Precise predictions for $t\bar{t}H$ production at the LHC

Swiss National Science Foundation

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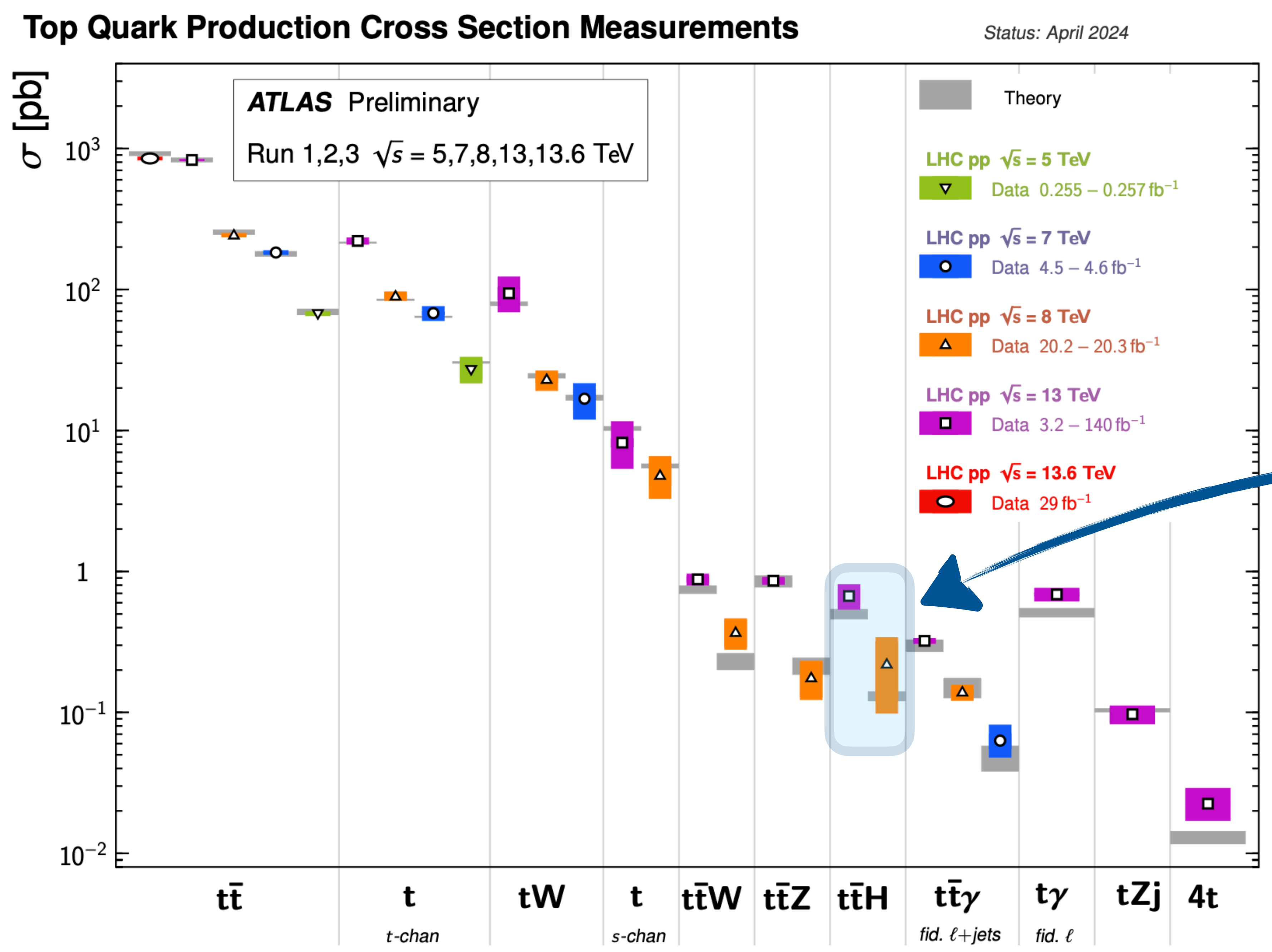
based on Phys.Rev.Lett. 130 (2023), JHEP 03 (2025) and arXiv 2503.15043

Why is $t\bar{t}H$ production interesting ?

motivations:

- the study of the Higgs boson is **one of the priorities** in the LHC experimental program, after its discovery in 2012
- the Higgs boson couplings to SM particles are proportional to their masses: **special role played by the top quark**!
- only about 1 % of the Higgs bosons are produced in association with a top-quark pair (first observation in 2018) but…
- the production mode $pp \to t\bar{t}H$ allows for a direct measurement of the **top-quark Yukawa coupling**

indirect model-dependent probes are for example the Higgs gluon-fusion production (via a top-quark loop) and $pp \to t\bar{t}t\bar{t}$ (at tree-level via diagrams featuring an off-shell Higgs propagator)



[ATL-PHYS-PUB-2024-006 (2024)]

the cross section is at least two orders of magnitude smaller than in the case of $t\bar{t}$ production but …
the process is crucial for characterising the interaction of top quarks with the Higgs sector

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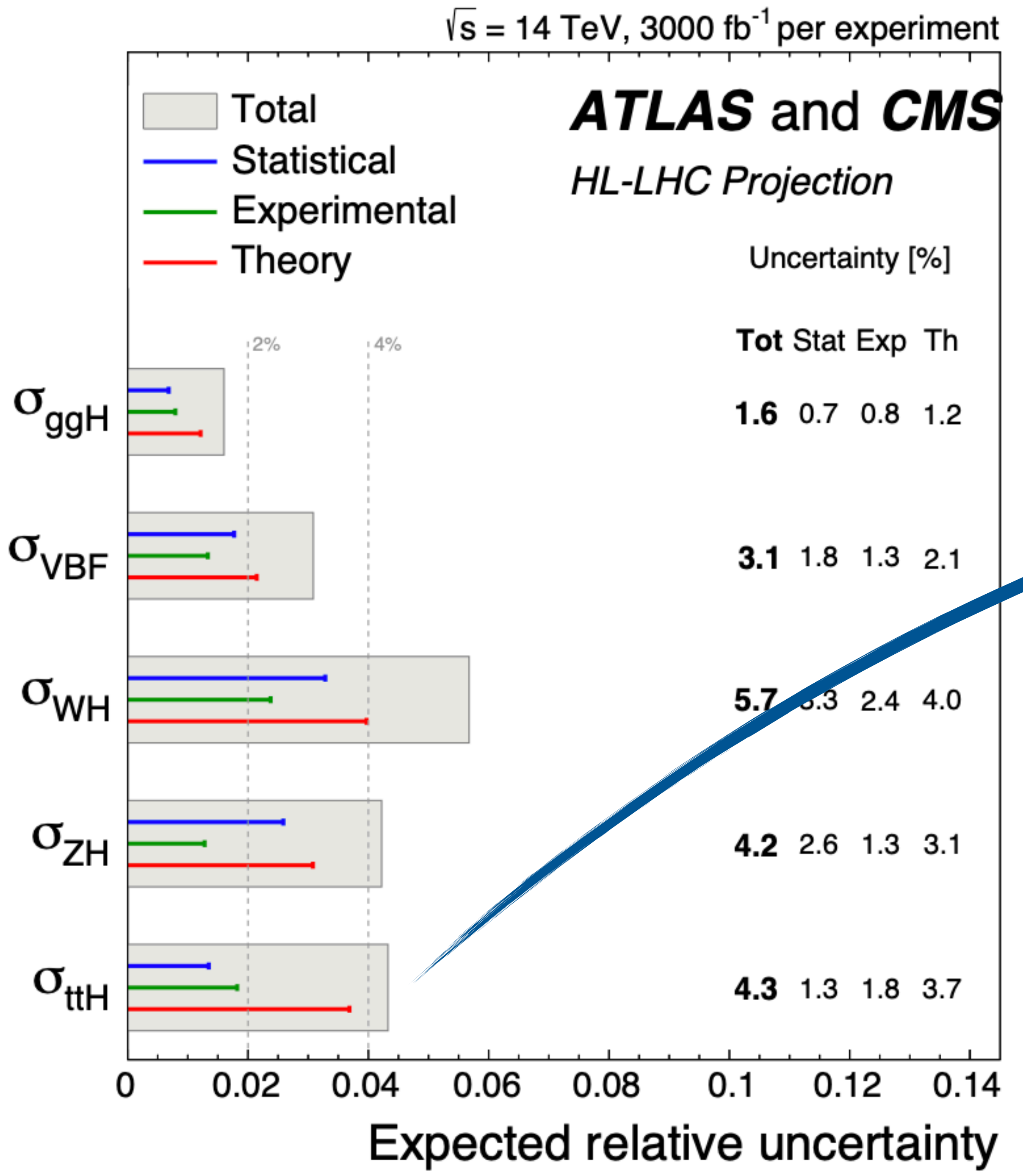

[Report from WG2 (2019)]

the current experimental accuracy is $\mathcal{O}(20\%)$ but, according to the HL-LHC projections, it is expected to go down to $\mathcal{O}(2\%)$

the extraction of the $t\bar{t}H(H \to b\bar{b})$ signal is limited by the theoretical uncertainties in the modelling of the backgrounds, mainly $t\bar{t}b\bar{b}$ and $t\bar{t}$ + light-flavour jets

moreover, NLO QCD + EW theory predictions equipped with NNLL soft-gluon resummation are affected by $\mathcal{O}(10\%)$ uncertainty

Theoretical predictions for $t\bar{t}H$

state of the art:

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas (2001,2003) [Reina, Dawson, Wackeroth, Jackson, Orr (2001,2003)]
- ☑ **NLO EW** corrections (*on-shell top quarks*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ **NLO QCD** corrections (*leptonically decaying top quarks*) [Denner, Feger (2015)] [Stremmer, Malgorzata (2022)]
- ☑ **NLO QCD + EW** corrections (*off-shell top quarks*) [Denner, Lang, Pellen, Uccirati (2017)]
- ☑ current predictions based on **NLO QCD + EW** corrections (*on-shell top quarks*), including **NNLL** soft-gluon resummation [Broggio et al.] [Kulesza et al.]

see Alessandro's talk!

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- ☑ **NNLO QCD** contributions for the **off-diagonal** partonic channels [Catani, Fabre, Grazzini, Kallweit (2021)]

- ☑ **complete NNLO QCD** predictions with approximated two-loop amplitudes [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]
- ☑ + complete set of **EW** corrections [Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2024)]
- ☑ + matched with **NNLL resummation** [LHC HWG arXiv 2503.15043] see also Alessandro's talk!

FOCUS OF THIS TALK!

Theoretical predictions for $t\bar{t}H$

state of the art:

☑ **complete NNLO QCD** predictions with approximated two-loop amplitudes

main bottleneck

Two-loop amplitudes for ttH production: the quark-initiated Nf-part
Bakul Agarwal, Gudrun Heinrich, Stephen P. Jones, Matthias Kerner, Sven Yannick Klein, Jannis Lang, Vitaly Magerya, Anton Olsson

One loop QCD corrections to $gg \to t\bar{t}H$ at $\mathcal{O}(\epsilon^2)$
Federico Buccioni, Philipp Alexander Kreer, Xiao Liu, Lorenzo Tancredi

Two-loop QCD amplitudes for $t\bar{t}H$ production from boosted limit
Guoxing Wang, Tianya Xia, Li Lin Yang, Xiaoping Ye

HOT TOPIC !!

Two-Loop Master Integrals for Leading-Color $pp \to t\bar{t}H$ Amplitudes with a Light-Quark Loop
F. Febres Cordero, G. Figueiredo, M. Kraus, B. Page, L. Reina

Our subtraction framework: q_T-slicing

[Catani, Grazzini (2007)]

