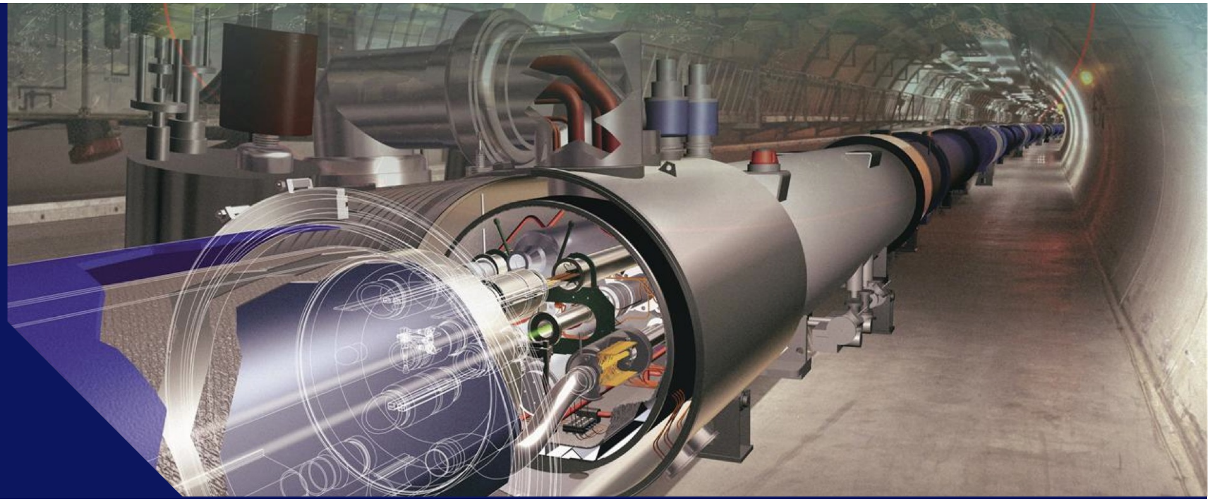
# SM@LHC 2025

Durham, UK

7 – 10 April 2025

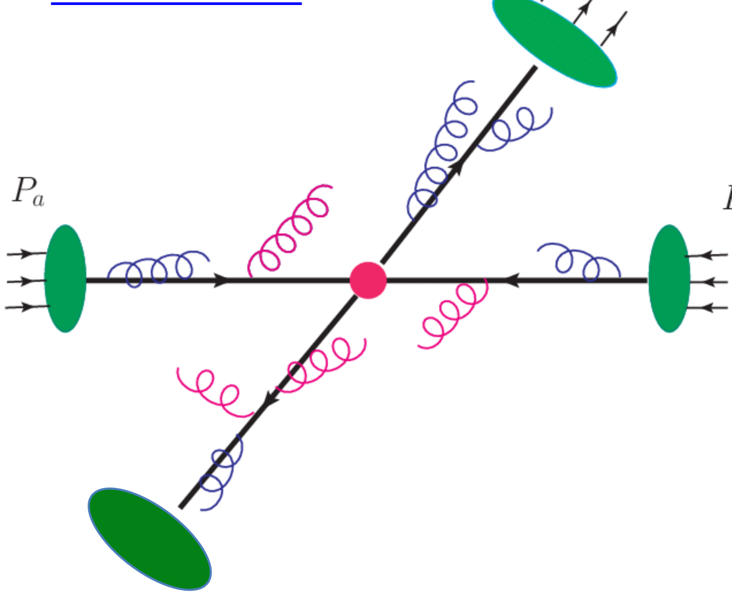


# Precise Determination of the Strong Coupling Constant from Dijet Cross Sections up to the Multi-TeV Range

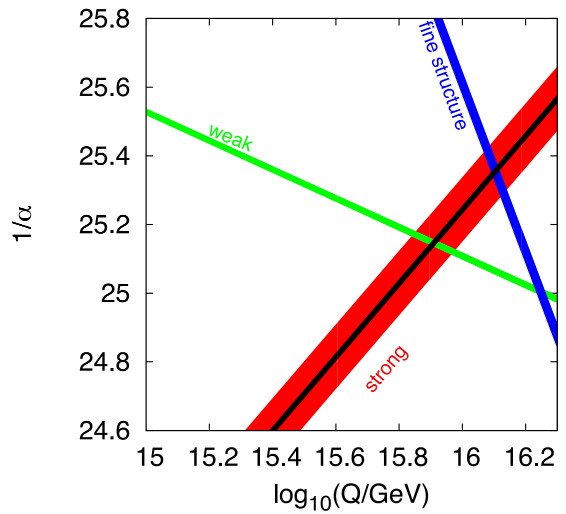
arXiv:[2412.21165](2412.21165)

