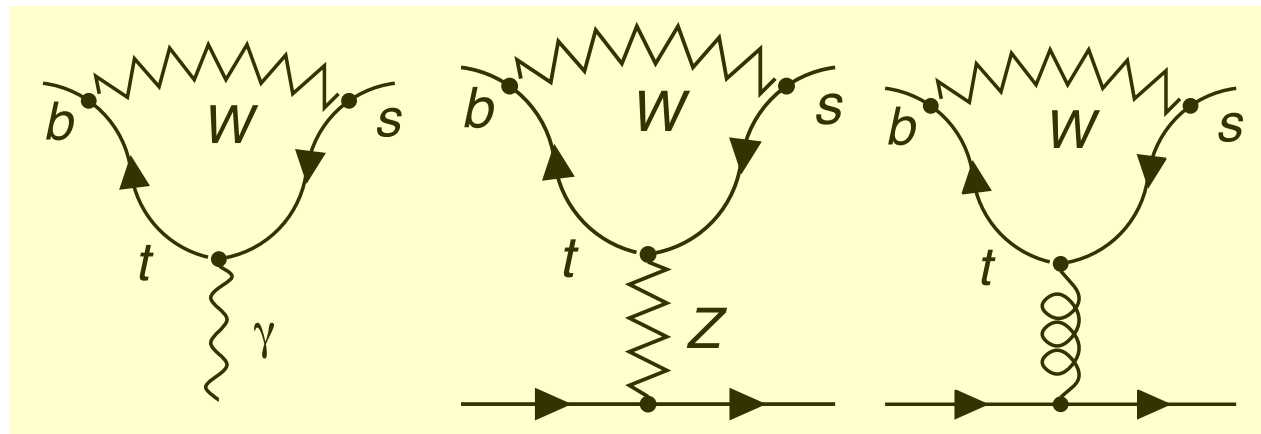
β and β_s are calculable in term of γ , the **KM phase**.



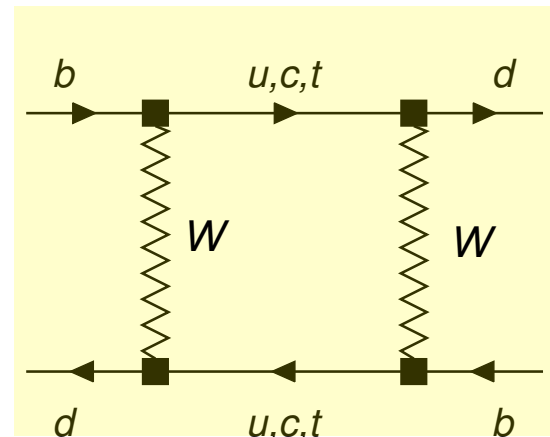
Flavour physics

- Best suited: **Flavour-changing neutral current (FCNC)** processes like $b \rightarrow s$, $b \rightarrow d$, $c \rightarrow u$, $\mu \rightarrow e$.

Penguins:

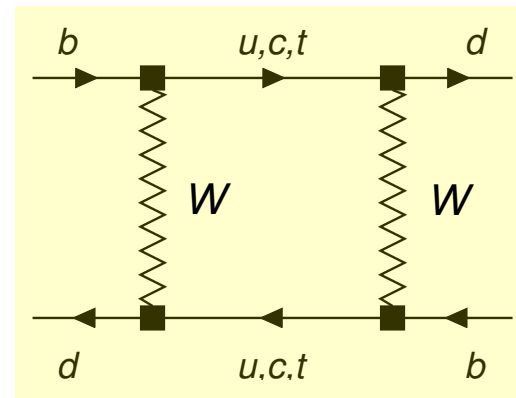


Box:



Flavour physics

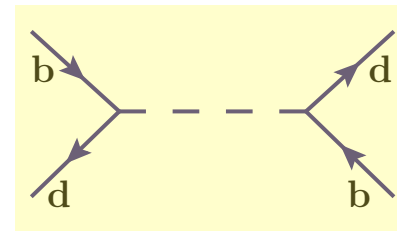
- In **FCNC** processes several suppression factors pile up:
 - electroweak loop
 - small elements of the Cabibbo-Kobayashi-Maskawa matrix $|V_{ts}| \simeq |V_{cb}| = 0.04$, $|V_{td}| = 0.01$
 - possible **GIM** suppression factor $(m_c^2 - m_u^2)/M_W^2$ (in Kaon FCNCs) or $(m_{\nu_3}^2 - m_{\nu_1}^2)/M_W^2$ (in $\mu \rightarrow e\gamma$)
 - possible helicity suppression by e.g. m_μ^2/M_W^2 in $B_{s,d} \rightarrow \mu^+\mu^-$



BSM mass reach

Flavour physics probes virtual effects of new heavy particles coupling to quarks, with a mass reach of

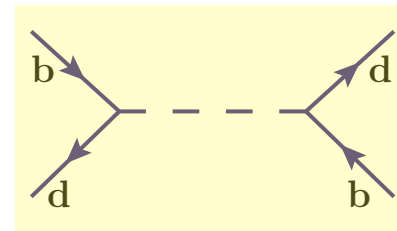
- a **few TeV** in tree-level decays like for $b \rightarrow c\tau\bar{\nu}$,
- a **few tens of TeV** in FCNC decays like $b \rightarrow s\ell^+\ell^-$, and
- a **few hundreds of TeV** in $B-\bar{B}$ mixing.



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⇒ The firm establishment of flavour BSM physics helps for the design of a future hadron collider and could establish a “**no-lose**” situation **for FCC-hh**.

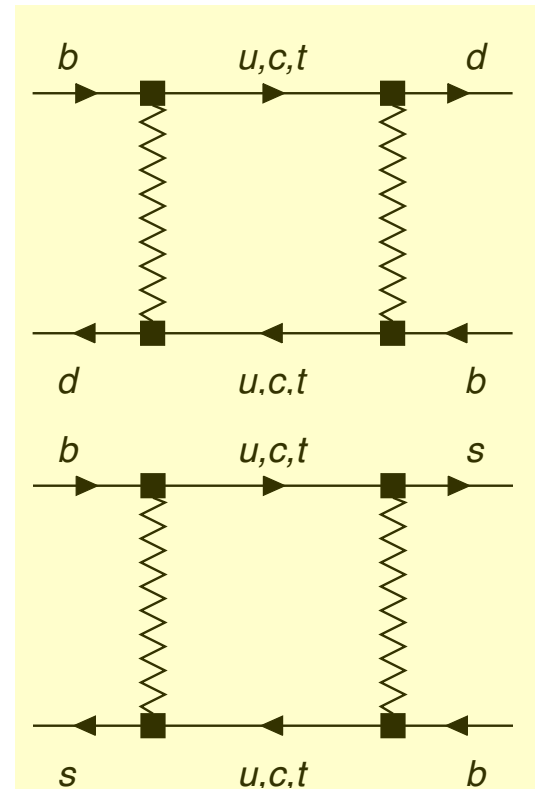
FCC-hh fans  **flavour physics**

flavour physicists  **FCC-ee: 10^{13} Z bosons are a perfect b factory!**

$B - \bar{B}$ mixing: β and β_s

$B-\bar{B}$ mixing

- $B_d-\bar{B}_d$ mixing and $B_s-\bar{B}_s$ mixing
- $\begin{pmatrix} |B_q\rangle \\ |\bar{B}_q\rangle \end{pmatrix}$ involves the 2×2
 - mass matrix M (from boxes with one or two tops)
 - decay matrix Γ (from box without top)
- $|\Gamma_{12}| \ll |M_{12}|$
- two mass eigenstates with mass difference
 - $\Delta m \simeq 2|M_{12}|$ equals $B-\bar{B}$ oscillation frequency