- cross section for the production of a triggered $Q\bar{Q}F$ final state at N^{k}LO

crucial to keep the mass of the heavy quark m_Q

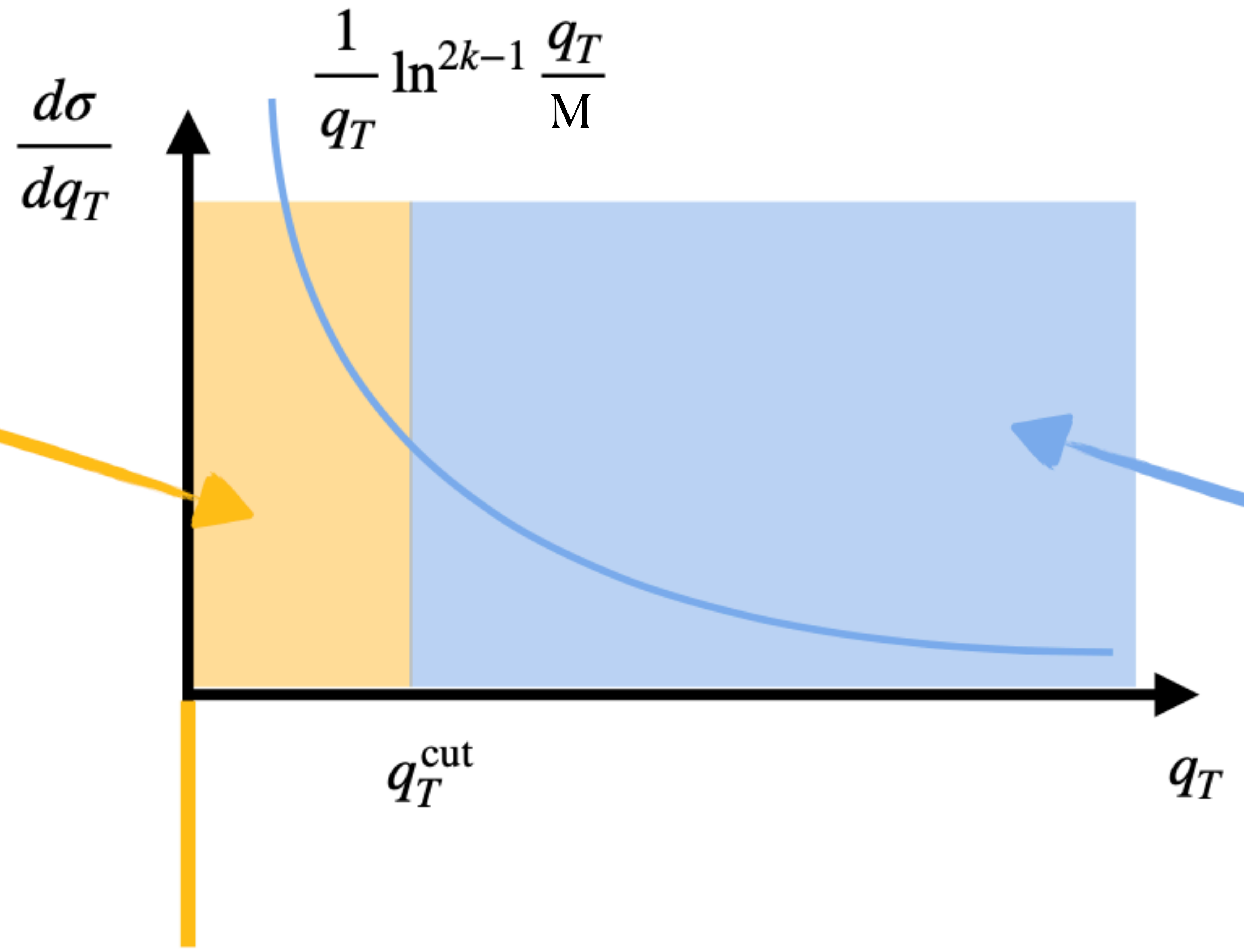
to complete an NNLO computation: crucial to construct an NNLO subtraction/slicing scheme and have all scattering amplitudes available

$$\sigma = \int_{<q_T^{\rm cut}} dq_T \frac{d\sigma}{dq_T} + \int_{>q_T^{\rm cut}} dq_T \frac{d\sigma}{dq_T}$$

all emissions are unresolved

we can exploit the QCD factorisation of the matrix elements in the singular soft and/or collinear limits

ingredients from q_T - resummation



1 emission is always resolved

the complexity of the calculation is reduced by 1 order

logarithmic IR sensitivity to the cut

q_T is the transverse momentum of the $Q\bar{Q}F$ system

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + [d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}]_{q_T>q_T^{\rm cut}} + \mathcal{O}((q_T^{\rm cut})^p)$$

Our subtraction framework: q_T-slicing [Catani, Grazzini (2007)]

▹ master formula at NNLO

$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\text{cut}}} + \mathcal{O}((q_T^{\text{cut}})^p)$$

☑ all required **tree-level** and **one-loop** matrix elements are known and can be evaluated with **automated tools** like OpenLoops2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]

☑ the remaining NLO-type singularities can be removed by applying a **local subtraction** method

[Catani, Seymour (1998)] [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

☑ **automatised numerical implementation** in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]

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☑ the hard-collinear coefficient receives contributions also from the **two-loop virtual amplitudes**

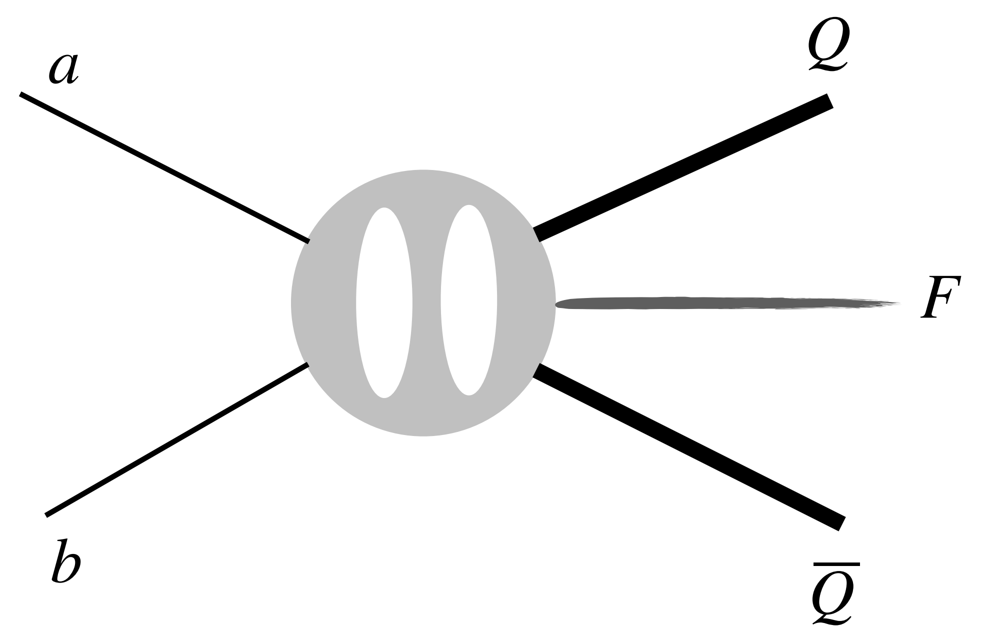
$$\mathcal{H}_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}^{(2)}(z_1, z_2)$$

where $$H^{(2)} = \left.\frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=Q}$$

UV renormalised and IR subtracted amplitude at scale μ_{IR}

[Ferroglia, Neubert, Pecjac, Yang (2009)]

Q is the invariant mass of the $Q\bar{Q}F$ system



main bottleneck: $2 \to 3$ and higher multiplicity two-loop amplitudes involving heavy loops and (many) external massive legs are currently out of reach. They require major breakthroughs

Our subtraction framework: q_T-slicing

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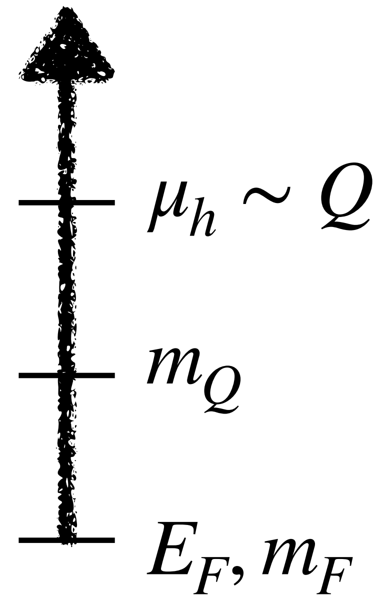
strategy: exploit the factorisation properties of QCD matrix elements in two different and rather complementary kinematic regimes

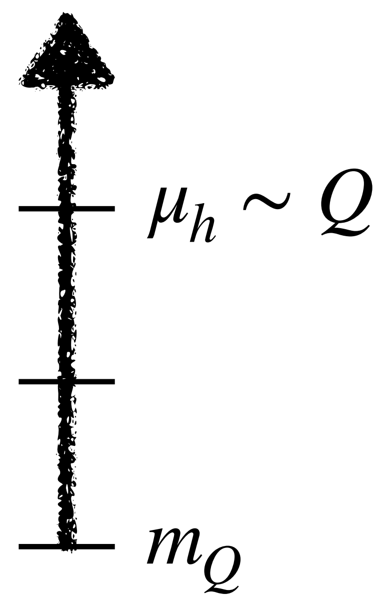
SOFT-BOSON approximation

1. soft limit for the external boson F ($E_F \to 0, m_F \to 0$)



MASSIFICATION

2. high-energy limit (ultra-relativistic quarks) ($m_Q \ll \mu_h$)



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strategy: exploit the factorisation properties of QCD matrix elements in two different and rather complementary kinematic regimes

disclaimer:
for $t\bar{t}H$, none of the two approximations is (a priori) justified in the bulk of the events.
The quality of the approximation must be carefully assessed

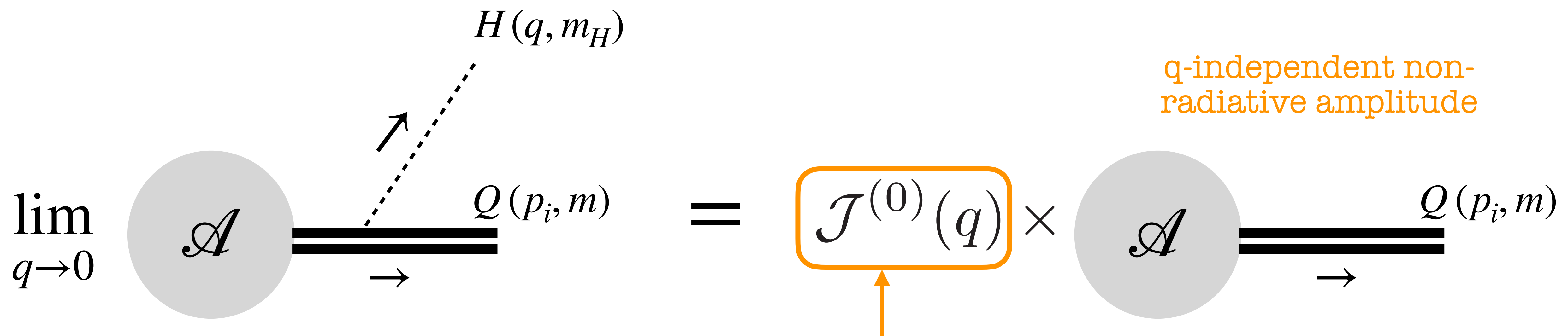
Soft Higgs-boson approximation

- We want to study the soft Higgs-boson limit for the amplitude associated with

$$a_1(p_1) + a_2(p_2) \to \mathcal{Q}(p_3, m)\overline{\mathcal{Q}}(p_4, m)....\mathcal{Q}(p_{N+1}, m)\overline{\mathcal{Q}}(p_{N+2}, m) + H(q, m_H)$$

one or more heavy-quark pairs with the same mass

- at tree-level, it is straightforward to show that the LP factorisation reads



$$\mathcal{J}^{(0)}(q) = \frac{m}{v} \sum_i \frac{m}{p_i \cdot q}$$ scalar eikonal current

- at bare level, the naïve factorisation formula holds true at all orders in α_s, due to the **abelian nature** of the Higgs boson

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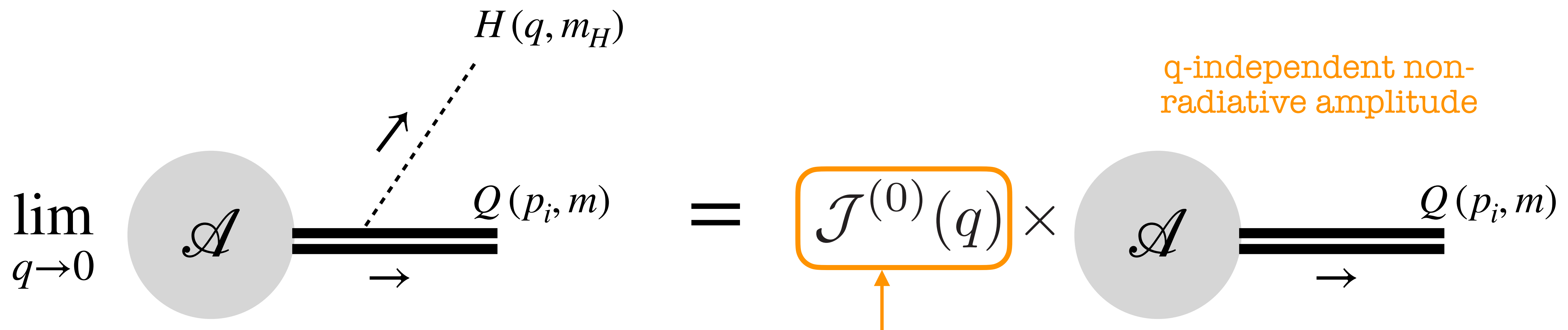

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scalar eikonal current

- at bare level, the naïve factorisation formula holds true at all orders in α_s, due to the **abelian nature** of the Higgs boson
- … but the renormalisation of the heavy-quark mass and wave function changes the overall normalisation by

up to two-loop order

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F\left(\alpha_s^{(n_l)}(\mu_R^2), \frac{\mu_R}{m}\right) = 1 + \frac{\alpha_s^{(n_l)}(\mu_R^2)}{2\pi}(-3C_F) + \left(\frac{\alpha_s^{(n_l)}(\mu_R^2)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2 - \frac{185}{12}C_FC_A + \frac{13}{6}C_F(n_l+n_h) - 3C_F\beta_0^{(n_l)}\ln\frac{\mu_R^2}{m^2}\right) + \mathcal{O}({\alpha_s^{(n_l)}}^3)$$

Soft Higgs-boson approximation

- **LP master formula** in the soft Higgs limit ($q \to 0$, $m_H \ll m$):

$$\mathcal{M}(p_1, p_2 ... p_N, q) \simeq F(\alpha_s(\mu_R); m/\mu_R) \, \frac{m}{v} \left(\sum_{i=1}^{N} \frac{m}{p_i \cdot q} \right) \mathcal{M}(p_1, p_2 ... p_N)$$

all-order UV renormalised amplitudes

- observations:
  - $F(\alpha_s(\mu_R); m/\mu_R)$ is perturbatively calculable, finite and gauge-independent
  - it can be derived by applying the so-called Higgs Low Energy theorems (LETs)

