# Quark Mass Effects in Higgs Boson Processes

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in collaboration with Michal Czakon, Felix Eschment, Marco Niggetiedt and Tom Schellenberger
based on [2312.09896] and [2407.12413]

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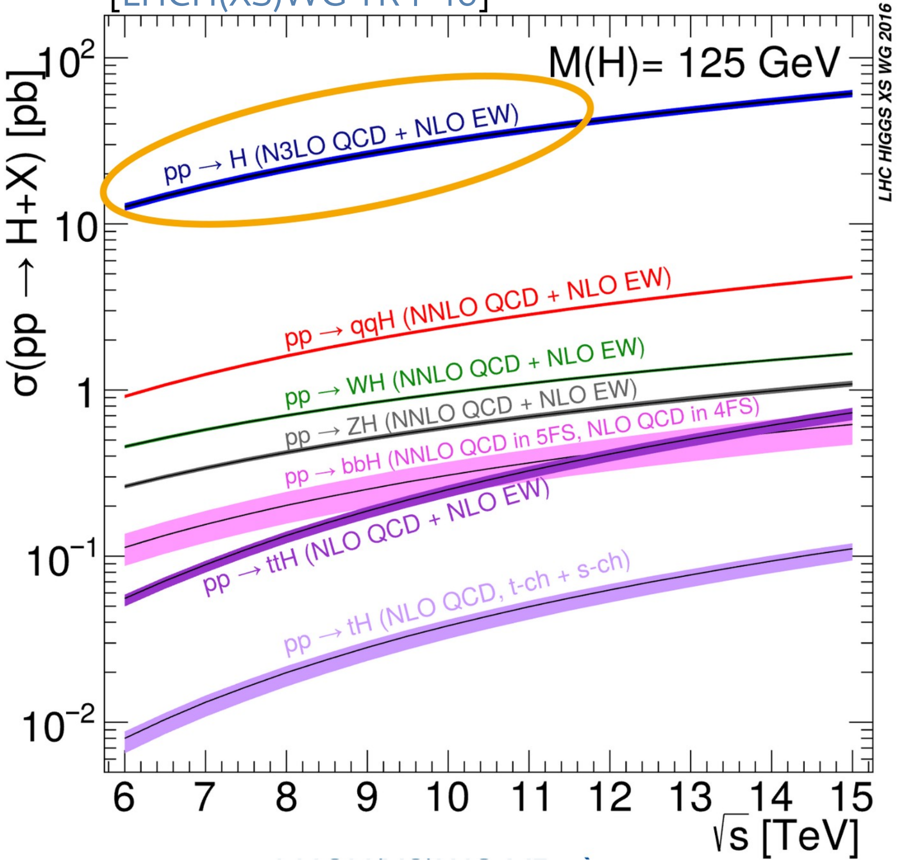
# Outline

- Introduction
- Corrections from quark masses through NNLO QCD
  - top and bottom-top-interference
  - MS vs. On-shell scheme
  - 4 vs 5 flavour scheme
- Summary

# Higgs-production at hadron colliders

[LHCH(XS)WG YR4' 16]




- Higgs production is dominated through gluon-fusion
- Experimental measurement

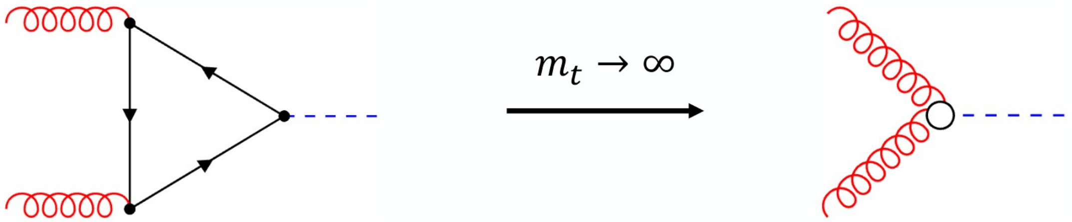
$$\sigma^{\text{exp.}}_{gg\to H} = 47.1 \pm 3.8\ \text{pb}$$

[CMS'22]

- HL LHC expects 2 % uncertainty
- Theory predictions need to keep up → Higher-order predictions crucial!

# HTL and HEFT

**Heavy Top Limit** (HTL or EFT):




$$\sigma_{gg\to H} = \sigma^{\mathrm{HTL}}_{gg\to H} + \mathcal{O}\left(\frac{m_H^2}{m_t^2}\right) \quad \text{for} \quad m_t \to \infty$$

**Higgs Effective Field Theory** (HEFT or rEFT): $\sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HEFT}} = \frac{\sigma^{\mathrm{LO}}}{\sigma^{\mathrm{LO}}_{\mathrm{HTL}}} \sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HTL}} \approx 1.064 \times \sigma^{\mathrm{N}^n\mathrm{LO}}_{\mathrm{HTL}}$

captures some of the top-quark mass effects for inclusive observables.
At higher loop-order questionable → needs full computation.
How to deal with other quark mass effects?