B oscillations and CP violation

- Consider **time-dependent CP asymmetry** for B_d decay into CP eigenstate f_{CP} :

$$a_{CP}(B_d(t) \rightarrow f_{CP}) \equiv \frac{\Gamma(\bar{B}_d(t) \rightarrow f_{CP}) - \Gamma(B_d(t) \rightarrow f_{CP})}{\Gamma(\bar{B}_d(t) \rightarrow f_{CP}) + \Gamma(B_d(t) \rightarrow f_{CP})}$$

tagged as $\bar{B}_d$ at $t = 0$

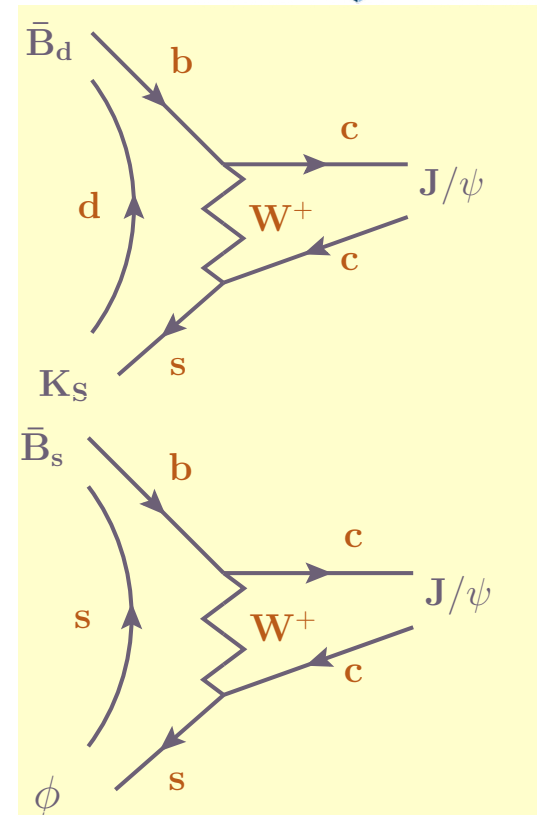
$$= -A_{CP}^{\text{dir}, B_d \rightarrow f_{CP}} \cos(\Delta mt) - A_{CP}^{\text{mix}, B_d \rightarrow f_{CP}} \sin(\Delta mt)$$

direct CP asymmetry

mixing-induced CP asymmetry

Mixing-induced CP violation

- Golden modes: only one decay amplitude:
 $A_{\text{CP}}^{\text{dir}} = 0$ and $A_{\text{CP}}^{\text{mix}} = \pm \sin(\phi_M - 2\phi_f)$,
 where ϕ_M is the CP phase of M_{12} (box diagram)
 and ϕ_f is the CP phase of the decay amplitude.
- $B_d \rightarrow J/\psi K_S$: $\phi_M = \arg[(V_{tb} V_{td}^*)^2] = 2\beta$
- $B_s \rightarrow J/\psi \phi$: $\phi_M = \arg[(V_{tb} V_{ts}^*)^2] = -2\beta_s$
- in both decays: $\phi_f = \arg(V_{cb} V_{cs}^*) = 0$, so that
 $A_{\text{CP}}^{\text{mix}} = \pm \sin(2\beta_{(s)})$

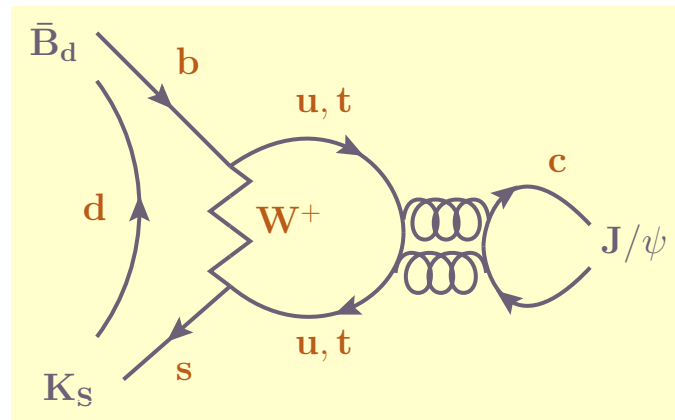


Penguin pollution

- $A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \psi K_S) = 0.709 \pm 0.011$ (all charmonium modes) (HFLAV)
1.6% precision!