[Shifman, Vainshtein, Voloshin, Zakharov (1979)]
[Kniehl, Spira (1995)]

we proved the relation with the soft limit of the scalar FF up to three-loop order
[Fael, Lange, Schönwald, Steinhauser (2022, 2023)]

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  - it can be derived by applying the so-called Higgs Low Energy theorems (LETs)
  - the IR singularity structure of the scattering amplitude is left changed
  - the non-radiative amplitude must be evaluated on a set of projected momenta (to preserve momentum conservation)
  - for the specific case of **$t\bar{t}H$ production**, the non-radiative amplitude is known up to two-loop order

[Bärnreuther, Czakon, Fiedler (2013)]

the soft factorisation formulae could provide a powerful cross check of future exact amplitude calculations, in this specific kinematic limit

Results: systematic uncertainties

[Phys.Rev.Lett. 130 (2023)]

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
$\sigma_{\rm LO}$	261.58	129.47	23055	2323.7
$\Delta\sigma_{\rm NLO,H}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\rm NLO,H}\|_{\rm soft}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\rm NNLO,H}\|_{\rm soft}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

- at NLO, difference of **5%** (**30%**) in $q\bar{q}$ (gg) channel
- at NNLO, the hard-virtual contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$ small!
- our prescription to provide a conservative uncertainty is:
  - ☑ apply the approximation at a **different subtraction scale** (vary μ_{IR} by a factor 2 around Q); add the two-loop shift based on the exact tree-level and one-loop $t\bar{t}H$ amplitudes
  - ☑ take into account the NLO discrepancy and multiply it by a **tolerance factor 3**
  - ☑ combine **linearly** the gg and $q\bar{q}$ channels

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FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO} , $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

it is clear that the quality of the final result depends on the size of the contribution we are approximating

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Results: total cross section

[Phys.Rev.Lett. 130 (2023)]

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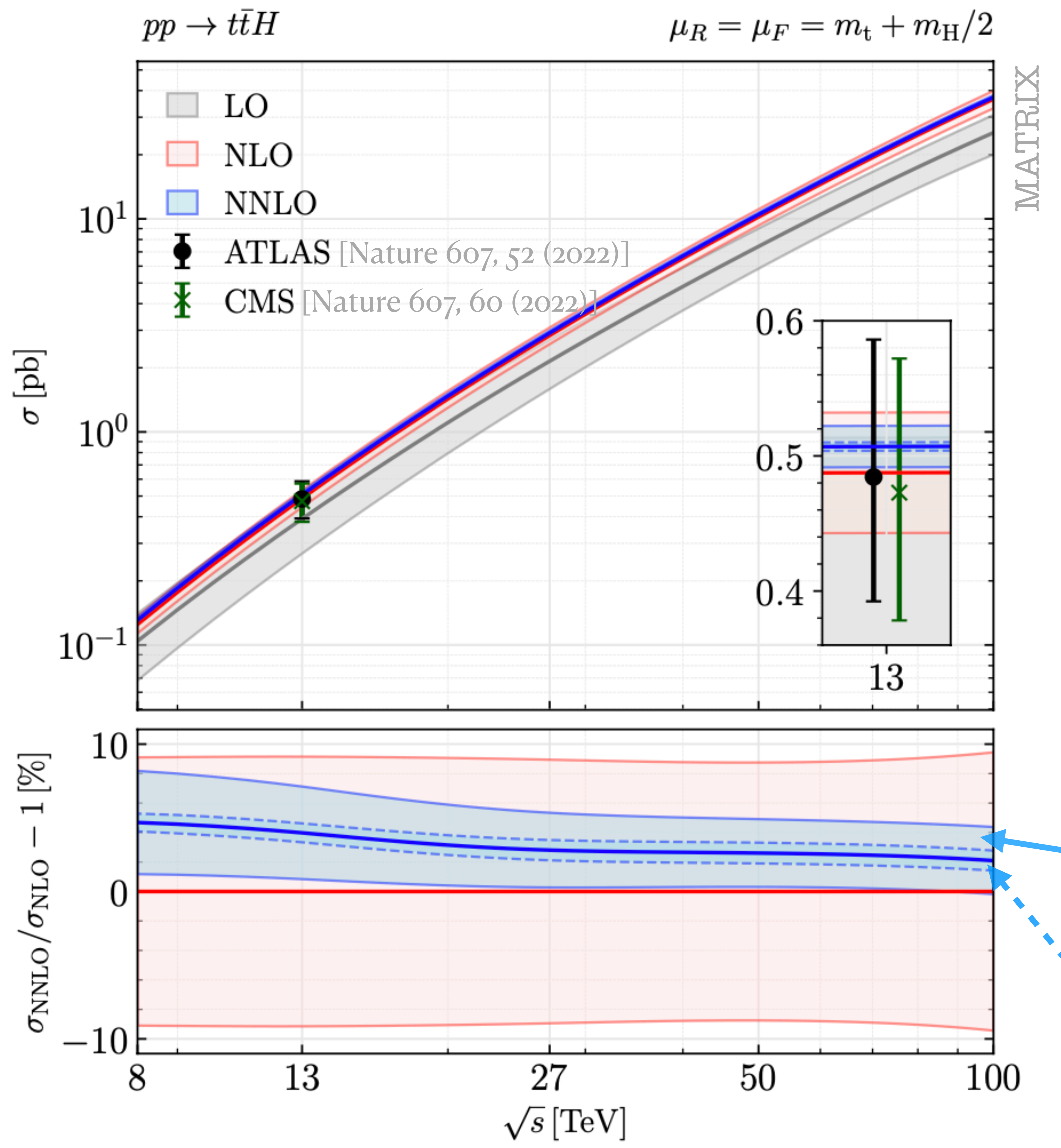


symmetrised 7-point scale variation

systematic + soft-approximation

σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

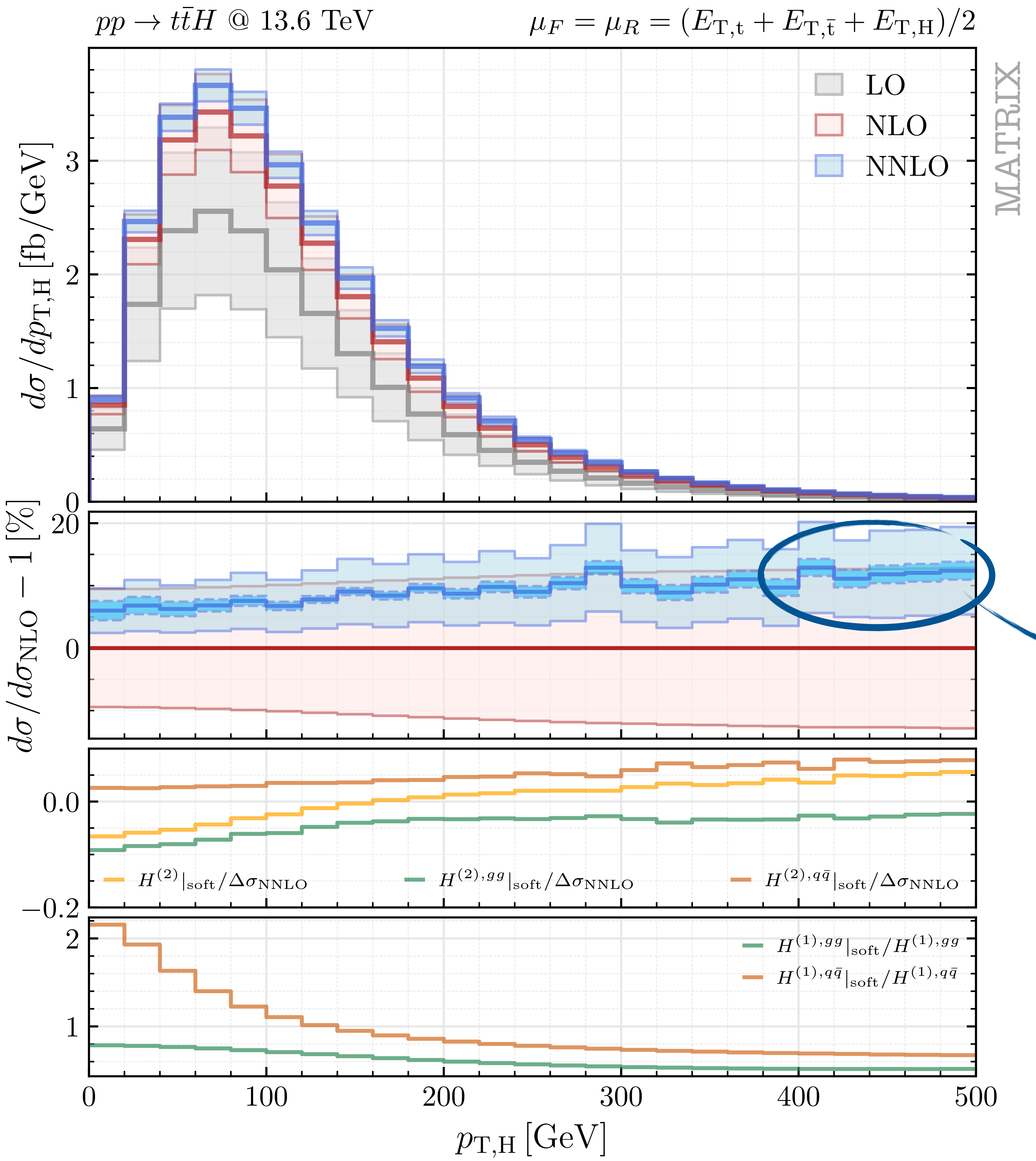
- at NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- at NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$

nice perturbative convergence with theory uncertainties at $\mathcal{O}(3\%)$

First differential results: "soft-based"

Higgs transverse momentum

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$



- significant reduction of the perturbative uncertainties
- soft-approximation uncertainty computed on a **bin-by-bin basis** (NLO discrepancy multiplied by a constant tolerance factor 3)

oversimplified procedure ...

- the systematic uncertainties seem to be under control, but are they trustable?

in the tail of the $p_{T,H}$ distribution, far from the region of validity of the soft-approximation, the systematic errors are "artificially" too small

First differential results: “soft-based”

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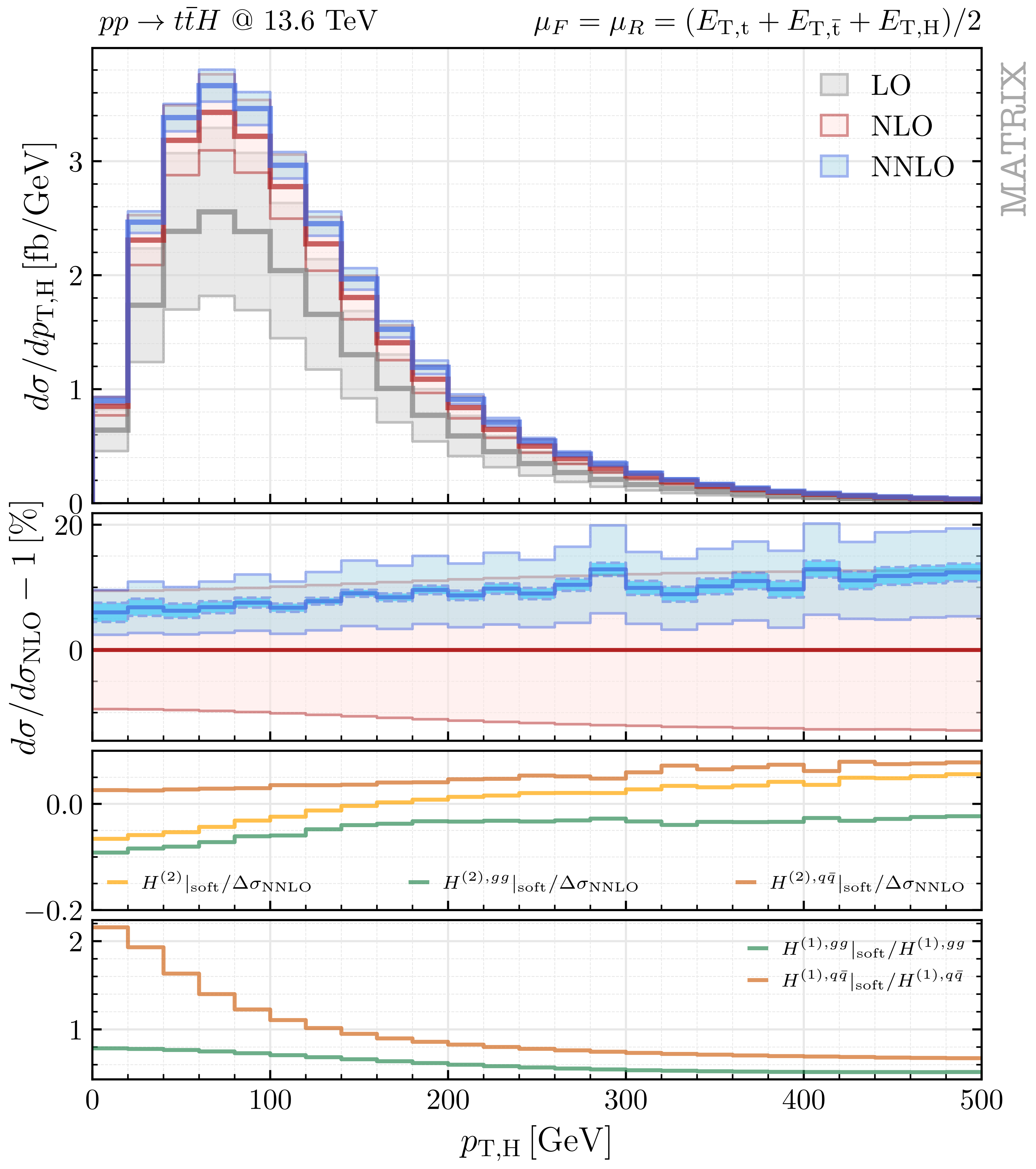