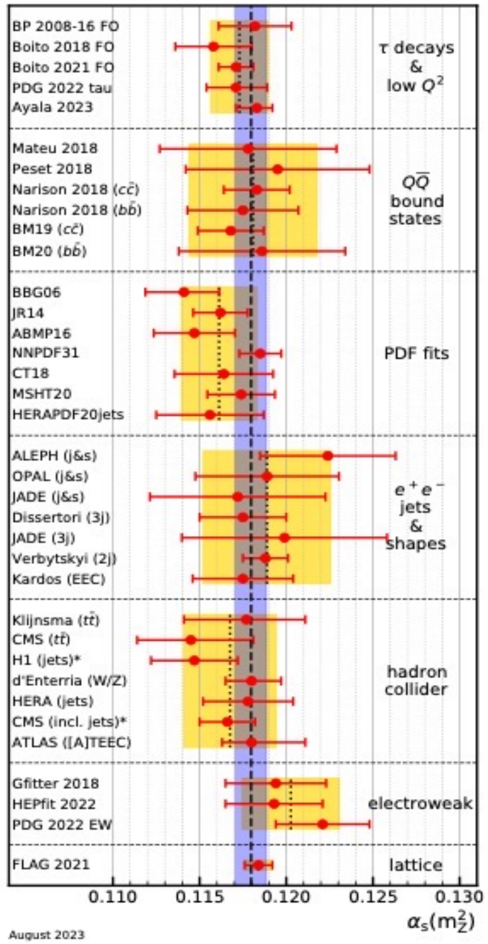

Claire Gwenlan, Oxford

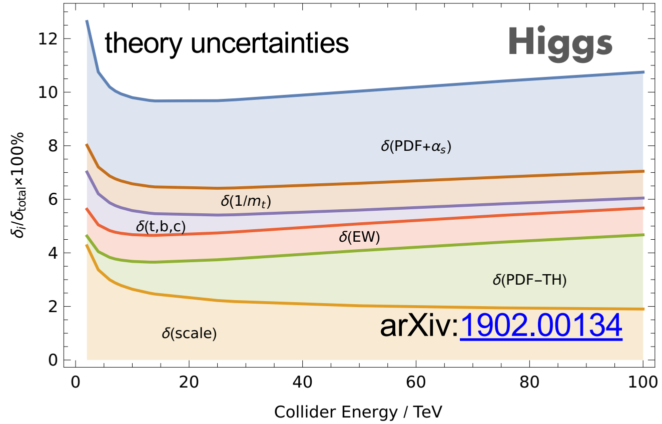
on behalf of NNLOJET and APPLfast

# Motivation




- **strong coupling αs** is least known coupling constant;
- **δαs/αs = $\mathcal{O}$(1%)**
- enters into the calculation of every process that involves the strong interaction

- **uncertainties on αs:**
- non-negligible uncertainty on many observables, EG. precision SM and Higgs measurements, BRs, …

**PDG24:** αs = 0.1180 ± 0.009

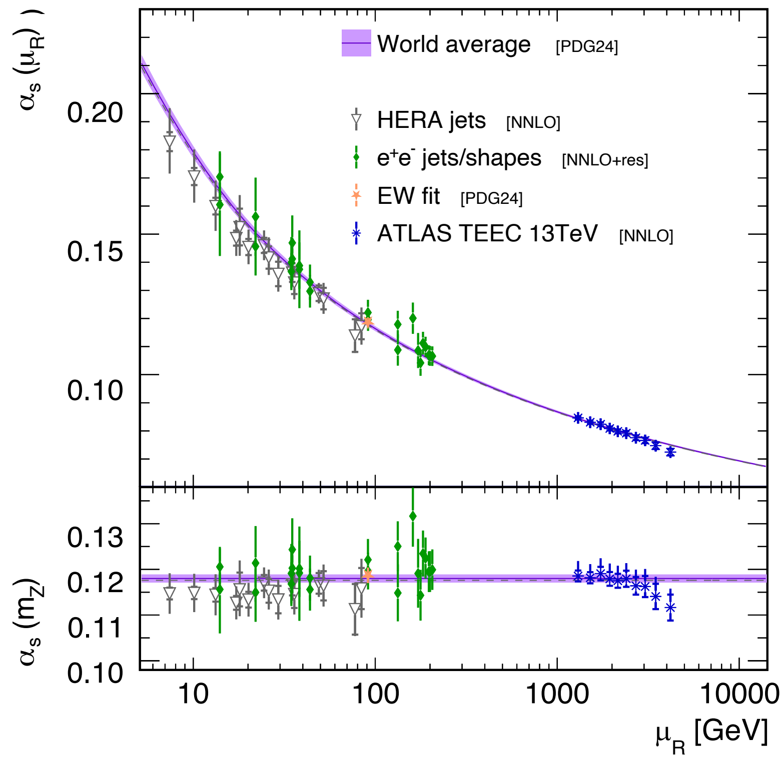
- **jet measurements provide direct constraints on αs**

