Energy correlators and α_s

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Outline

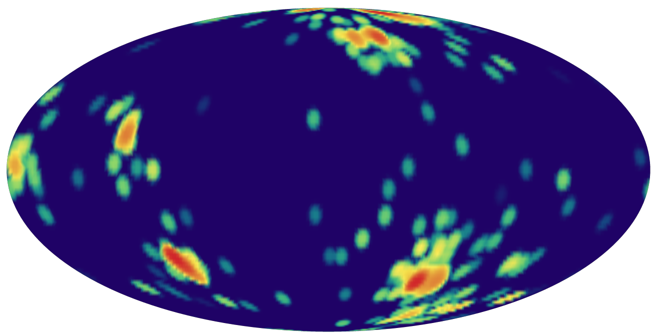
1. Energy-Energy Correlator
2. Higher-point energy correlators
3. Strong coupling determination
4. Conclusions

1. Energy-Energy Correlator (EEC)

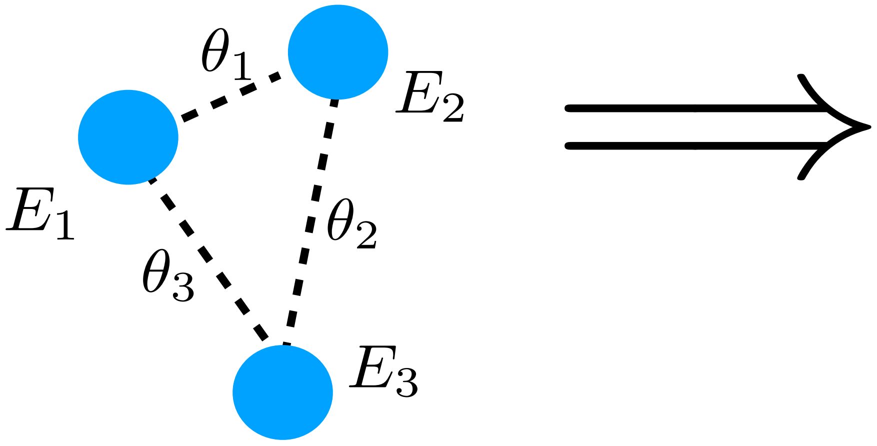
Introduction to energy correlators

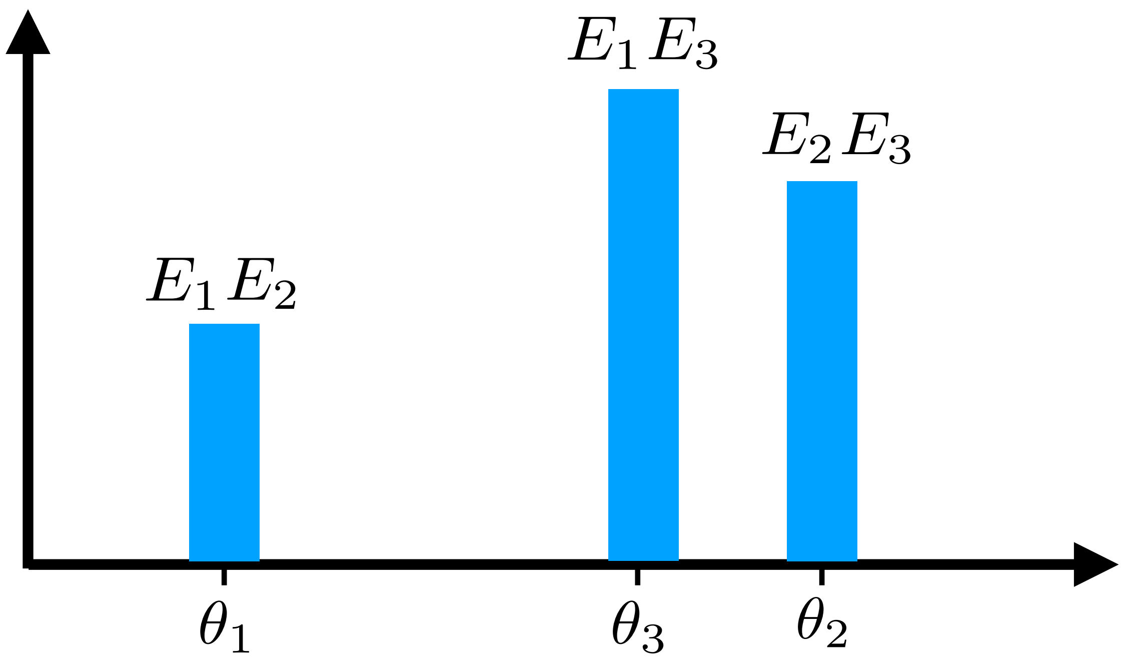
- Event shapes, like thrust, describe it through one number.
- Energy-Energy Correlator probes **correlations** in energy flow:

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{(\sum_k E_k)^2} \delta(\theta - \theta_{ij})$$



[Basham, Brown, Ellis, Love]





Why the hype?

Recent interest in energy correlators has been driven by:

- Natural **separation** of physics at different scales.
- Suppression of soft contamination (no grooming).
- **Simpler** theoretical description, improving interpretation.

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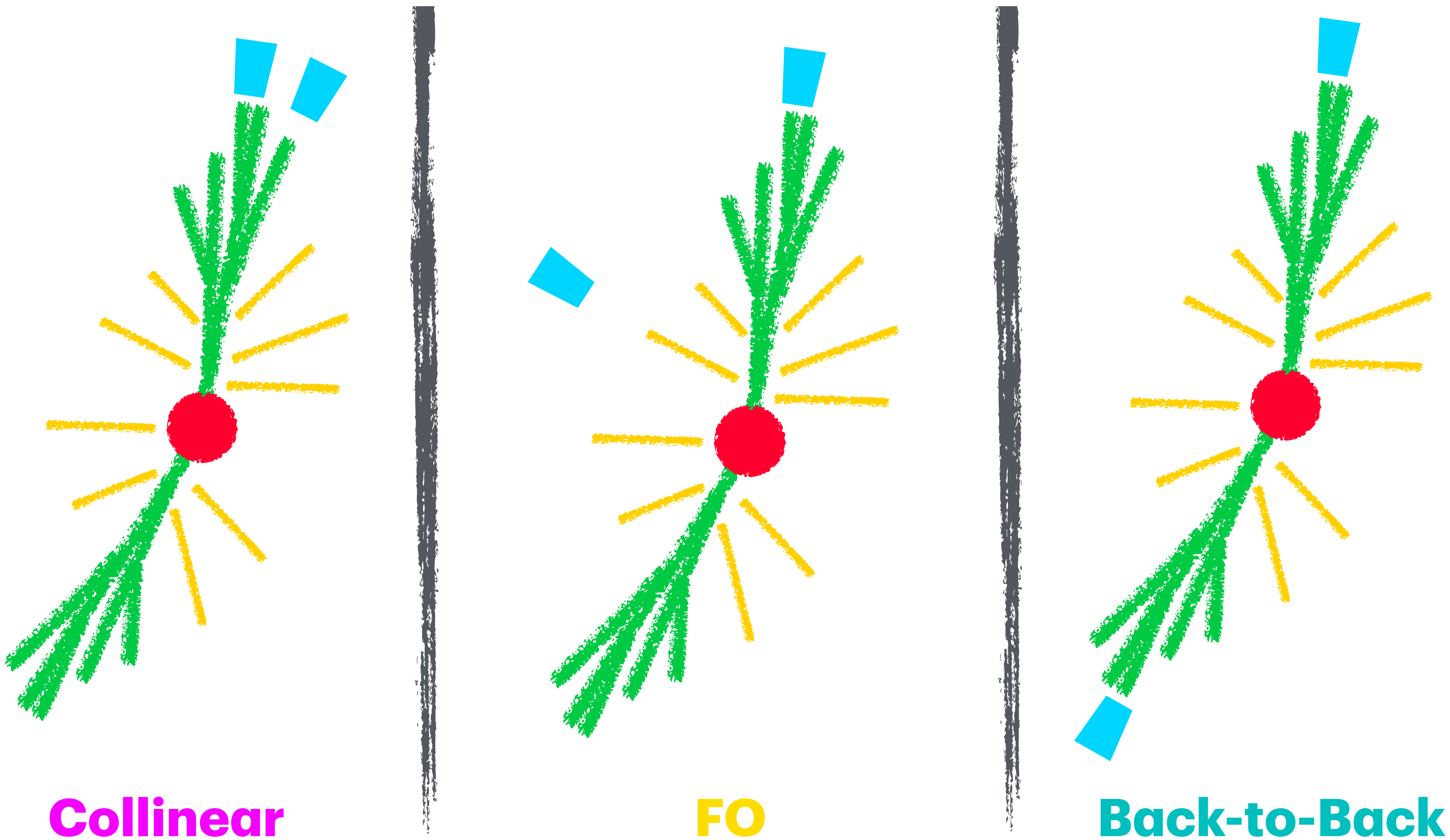
Wide range of applications:

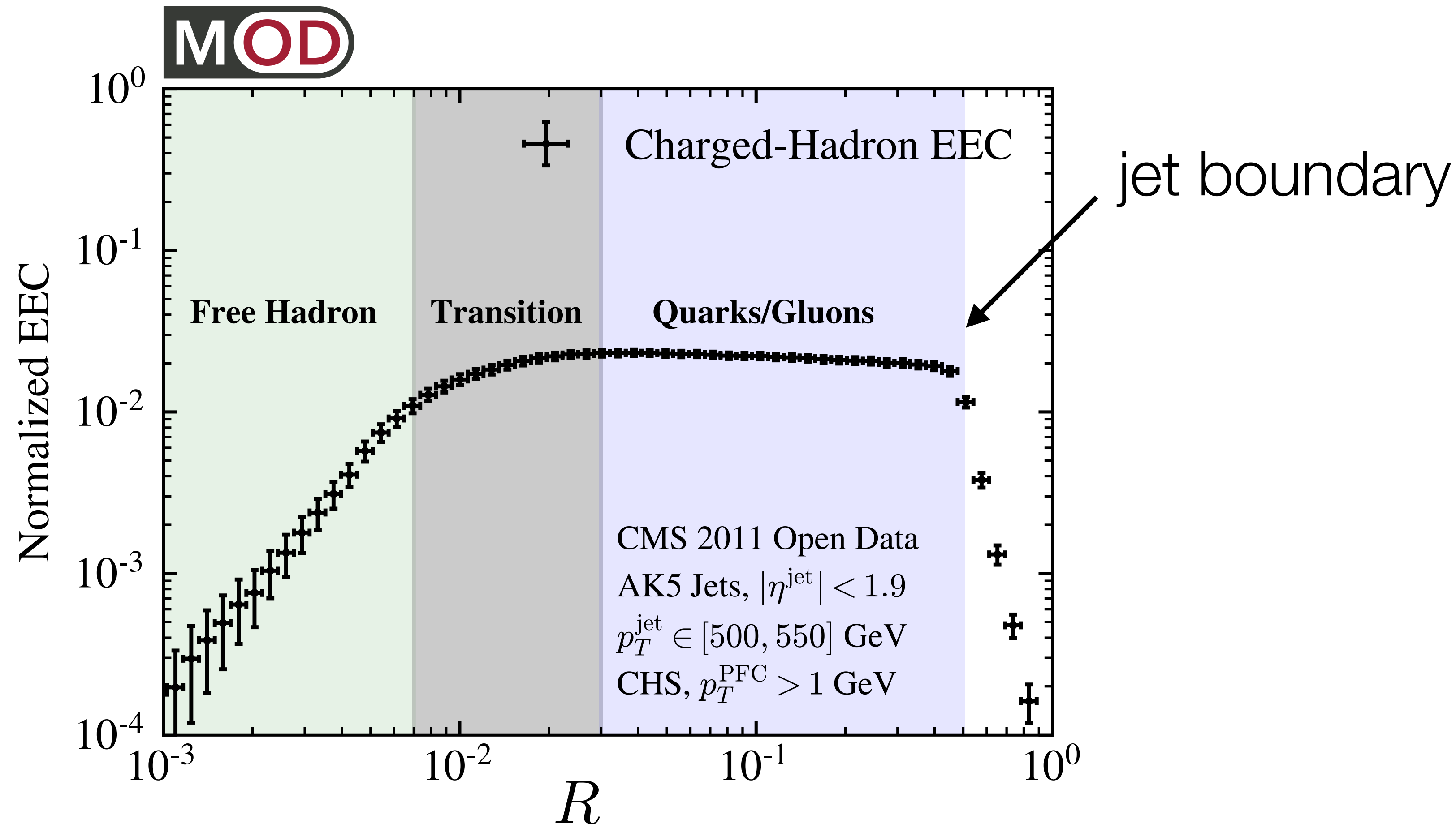
- Strong coupling determination (this talk).
- Top quark mass determination → see A. Pathak's talk.
- Probing quark-gluon plasma.
- Dead cone for heavy quarks…

Different physics at different angles

- Collinear: power-law scaling, determined by DGLAP evolution.
- Back-to-back: Sudakov, described by TMD factorization.



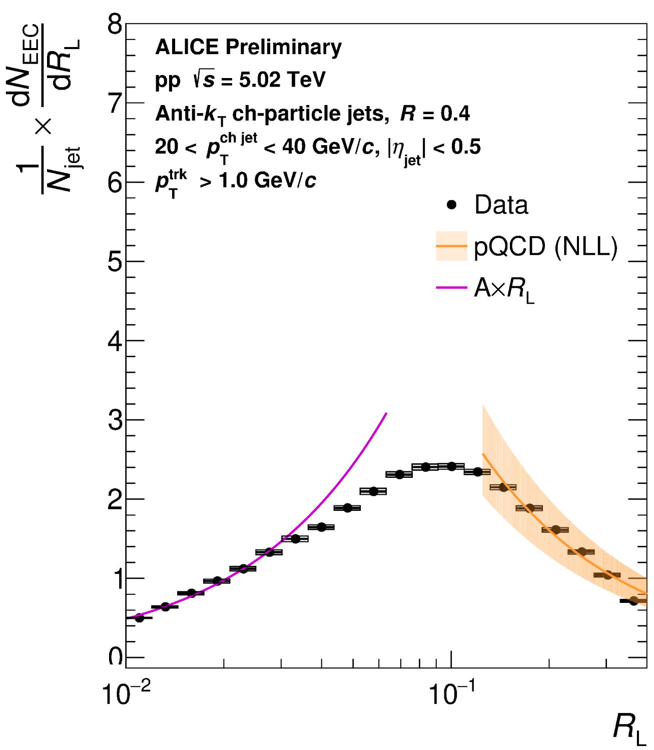
Collinear region

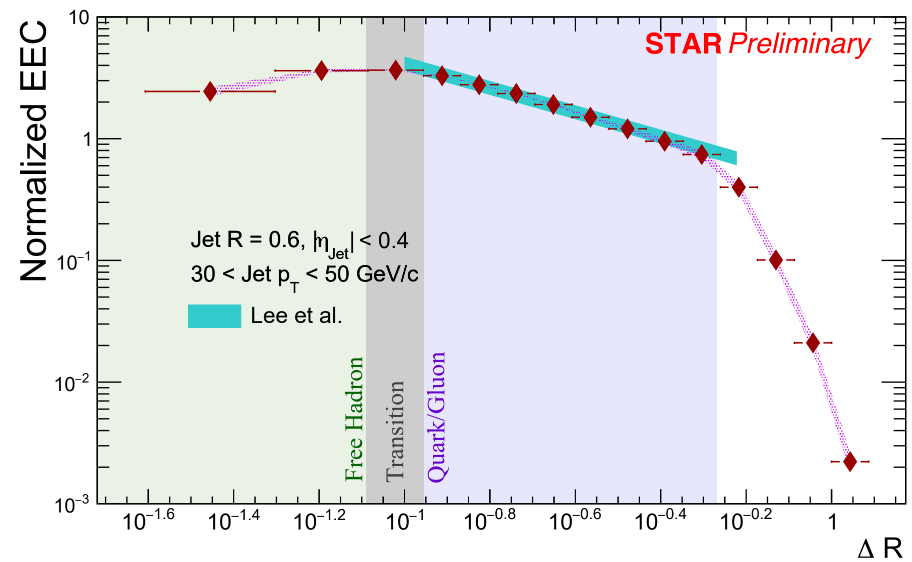


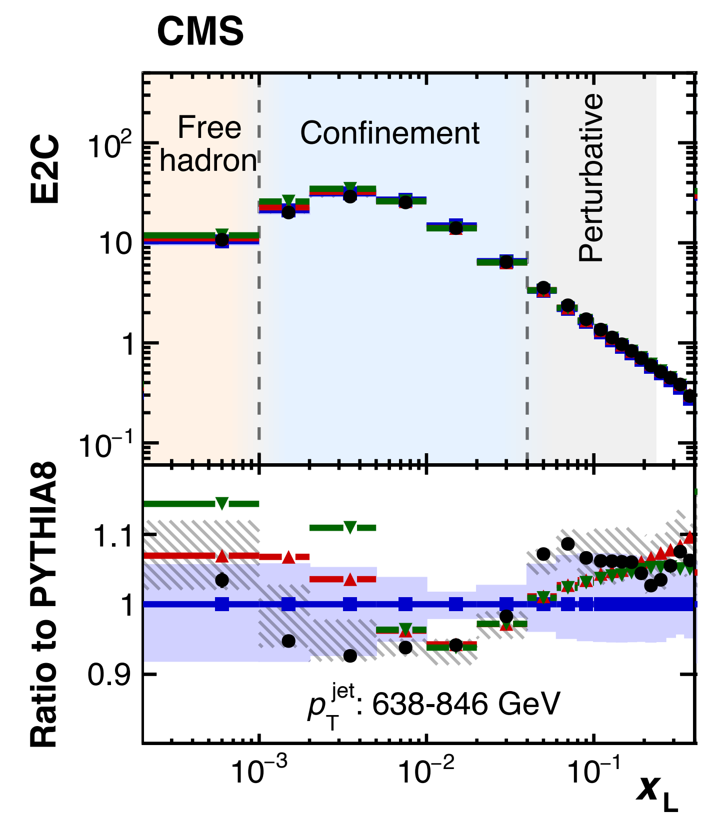
[Komiske, Moult, Thaler, Zhu]

- At the LHC, $(E, \theta) \to (p_T, R)$.
- Perturbative region: $\sim R^\gamma$ with γ set by DGLAP.
- Nonperturbative region: $\sim R^2$, free hadron gas.