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to make our predictions more robust at the differential level we “combine” the SOFT-HIGGS APPROXIMATION with a HIGH-ENERGY expansion

Mass factorisation or massification

original formulation

[Penin (2006)]
[Moch, Mitov (2007)]

- **idea**: reconstruct the massive amplitudes, in the ultra-relativistic quark limit $m \ll Q$, up to power corrections $\mathcal{O}(m^2/Q^2)$
- If contributions from **heavy-quark loops** are **neglected**, the master formula is

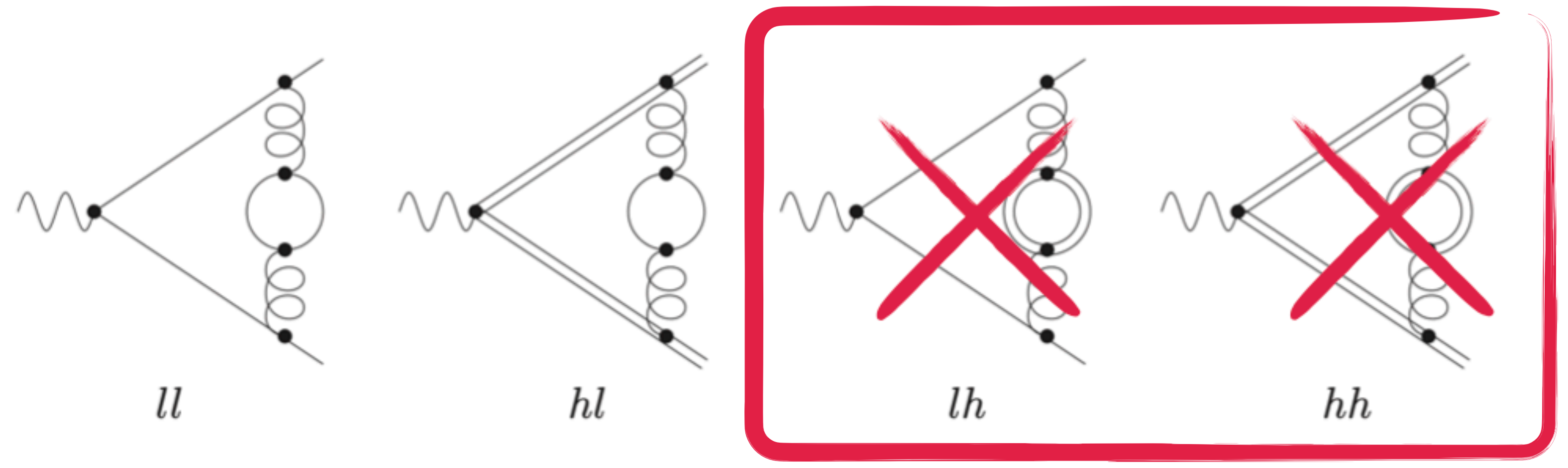
we are "dressing" n_Q external quarks with a mass m

$$|\mathcal{M}_m\rangle = \left(Z_{[Q]}^{(m|0)}\left(\alpha_s^{(n_l)}, \frac{\mu^2}{m^2}, \epsilon\right)\right)^{n_Q/2} |\mathcal{M}\rangle$$

all-order UV renormalised amplitudes in $\overline{\mathrm{MS}}$ scheme with n_l running quarks

universal, perturbatively computable, ratio between massive and massless FFs

$$Z_{[Q]}^{(m|0)}\left(\alpha_s^{(n_l)}, \frac{\mu^2}{m^2}, \epsilon\right) = \mathcal{F}^{[Q\overline{Q}\to F]}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s^{(n_l)}(\mu^2), \epsilon\right)\left(\mathcal{F}_0^{[q\bar{q}\to F]}\left(\frac{Q^2}{\mu^2}, \alpha_s^{(n_l)}(\mu^2), \epsilon\right)\right)^{-1}$$



the mass "screens" collinear singularities

1. all ϵ poles, n_h-independent logarithms of the mass and finite terms of the massive amplitude are predicted
2. it can be viewed as a change in regularisation scheme

Mass factorisation or massification

generalised formulation

[Wang et al. (2023)]

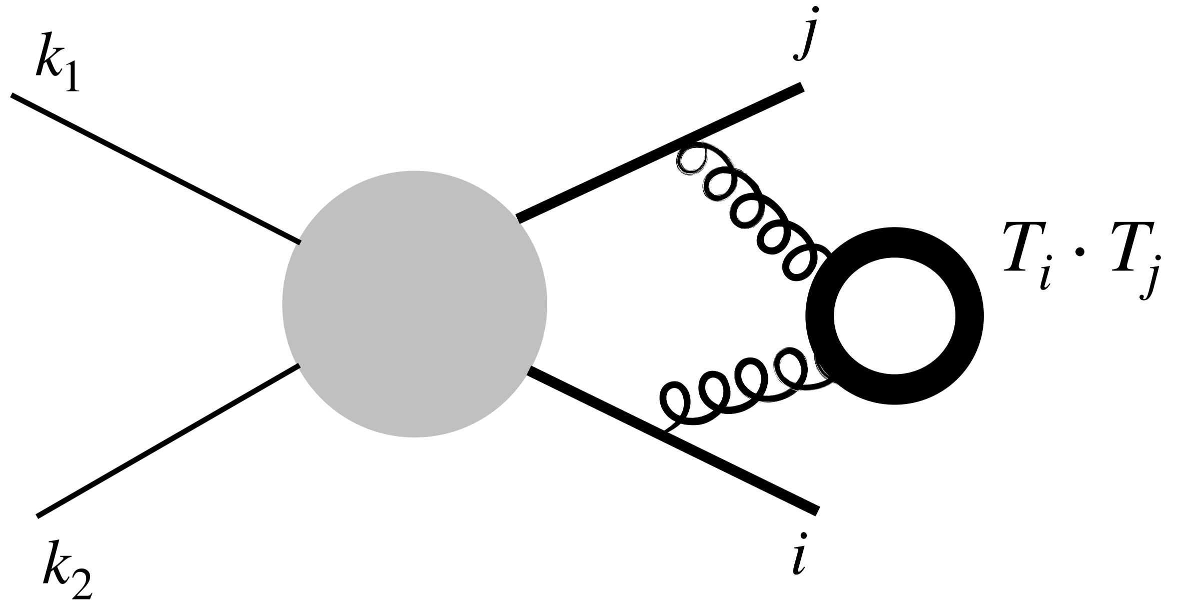
- If contributions from **heavy-quark loops** are **included**, a non-trivial soft function emerges starting from α_s^2
- the master formula gets modified as

[Becher, Melnikov (2007)]
[Engel et al. (2019)]

$$|\mathcal{M}_m\rangle = \prod_i \left(Z_{[i]}^{(m|0)} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon \right) \right)^{1/2} \mathbf{S} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) |\mathcal{M}\rangle$$

all-order UV renormalised amplitudes in $\overline{\text{MS}}$ scheme with $n_f = n_l + n_h$ running quarks

process-dependent **SOFT** function, operator in colour space, it starts contributing at two-loop order



$$\mathbf{S} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) = 1 + \left(\frac{\alpha_s^{(n_f)}(\mu^2)}{4\pi} \right)^2 n_h \sum_{i>j} (-\mathbf{T}_i \cdot \mathbf{T}_j) \, S^{(2)} \left(\frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) + \mathcal{O}(\alpha_s^3)$$

with $$S^{(2)} \left(\frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) = T_R \left(\frac{\mu^2}{m^2} \right)^{2\epsilon} \left(-\frac{4}{3\epsilon^2} + \frac{20}{9\epsilon} - \frac{112}{27} - \frac{4\zeta_2}{3} \right) \ln \left(\frac{-s_{ij}}{m^2} \right)$$

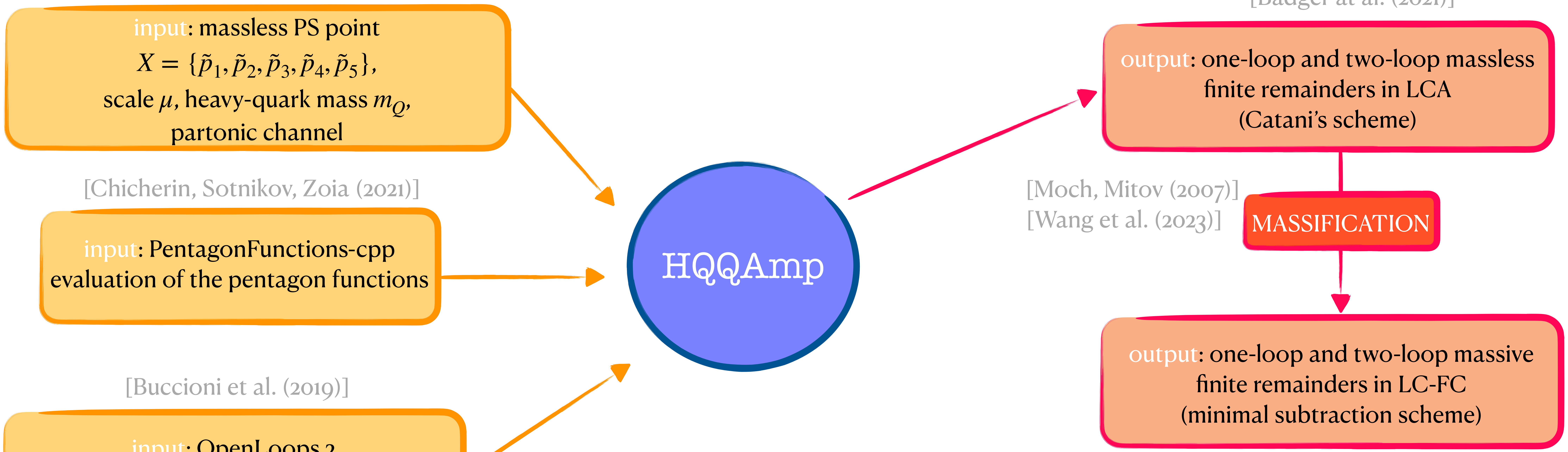
for the specific case of $\mathrm{Q\overline{Q}H}$ production we can reconstruct the massive amplitudes, up to power corrections in the heavy-quark mass, by exploiting the corresponding (known) massless amplitudes

[Badger et al. (2021, 2024)]

HQQAmp: a massive C++ implementation

- idea: implement the one-loop and two-loop massless amplitudes of [Badger at al. (2021)] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**
- different workflow and possibility of choosing the **precision** for the MIs and relative coefficients

$$q/g(p_1) + \bar{q}/g(p_2) \to H(p_3) + \mathcal{Q}(p_4) + \overline{\mathcal{Q}}(p_5)$$



evaluation time per phase space point:
$\mathcal{O}(2-3s)$ for both partonic channels
[quadruple (double) precision for the coefficients (MIs)]

cross-checked against an independent implementation for $b\bar{b}H$

see Christian's talk!