# existing αs determinations at NNLO using jet data

- **previous NNLO extractions using HERA and LHC inclusive jet or dijet data**
- **HERA – αs(MZ)**
- HERA inclusive jets and dijets (arXiv:2112.01120)
- **LHC – αs(MZ)**
- CMS inclusive jets (arXiv:2111.10431)
- CMS dijets (arXiv: 2207.13735 , 2312.16669)
- and at **aN3LO** using LHC jets (MSHT, arXiv:2404.02964)

- **running of αs also probed:**
- **HERA**: H1 inclusive and dijets (arXiv:1709.07251), HERA (1906.05303) and ZEUS (2309.02889) inclusive jets
- **LHC**: CMS inclusive jets at √s=2.76, 7, 8 and 13 TeV, covering energy scales between 100 and 1600 GeV (arXiv:2412.16665) **(not shown opposite)**

- **as well as in** e+e– at LEP; and using transverse energy-energy correlations at ATLAS, probing scales up to 4.2 TeV (arXiv:2301.09351)



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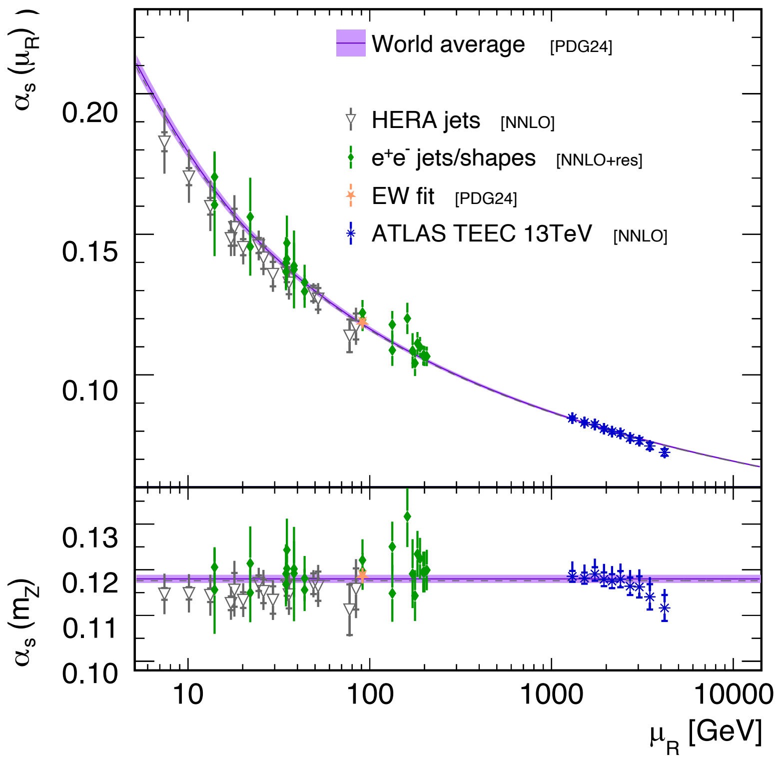



- **this work – new αs extraction:**
- incorporating **multiple LHC dijet data sets** from ATLAS and CMS, supplemented with HERA dijets, extending precision and range
- **full NNLO QCD theory** (with full colour contributions) for the first time

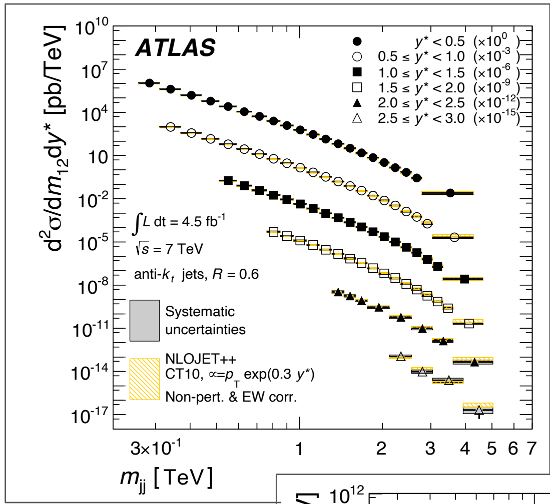
arXiv:2412.21165

# LHC dijet data

| Data | $\sqrt{s}$ [TeV] | d$\sigma$ | $R$ | $\mathcal{L}$ |
|---|---|---|---|---|
| ATLAS | 7 | $\frac{d^2\sigma}{dm_{jj}dy^*}$ | 0.6 | $4.5\,\mathrm{fb}^{-1} \pm 1.8\,\%$ |
| CMS | 7 | $\frac{d^2\sigma}{dm_{jj}dy_{max}}$ | 0.7 | $5.0\,\mathrm{fb}^{-1} \pm 2.2\,\%$ |
| CMS | 8 | $\frac{d^3\sigma}{d\langle p_T\rangle_{1,2}dy^*dy_b}$ | 0.7 | $19.7\,\mathrm{fb}^{-1} \pm 2.6\,\%$ |
| ATLAS | 13 | $\frac{d^2\sigma}{dm_{jj}dy^*}$ | 0.4 | $3.2\,\mathrm{fb}^{-1} \pm 2.1\,\%$ |
| CMS | 13 | $\frac{d^2\sigma}{dm_{jj}dy_{max}}$ | 0.8 | $33.5\,\mathrm{fb}^{-1} \pm 1.2\,\%$ |
| CMS | 13 | $\frac{d^3\sigma}{dm_{jj}dy^*dy_b}$ | 0.8 | $29.6\,\mathrm{fb}^{-1} \pm 1.2\,\%$ |