- $A_{\text{CP}}^{\text{mix}}(B_s \rightarrow J/\psi \phi) = -0.060 \pm 0.014$ (HFLAV)

- Yet these $b \rightarrow c\bar{c}s$ modes are not completely golden: **Penguin pollution**, second amplitude with $V_{ub}V_{us}^*$ interferes with tree amplitude.



- $\frac{|V_{ub}V_{us}|}{|V_{cb}V_{cs}|} = 0.022$. Relevant with today's precision?

Penguin pollution

- Two ways to estimate penguin pollution:

- Use $b \rightarrow c\bar{c}d$ data to constrain penguin/tree ratio, employing

$$\frac{|V_{ub}V_{ud}|}{|V_{cb}V_{cd}|} = 0.5 \gg 0.02 \text{ and using approximate U-spin}$$

symmetry (linking strange to down) of QCD.

M. Barel, K. De Bruyn, R. Fleischer, E. Malami, *J.Phys.G* 48 (2021) 6, 065002

Works for $B_d \rightarrow J/\psi K_S$, but not for $B_s \rightarrow J/\psi \phi$.

- Calculate penguin pollution in QCD with soft-collinear factorisation for both $b \rightarrow c\bar{c}d$ and $b \rightarrow c\bar{c}s$, see

Ph. Frings, UN, M. Wiebusch, Phys.Rev.Lett. 115 (2015) 6, 061802

Penguin pollution starts to matter

■ Measurements (HFLAV):

$$A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \psi K_S) = 0.709 \pm 0.011, \quad A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \psi K_S) = 0.004 \pm 0.012$$

$$A_{\text{CP}}^{\text{mix}}(B_s \rightarrow J/\psi \phi) = -0.060 \pm 0.014, \quad A_{\text{CP}}^{\text{dir}}(B_s \rightarrow J/\psi K^+ K^-) = 0.004 \pm 0.009$$

■ $A_{\text{CP}}^{\text{mix}} = \pm \sin(2\beta_{(s)}) + \Delta A_{\text{CP}}^{\text{mix}}$ with penguin pollution $\Delta A_{\text{CP}}^{\text{mix}}$.

Decay:	$B_d \rightarrow J/\psi K_S$	$B_d \rightarrow \psi(2S) K_S$	$B_d \rightarrow J/\psi \pi^0$
$\max(\Delta\phi_M^d)$	0.68°	0.74°	N/A
$\max(\Delta A_{\text{CP}}^{\text{mix}})$	$0.86 \cdot 10^{-2}$	$0.94 \cdot 10^{-2}$	0.18
$\max(A_{\text{CP}}^{\text{dir}})$	$1.33 \cdot 10^{-2}$	$1.33 \cdot 10^{-2}$	0.29

$b \rightarrow c\bar{c}d$ mode

polarisation

Decay:	$B_s \rightarrow (J/\psi \phi)^0$	$B_s \rightarrow (J/\psi \phi)^\perp$	$B \rightarrow (J/\psi \phi)^\parallel$	$B_s \rightarrow J/\psi K_S$
$\max(\Delta\phi_M^s)$	0.97°	1.22°	0.99°	N/A
$\max(\Delta A_{\text{CP}}^{\text{mix}})$	$1.70 \cdot 10^{-2}$	$2.13 \cdot 10^{-2}$	$1.73 \cdot 10^{-2}$	26
$\max(A_{\text{CP}}^{\text{dir}})$	$1.89 \cdot 10^{-2}$	$2.35 \cdot 10^{-2}$	$1.92 \cdot 10^{-2}$	27

Direct CP violation in $B \rightarrow J/\psi\pi$

■ Isospin symmetry: $A_{\text{CP}}^{\text{dir}}(B_d \rightarrow J/\psi\pi^0) \simeq A_{\text{CP}}^{\text{dir}}(B^+ \rightarrow J/\psi\pi^+)$