Jet-based EEC measurements







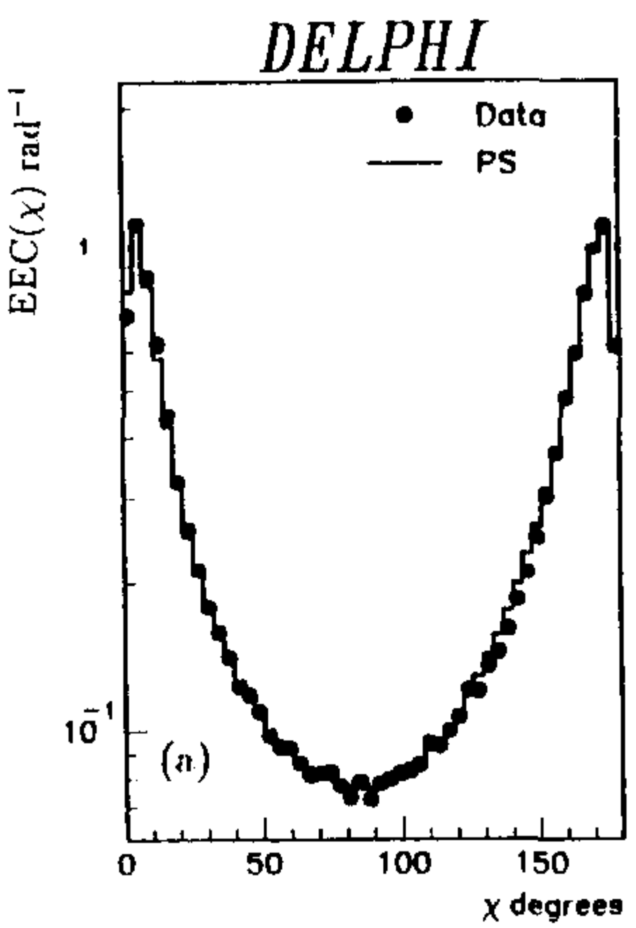
- Scaling of EEC in perturbative and nonperturbative regimes observed by ALICE, STAR and CMS over **wide** energy range

(Note factor R difference compared to the previous slide.)

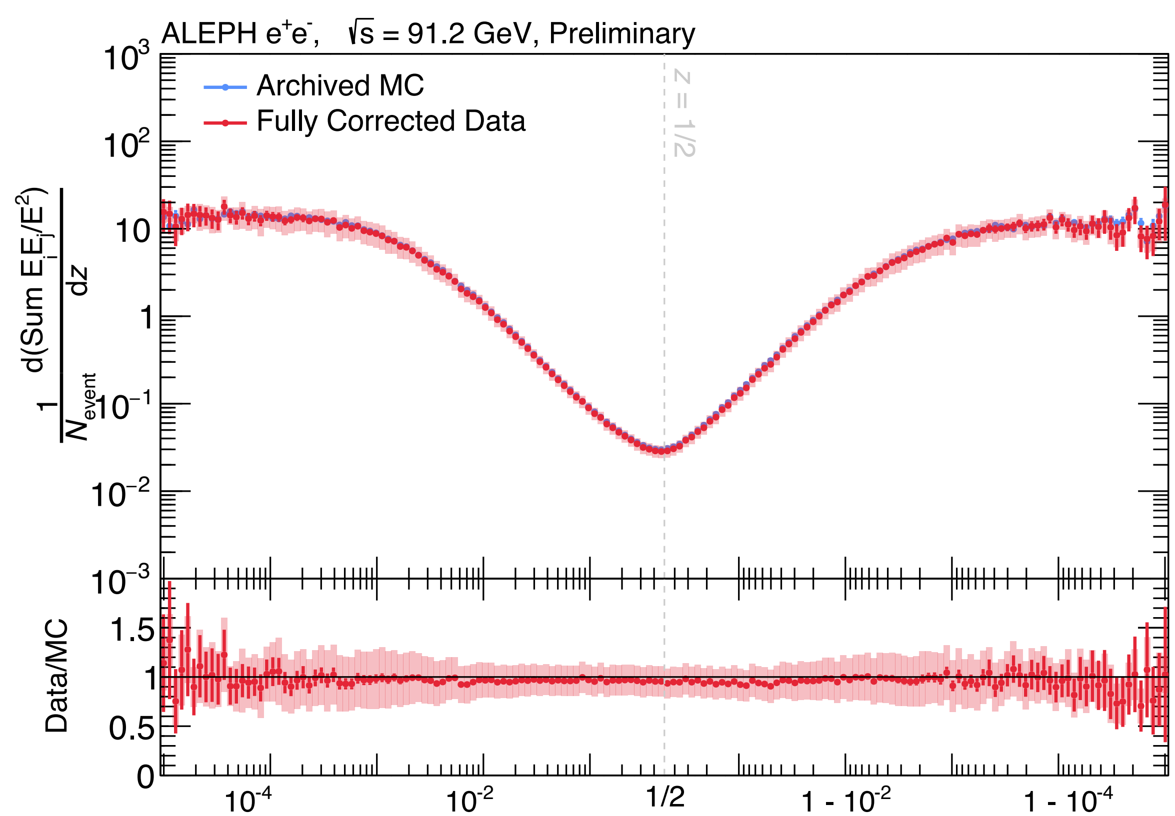
Superior resolution from tracks

- Resolution of charged-particle tracks essential for probing energy correlators in collinear and back-to-back limit.

All particles:



Charged particles:



$z \equiv \frac{1}{2}(1 - \cos\theta)$

[Y.-C. Chen's talk at Hard Probes 2024]

Track-based predictions for EEC

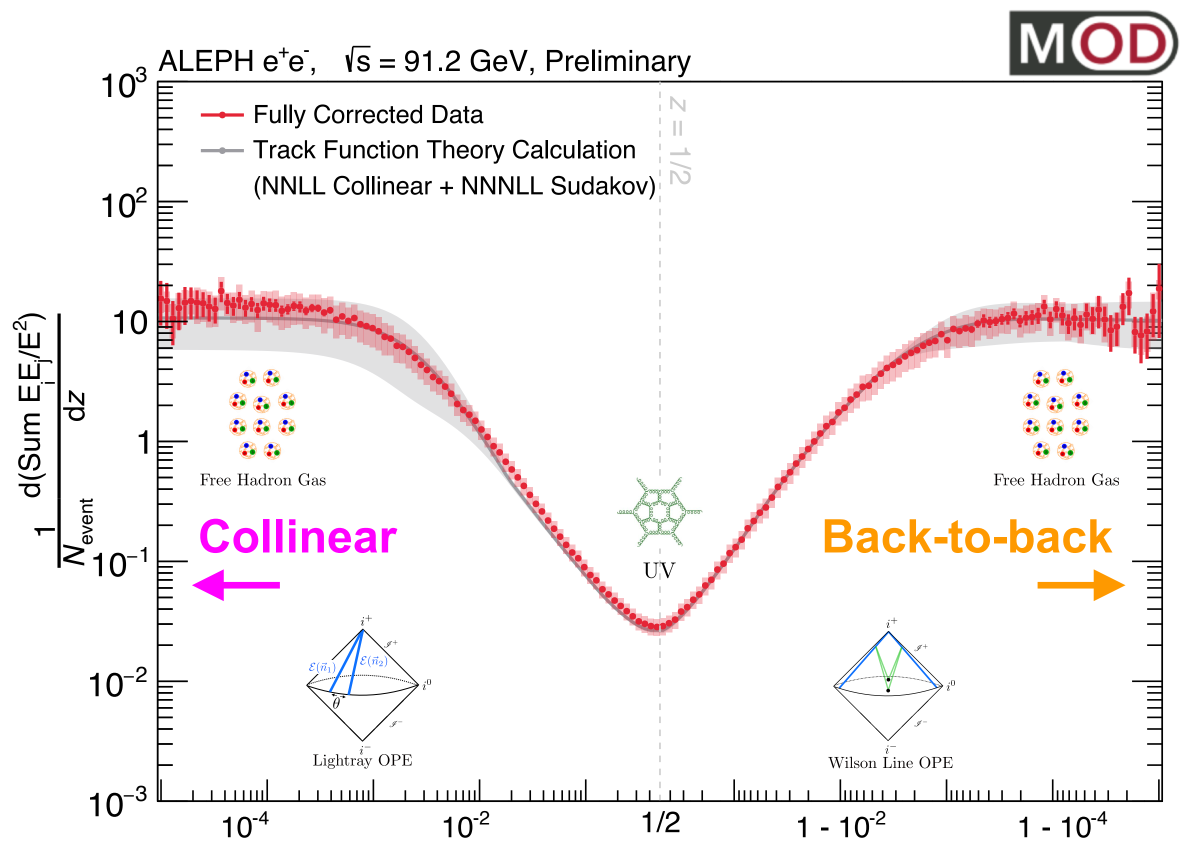
- Track-based predictions using track functions (moments).
  [Chang, Procura, Thaler, WW; Chen, Moult, Zhang, Zhu; Li, Moult, Schrijnder van Velzen, WW, Zhu]

- Comparison to archived ALEPH data:

Charged particles:

Predictions include:

- NNLL in collinear
- NNNLL in back-to-back
- Collins-Soper kernel
- Nonperturbative Ω_1



$z \equiv \frac{1}{2}(1 - \cos\theta)$

[Y.-C. Chen's talk at Hard Probes 2024 - theory input: Jaarsma, Li, Moult, WW, Zhu]

Transverse EEC (TEEC)

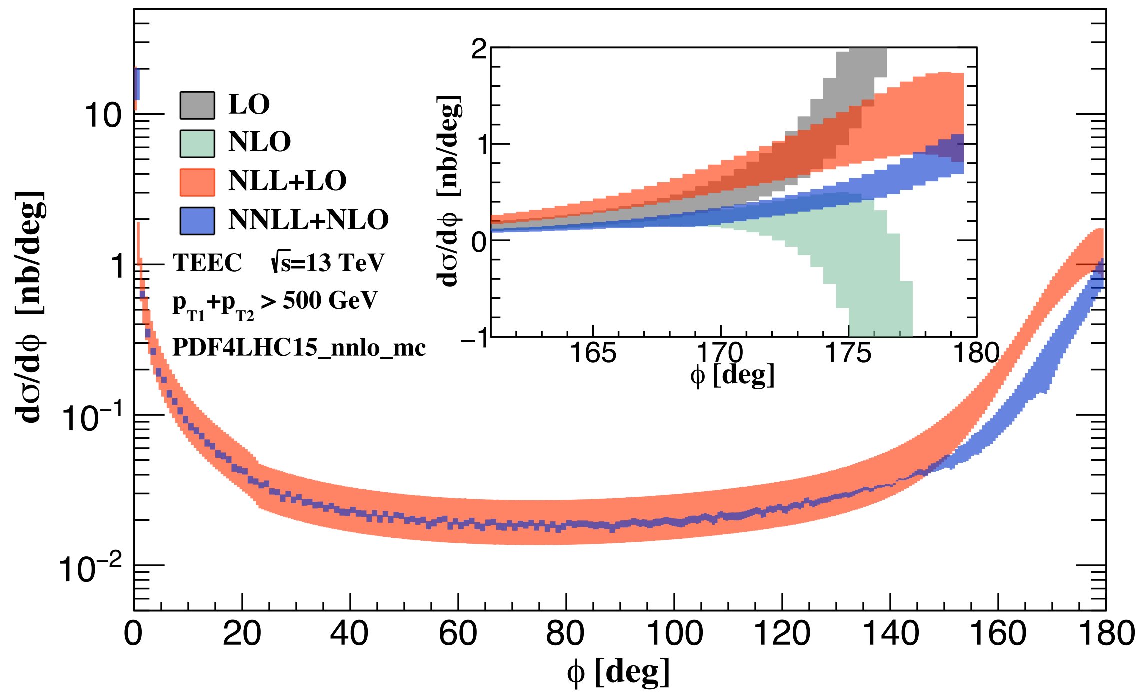
- Study back-to-back in pp, by restricting to **transverse plane:**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\phi} = \int \mathrm{d}\sigma \sum_{i,j} \frac{E_{T,i} E_{T,j}}{(\sum_k E_{T,k})^2}\, \delta(\cos\phi - \cos\phi_{ij})$$

[Ali, Pietarinen, Stirling]

(At the LHC, jets instead of particles are used.)

- High precision available:



[Gao, Li, Moult, Zhu]

2. Higher-point energy correlators

Projected N-point energy correlator (ENC)

- Project onto largest angle θ_L:

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}\theta_L} = \int \mathrm{d}\sigma \sum_{i,j,k,\ldots} \frac{E_i E_j E_k \cdots}{(\sum_m E_m)^N} \, \delta(\theta_L - \max\{\theta_{ij}, \theta_{ik}, \cdots\})$$

[Chen, Moult, Zhang, Zhu]

- Uncertainties reduced in ratio ENC/EEC.