# Precision predictions for Higgs production in gluon-fusion

[LHCH(XS)WG YR4' 16]

Immense community effort to achieve precise theory predictions

$$\boxed{\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.}$$

| | | | | |
|---|---|---|---|---|
| $48.58\,\mathrm{pb} =$ | $16.00\,\mathrm{pb}$ | $(+32.9\%)$ | (LO, rEFT) | [Georgi, Glashow, Machacek, Nanopoulos'78] |
| | $+\,20.84\,\mathrm{pb}$ | $(+42.9\%)$ | (NLO, rEFT) | [Dawson '91][Djouadi, Spira Zerwas '91] |
| | $-\ \ 2.05\,\mathrm{pb}$ | $(-4.2\%)$ | $((t, b, c)$, exact NLO) | [Graudenz, Spira, Zerwas '93] |
| | $+\ \ 9.56\,\mathrm{pb}$ | $(+19.7\%)$ | (NNLO, rEFT) | [Ravindran, Smith, Van Neerven '02] [Harlander, Kilgore '02][Anastasiou, Melnikov '02] |
| | $+\ \ 0.34\,\mathrm{pb}$ | $(+0.7\%)$ | (NNLO, $1/m_t$) | [Harlander, Ozeren'09][Pak, Rogal, Steinhauser'10] [Harlander, Mantler, Marzani, Ozeren '10] |
| | $+\ \ 2.40\,\mathrm{pb}$ | $(+4.9\%)$ | (EW, QCD-EW) | [Aglietti, Bonciani, Degrassi,Vicini'04] [Actis, Passarino, Sturm, Uccirati'08] [Anastasiou, Boughezal, Petriello'09] |
| | $+\ \ 1.49\,\mathrm{pb}$ | $(+3.1\%)$ | ($\mathrm{N^3LO}$, rEFT) | [Anastasiou, Duhr, Dulat, Herzog, Mistlberger'15] |

# Remaining theory uncertainties

[LHCH(XS)WG YR4' 16]

N4LO approximation
[Das, Moch, Vogt '20]

aN3LO PDFs
→ see Roy's talk

**Exact top-mass dependence through NNLO QCD**
[Czakon, Harlander, Klappert, Niggetiedt'21]

Input parameters

| | |
|---|---|
| $\sqrt{S}$ | 13 TeV |
| $m_h$ | 125 GeV |
| PDF | PDF4LHC15_nnlo_100 |
| $\alpha_s(m_Z)$ | 0.118 |
| $m_t(m_t)$ | 162.7 GeV ($\overline{\text{MS}}$) |
| $m_b(m_b)$ | 4.18 GeV ($\overline{\text{MS}}$) |
| $m_c(3GeV)$ | 0.986 GeV ($\overline{\text{MS}}$) |
| $\mu = \mu_R = \mu_F$ | 62.5 GeV ($= m_H/2$) |

| $\delta$(scale) | $\delta$(trunc) | $\delta$(PDF-TH) | $\delta$(EW) | $\delta(t, b, c)$ | $\delta(1/m_t)$ |
|---|---|---|---|---|---|
| $^{+0.10\text{ pb}}_{-1.15\text{ pb}}$ | $\pm 0.18$ pb | $\pm 0.56$ pb | $\pm 0.49$ pb | $\pm 0.40$ pb | $\pm 0.49$ pb |
| $^{+0.21\%}_{-2.37\%}$ | $\pm 0.37\%$ | $\pm 1.16\%$ | $\pm 1\%$ | $\pm 0.83\%$ | $\pm 1\%$ |

N3LO HEFT
[Mistlberger'18]

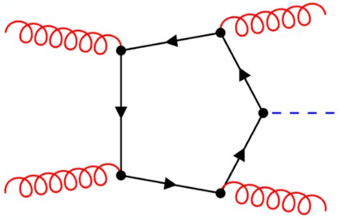
Improved QCD-EW predictions
[Bonetti, Melnikov, Trancredi'18] [Anastasiou et al '19]
[Bonetti et al. '20][Bechetti et al. '21] [Bonetti, Panzer, Trancredi '22]

**Bottom-top-interference**
[Czakon, Eschment, Niggetiedt, Poncelet, Schellenberger `23,`24]

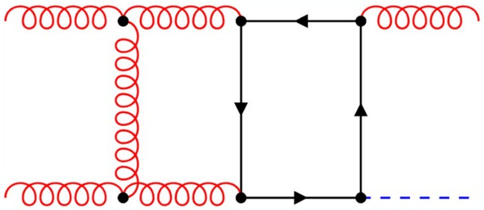
NNLO PS matching → Christian

# NNLO QCD ingredients

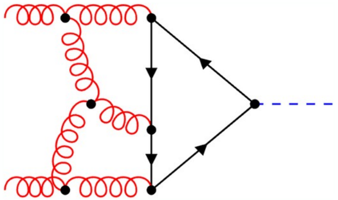
Double real (one-loop)



Real virtual (two-loop)



Double virtual (three-loop)