**used for additional fit variant**

(2D and 3D cannot be used simultaneously due to unknown correlations)

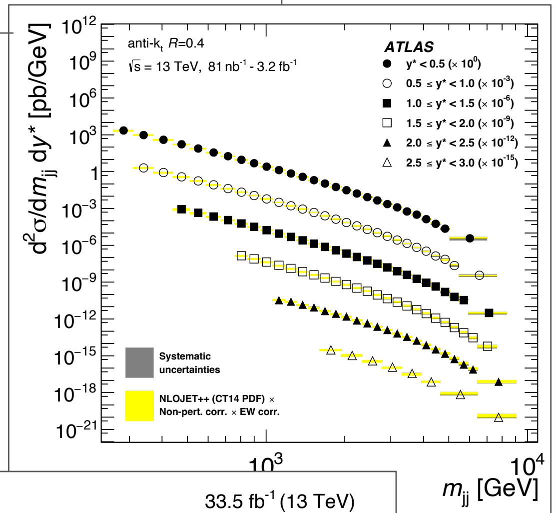


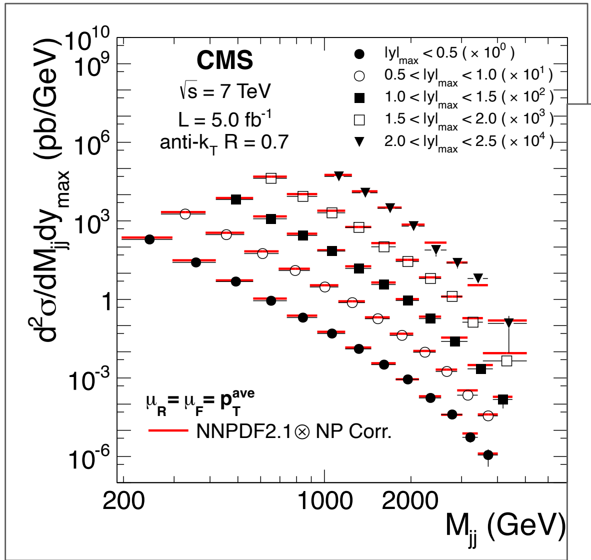

arXiv:1312.3524

$y^* = |y_1 - y_2|/2$

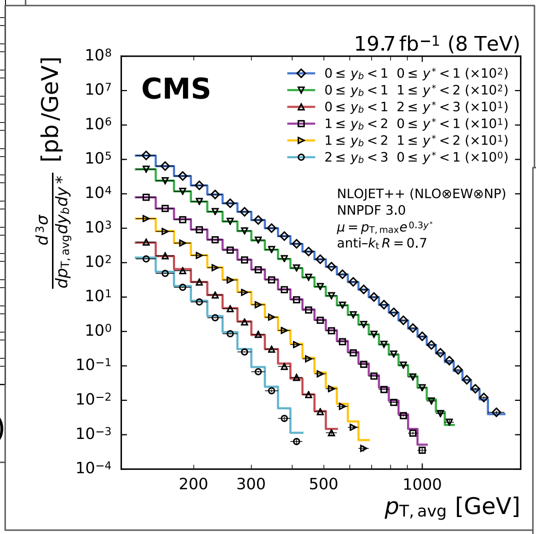
$y_b = |y_1 + y_2|/2$

arXiv:1711.02692



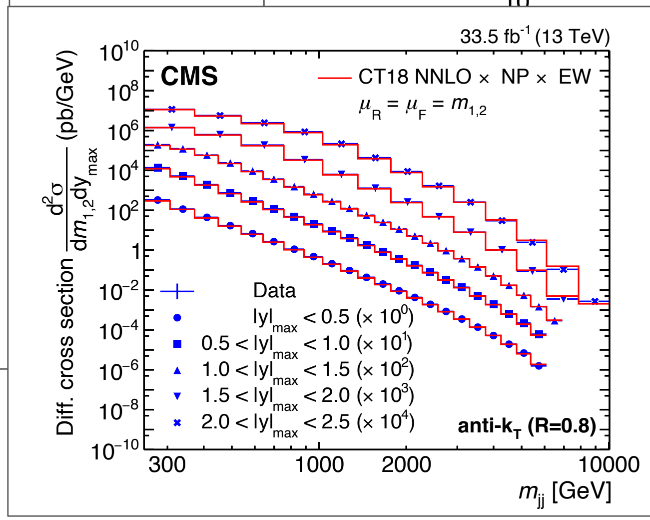




arXiv:1212.6660




arXiv:1705.02628

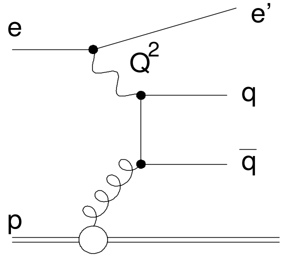
arXiv:2312.16669

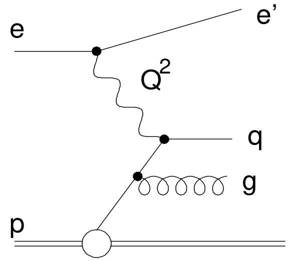



selection: **ymax** or **y*** = **|y1-y2|/2 < 2** ; **yb** = **|y1+y2|/2 < 1** :