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[Chen, Moult, Zhang, Zhu]

- Uncertainties reduced in ratio ENC/EEC.
- Perturbative region has power-law scaling $\sim R_L^{\gamma(N)}$ with at LL

$$\gamma(N) \sim \int_0^1 dx\, x^N P(x)$$

- This scaling follows from:

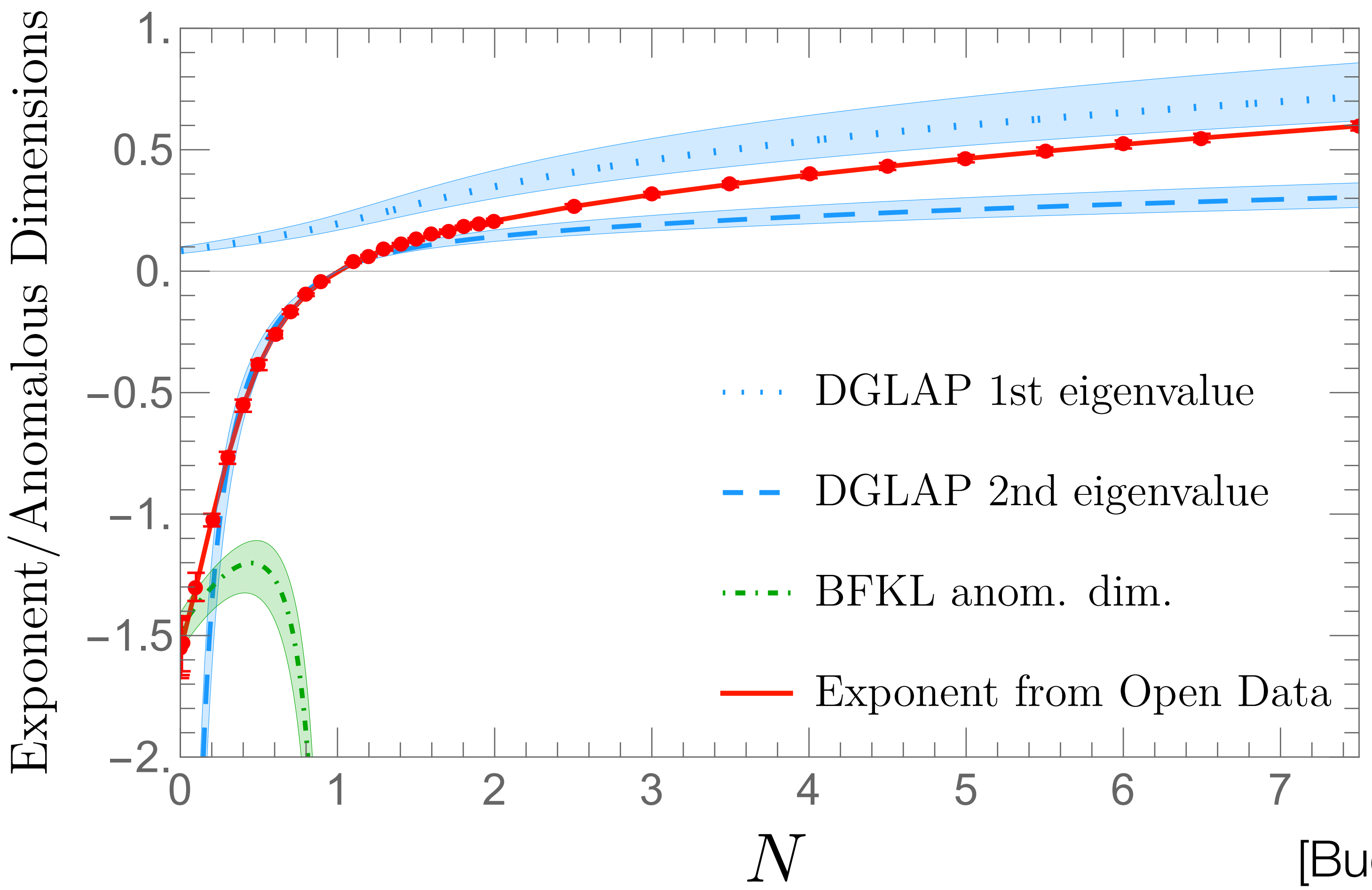
$$\int^{R_L} dR_L' \frac{d\sigma^{[N]}}{dR_L'} = \int_0^1 dx\, x^N \vec{H}\left(x, \frac{Q}{\mu}\right) \cdot \vec{J}^{[N]}\left(\ln \frac{R_L x Q}{\mu}\right)$$

[Dixon, Moult, Zhu; Chen, Moult, Zhang, Zhu]

hard scattering

jet formation

Power-law scaling as function of N



[Budhraja, Chen, WW]

- Fit power-law exponent of ENC in CMS open data.
- Due to quark/gluon mixing not just one power law
  → plot both eigenvalues.
- Interestingly, approaches BFKL for $N \to 0$.
  [Chang, Chen, Kravchuk, Simmons-Duffin, Zhu]

Better parametrization

- Computation time is an issue: $\mathcal{O}(M^N)$ for M particles.
(or $\mathcal{O}(2^{2M})$ for non-integer N)

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- Computation time is an issue: $\mathcal{O}(M^N)$ for M particles.
  (or $\mathcal{O}(2^{2M})$ for non-integer N)

- Isolate a special point s and **only** consider the distance to it:

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1} = \int \mathrm{d}\sigma \sum_{s} z_s \sum_{i_1, i_2,, \ldots} z_{i_1} z_{i_2} \cdots \delta(R_1 - \max\{R_{s i_1}, R_{s i_2}, \cdots\})$$

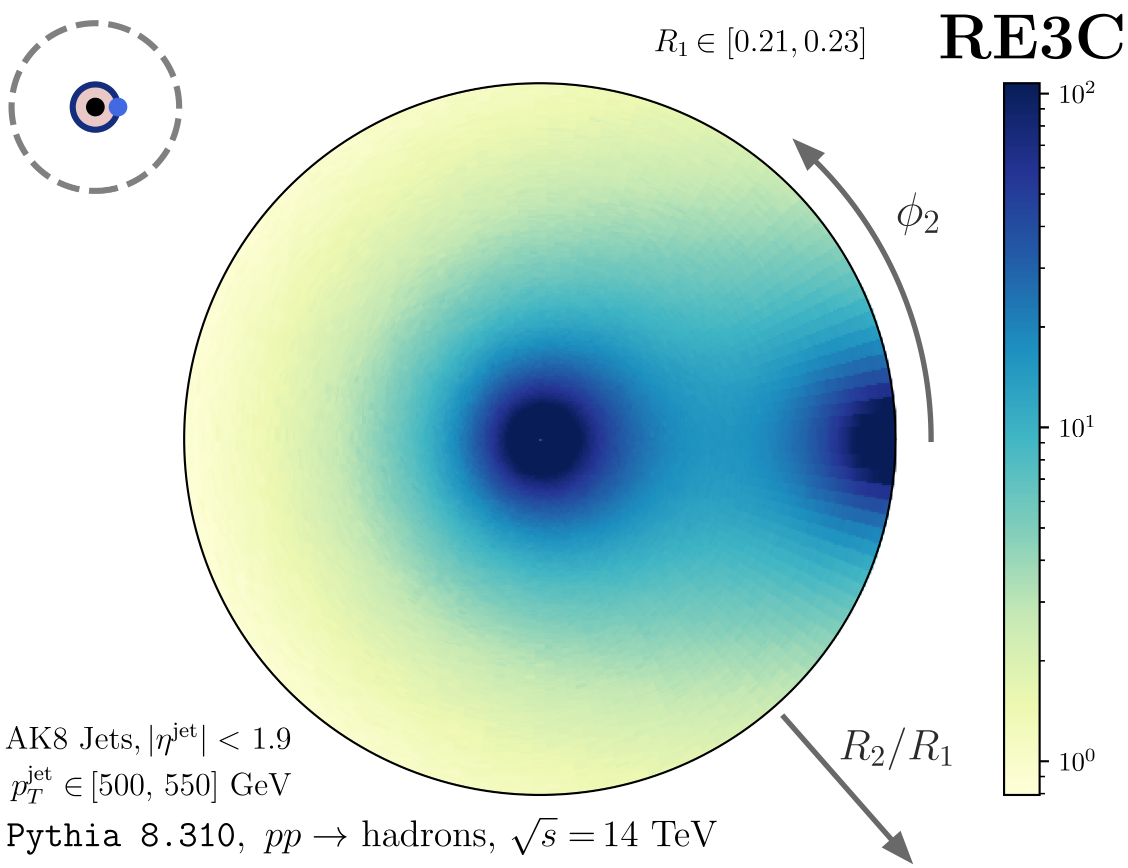
[Alipour-fard, Budhraja, Thaler, WW]

- $R_1 \leq R_L \leq 2R_1$, so R_1 is good measure of overall scale.

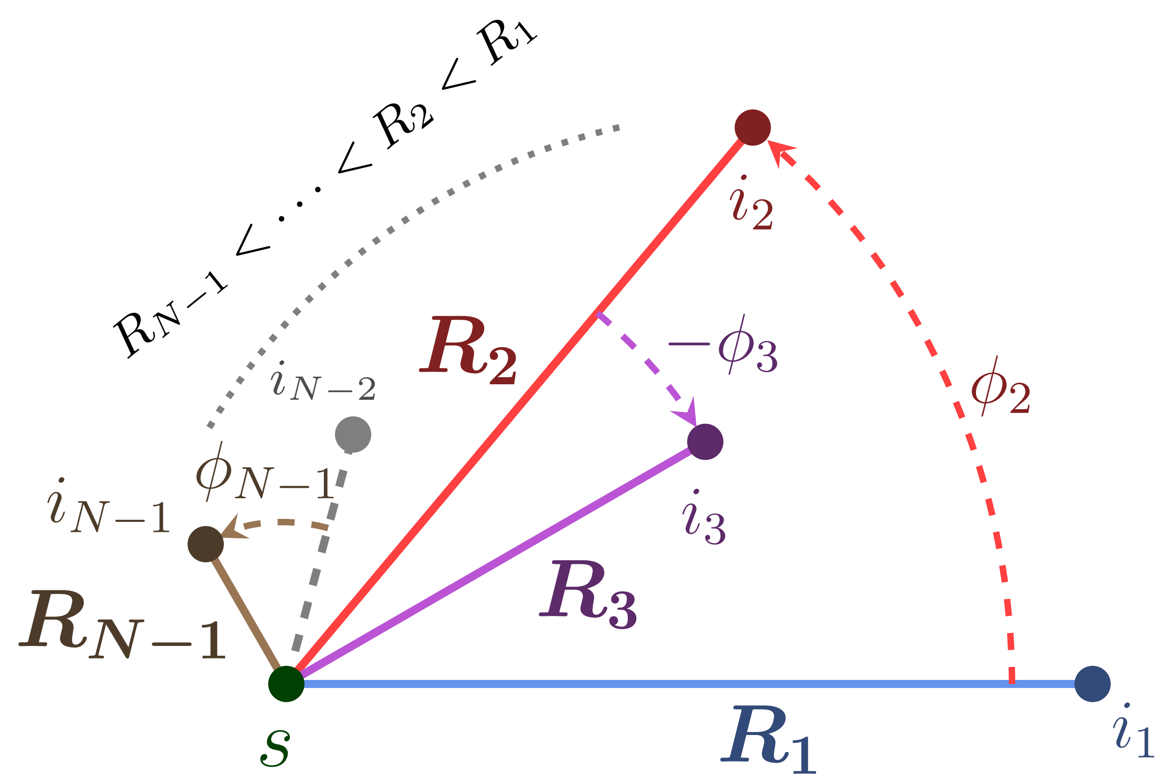
- First difference is an NNLL effect → $R_L = R_1[1 + \mathcal{O}(\alpha_s)]$.

- Time is $\mathcal{O}(M^2 \ln M)$ for projected correlator for all N!

Better parametrization and visualization

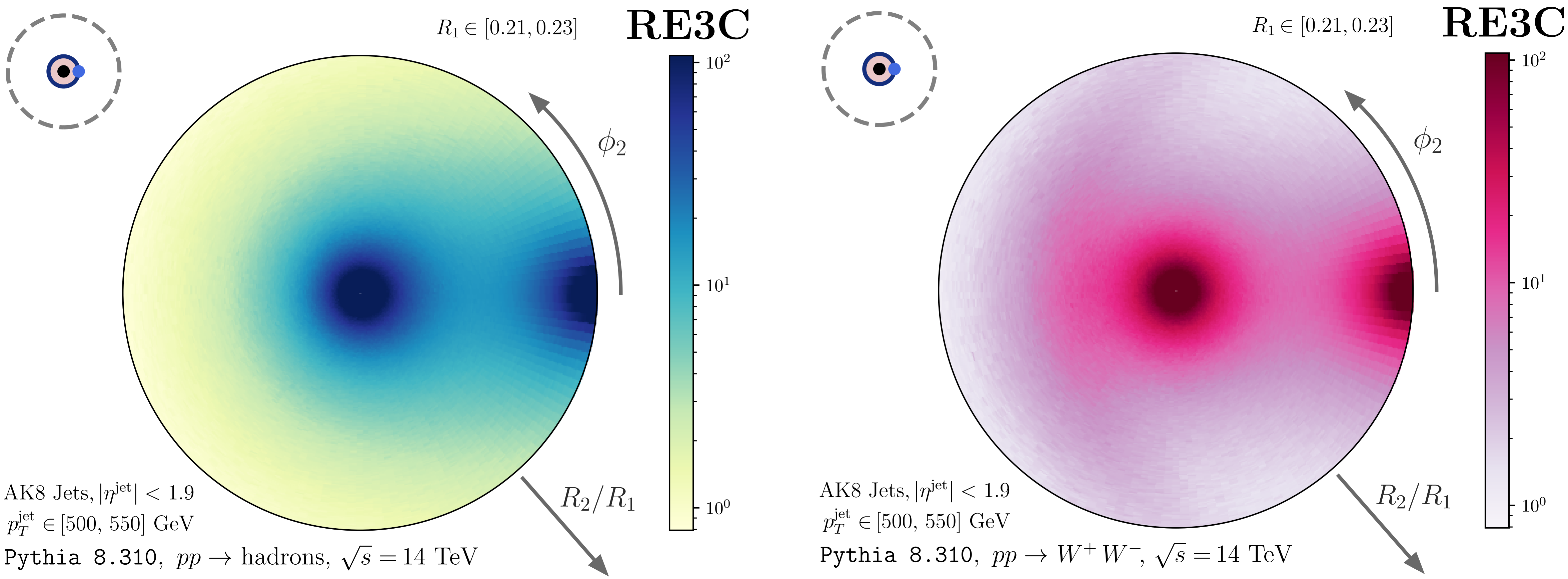


[Alipour-fard, Budhraja, Thaler, WW]



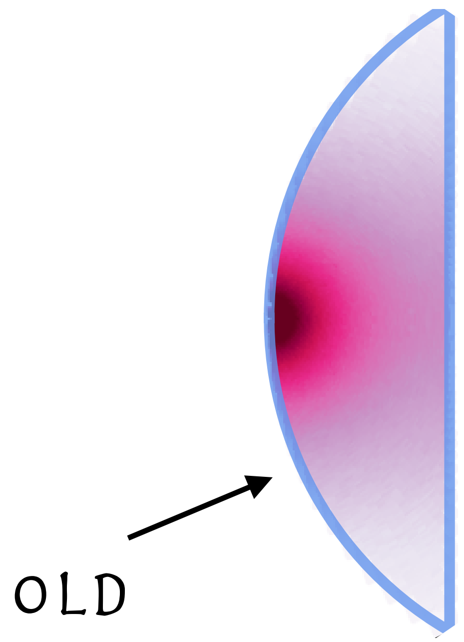
- Resolve three particles using polar coordinates around special point (non-redundant).