Amplitudes [Del Duca et al ’01][Budge et al. ‘20]
taken from MCFM [Campbell, Ellis ‘99]
+ QCDLoop [Carrazza, Ellis, Zanderighi’16]

Numerical solution with DEQ for masters
→ numerical interpolation grid
→ improved stability through IR subtraction

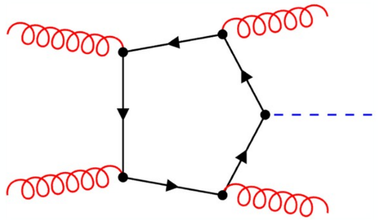
$$\langle M^{(1)}_{\mathrm{exact}}|M^{(2)}_{\mathrm{exact}}\rangle\big|_{\mathrm{regulated}} \equiv \langle M^{(1)}_{\mathrm{exact}}|M^{(2)}_{\mathrm{exact}}\rangle - \left[\langle M^{(1)}_{\mathrm{HEFT}}|M^{(2)}_{\mathrm{HEFT}}\rangle + \frac{8\pi\alpha_s}{\hat{t}}\left\langle P^{(0)}_{gg}\left(\frac{\hat{s}}{\hat{s}+\hat{u}}\right)\right\rangle\langle F^{(1)}|(F^{(2)}_{\mathrm{exact}} - F^{(2)}_{\mathrm{HEFT}})\rangle\right]$$

[Czakon, Harlander, Klappert, Niggetiedt’21]

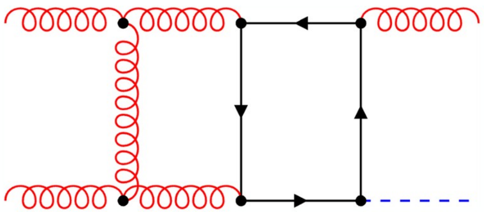
Asymptotic expansion in $\dfrac{m_q^2}{m_H^2}$ [Czakon, Niggetiedt’20] [Niggetiedt, Usovitsch’23]

# NNLO QCD ingredients

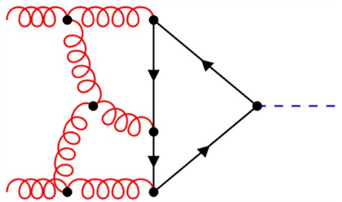
Double real (one-loop)



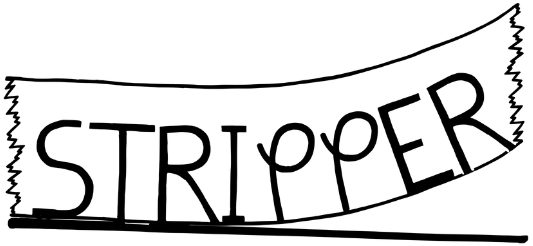
Real virtual (two-loop)



Double virtual (three-loop)



Implemented in




[Czakon'10][Czakon Heymes'14]
[Czakon, van Hameren, Mitov, Poncelet'19]

extensions to deal with arbitrary loop-induced processes through NNLO QCD
(just needs the loop amplitudes as inputs)

# Finite top-quark effects at two-loops

[Czakon, Harlander, Klappert, Niggetiedt'21]

| channel | $\sigma^{\text{NNLO}}_{\text{HEFT}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$ | $(\sigma^{\text{NNLO}}_{\text{exact}}-\sigma^{\text{NNLO}}_{\text{HEFT}})$ [pb] $\mathcal{O}(\alpha_s^3)$ | $\mathcal{O}(\alpha_s^4)$ | $(\sigma^{\text{NNLO}}_{\text{exact}}/\sigma^{\text{NNLO}}_{\text{HEFT}}-1)$ [%] |
|---|---|---|---|---|
| | | $\sqrt{s}=8\,\text{TeV}$ | | |
| $gg$ | $7.39+8.58+3.88$ | $+0.0353$ | $+0.0879\pm0.0005$ | $+0.62$ |
| $qg$ | $0.55+0.26$ | $-0.1397$ | $-0.0153\pm0.0002$ | $-19$ |
| $qq$ | $0.01+0.04$ | $+0.0171$ | $-0.0191\pm0.0002$ | $-4$ |
| total | $7.39+9.14+4.18$ | $-0.0873$ | $+0.0535\pm0.0006$ | $-0.16$ |
| | | $\sqrt{s}=13\,\text{TeV}$ | | |
| $gg$ | $16.30+19.64+8.76$ | $+0.0345$ | $+0.2431\pm0.0020$ | $+0.62$ |
| $qg$ | $1.49+0.84$ | $-0.3696$ | $-0.0408\pm0.0005$ | $-18$ |
| $qq$ | $0.02+0.10$ | $+0.0322$ | $-0.0501\pm0.0006$ | $-15$ |
| total | $16.30+21.15+9.70$ | $-0.3029$ | $+0.1522\pm0.0021$ | $-0.32$ |

→ top-quark mass small effect
→ HEFT rescaling captures already the largest impact on the inclusive cross-section
→ quark-channels have large $E_{\text{CMS}}$ dependence