**367** LHC cross sections used in **$\alpha_s$** determination

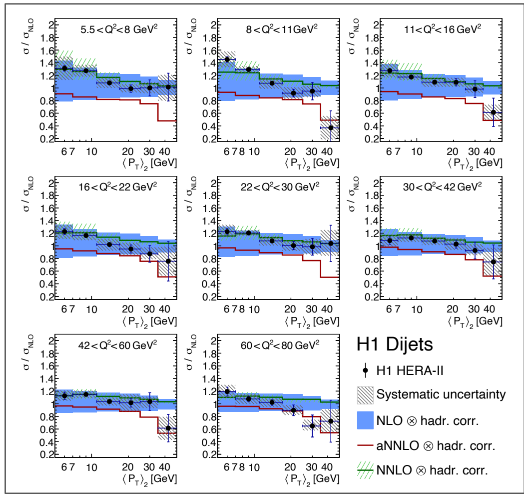
# HERA dijet data

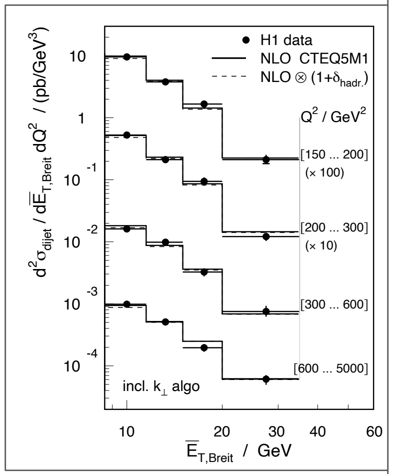


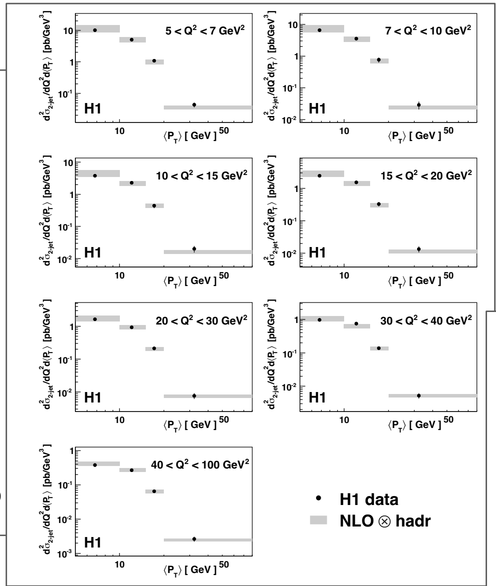

| Data set | $\sqrt{s}$ [GeV] | Cuts |
|---|---|---|
| H1 300 GeV high-$Q^2$ | 300 | – |
| H1 HERA-I low-$Q^2$ | 320 | $\mu > 2m_b$ |
| H1 HERA-II low-$Q^2$ | 320 | $\mu > 2m_b$ |
| H1 HERA-II high-$Q^2$ | 320 | – |
| ZEUS HERA-I+II high-$Q^2$ | 320 | $\langle p_T \rangle_{1,2} > 15\,\text{GeV}$ |