Better parametrization and visualization



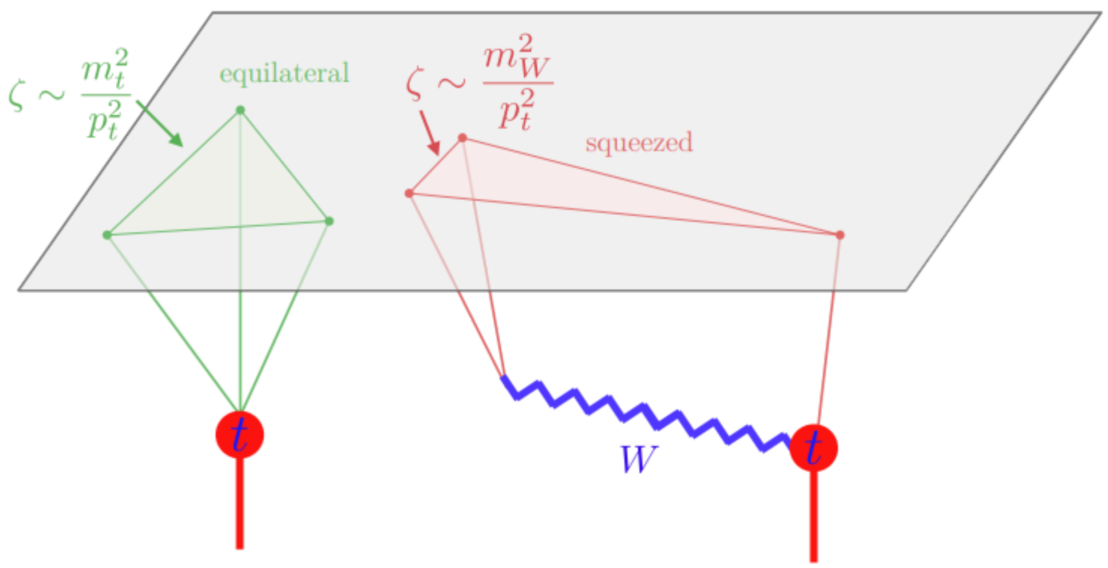
[Alipour-fard, Budhraja, Thaler, WW]

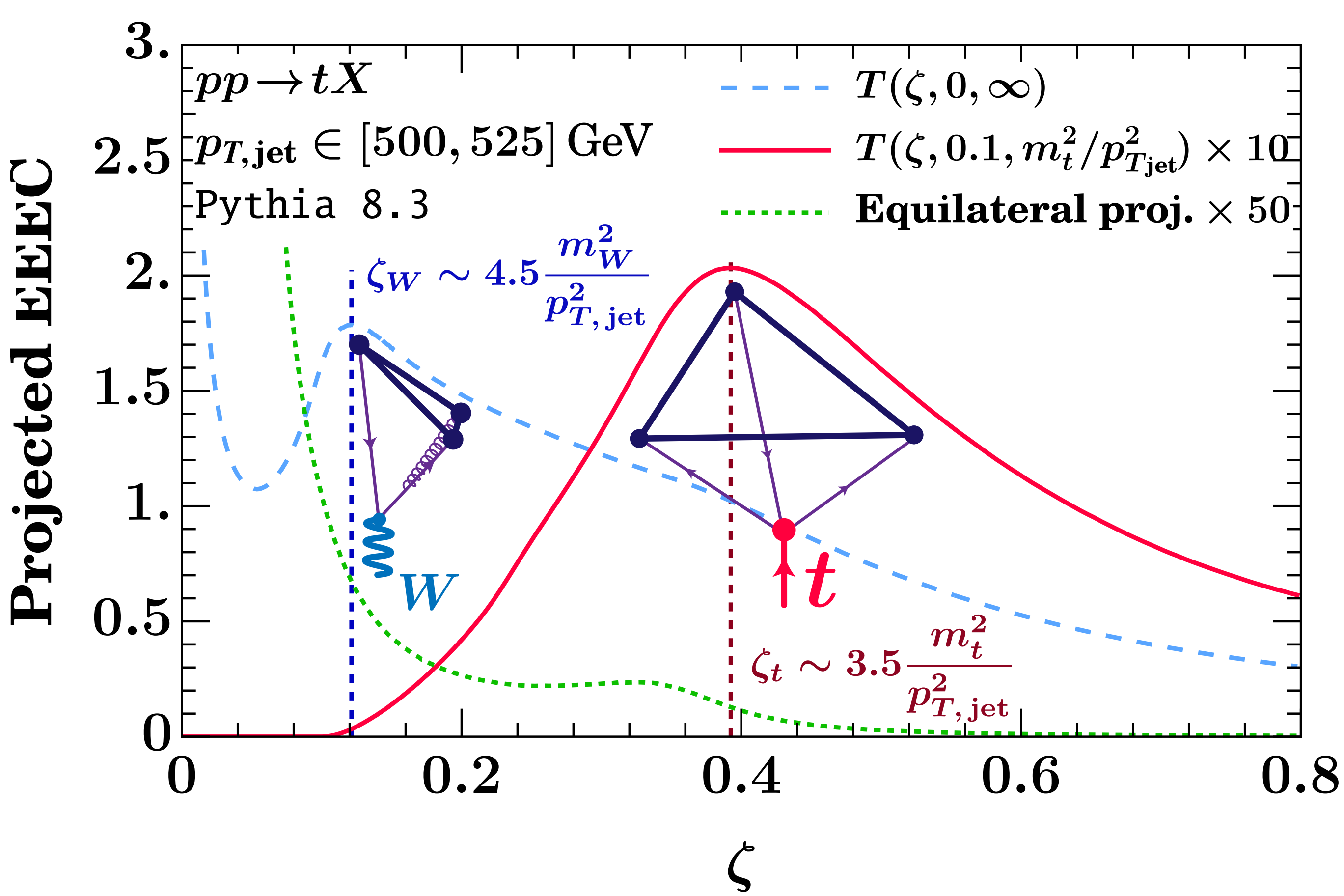
- Resolve three particles using polar coordinates around special point (non-redundant).
- Qualitative difference between QCD and W jets, not visible in old parametrization.



Top quark mass determination

- Existing approaches offer **either** good theoretical control **or** good sensitivity to top quark mass → try energy correlators.
- Convert the top quark peak position into a mass using *W*.

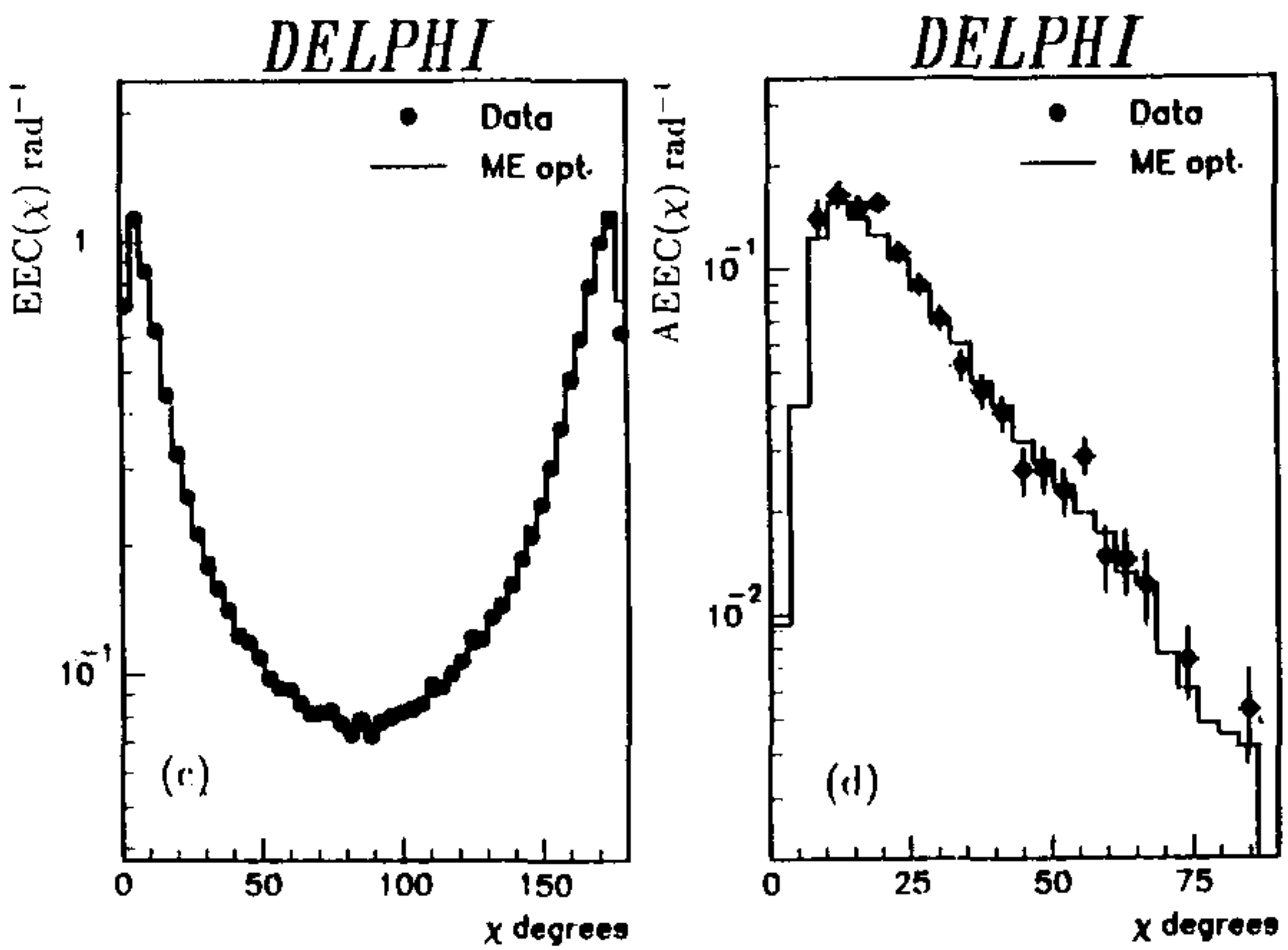




[Holguin, Moult, Pathak, Procura, Schofbeck, Schwarz]

3. Strong coupling determination

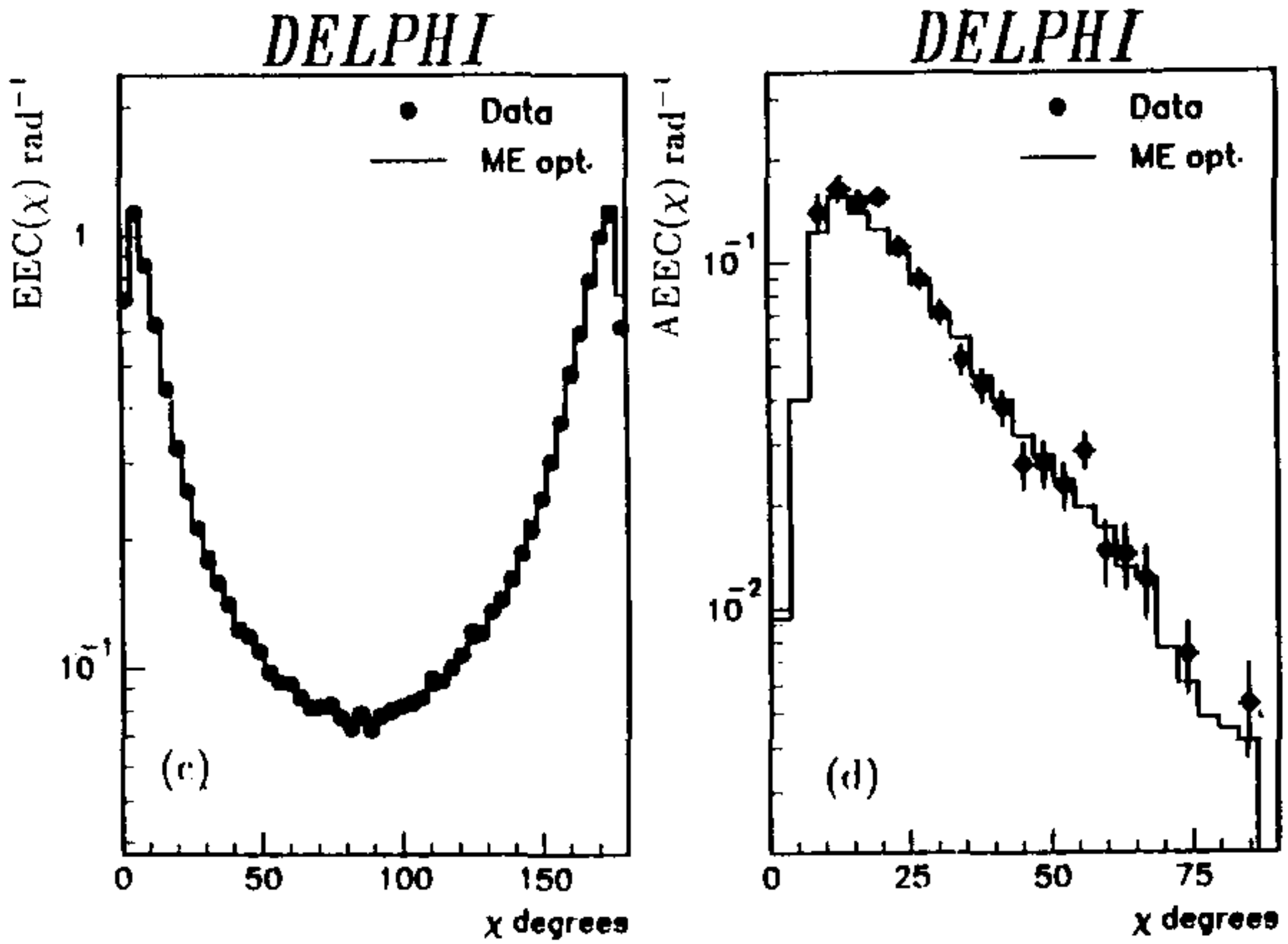
Strong coupling from AEEC at LEP



$$\mathrm{AEEC}(\chi) = \mathrm{EEC}(180^{\mathrm{o}} - \chi) - \mathrm{EEC}(\chi)$$

1990

Strong coupling from AEEC at LEP



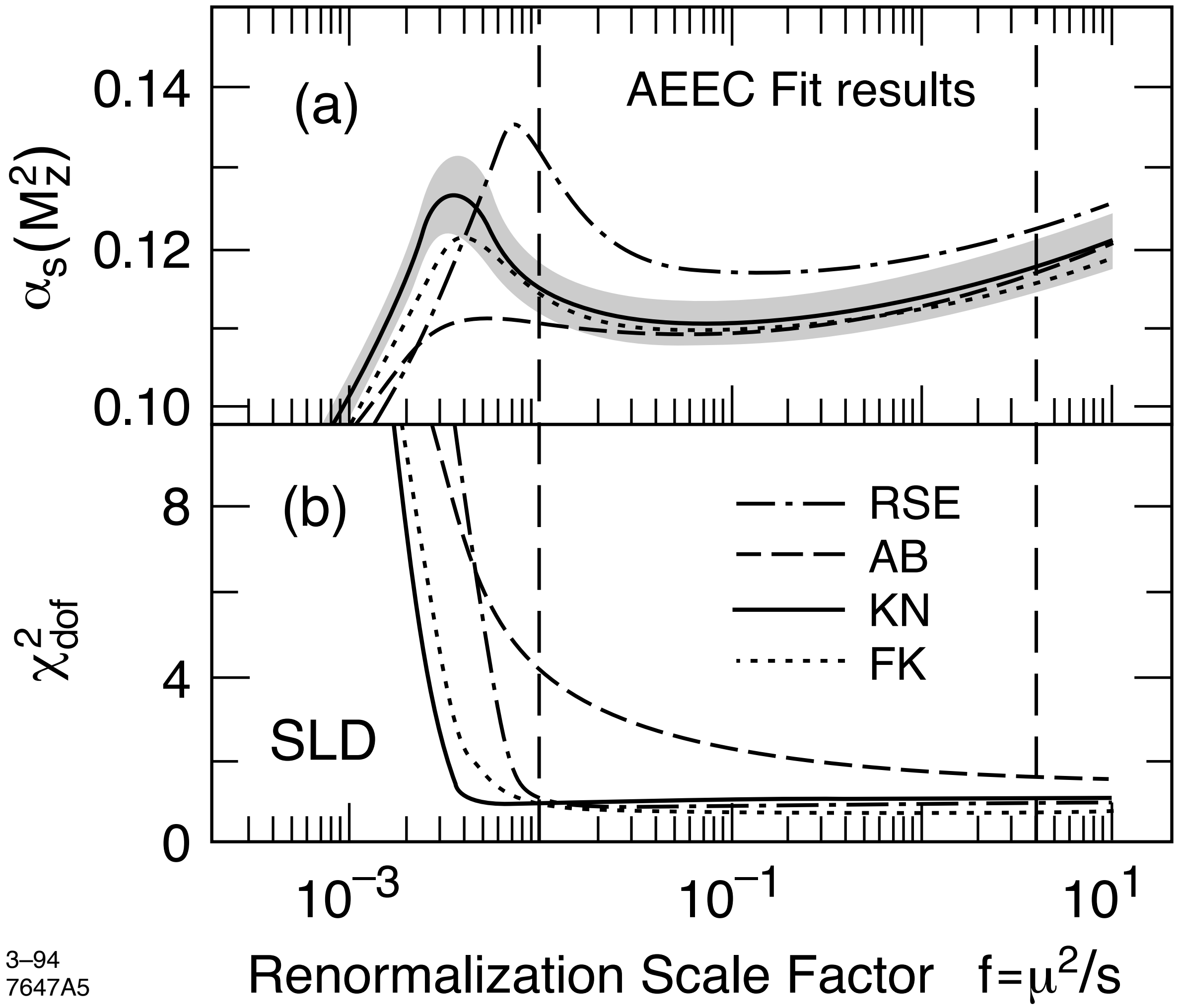
Uncertainties:

- Theory: vary scale $\mu^2 = f s$ with f between 0.004 and 1.
- Systematic: vary fragmentation parameters.