■ LHCb, *Phys.Rev.Lett.* 134 (2025) 10, 101801:

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow J/\psi\pi^+) - A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \psi K^+) = (1.29 \pm 0.49 \pm 0.08) \times 10^{-2}$$

deducing $A_{\text{CP}}^{\text{dir}}(B_d \rightarrow J/\psi\pi^+) = (1.51 \pm 0.51 \pm 0.08) \times 10^{-2}$

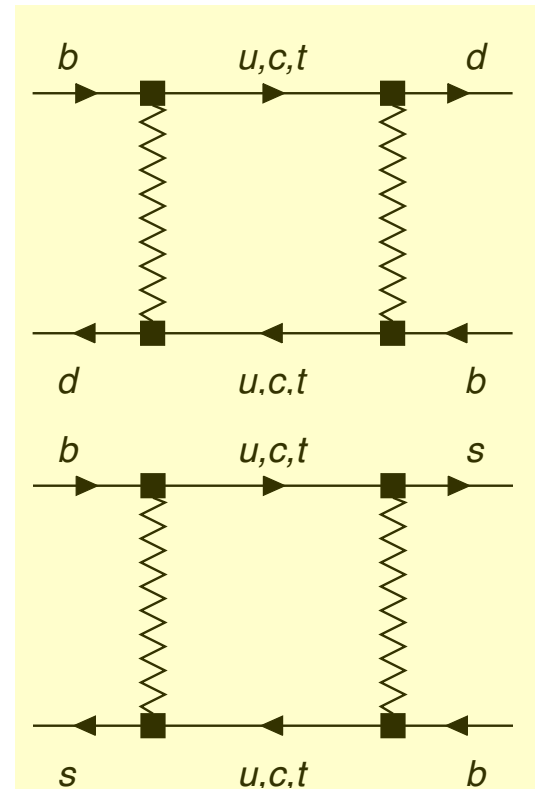
which is far more precise than our upper bound from 2015:

$$|A_{\text{CP}}^{\text{dir}}(B \rightarrow J/\psi\pi)| \leq 0.29$$

$B - \bar{B}$ mixing: width difference

Width difference

- mass matrix M (from boxes with one or two tops)
- decay matrix Γ (from box without top)
- $|\Gamma_{12}| \ll |M_{12}|$
- two mass eigenstates with
 - mass difference $\Delta m \simeq 2 |M_{12}|$, equal to $B-\bar{B}$ oscillation frequency
 - decay width difference $\Delta\Gamma = -\Delta m \operatorname{Re} \frac{\Gamma_{12}}{M_{12}}$



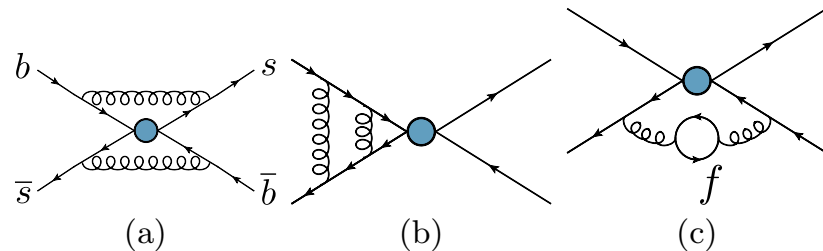
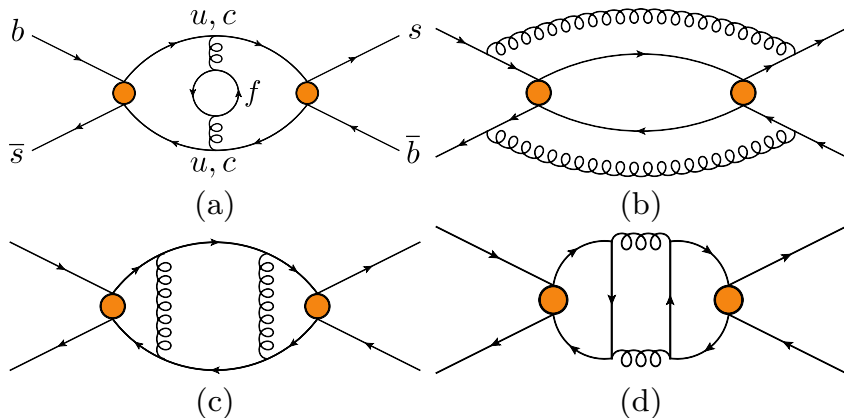
Width difference