arXiv:1611.03421



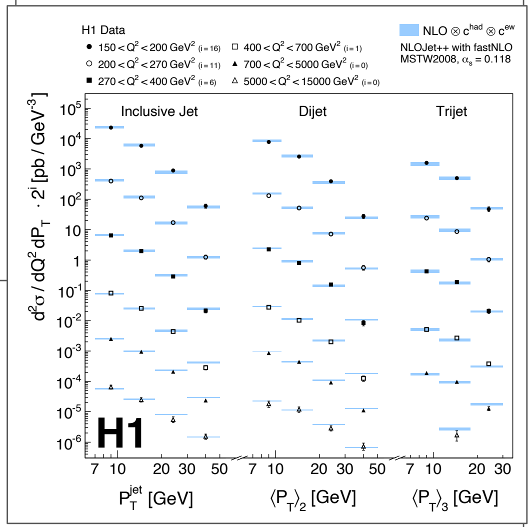



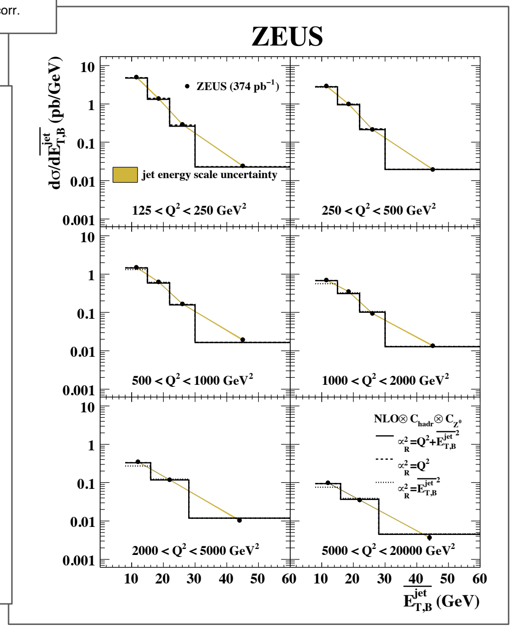

arXiv:0010054




arXiv:0911.5678

arXiv:1010.6167




arXiv:1406.4709

all HERA dijet measurements have been used in previous NLO and/or NNLO **$\alpha_s$** extractions; well understood data and theory

**486** HERA+LHC cross sections in combined **$\alpha_s$** fit

*methodology as used in:*
arXiv:1709.07251
arXiv:1906.05303

# $\alpha_s$ fit methodology

- **$\alpha_s$** determined in $X^2$ minimisation (MIGRAD)
- **$\alpha_s(M_Z)$** is a free parameter to NNLO theory calculation **$\sigma$**i

$$\chi^2 = \sum_{i,j} \log \frac{\varsigma_i}{\sigma_i} \left(V_{\text{exp}} + V_{\text{NP}} + V_{\text{NNLOstat}} + V_{\text{PDF}}\right)^{-1}_{ij} \log \frac{\varsigma_j}{\sigma_j}$$

| | |
|---|---|
| $\zeta_i$ | LHC or HERA jet data |
| $\sigma_i$ | NNLO theory |
| V | covariance matrices |

- **QCD** cross section sensitive to **$\alpha_s$**

$$\mathrm{d}\sigma = \sum_{a,b} \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} f_a(x_1, \mu_\mathrm{F}) f_b(x_2, \mu_\mathrm{F}) \mathrm{d}\hat{\sigma}_{ab}(\mu_\mathrm{R}, \mu_\mathrm{F}) \,.\, c_{\mathrm{NP}} \,.\, c_{\mathrm{EW}}$$

**non-pertubative (NP) and electroweak (EW) correction factors**

**pdfs**

**matrix element (ME) / partonic cross section**

where both the **pdfs** and **ME** depend on **$\alpha_s$**:

$$\mathrm{d}\hat{\sigma}_{ab}(\alpha_\mathrm{s}) = \left(\frac{\alpha_\mathrm{s}(\mu)}{2\pi}\right)^2 \mathrm{d}\hat{\sigma}_{ab,\mathrm{LO}} + \left(\frac{\alpha_\mathrm{s}(\mu)}{2\pi}\right)^3 \mathrm{d}\hat{\sigma}_{ab,\mathrm{NLO}} + \left(\frac{\alpha_\mathrm{s}(\mu)}{2\pi}\right)^4 \mathrm{d}\hat{\sigma}_{ab,\mathrm{NNLO}} + \mathcal{O}(\alpha_\mathrm{s}^5(\mu))$$