- Extract from **integral** of $\mathrm{AEEC}(\chi) = \mathrm{EEC}(180^{\mathrm{o}} - \chi) - \mathrm{EEC}(\chi)$ between $28.8^{\mathrm{o}} < \chi < 90^{\mathrm{o}}$, comparing to $\mathcal{O}(\alpha_s^2)$ [Ellis, Ross, Terrano]
- Result: $\alpha_s(m_Z) = 0.106^{+0.003}_{-0.003}$ (stat.) $^{+0.003}_{-0.003}$ (syst.) $^{+0.003}_{-0.000}$ (theory)

1990

Strong coupling from (A)EEC at SLC

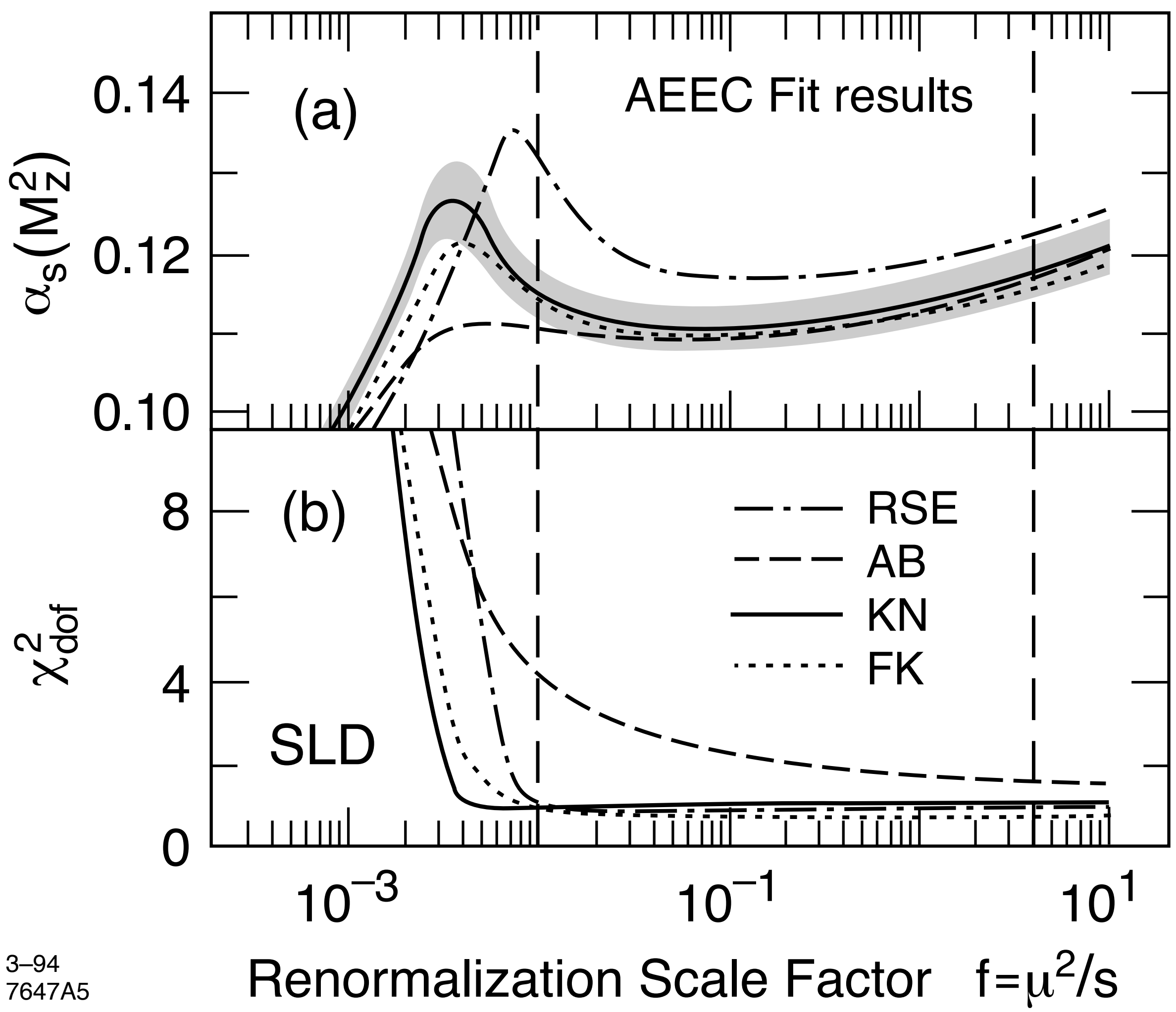


Uncertainties:

- Theory $\mathcal{O}(\alpha_s^2)$:
  - scale uncertainty (again using f)
  - differences between 4 calculations
  - fragmentation
- Systematic:
  - event selection cuts
  - tracking
  - Monte Carlo statistics

1994

Strong coupling from (A)EEC at SLC



Uncertainties:

- Theory $\mathcal{O}(\alpha_s^2)$:
  - scale uncertainty (again using f)
  - differences between 4 calculations
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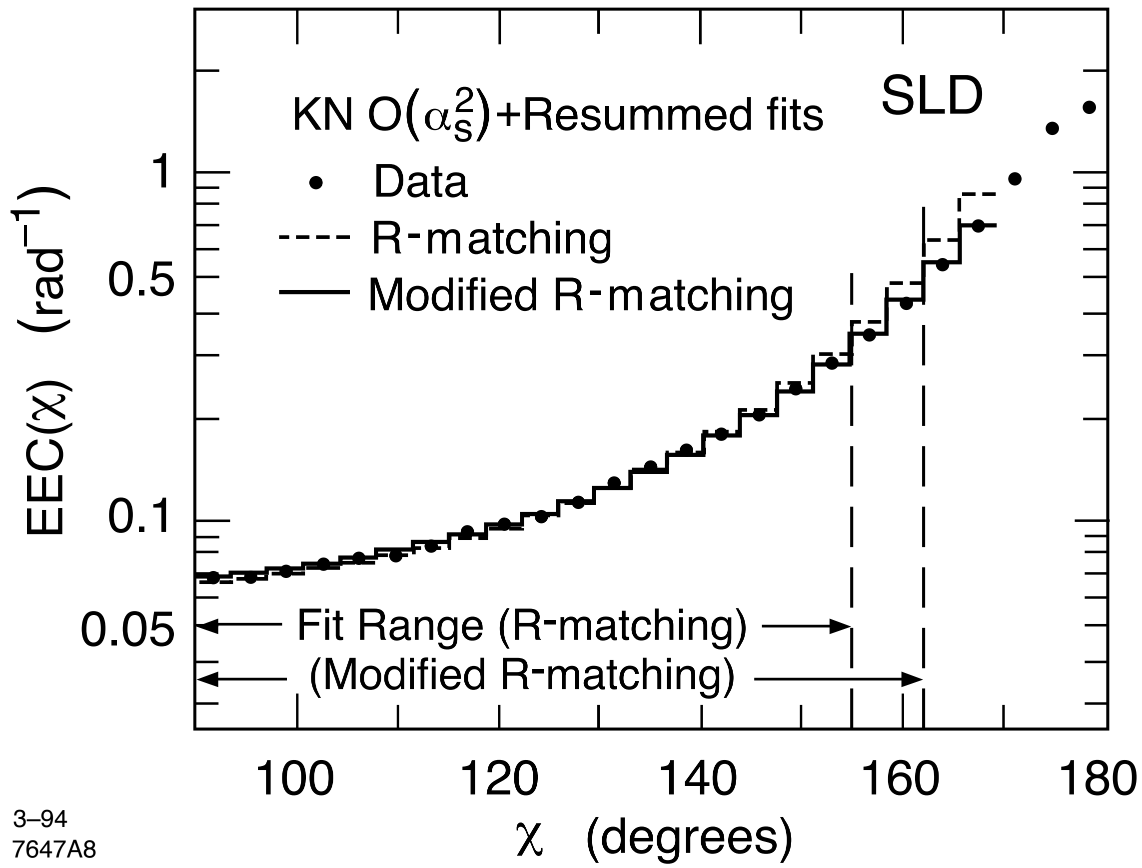
- Systematic:
  - event selection cuts
  - tracking
  - Monte Carlo statistics

- AEEC vs. EEC: smaller scale uncertainty, fragmentation similar
- Fit AEEC between $21.6^{\circ} < \theta < 79.2^{\circ}$:

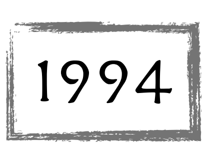
$$\alpha_s(m_Z) = 0.116^{+0.002}_{-0.002}\,(\text{stat.})\,^{+0.004}_{-0.004}\,(\text{syst.})\,^{+0.006}_{-0.006}\,(\text{theory})$$

1994

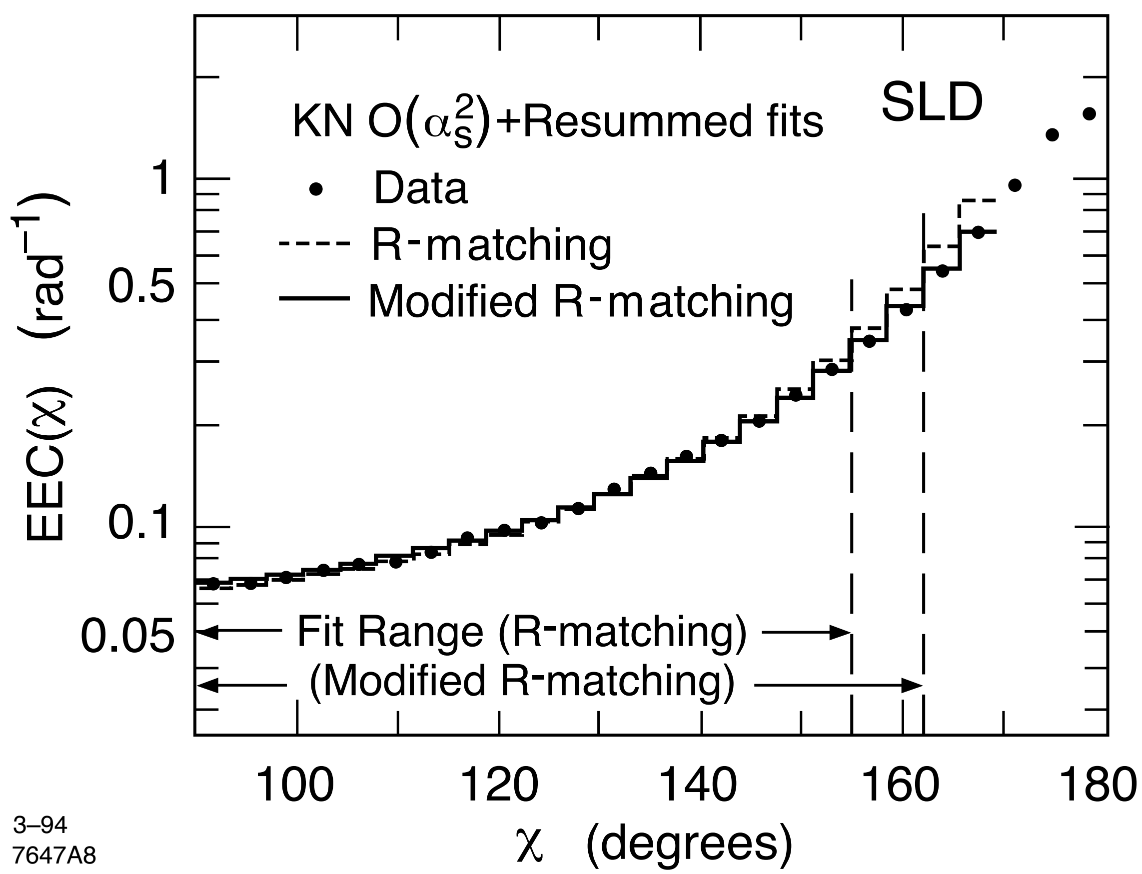
Strong coupling from EEC at SLC + resummation

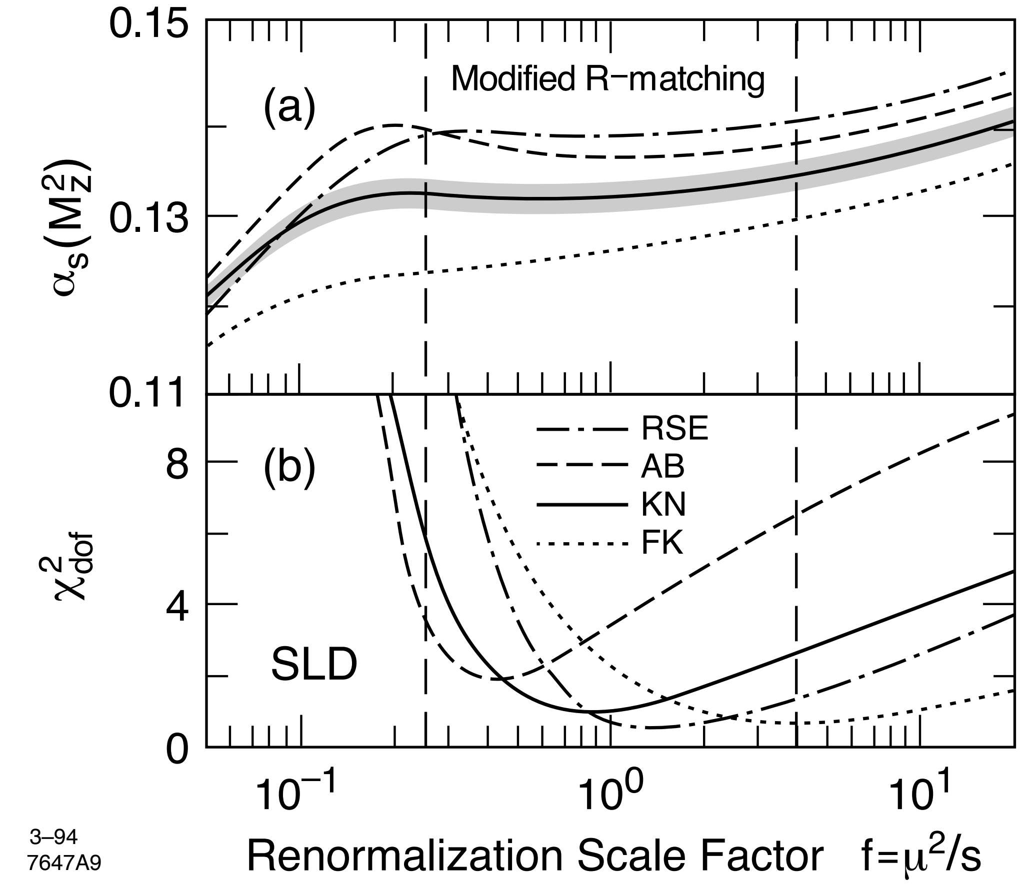


- Include NLL resummation in the back-to-back region



Strong coupling from EEC at SLC + resummation





- Include NLL resummation in the back-to-back region
- Fit EEC, averaging 2 matching schemes, **now** vary $\frac{1}{4} \leq f \leq 4$:

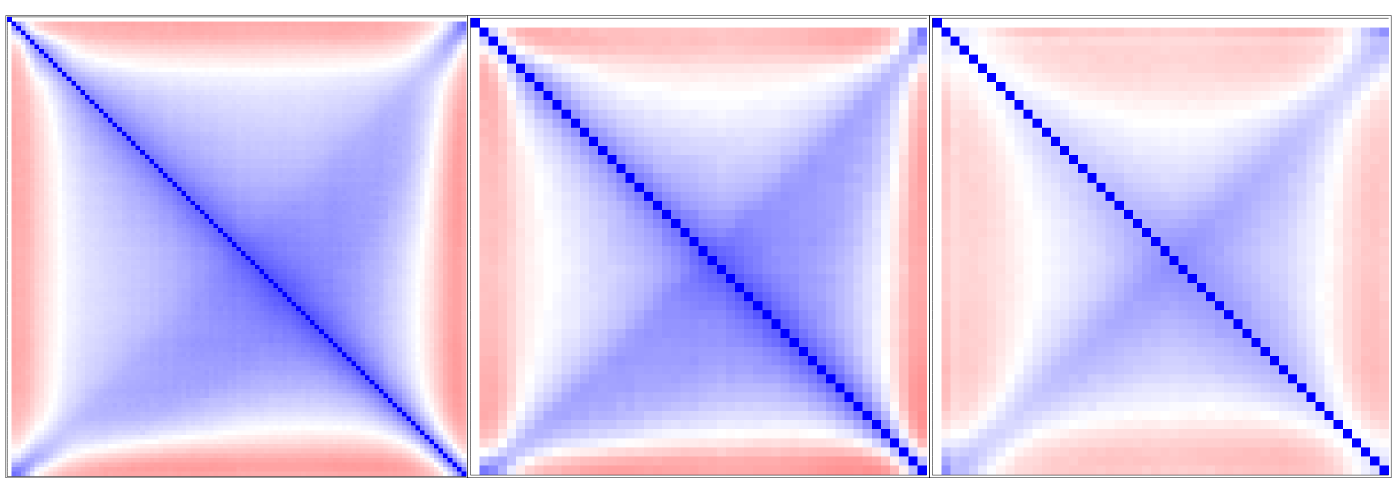
$$\alpha_s(m_Z) = 0.130^{+0.002}_{-0.002}\,\text{(stat.)}\,^{+0.002}_{-0.003}\,\text{(syst.)}\,^{+0.007}_{-0.007}\,\text{(theory)}$$

1994

Global fit of EEC

- Theory input: NNLO + NNLL, b-quark mass effects.
- Hadronization modelled using Sherpa with Lund string or cluster fragmentation, and Herwig.
- Correlations between bins assessed using Monte Carlo.