- QCD corrections to M_{12} are governed by $\alpha_s(m_t)$. $\Rightarrow$ NLO sufficient
- QCD corrections to Γ_{12} are governed by $\alpha_s(m_b)$. $\Rightarrow$ need NNLO.
- $B_s - \bar{B}_s$ system: Precise measurement (HFLAV):

$$\Delta\Gamma_s = (0.082 \pm 0.005) \text{ ps}^{-1}$$
- while in the $B_d - \bar{B}_d$ system:

$$\Delta\Gamma_d = (0.001 \pm 0.007) \text{ ps}^{-1}$$

$B_s - \bar{B}_s$ width difference



Sample 3-loop diagrams.

Orange dot: effective four-quark operators describing W exchange

Effective operators with local $\bar{s}b\bar{s}b$ interaction.

Perturbative calculation: Wilson coefficient.

To predict $\Delta\Gamma$ we also need hadronic matrix element à la $\langle \bar{B}_s | \bar{s}b\bar{s}b | B_s \rangle$.

$B_s - \bar{B}_s$ width difference



Renormalisation scale
dependence:

$$\frac{\Delta\Gamma_s}{\Delta m_s}$$

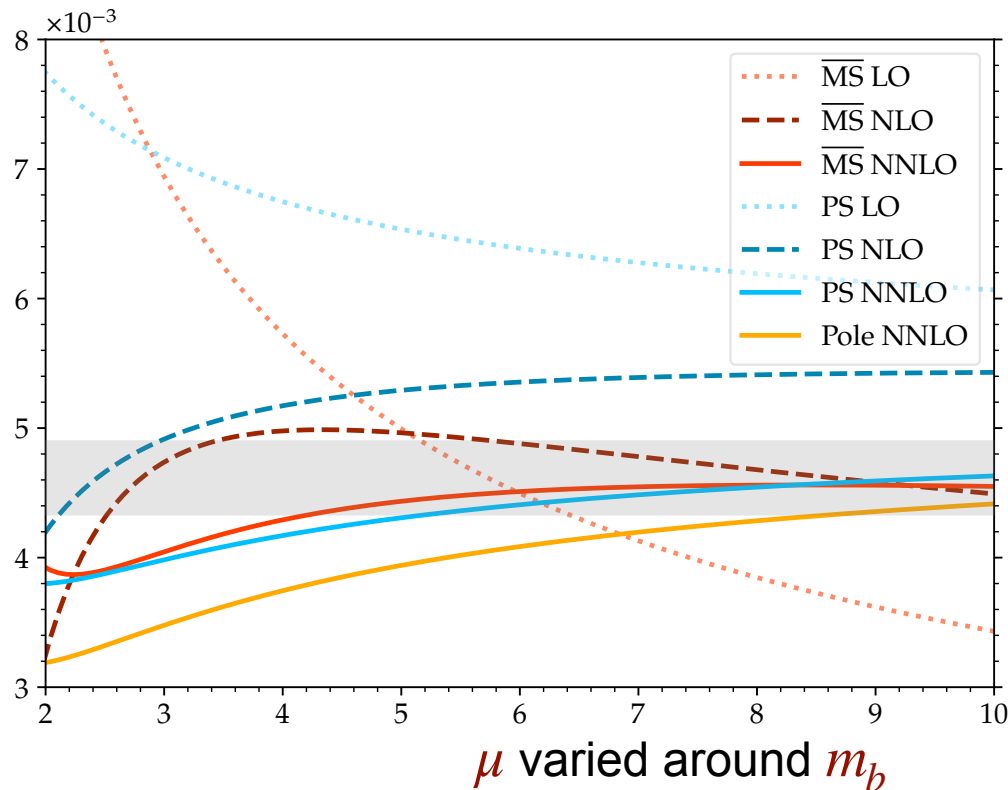
Other sources of uncertainties

$\frac{\alpha_s}{m_b}$ corrections missing

matrix elements of $\frac{1}{m_b}$

suppressed operators