HQQAmp: a massive C++ implementation

- idea: implement the one-loop and two-loop massless amplitudes of [Badger at al. (2021)] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**
- different workflow and possibility of choosing the **precision** for the MIs and relative coefficients

$$
\begin{aligned}
|\mathcal{M}_m^{\text{fin}}\rangle &= \mathbf{Z}^{-1}_{m\ll\mu_h}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon\right) Z_{[\mathcal{Q}]}^{(m|0)}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon\right) Z_{[c]}^{(m|0)}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon\right) \\
&\times \mathbf{S}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon\right) \mathbf{Z}_{(m=0)}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \epsilon\right) |\mathcal{M}_{(m=0)}^{\text{fin}}\rangle + \mathcal{O}\left(\frac{m}{\mu_h}\right) \\
&= \mathcal{F}_{[c]}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \frac{\mu^2}{s_{ij}}\right) |\mathcal{M}_{(m=0)}^{\text{fin}}\rangle + \mathcal{O}\left(\frac{m}{\mu_h}\right)
\end{aligned}
$$

it is an operator in colour space and it encodes all mass logarithms!

$$
\begin{aligned}
|\mathcal{M}_m^{(1),\text{fin}}\rangle &= |\mathcal{M}_{(m=0)}^{(1),\text{fin}}\rangle + \mathcal{F}_{[c]}^{(1)}\,|\mathcal{M}_{(m=0)}^{(0)}\rangle \\
|\mathcal{M}_m^{(2),\text{fin}}\rangle &= |\mathcal{M}_{(m=0)}^{(2),\text{fin}}\rangle + \mathcal{F}_{[c]}^{(1)}\,|\mathcal{M}_{(m=0)}^{(1),\text{fin}}\rangle + \mathcal{F}_{[c]}^{(2)}\,|\mathcal{M}_{(m=0)}^{(0)}\rangle
\end{aligned}
$$

massless two-loop contribution in LCA. All remaining terms are “promoted” to FC

no need to implement the higher ϵ orders of the massless one-loop amplitude

[Badger at al. (2021)]

output: one-loop and two-loop massless finite remainders in LCA (Catani's scheme)

[Wang et al. (2023)] MASSIFICATION

output: one-loop and two-loop massive finite remainders in LC-FC (minimal subtraction scheme)

Yukawa renormalised ON-SHELL

N.B. application of the massification directly on the finite remainders

Quality of both approximations at NLO

[JHEP 03 (2025)]

setup: NNLO NNPDF40, $m_H = 125.09GeV$, $m_t = 172.5GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

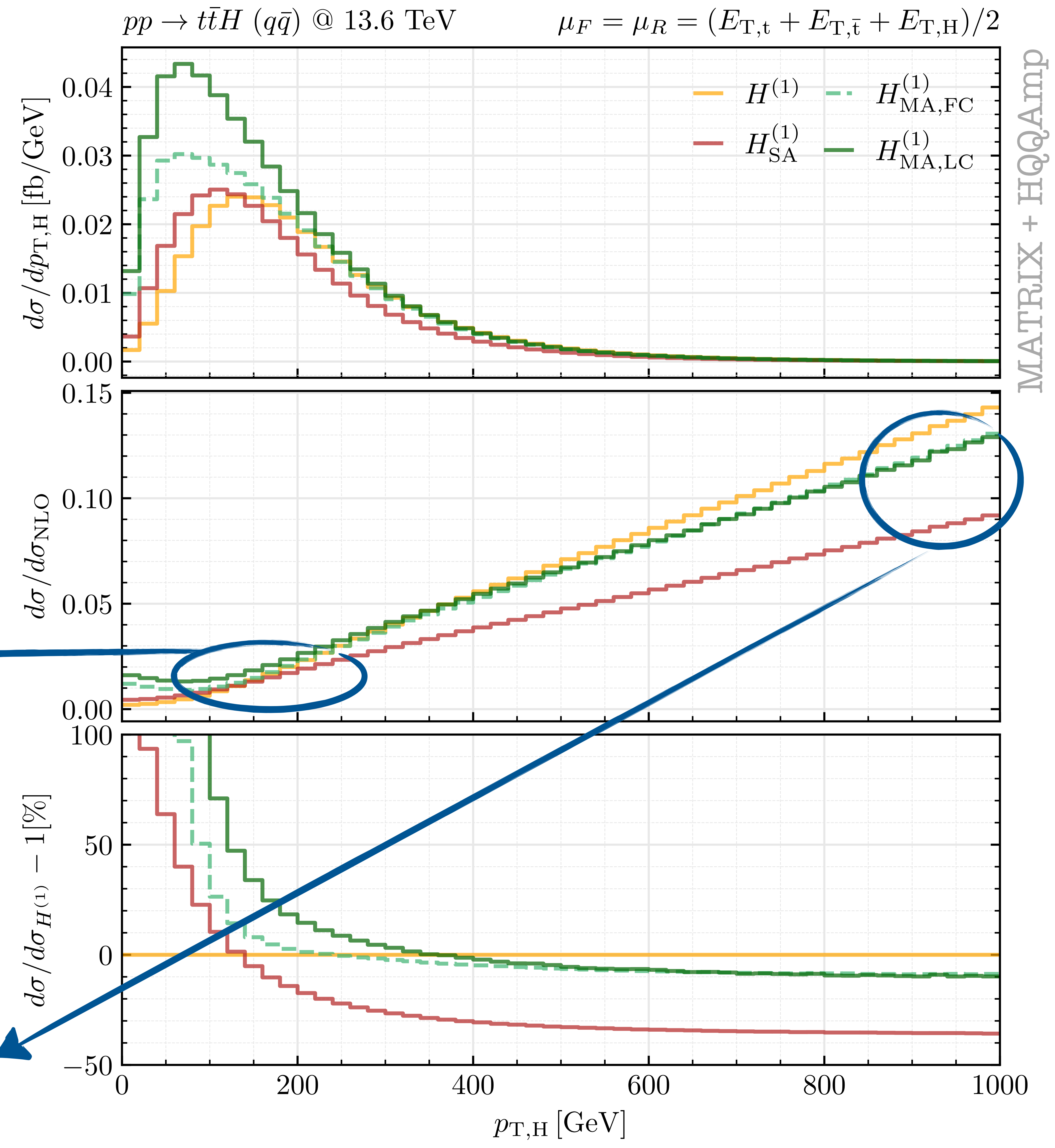
different setup!

around the peak:

1. FC-FC massification and soft approximation are nearly equivalent
2. LC-FC massification overestimates the exact result by almost a factor of 2

in the high-p_T tail:

1. missing subleading colour contributions are less relevant
2. soft approximation underestimates the exact result: $\mathcal{O}(2\%)$ difference of the NLO cross section

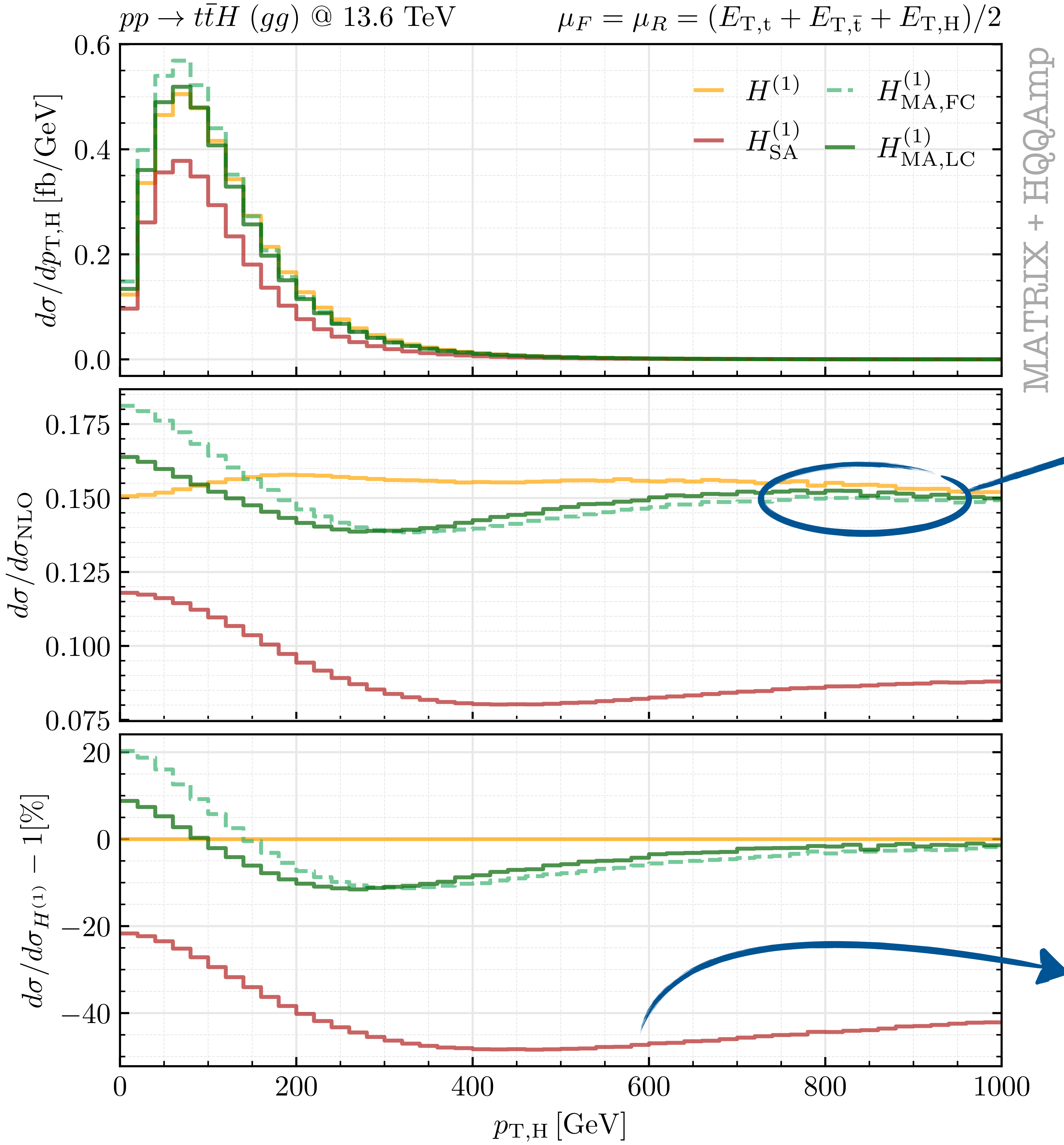


Quality of both approximations at NLO

[JHEP 03 (2025)]

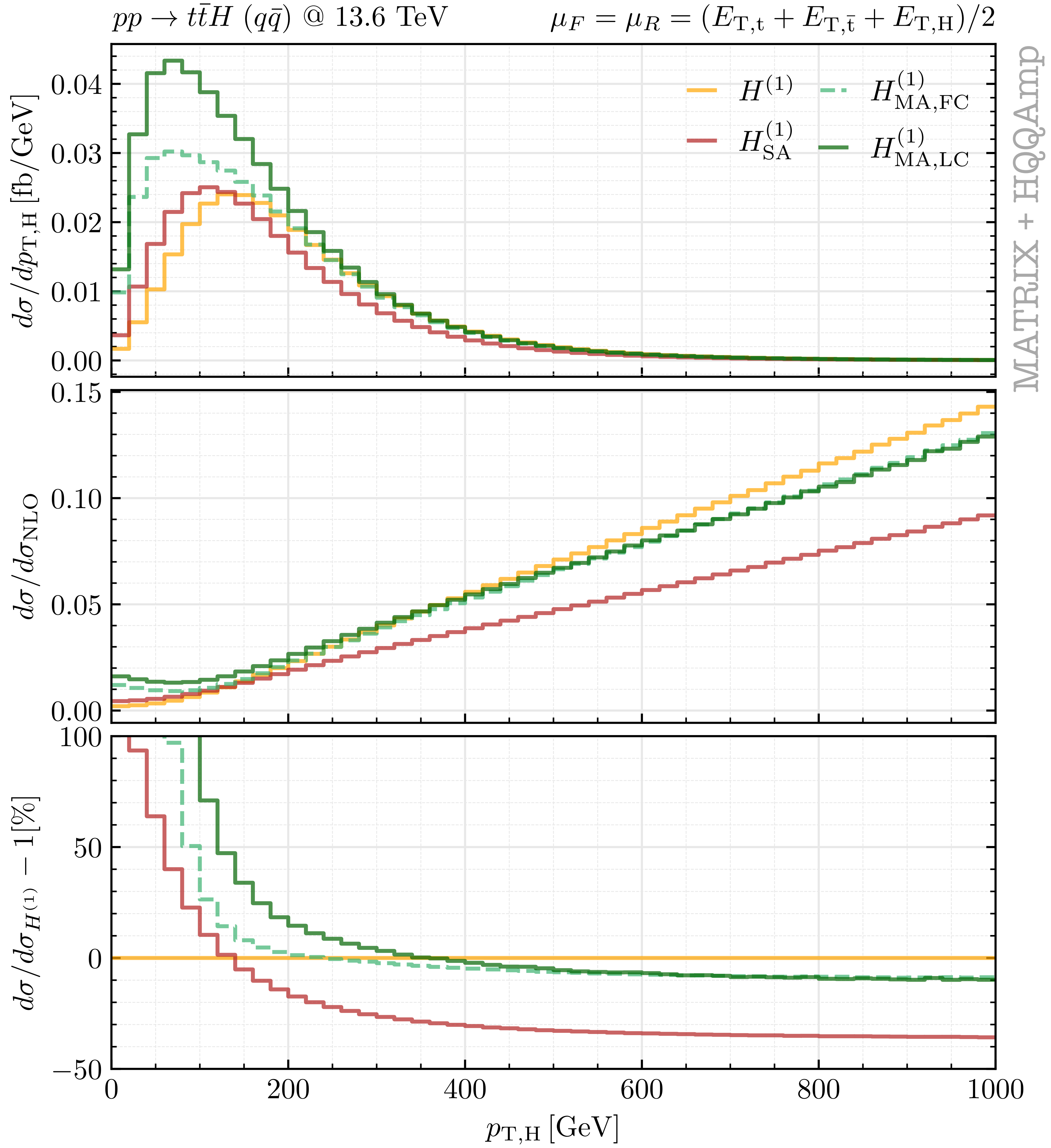
setup: NNLO NNPDF40, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

different setup!



massified results are in good agreement with the exact one-loop, with negligible effects on the NLO cross section in the tail

soft-approximated result is systematically below the exact one-loop, with effects of $\mathcal{O}(8\%)$ of the NLO cross section in the tail



First differential results: "best" $H^{(2)}$ prediction
[JHEP 03 (2025)]

setup: NNLO NNPDF40, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

H1-based error

$$\delta_{\rm SA}^{H^{(1)}} = 2 \times \left| \frac{\sigma_{H^{(1)}_{\rm SA}}}{\sigma_{H^{(1)}}} - 1 \right| \times \max\left(|\sigma_{H^{(2)}_{\rm SA}}|, |\sigma_{H^{(2)}_{\rm MA}}| \right)$$

$$\delta_{\rm MA}^{H^{(1)}} = 2 \times \max\left(\left| \frac{\sigma_{H^{(1)}_{\rm MA,FC}}}{\sigma_{H^{(1)}}} - 1 \right|, \left| \frac{\sigma_{H^{(1)}_{\rm MA,LC}}}{\sigma_{H^{(1)}}} - 1 \right| \right) \times \max\left(|\sigma_{H^{(2)}_{\rm SA}}|, |\sigma_{H^{(2)}_{\rm MA}}| \right)$$