Experiment	$\sqrt{s}$, GeV, data	$\sqrt{s}$, GeV, MC	Events
SLD [47]	91.2(91.2)	91.2	60000
OPAL [50]	91.2(91.2)	91.2	336247
OPAL [51]	91.2(91.2)	91.2	128032
L3 [48]	91.2(91.2)	91.2	169700
DELPHI [49]	91.2(91.2)	91.2	120600
TOPAZ [52]	59.0 – 60.0(59.5)	59.5	540
TOPAZ [52]	52.0 – 55.0(53.3)	53.3	745
TASSO [53]	38.4 – 46.8(43.5)	43.5	6434
TASSO [53]	32.0 – 35.2(34.0)	34.0	52118
PLUTO [58]	34.6(34.6)	34.0	6964
JADE [54]	29.0 – 36.0(34.0)	34.0	12719
CELLO [57]	34.0(34.0)	34.0	2600
MARKII [56]	29.0(29.0)	29.0	5024
MARKII [56]	29.0(29.0)	29.0	13829
MAC [55]	29.0(29.0)	29.0	65000
TASSO [53]	21.0 – 23.0(22.0)	22.0	1913
JADE [54]	22.0(22.0)	22.0	1399
CELLO [57]	22.0(22.0)	22.0	2000
TASSO [53]	12.4 – 14.4(14.0)	14.0	2704
JADE [54]	14.0(14.0)	14.0	2112



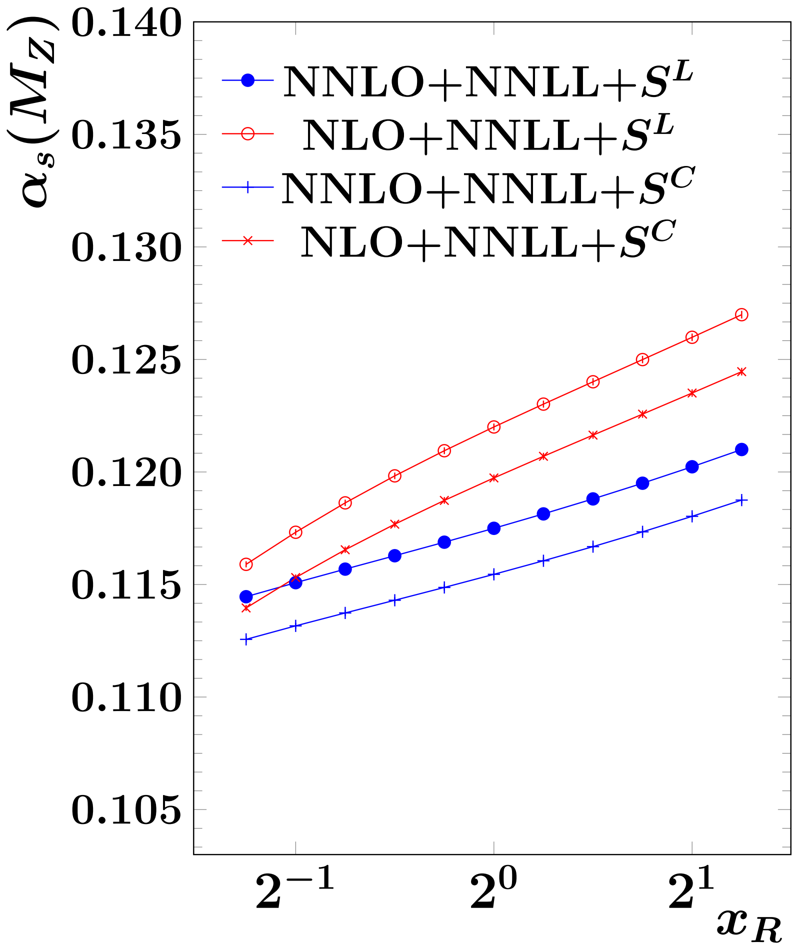
Color to correlation coefficient	-100	-80	-60	-40	-20	0 or N/A	20	40	60	80	100

2018

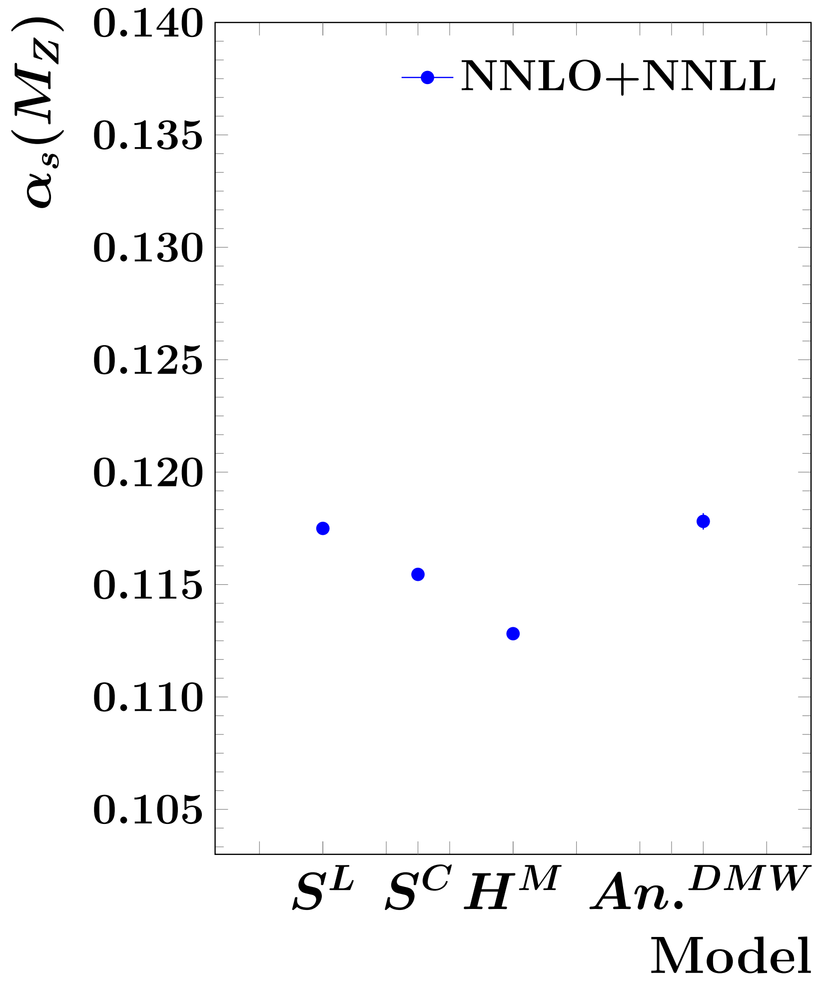
[Kardos, Kluth, Somogyi, Tulipant, Verbytskyi]

Global fit of EEC

Renormalization scale:



Hadronization model:



- Fit EEC between $60^o < \theta < 160^o$:

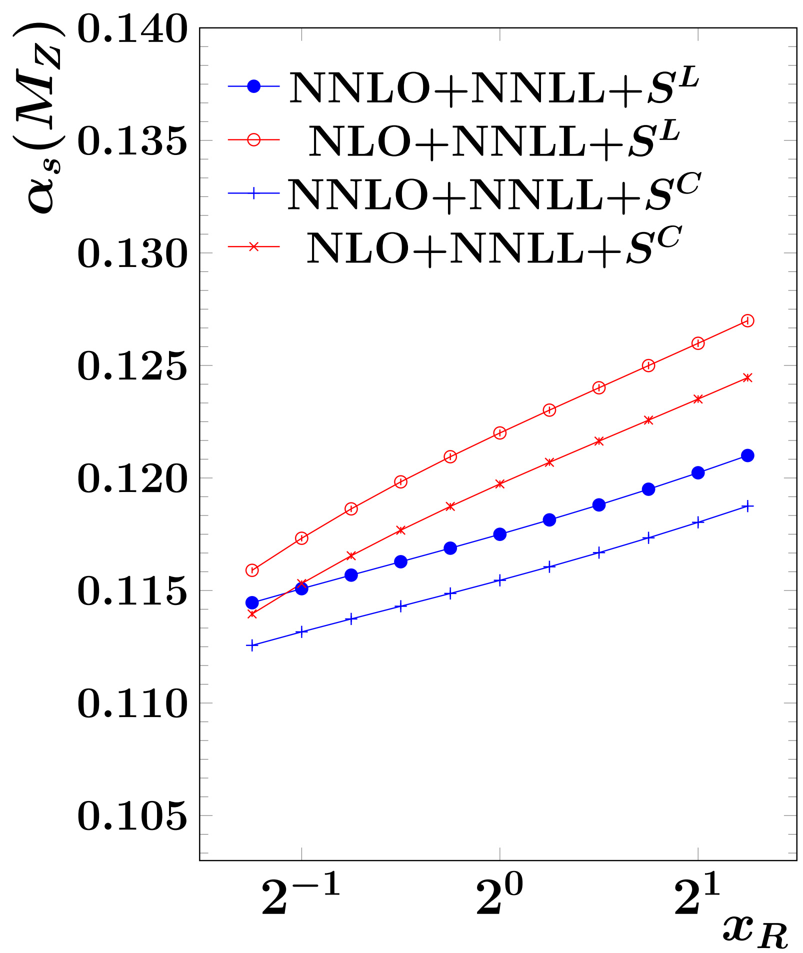
$$\alpha_s(m_Z) = 0.1175^{+0.0002}_{-0.0002}\,\text{(exp.)}\;^{+0.0010}_{-0.0010}\,\text{(hadr.)}\;^{+0.0026}_{-0.0026}\,\text{(ren.)}\;^{+0.0008}_{-0.0008}\,\text{(res.)}$$

2018

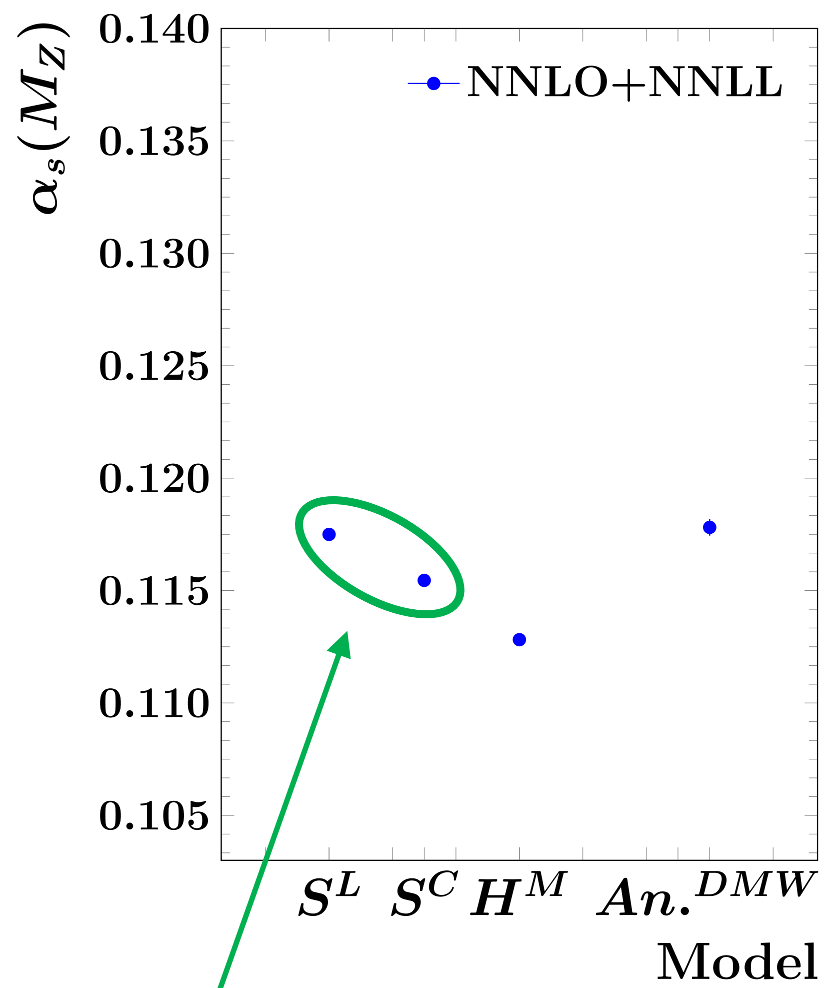
[Kardos, Kluth, Somogyi, Tulipant, Verbytskyi]

Global fit of EEC

Renormalization scale:



Hadronization model:



- Fit EEC between $60^o < \theta < 160^o$:

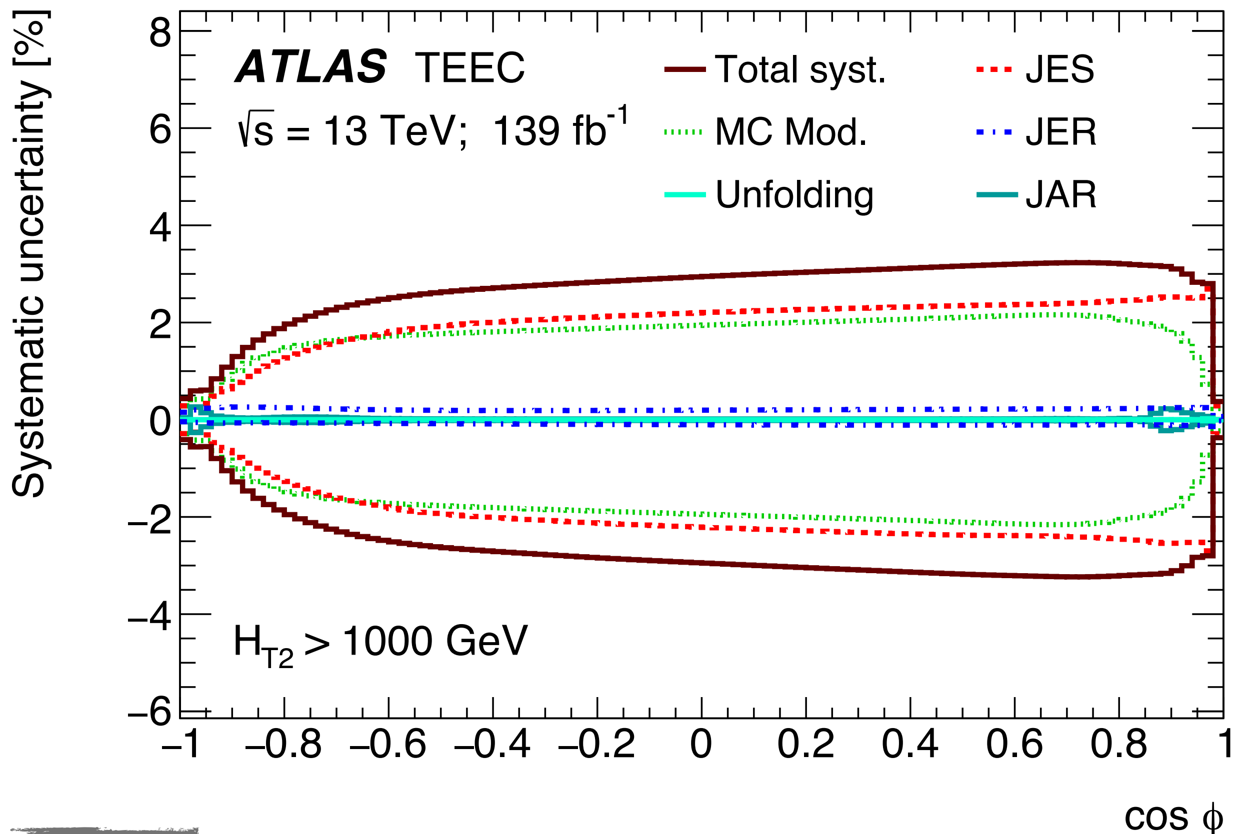
$$\alpha_s(m_Z) = 0.1175^{+0.0002}_{-0.0002}\,(\text{exp.})\,{}^{+0.0010}_{-0.0010}\,(\text{hadr.})\,{}^{+0.0026}_{-0.0026}\,(\text{ren.})\,{}^{+0.0008}_{-0.0008}\,(\text{res.})$$