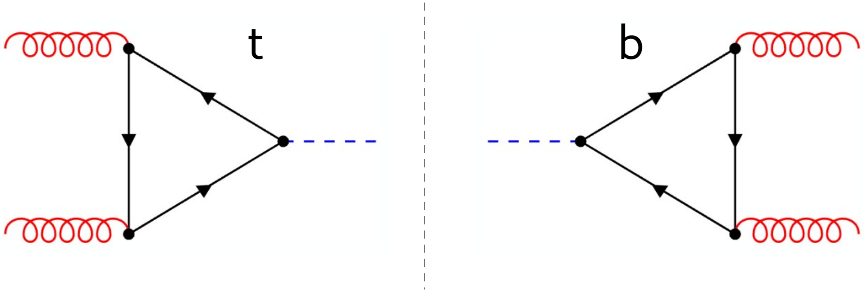
# Bottom-quark mass effects

Quark mass effects are power suppressed, tops: $\frac{m_H^2}{4m_t^2}$ 'light' quarks: $\frac{m_b^2}{m_H^2}$

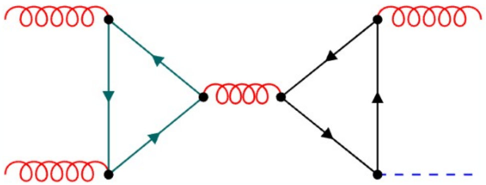
Largest contribution from bottom/charm quarks → interference terms

However there might be logarithmic enhancements
→ rendering the contributions quite sizeable

$$\log^2\left(\frac{m_b^2}{m_H^2}\right)$$




Re-use [Czakon, Harlander, Klappert, Niggetiedt'21],
(needs re-evaluations with different mass)
conceptually new terms simple:



→ factorise in one-loop amps

# Mass renormalisation scheme dependence I

→ top-quark mass is of the same scale as the process → on-shell good

(and also easier to compute, running masses require several numerical grids)

→ bottom-quark mass is a very small scale compared to process

→ a short-distance mass probably better!

| Order | $(\sigma_t^{\overline{\text{MS}}} - \sigma_t^{\text{OS}})$ [pb] |
|---|---|
| $\sqrt{s} = 13$ TeV | |
| $\mathcal{O}(\alpha_s^2)$ | $-0.04$ |
| LO | $-0.04^{+0.12}_{-0.17}$ |
| $\mathcal{O}(\alpha_s^3)$ | $+0.02$ |
| NLO | $-0.02^{+0.14}_{-0.30}$ |
| $\mathcal{O}(\alpha_s^4)$ | $+0.01$ |
| NNLO | $-0.01^{+0.12}_{-0.24}$ |

$$Z_m^{\overline{\text{MS}}}\bar{m} = Z_m^{\text{OS}} m^{\text{OS}} \to m^{\text{OS}} = \bar{m}\left(1 + c_1\frac{\alpha_s}{\pi} + c_2\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right)$$

$$\begin{aligned}
\mathcal{M}^{\overline{\text{MS}}} &= \mathcal{M}^{\text{OS}} + \delta\mathcal{M} \\
\delta\mathcal{M}^{(1)} &= \overline{m}c_1\frac{\alpha_s}{\pi}\frac{\mathrm{d}\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m}\bigg|_{m=\overline{m}} \\
\delta\mathcal{M}^{(2)} &= \overline{m}\left[c_1\frac{\alpha_s}{\pi}\frac{\mathrm{d}\mathcal{M}^{\text{OS},(1)}}{\mathrm{d}m}\bigg|_{m=\overline{m}} + c_2\left(\frac{\alpha_s}{\pi}\right)^2\frac{\mathrm{d}\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m}\bigg|_{m=\overline{m}}\right] + \frac{1}{2}\left(\overline{m}c_1\frac{\alpha_s}{\pi}\right)^2\frac{\mathrm{d}^2\mathcal{M}^{\text{OS},(0)}}{\mathrm{d}m^2}\bigg|_{m=\overline{m}}
\end{aligned}$$

# Mass renormalisation scheme dependence II

| Order | $\sigma_{t\times b}$ [pb] | | |
|---|---|---|---|
| | $\sqrt{s} = 13$ TeV | | |
| | 5FS<br>$m_t = 173.06$ GeV<br>$\overline{m}_b(\overline{m}_b) = 4.18$ GeV | 5FS<br>$m_t = 173.06$ GeV<br>$m_b = 4.78$ GeV | 5FS<br>$\overline{m}_t(\overline{m}_t) = 162.7$ GeV<br>$\overline{m}_b(\overline{m}_b) = 4.18$ GeV |
| $\mathcal{O}(\alpha_s^2)$ | $-1.11$ | $-1.98$ | $-1.12$ |
| LO | $-1.11^{+0.28}_{-0.43}$ | $-1.98^{+0.38}_{-0.53}$ | $-1.12^{+0.28}_{-0.42}$ |
| $\mathcal{O}(\alpha_s^3)$ | $-0.65$ | $-0.44$ | $-0.64$ |
| NLO | $-1.76^{+0.27}_{-0.28}$ | $-2.42^{+0.19}_{-0.12}$ | $-1.76^{+0.27}_{-0.28}$ |
| $\mathcal{O}(\alpha_s^4)$ | $+0.02$ | $+0.43$ | $-0.02$ |
| NNLO | $-1.74(2)^{+0.13}_{-0.03}$ | $-1.99(2)^{+0.29}_{-0.15}$ | $-1.78(1)^{+0.15}_{-0.03}$ |