μ_{IR}-variation error

$$\delta_{\rm SA}^{\mu_{\rm IR}} = \max\left(\left| \sigma_{H^{(2)}_{\rm SA}(\tilde{Q}/2)} + (Q/2 \to Q) - \sigma_{H^{(2)}_{\rm SA}} \right|, \left| \sigma_{H^{(2)}_{\rm SA}(2\tilde{Q})} + (2Q \to Q) - \sigma_{H^{(2)}_{\rm SA}} \right| \right)$$

$$\delta_{\rm MA}^{\mu_{\rm IR}} = \max\left(\left| \sigma_{H^{(2)}_{\rm MA}(\tilde{Q}/2)} + (Q/2 \to Q) - \sigma_{H^{(2)}_{\rm MA}} \right|, \left| \sigma_{H^{(2)}_{\rm MA}(2\tilde{Q})} + (2Q \to Q) - \sigma_{H^{(2)}_{\rm MA}} \right| \right)$$

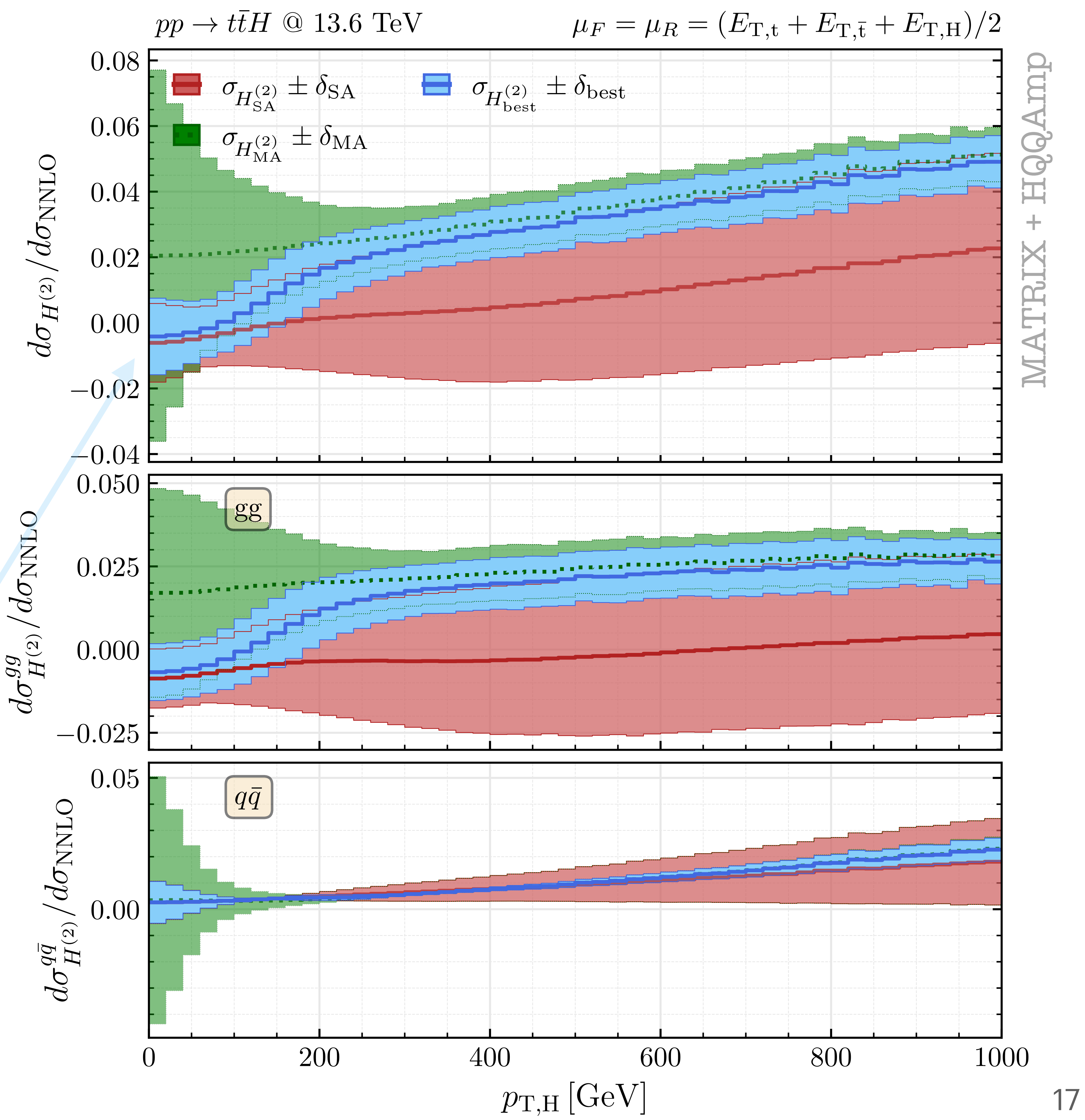
the final systematic error on each approximation and for each partonic channel is obtained by taking the maximum between $\delta^{\mu_{\rm IR}}$ and $\delta^{H^{(1)}}$

"best" for each partonic channel:

$$\sigma_{H^{(2)}_{\rm best}} = \frac{1}{\omega_{\rm SA} + \omega_{\rm MA}} \left(\omega_{\rm SA}\sigma_{H^{(2)}_{\rm SA}} + \omega_{\rm MA}\sigma_{H^{(2)}_{\rm MA}} \right)$$

$$\delta_{\rm best} = \left(\frac{1}{\omega_{\rm SA} + \omega_{\rm MA}} \right)^{1/2}$$

the errors on each channel are finally combined quadratically

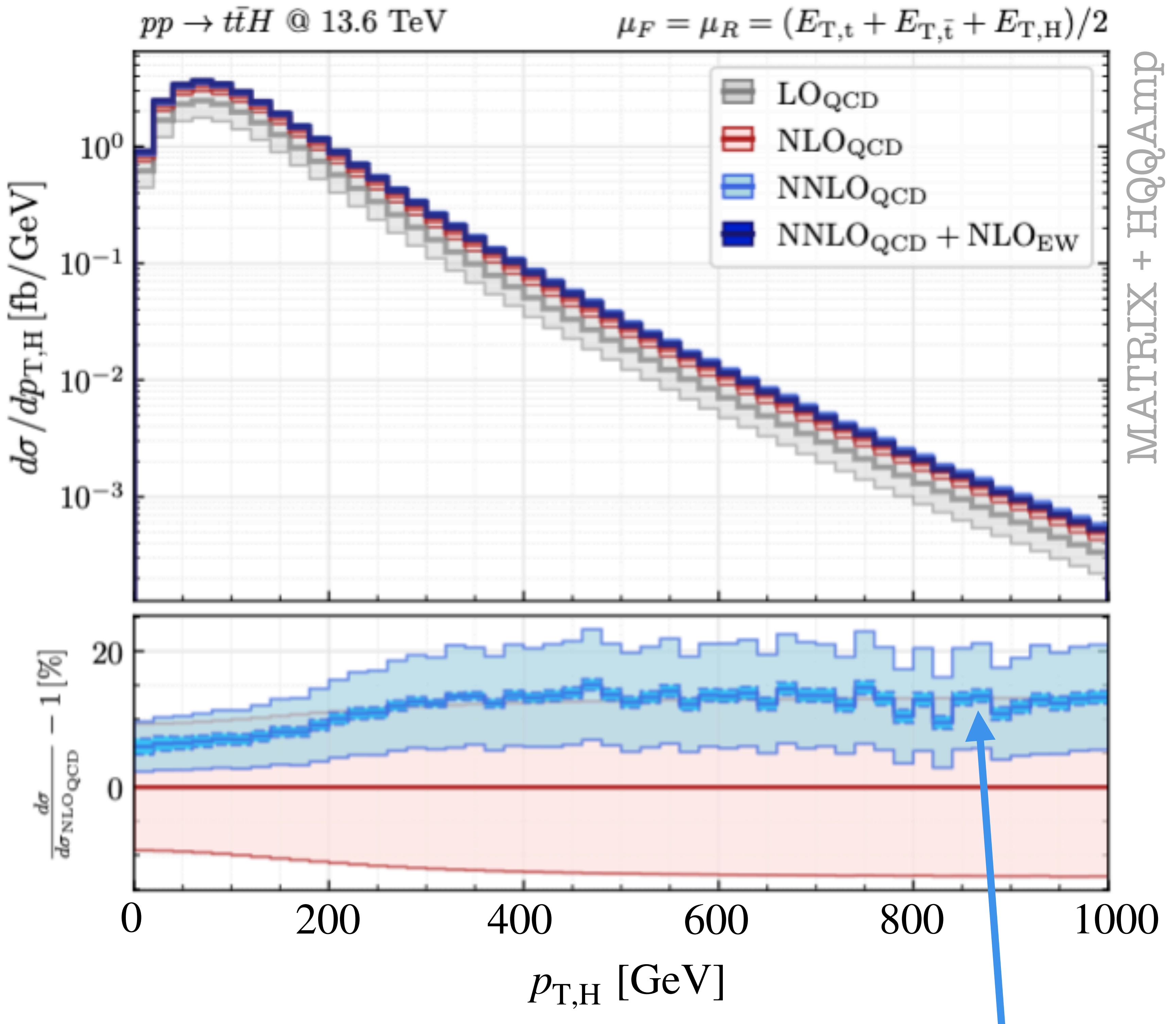


1. the "best" prediction nicely interpolates between the two limits
2. the associated error does not vary strongly over the $p_{T,H}$ range
3. the individual soft and massified predictions have overlapping error bands

NNLO QCD + EW predictions

[JHEP 03 (2025)]

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$



systematic error associated with the "best" prediction for the double-virtual contribution

total XS at fixed scale $\mu_R = \mu_F = m_t + m_H/2$

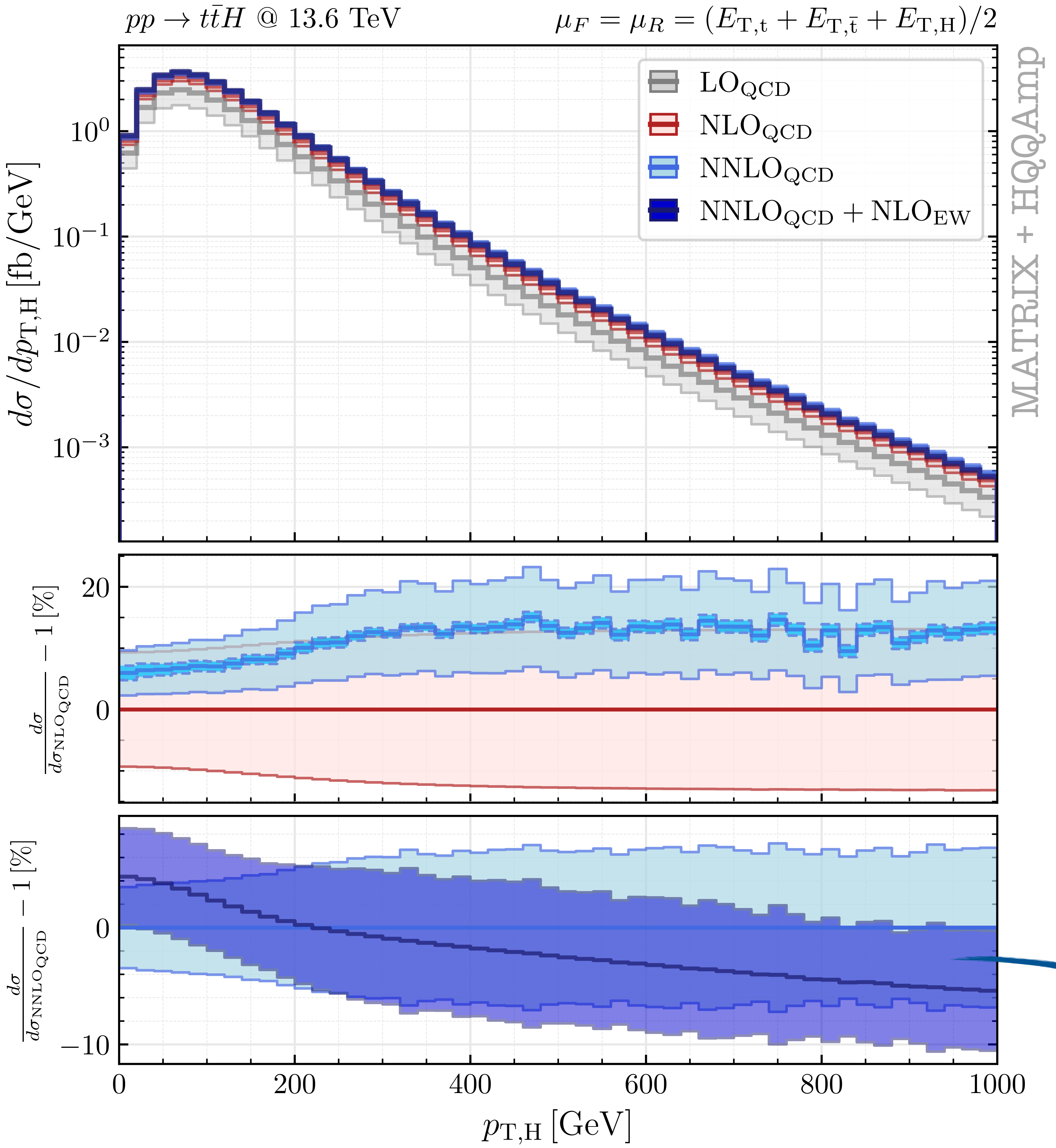
$\sqrt{s} = 13.6$TeV	σ [fb]
LO_{QCD}	$423.9 \; ^{+30.7\%}_{-21.9\%}$(scale)
NLO_{QCD}	$528.9 \; ^{+5.7\%}_{-9.0\%}$(scale)
$NNLO_{QCD}$	$550.7(5) \; ^{+0.9\%}_{-3.1\%}$(scale) $\pm 0.9\%$(approx)
$NNLO^{soft}_{QCD}$	$548.7(5) \; ^{+0.8\%}_{-3.0\%}$(scale) $\pm 0.6\%$(approx)