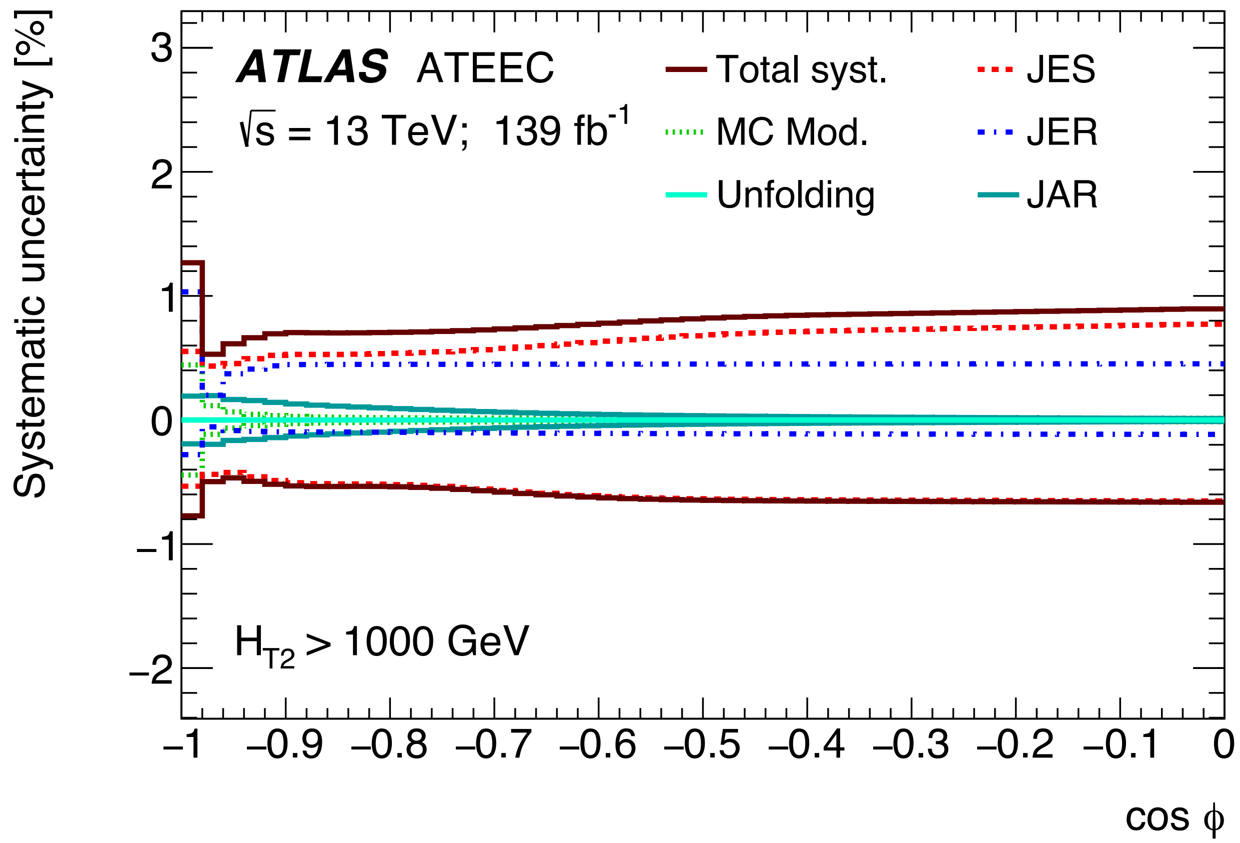
2018

[Kardos, Kluth, Somogyi, Tulipant, Verbytskyi]

Strong coupling from (A)TEEC

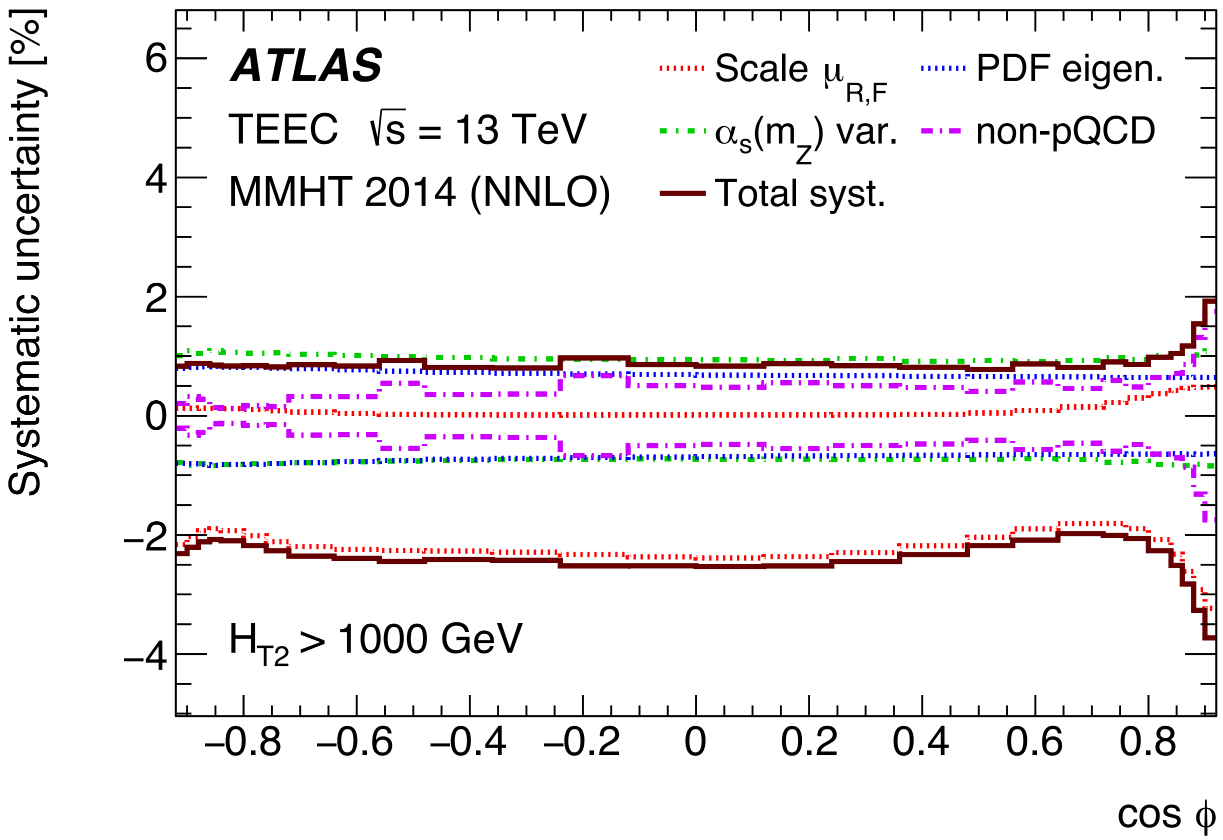
- TEEC not measured with particles but on **anti-k_T R=0.4 jets** with $p_T > 100$ GeV. (For two leading jets, $H_{T2} > 800$ GeV.)
- Experimental uncertainties:
  - Jet Energy Scale and Monte Carlo Modeling dominate

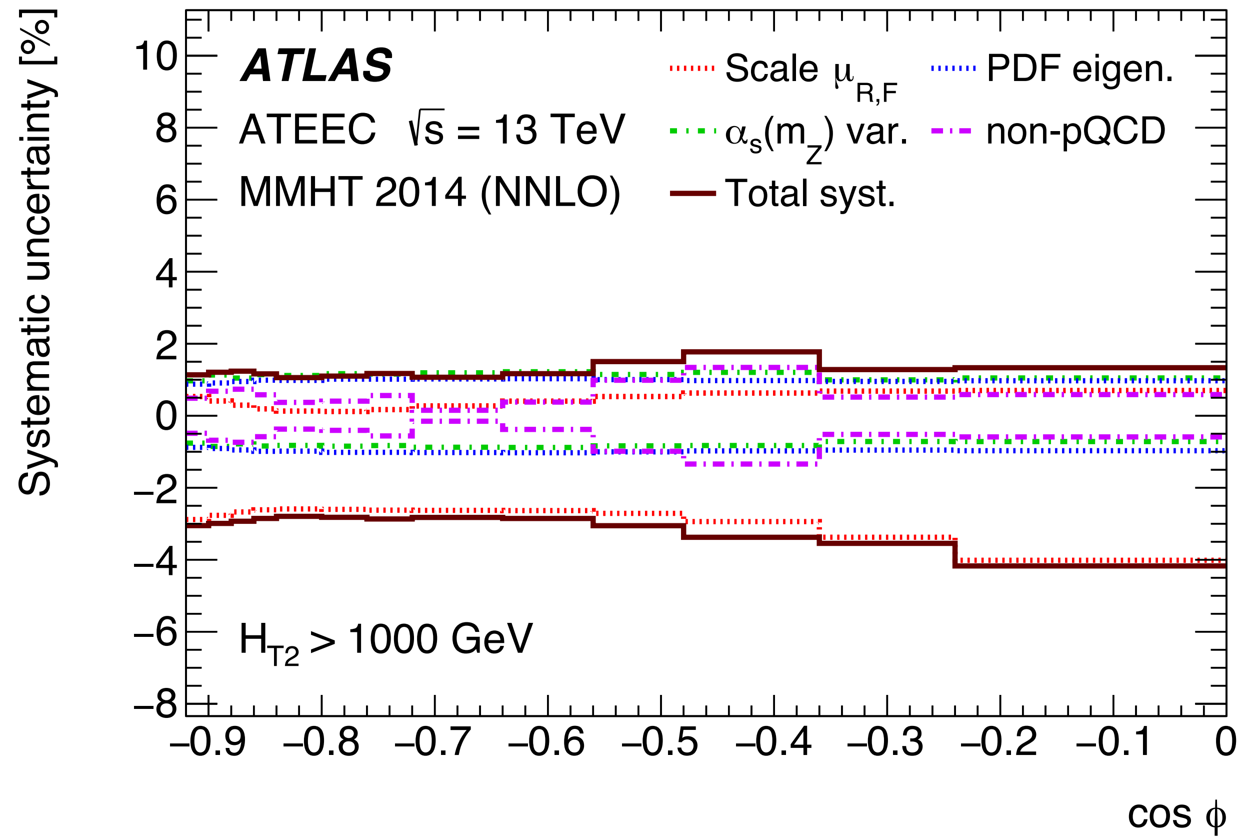




2023

Strong coupling from (A)TEEC



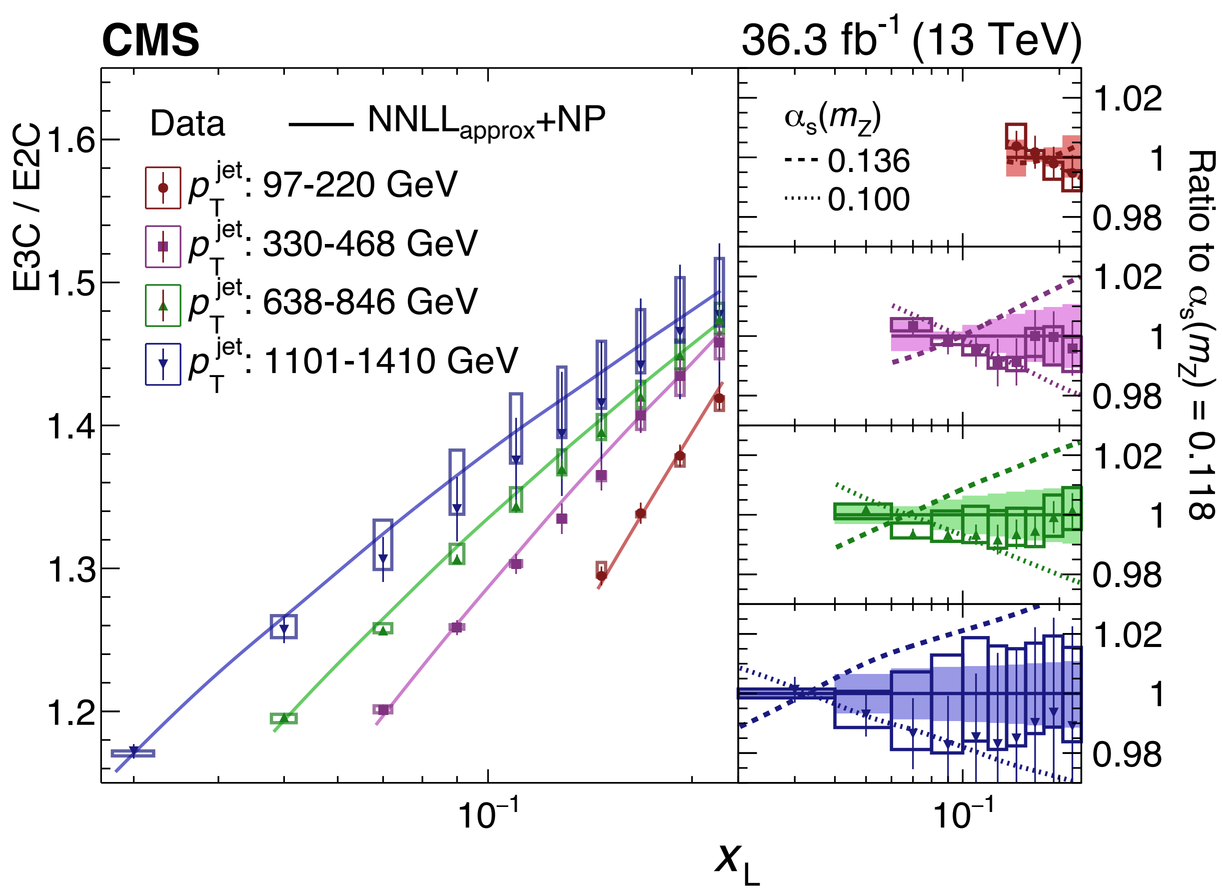


- Theoretical uncertainties
  - Scale uncertainty reduced by factor 3 by NNLO [Czakon, Mitov, Poncelet; …]
  - Varying $0.117 < \alpha_s(m_Z) < 0.119$ is shown for comparison
  - Fit: $\alpha_s(m_Z) = 0.1175^{+0.0006}_{-0.0006}$ (exp.) ${}^{+0.0034}_{-0.0017}$ (theory) TEEC

    $\alpha_s(m_Z) = 0.1185^{+0.0009}_{-0.0009}$ (exp.) ${}^{+0.0025}_{-0.0012}$ (theory) ATEEC