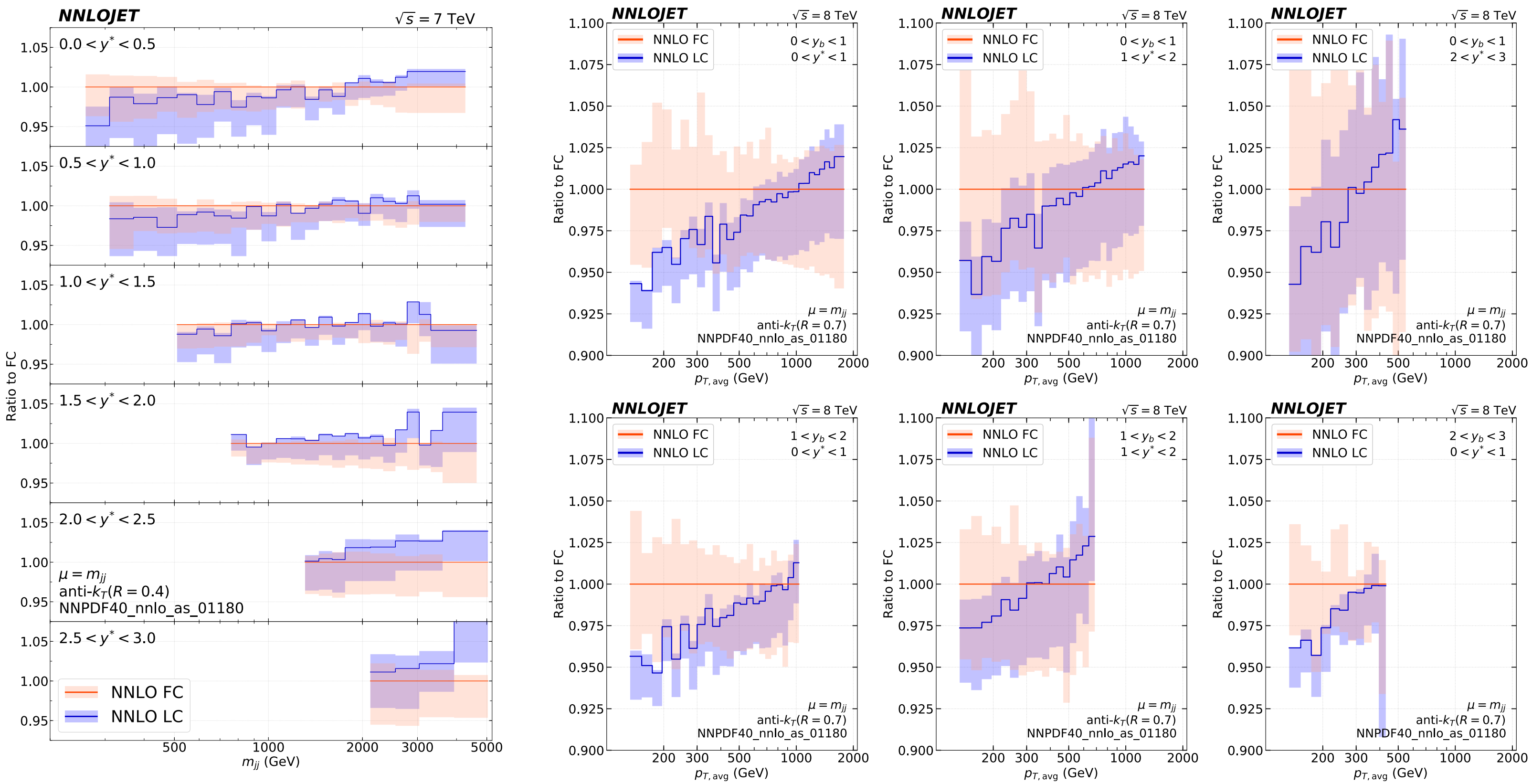
$$f_a(x, \mu, \alpha_\mathrm{s}) = (\Gamma(\mathcal{P}, \mu, \mu_0, \alpha_\mathrm{s}) \otimes f_{\mu_0})_a$$

where Γ are the DGLAP kernels evaluated at 3-loop with 5 active flavours using Apfel++

- **$\alpha_s$** dependence of the **pdfs** accounted for by fixing x-dependence at a start scale of $\mu_0$ (pdf≡$f_{\mu 0}$) and evolving using DGLAP
- **PDF4LHC21** used for nominal fit

# NNLO calculations

- **NNLO** parton level cross section obtained from **NNLOJET**
- **(pp)** arXiv:1905.09047, 1705.10271, 1801.06415; **(ep)** arXiv:1703.05977, 1606.03991
- **includes sub-leading colour contributions** (arXiv:1907.12911, 2204.10173, 2208.02115);
  **used in this work for the first time in $\alpha_s$ determination with LHC jet data**



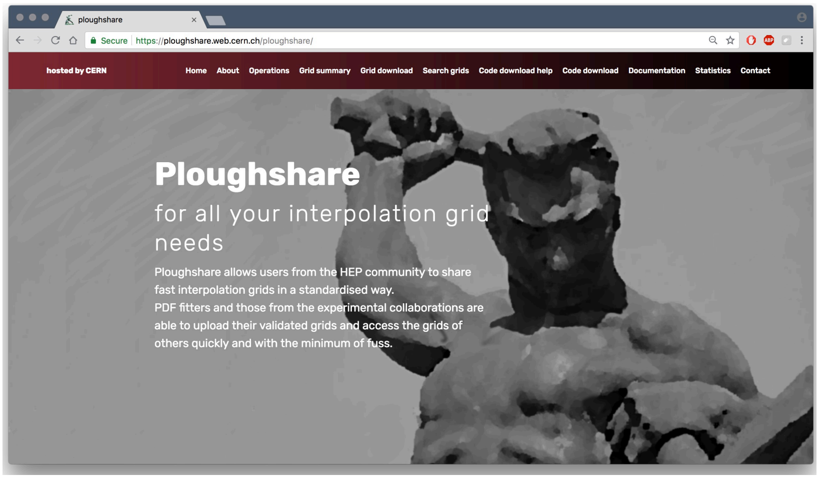

ATLAS 7 TeV and CMS 8TeV (triple differential) dijet cross section phase space (arXiv:2204.10173)