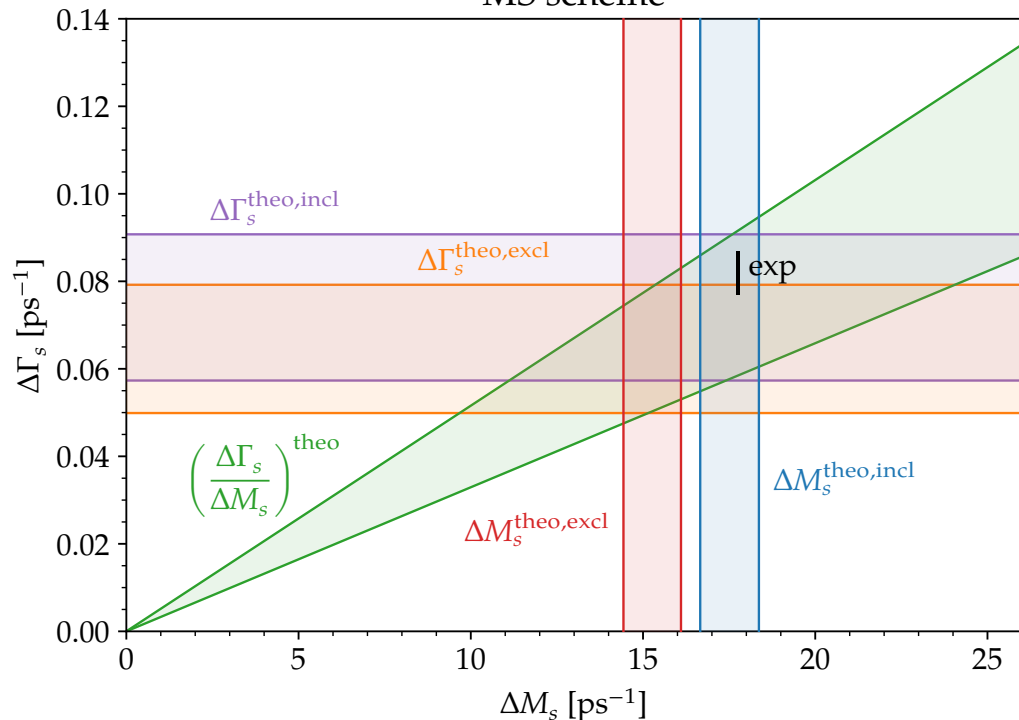
M. Gerlach, UN, V. Shtabovenko, M. Steinhauser,
Phys. Rev. Lett. 129 (2022) 10, 102001.



$B_s - \bar{B}_s$ width difference



$\overline{\text{MS}}$ scheme



Prediction of Δm_s and $\Delta \Gamma_s$ suffer from the “ V_{cb} puzzle”.

Their ratio is essentially unaffected by uncertainties in CKM elements.

Summary and Outlook

Summary and Outlook

- Flavour physics probes **BSM physics** associated with very heavy particles and is instrumental to formulate a physics case for **future hadron colliders**.
- The current experimental precision in $A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \psi K_S)$ and $A_{\text{CP}}^{\text{mix}}(B_s \rightarrow J/\psi \phi)$ calls for a better understanding of penguin pollution.
- The width difference $\Delta\Gamma$ between the mass eigenstates of neutral **B** mesons is known to **NNLO** in QCD.
- For further information on **B** lifetime calculations and measurements see [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, *Eur.Phys.J.ST* 233 \(2024\) 2, 359-390](#)