2023

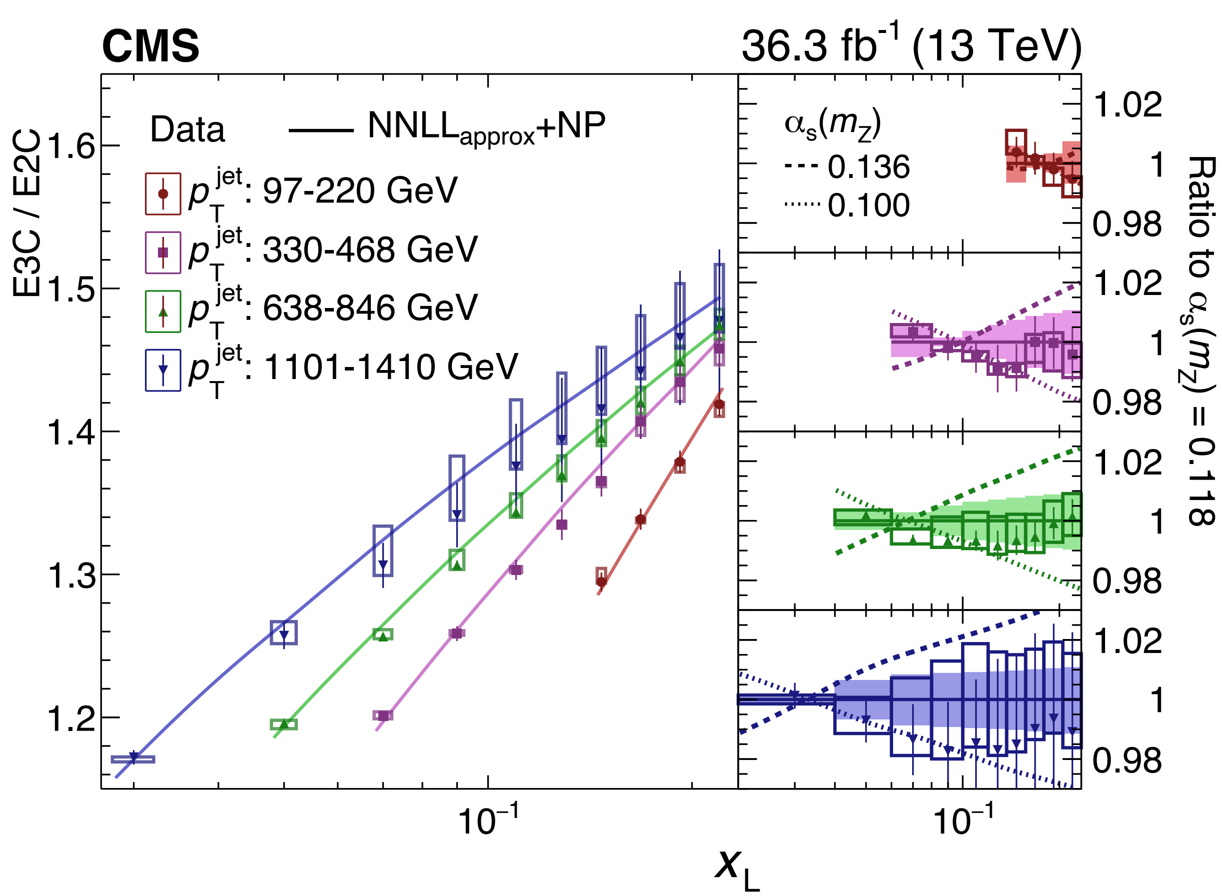
Strong coupling from E3C/EEC in jets

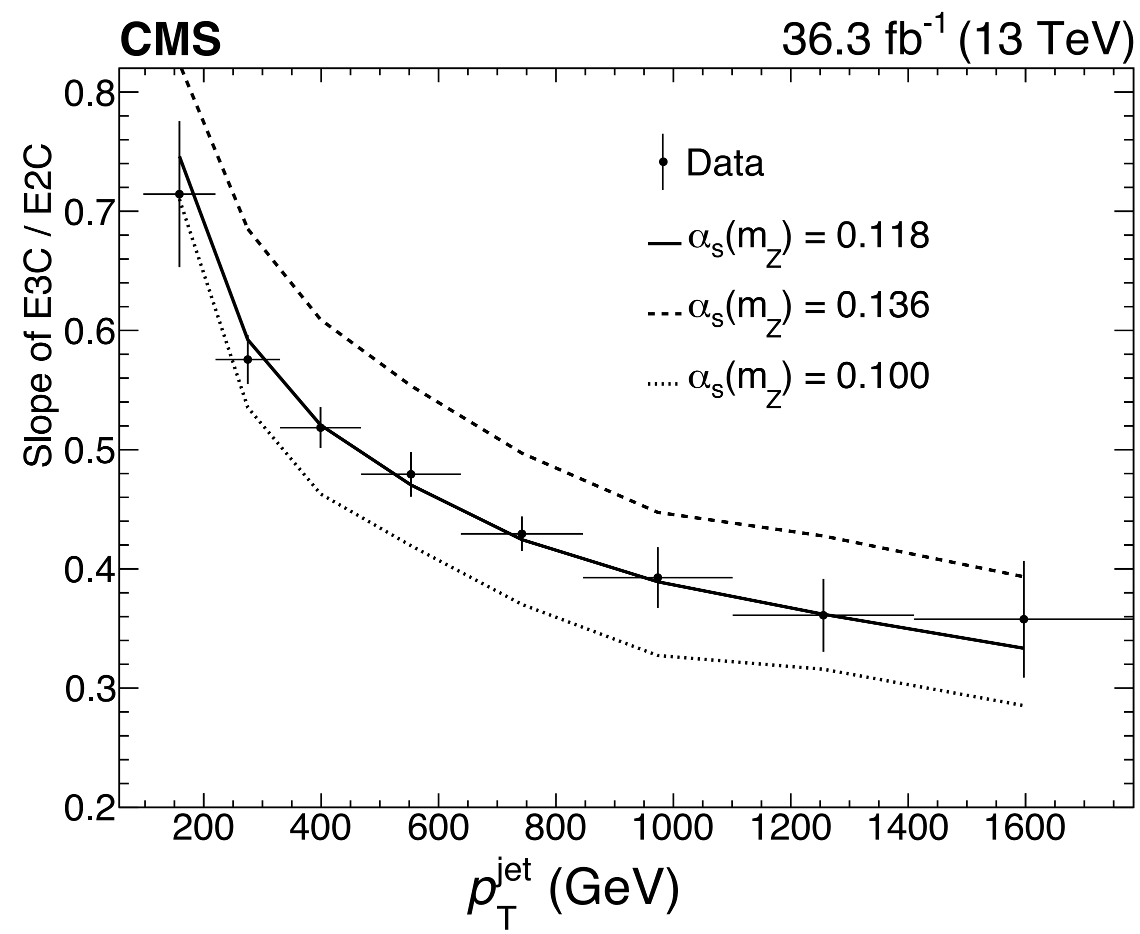


- Extract $\alpha_s(m_Z)$ from slope of E3C/EEC, compare to NLO+NNLL. E.g. effect of hadronization and UE: 5-40% → **0-3% in ratio.**

2024

Strong coupling from E3C/EEC in jets





- Extract $\alpha_s(m_Z)$ from slope of E3C/EEC, compare to NLO+NNLL. E.g. effect of hadronization and UE: 5-40% → **0-3% in ratio.**
- Best fit $\alpha_s(m_Z) = 0.1229^{+0.0014}_{-0.0012}$ (stat.) $^{+0.0023}_{-0.0036}$ (syst.) $^{+0.0030}_{-0.0033}$ (theory) is **most precise** measurement from jet substructure.

2024

Nonperturbative corrections from field theory

- Back to e^+e^-: jets have same behavior in collinear limit.

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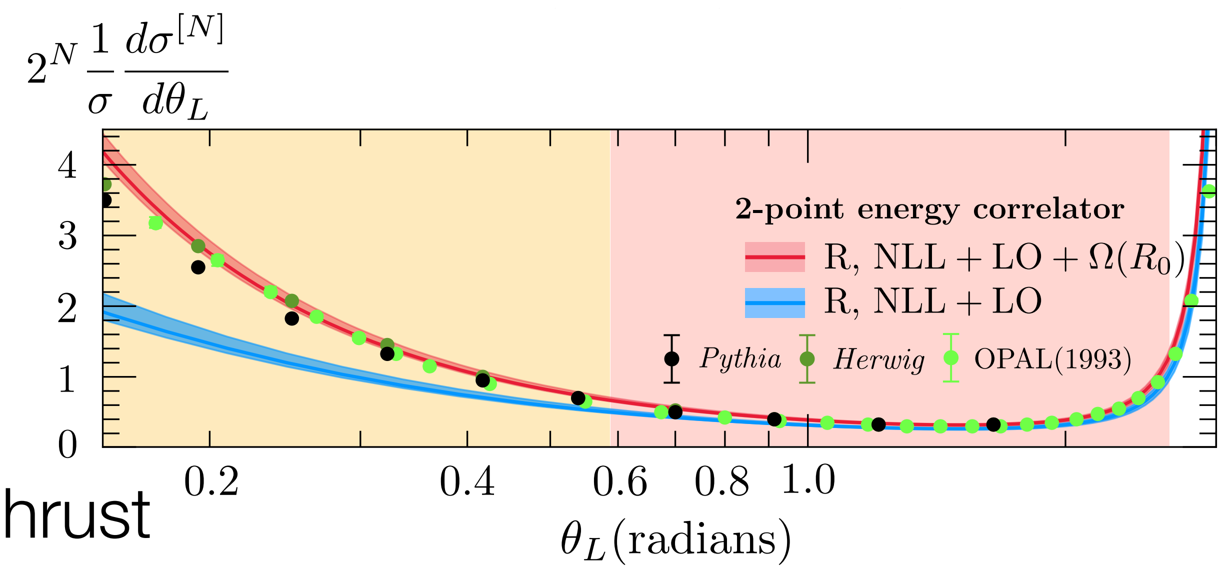
from thrust

[Lee, Pathak, Stewart, Sun]



- Does not cancel in ratio (but does in the asymmetry).

[See also Schindler, Stewart, Sun; Chen, Monni, Xu, Zhu]

2024

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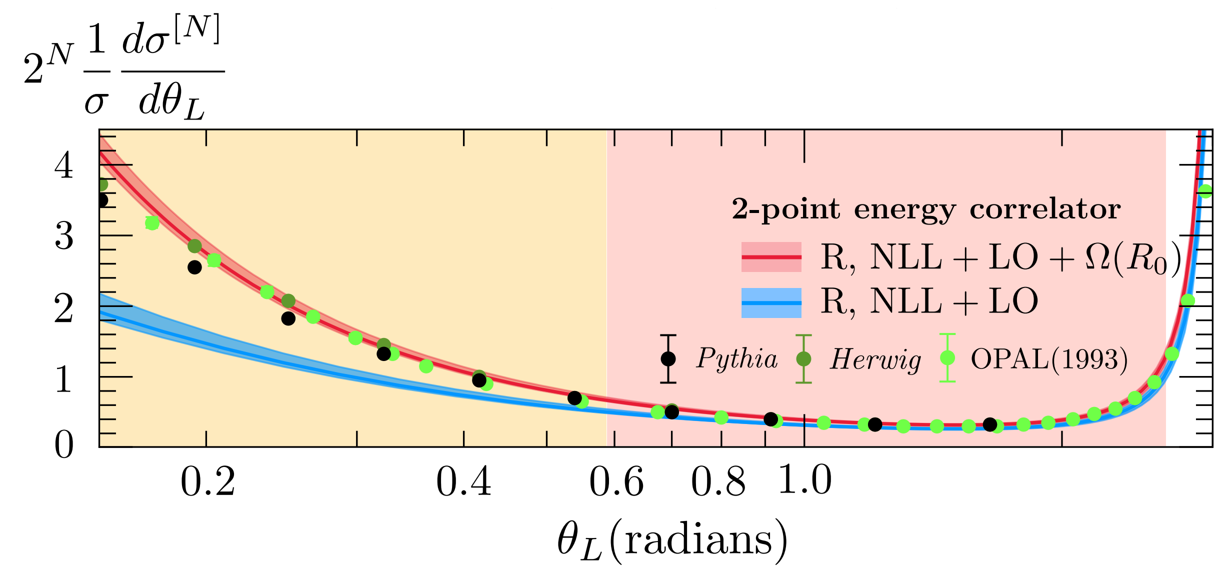

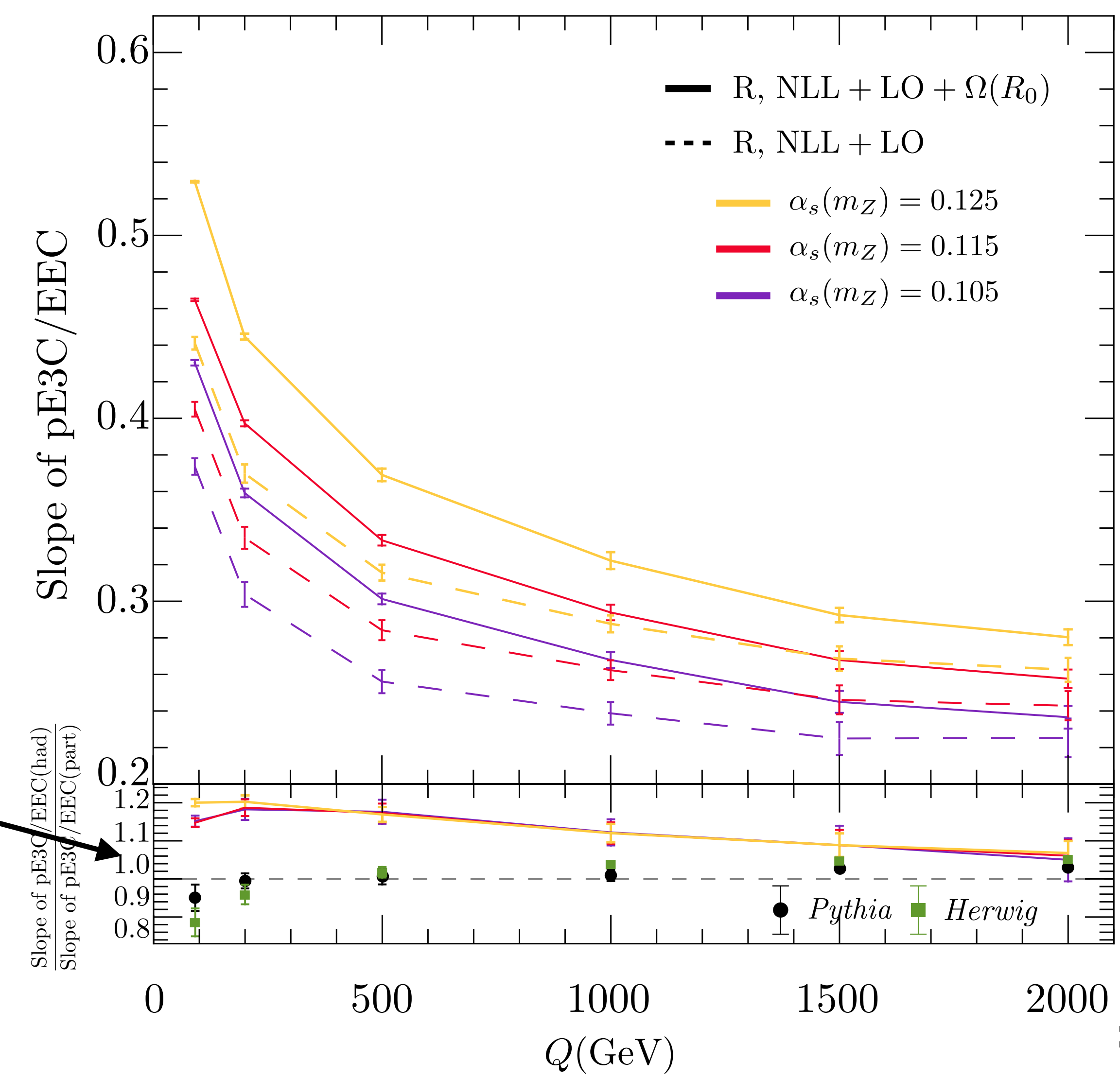

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Conclusions

- Energy correlators separate scales, suppress soft radiation, simple(r) theory → applications: α_s, m_{top}, QGP, …
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Thank you!