- NNLO QCD predictions based on the soft-approximated and "best" double virtual are **fully compatible**: difference of 0.4 %
- the **systematic uncertainty** based on the refined prescription is **slightly larger**: $\mathcal{O}(0.9\%)$ instead of $\mathcal{O}(0.6\%)$ of the NNLO cross section

NNLO QCD + EW predictions

[JHEP 03 (2025)]

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$

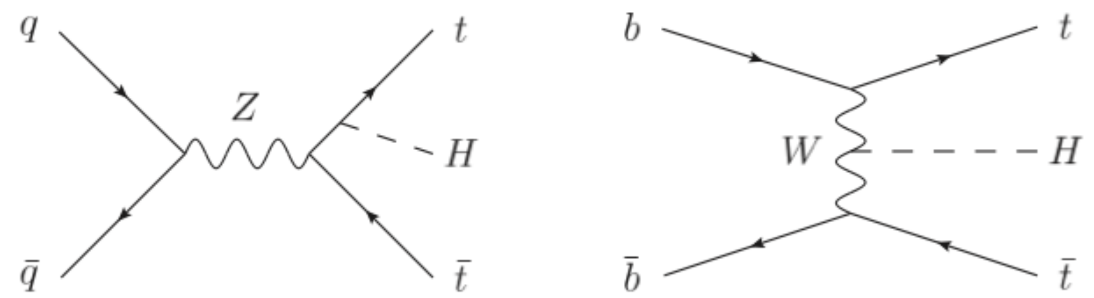


positive (negative) subdominant LO and NLO corrections in the small (large) $p_{T,H}$ region

total XS at fixed scale $\mu_R = \mu_F = m_t + m_H/2$

$\sqrt{s} = 13.6$TeV	σ [fb]
$\mathrm{LO_{QCD}}$	423.9 $^{+30.7\%}_{-21.9\%}$(scale)
$\mathrm{NLO_{QCD}}$	528.9 $^{+5.7\%}_{-9.0\%}$(scale)
$\mathrm{NNLO_{QCD}}$	550.7(5) $^{+0.9\%}_{-3.1\%}$(scale) $\pm0.9\%$(approx)
$\mathrm{NNLO_{QCD} + NLO_{EW}}$	562.3(5) $^{+1.1\%}_{-3.2\%}$(scale) $\pm0.9\%$(approx)

- inclusion of **all subdominant LO** ($\mathcal{O}(\alpha_s\alpha^2)$, $\mathcal{O}(\alpha^3)$) **and NLO** ($\mathcal{O}(\alpha_s^2\alpha^2)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha^4)$) contributions: +2 % at the cross section level



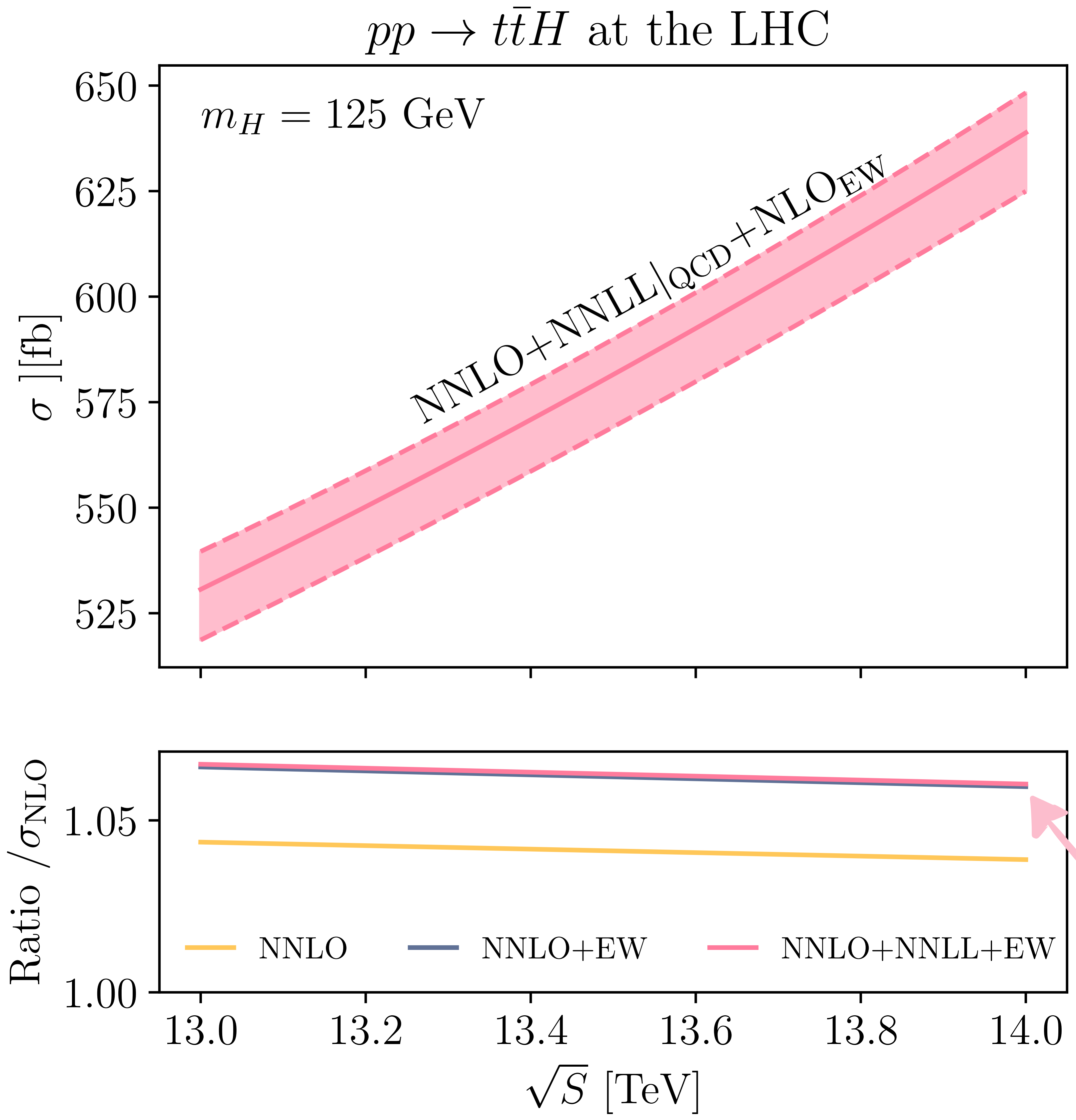
+ photon-induced diagrams

non-negligible impact compared to NNLO scale-variation bands

NNLO QCD + EW + NNLL predictions

[LHC HWG arXiv 2503.15043]

setup: NNLO PDF4LHC21_40_pdfas, $m_t = 172.5 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$



$\mathcal{O}(+4\%)$ for the NNLO QCD corrections and $\mathcal{O}(+2\%)$ for the EW ones, roughly independent on the collider energy and Higgs mass

- according to the recommendations of the LHC HWG, we have recently provided state-of-the-art predictions for the $t\bar{t}H$ total cross section by matching our **fixed-order NNLO** predictions with **soft-gluon resummation up to NNLL**

$$\sigma_{\text{NNLO+NNLL}} = \frac{\sigma^{\text{SCET}}_{\text{NNLO+NNLL}} + \sigma^{\text{dQCD}}_{\text{NNLO+NNLL}}}{2}$$

- extensive comparisons between the two resummation approaches

SCET: [Broggio et al.] dQCD: [Kulesza et al.] see Alessandro's talk!

- inclusion of the subleading LO and NLO contributions

$$\sigma_{\text{NNLO+NNLL+EW}} = \sigma_{\text{NNLO+NNLL}} + \sum_{i=2}^{3} \sigma_{\text{LO,i}} + \sum_{j=2}^{4} \sigma_{\text{NLO,j}}$$

resummation effects:

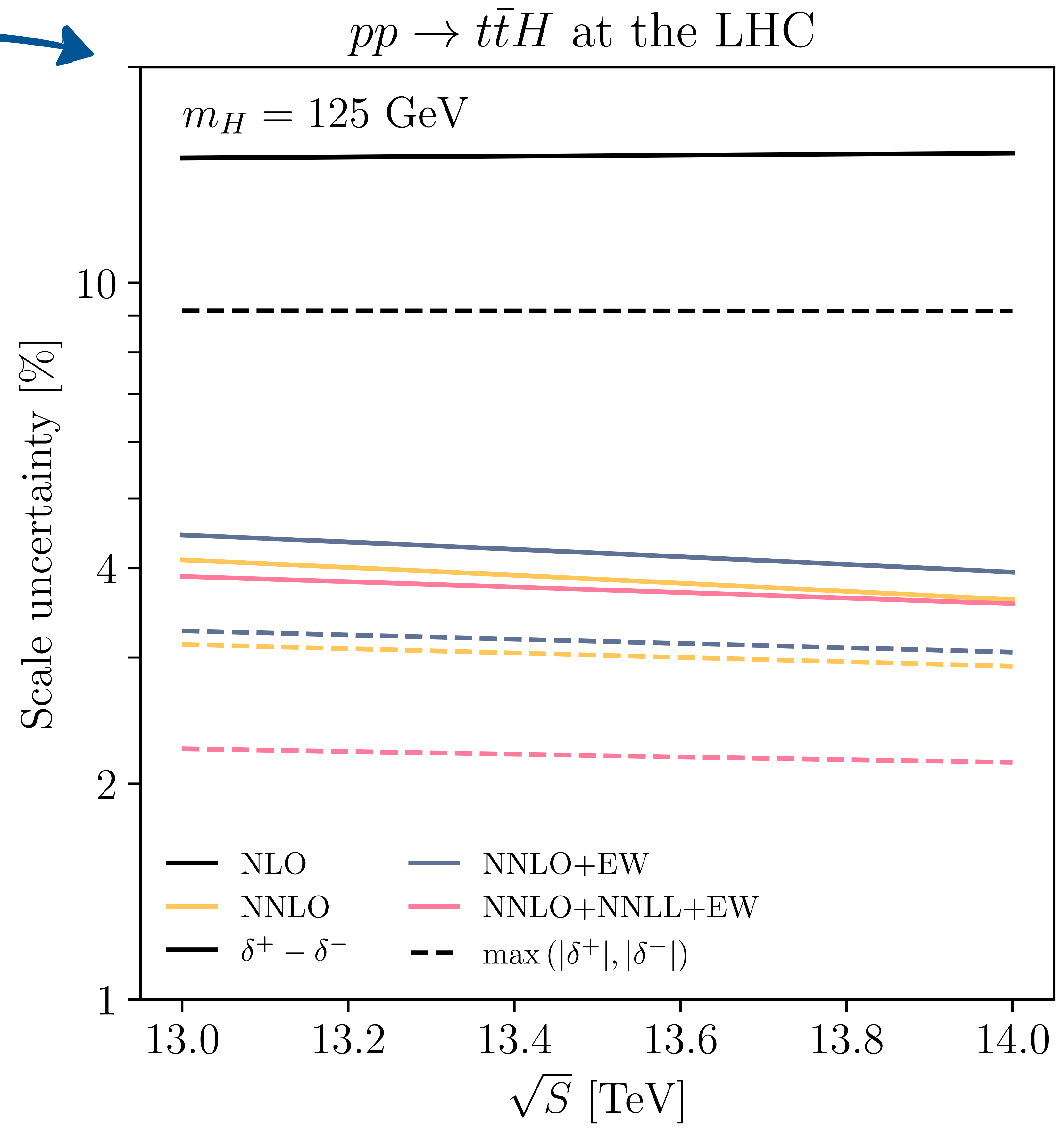
1. regardless of the framework, the central prediction is affected by only 0.1%
2. improved stability under variations of the central scale
3. the scale dependence is further reduced to $\mathcal{O}(1.5-2\%)$

NNLO QCD + EW + NNLL predictions

[LHC HWG arXiv 2503.15043]

setup: NNLO PDF4LHC21_40_pdfas, $m_t = 172.5 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- different sources of **theoretical uncertainties**:
  - missing higher orders estimated via 7-point scale variation
  - PDF and α_s uncertainties: $\Delta_{\text{PDF}} = 2.2\,\%$ and $\Delta_{\alpha_s} = 1.7\,\%$
  - approximation of the double virtual: $\Delta_{\text{virt}} = 0.9\,\%$
  - numerical and q_T-extrapolation: $\mathcal{O}(0.3\%)$
  - ambiguities in the resummation approach: $\mathcal{O}(0.1\%)$
  - uncertainties related to the m_t value and renormalisation scheme: negligible



all quoted theoretical uncertainties have a negligible dependence on the collider energy and m_H value considered in our work

Conclusions

- As the LHC has entered its "precision" phase, more accurate theoretical predictions are of paramount importance
- the current frontier is represented by NNLO corrections for $2 \to 3$ processes with **several massive external legs**

main bottleneck: two-loop amplitudes

- the associated production of a Higgs boson with a top-quark pair ($t\bar{t}H$) belongs to this category and it is crucial for the measurement of the top-Yukawa coupling
- strategy: develop physically motivated, reasonable and reliable **approximations** for the double-virtual contribution

SOFT-BOSON APPROXIMATION   MASSIFICATION

- we have "updated" our previous prediction for the NNLO QCD total cross section by designing a **more solid estimate** of the double-virtual contribution based on both approximations
- the quantitative impact of the genuine two-loop contribution, in our framework, is relatively **small** (~1% on σ_{NNLO})
- thus, we have achieved **good control** of the systematic errors and a **reduction** of the perturbative uncertainties
- we have shown **differential results** for the Higgs transverse momentum
- we have included the full tower of **EW corrections** and matched our fixed-order results with the **NNLL soft-gluon resummation**, in accordance with the recommendations of the LHC HWG

state-of-the-art predictions for $t\bar{t}H$!!