Pure top-quark mass effects

| Order | $\sigma_{\mathrm{HEFT}}$ [pb] | $(\sigma_t - \sigma_{\mathrm{HEFT}})$ [pb] |
|---|---|---|
| $\mathcal{O}(\alpha_s^2)$ | $+16.30$ | – |
| LO | $16.30^{+4.36}_{-3.10}$ | – |
| $\mathcal{O}(\alpha_s^3)$ | $+21.14$ | $-0.303$ |
| NLO | $37.44^{+8.42}_{-6.29}$ | $-0.303^{+0.10}_{-0.17}$ |
| $\mathcal{O}(\alpha_s^4)$ | $+9.72$ | $+0.147(1)$ |
| NNLO | $47.16^{+4.21}_{-4.77}$ | $-0.156(1)^{+0.13}_{-0.03}$ |

→ bottom quark in OS alternating, MS convergent

→ MS/OS irrelevant for top-quark (unsurprising)

→ agreement between MS and OS at NNLO QCD

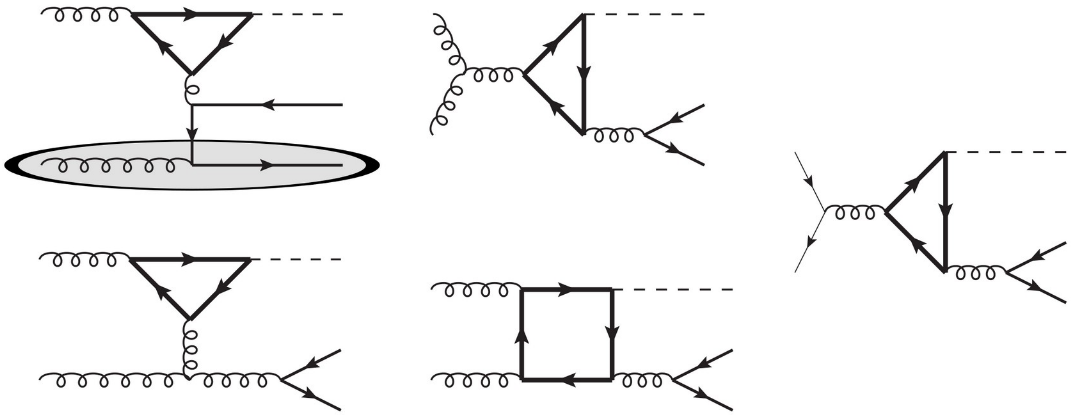
→ size of correction of top-bottom-quark interference **larger then top-quark mass effect**!

# Flavour number scheme I

<u>5 flavour scheme (5FS)</u>

→ keeping external bottoms massless

→ **keep bottom in loops coupled to Higgs massive** (gauge-invariant but slightly inconsistent)



<u>4 flavour scheme (4FS)</u>

→ requires to include massive b-quarks in the final state

→ 4 FS PDFs

→ bottom mass regulates IR singularities, cancellation of $\log m_b$ in inclusive observables

# Flavour number scheme II

| Order | $\sigma_{\mathrm{HEFT}}$ [pb] | | | | |
|---|---|---|---|---|---|
| | $\sqrt{s} = 13$ TeV | | | | |
| | 5FS | 4FS $m_b = 0.01$ GeV | 4FS $m_b = 0.1$ GeV | 4FS $m_b = 4.78$ GeV | 4FS $\overline{m}_b(\overline{m}_b) = 4.18$ GeV |
| $\mathcal{O}(\alpha_s^2)$ | $+16.30$ | $+16.27$ | $+16.27$ | $+16.27$ | $16.27$ |
| LO | $16.30^{+4.36}_{-3.10}$ | $16.27^{+4.63}_{-3.22}$ | $16.27^{+4.63}_{-3.22}$ | $16.27^{+4.63}_{-3.22}$ | $16.27^{+4.63}_{-3.22}$ |
| $\mathcal{O}(\alpha_s^3)$ | $+21.14$ | $+20.08(3)$ | $+20.08(3)$ | $+20.08(3)$ | $+20.08(3)$ |
| NLO | $37.44^{+8.42}_{-6.29}$ | $36.35(3)^{+8.57}_{-6.32}$ | $36.35(3)^{+8.57}_{-6.32}$ | $36.35(3)^{+8.57}_{-6.32}$ | $36.35(3)^{+8.57}_{-6.32}$ |
| $\mathcal{O}(\alpha_s^4)$ | $+9.72$ | $+10.8(4)$ | $+11.1(4)$ | $+9.5(2)$ | $+9.6(2)$ |
| NNLO | $47.16^{+4.21}_{-4.77}$ | $47.2(4)^{+5.4}_{-5.4}$ | $47.5(4)^{+5.4}_{-5.5}$ | $45.9(2)^{+4.3}_{-4.9}$ | $46.0(2)^{+4.4}_{-5.0}$ |

→ computation in HEFT for more stability
→ vanishing b-mass limit consistent with 5FS scheme
→ finite b-mass effect ~3% consistent with prior estimates [Pietrulewicz, Stahlhofen’23]

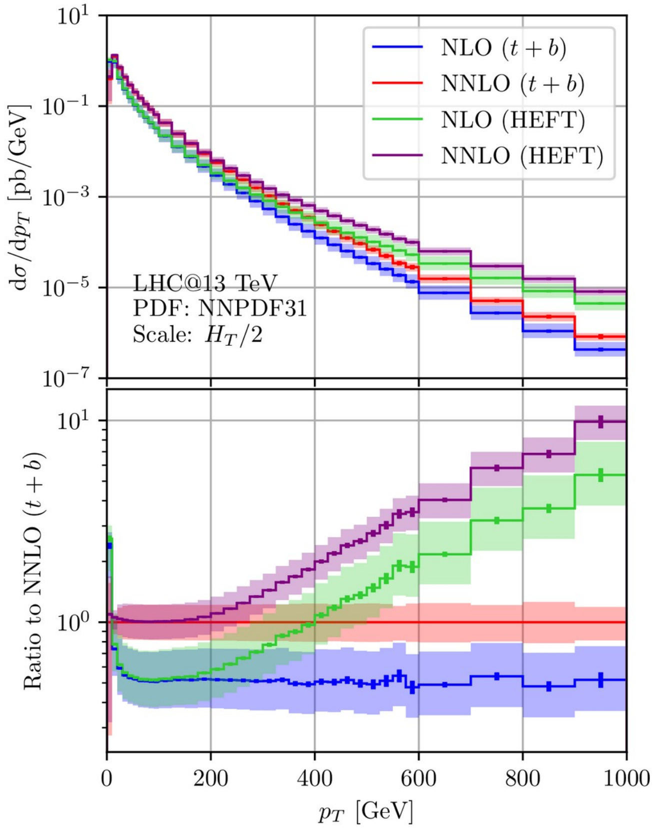
# Flavour number scheme III

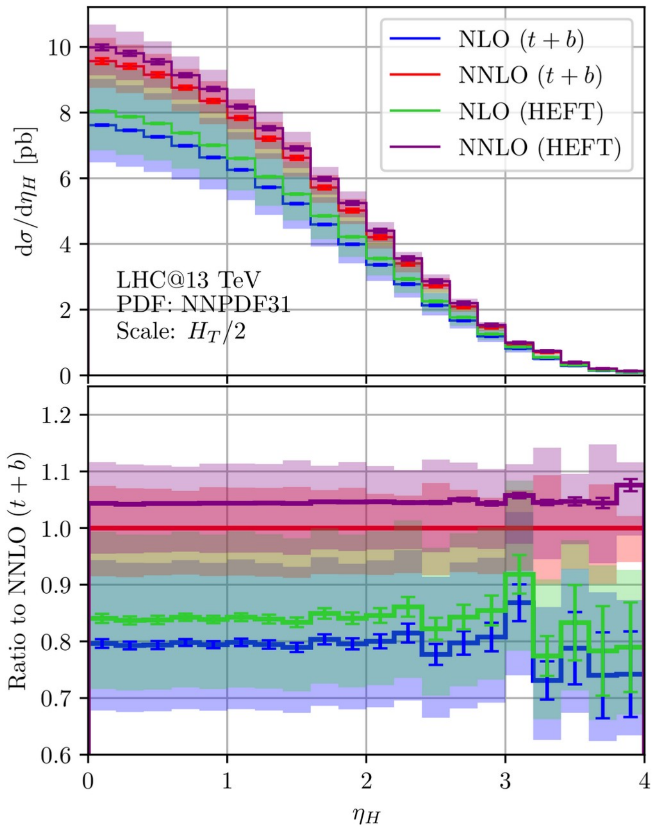
Impact on the exact top-bottom interferences:

| Order | $\sigma_{t\times b}$ [pb] | | | |
|---|---|---|---|---|
| | $\sqrt{s} = 13$ TeV | | | |
| | 5FS<br>$m_t = 173.06$ GeV<br>$\overline{m}_b(\overline{m}_b) = 4.18$ GeV | 5FS<br>$m_t = 173.06$ GeV<br>$m_b = 4.78$ GeV | 5FS<br>$\overline{m}_t(\overline{m}_t) = 162.7$ GeV<br>$\overline{m}_b(\overline{m}_b) = 4.18$ GeV | 4FS<br>$m_t = 173.06$ GeV<br>$\overline{m}_b(\overline{m}_b) = 4.18$ GeV |
| $\mathcal{O}(\alpha_s^2)$ | $-1.11$ | $-1.98$ | $-1.12$ | $-1.15$ |
| LO | $-1.11^{+0.28}_{-0.43}$ | $-1.98^{+0.38}_{-0.53}$ | $-1.12^{+0.28}_{-0.42}$ | $-1.15^{+0.29}_{-0.45}$ |
| $\mathcal{O}(\alpha_s^3)$ | $-0.65$ | $-0.44$ | $-0.64$ | $-0.66$ |
| NLO | $-1.76^{+0.27}_{-0.28}$ | $-2.42^{+0.19}_{-0.12}$ | $-1.76^{+0.27}_{-0.28}$ | $-1.81^{+0.28}_{-0.30}$ |
| $\mathcal{O}(\alpha_s^4)$ | $+0.02$ | $+0.43$ | $-0.02$ | $-0.02$ |
| NNLO | $-1.74(2)^{+0.13}_{-0.03}$ | $-1.99(2)^{+0.29}_{-0.15}$ | $-1.78(1)^{+0.15}_{-0.03}$ | $-1.83(2)^{+0.14}_{-0.03}$ |

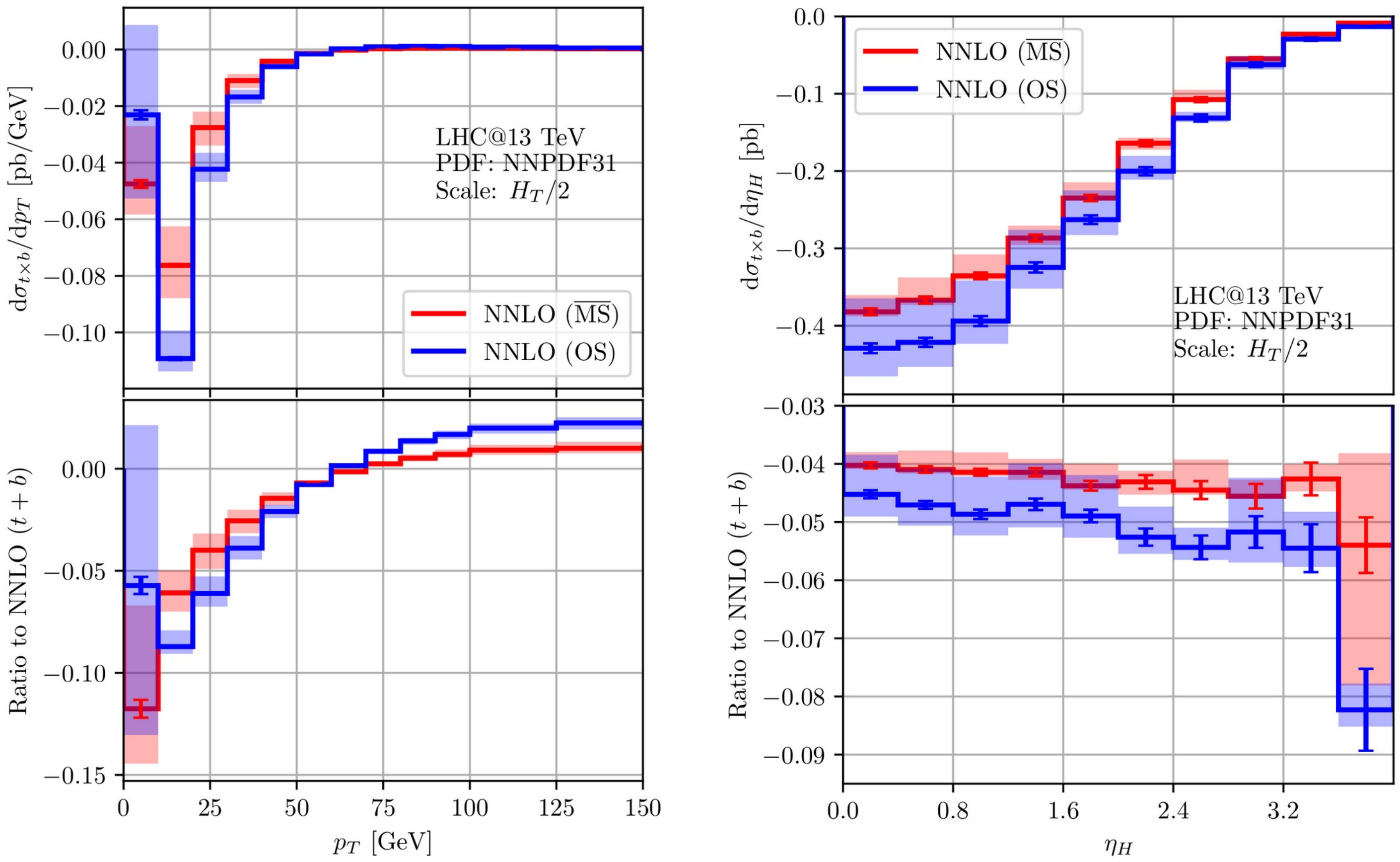
Small effect on small contribution → use 5FS for the interferences