Thank you for your attention!

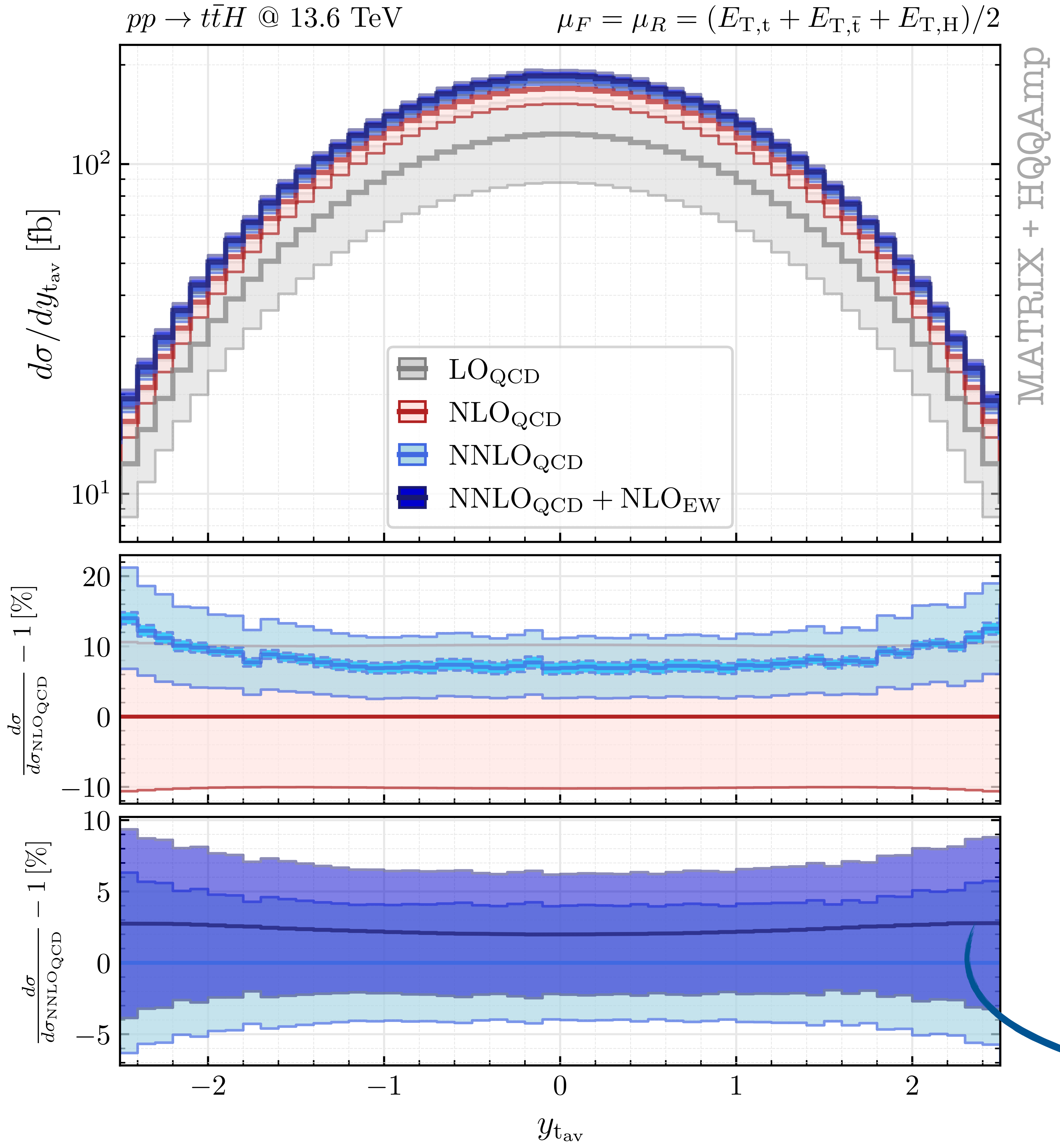
Backup slides

NNLO QCD + EW predictions

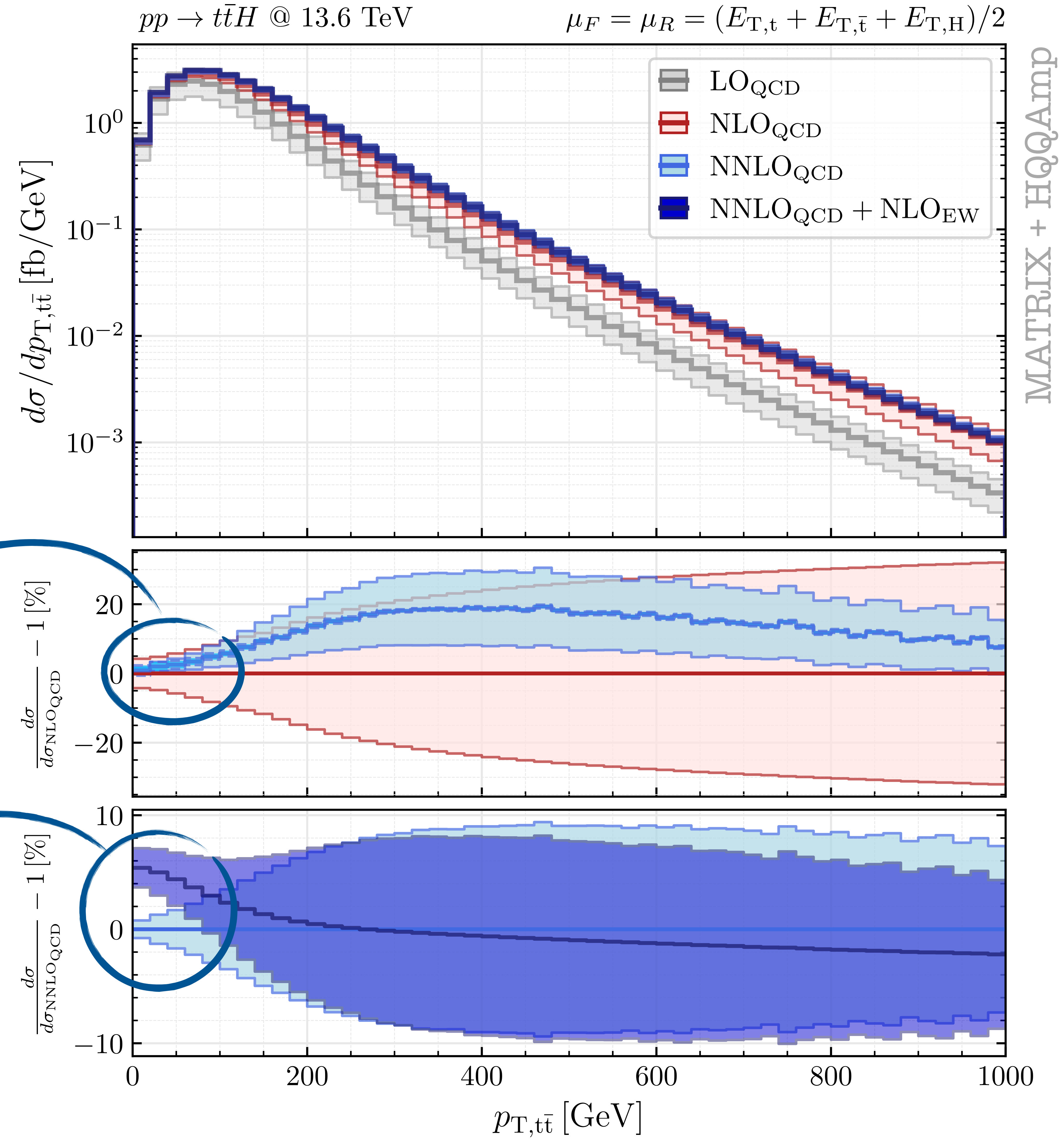
more distributions ...

[JHEP 03 (2025)]

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$



constant shift



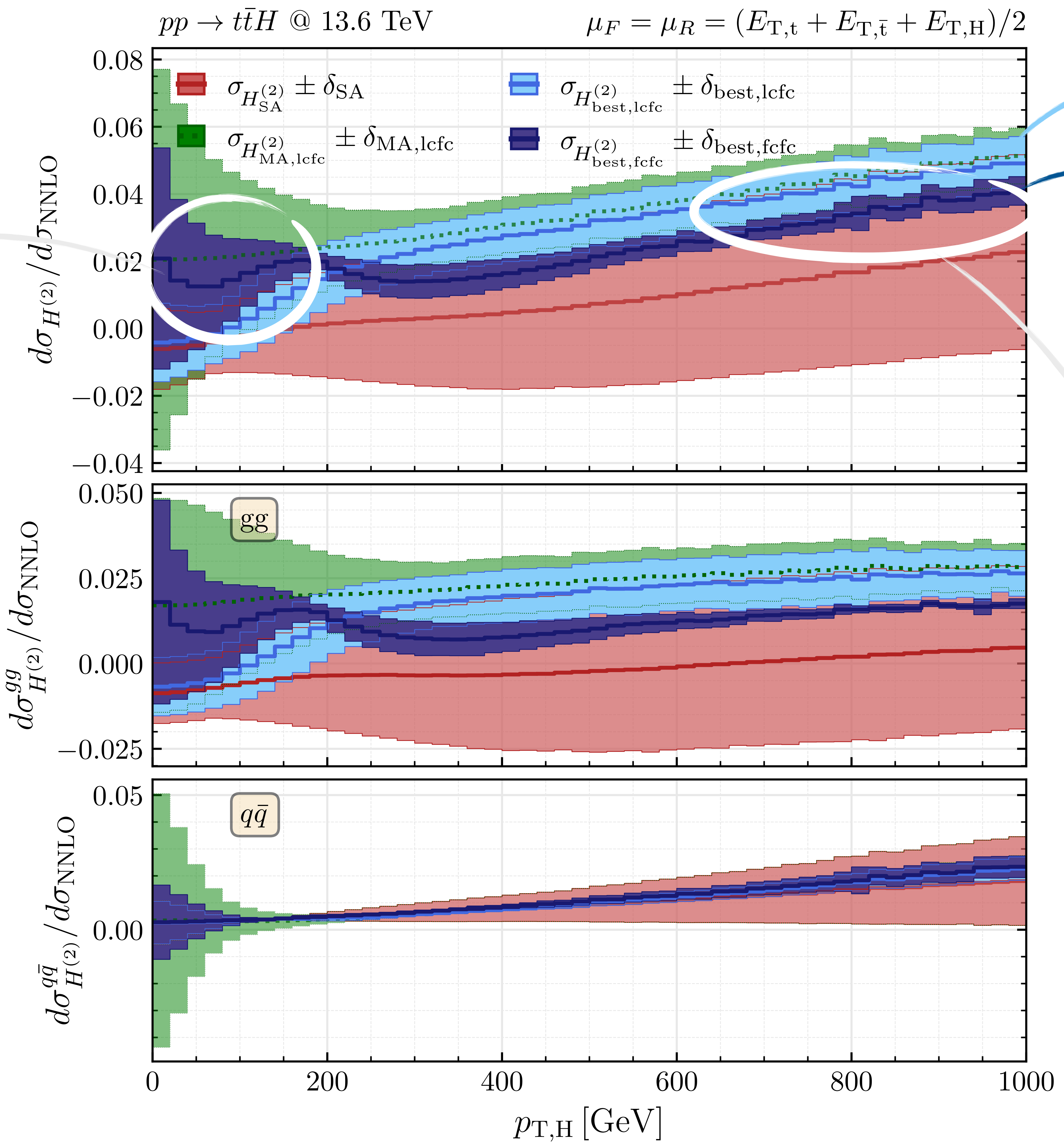
extreme reduction of the scale uncertainties

no overlapping bands

“best” $H^{(2)}$ prediction: impact of SLC in MA

setup: NNLO NNPDF40, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

“correlated” H1-based error



our “best” prediction [JHEP 03 (2025)]

[Badger et al. (2024)]

inclusion of the SLC in the two-loop MASSLESS $pp \to b\bar{b}H$ amplitudes, entering the construction of the massified (MA) result

1. huge effects of the SLC massless terms on the MA result
2. non-predictivity of the massification in this region, reflected by corresponding huge systematic errors
3. “artificial” blow-up of the SA error, due to our correlated H1-based error estimate

1. negative ($\mathcal{O}(-30\%)$) impact on the hard-virtual contribution of the SLC two-loop massless terms, included in the MA result
2. sensible reduction of the μ_{IR}-variation errors

… still to be further investigated