# Differential distributions



# Differential distributions → top-bottom interference



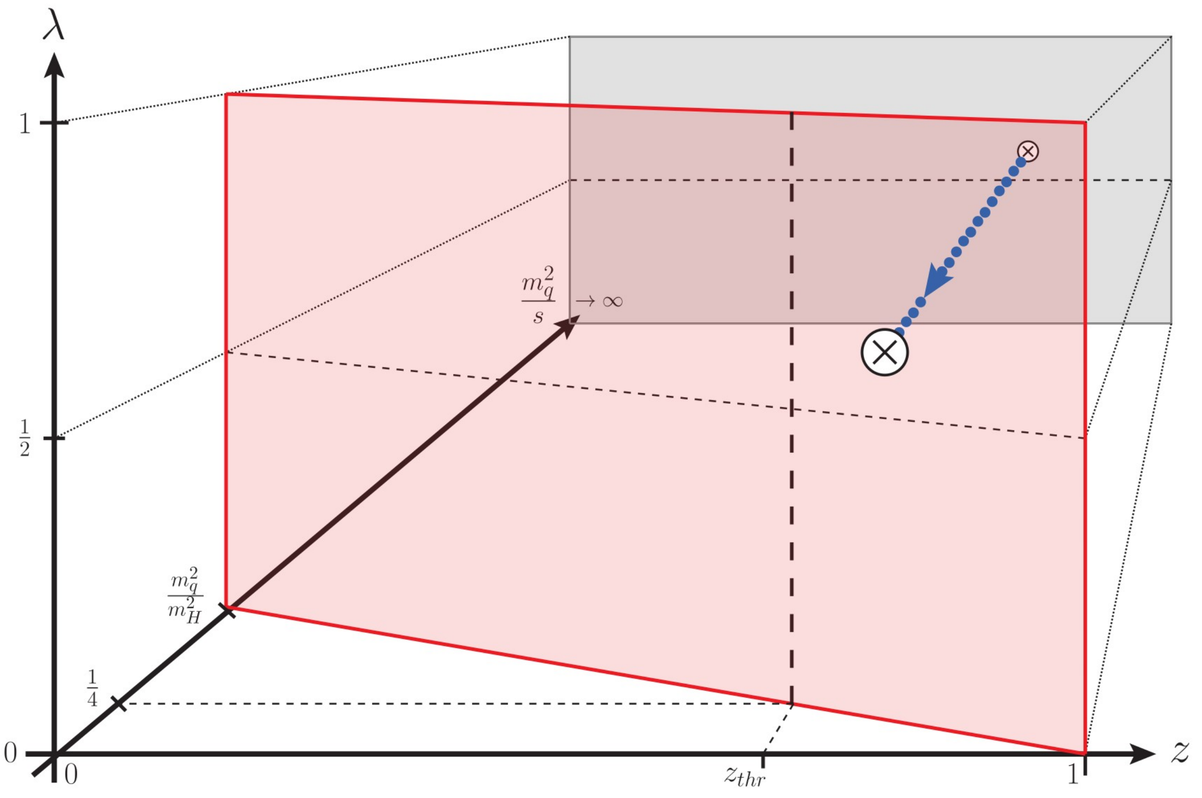
# Summary

- Complete analysis of top-bottom-interference effects through NNLO QCD
- Addresses one of the leading uncertainties
  → higher-order corrections stabilize the theory uncertainties (scheme&scale dependence)
- Study of renormalisation scheme dependence
- Differential distributions!

$$\boxed{\sigma_{ggH} = 48.81(1)^{+0.65}_{-2.02}(\mathrm{N^3LO\ HEFT}) - 0.16^{+0.13}_{-0.03}(\mathrm{NNLO}\ t) - 1.74(2)^{+0.13}_{-0.03}(\mathrm{NNLO}\ t\times b)\ \mathrm{pb.}}$$

# Backup

# Real-virtual numerics



| Grid mass | Value [GeV] | Approximate ratio $m_{b,t}^2/m_H^2$ | Relative error [‰] |
|---|---|---|---|
| $m_b$ | 4.78 | $\frac{1}{684}$ | 0.2 |
| $\overline{m}_b(m_H)$ | 2.789 | $\frac{1}{2011}$ | 0.9 |
| $\overline{m}_b(m_H/2)$ | 2.961 | $\frac{1}{1782}$ | 0.2 |
| $\overline{m}_b(m_H/4)$ | 3.170 | $\frac{1}{1557}$ | 1.0 |
| $\overline{m}_b^{\min}$ | 1.67 | $\frac{1}{5602}$ | 0.1 |
| $m_t$ | 172.4 | $\frac{23}{12}$ | 4 |
| $\overline{m}_t(m_H)$ | 166.1 | $\frac{136}{77}$ | 0.0 |
| $\overline{m}_t(m_H/2)$ | 176.2 | $\frac{149}{75}$ | 0.0 |
| $\overline{m}_t(m_H/4)$ | 188.2 | $\frac{213}{94}$ | 0.0 |