# APPLfast

- $\alpha_s$ fit algorithm requires (re-)calculation of cross section predictions multiple times with different values of $\alpha_s(M_Z)$ (and corresponding pdfs)
- **NNLOJET** interfaced to **APPLfast** library (arXiv:1906.05303, 2207.13735) for **fast reproduction of cross section calculations** with different $\alpha_s(M_Z)$ and **pdfs**
- **APPLfast:** generic interface between NNLOJET and both **APPLGRID** (EPJC66 (2010) 503) and **fastNLO** (arXiv:1208.3641)
- → NNLO pQCD coefficients stored on "interpolation grids", independently of $\alpha_s(M_Z)$ and **pdfs**

- resulting interpolation grids for the HERA and LHC dijet datasets have typically **sub-permille accuracy** in reproducing full calculation

- all interpolation grids will be made available on Ploughshare (in both APPLGRID or fastNLO format)




ploughshare.web.cern.ch

# $\alpha_s$ fit uncertainties

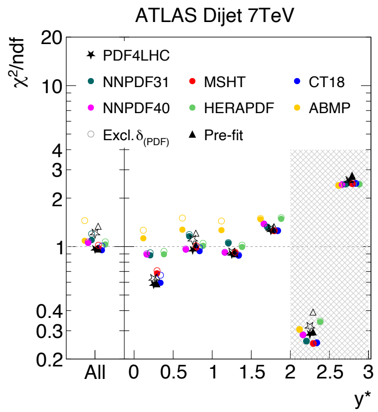
- **$\alpha_s(M_Z)$ fit uncertainties:** $(\mathrm{fit,PDF})$ $$\chi^2 = \sum_{i,j} \log\frac{\varsigma_i}{\sigma_i}\,(V_{\mathrm{exp}} + V_{\mathrm{NP}} + V_{\mathrm{NNLOstat}} + V_{\mathrm{PDF}})^{-1}_{ij} \log\frac{\varsigma_j}{\sigma_j}$$
- encoded in covariance matrices V, and consist of:
- **experimental (exp)**, including statistical and systematic correlations as reported by the experiment collaborations
- **non-perturbative (NP)**, provided by the experimental collaborations (evaluated using different MC models)
- **NNLO statistical**, arising from the MC integration in NNLOJET (typically at the percent level or less)
- **pdf uncertainties**, obtained from the relevant pdf set and evaluated at **$\mu_0$**

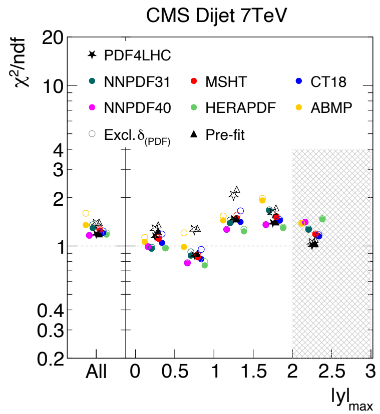
- **starting scale of pdf evolution:** $(\mu_0)$
- varied by factors of 0.5 and 2 about the nominal of **$\mu_0$=90 GeV**
- also covers uncertainty arising from an **$\alpha_s(M_Z)$** variation in the original pdf

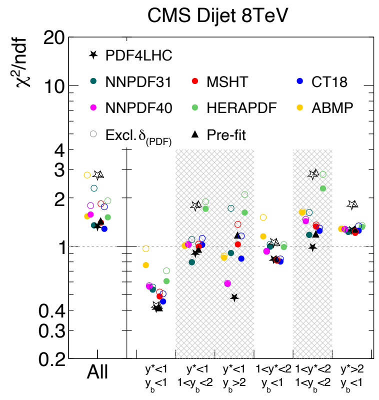
- **scale uncertainties:** $(\mu_{\mathrm{R}},\mu_{\mathrm{F}})$
- **renormalisation ($\mu_R$)** and **factorisation scale ($\mu_F$)** varied independently by factors of 0.5, 1 or 2, **7-point scale variation** ( omits variations (0.5,2) and (2,0.5) )
- nominal scales: **LHC**: $\mu_R = \mu_F =$ **mjj** ; **HERA**: $\mu_R = \mu_F = \sqrt{Q^2 + \langle p_T\rangle^2_{1,2}}$
- scale variations for **LHC** and **HERA** considered **uncorrelated** (uncerts added in quadrature for **HERA+LHC** fits)

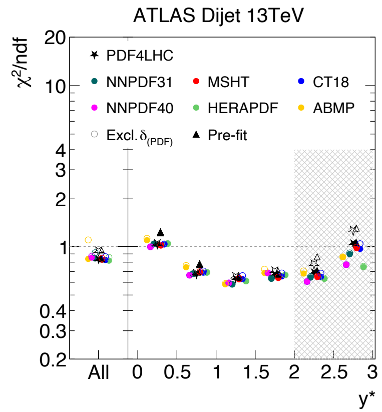
# consistency study

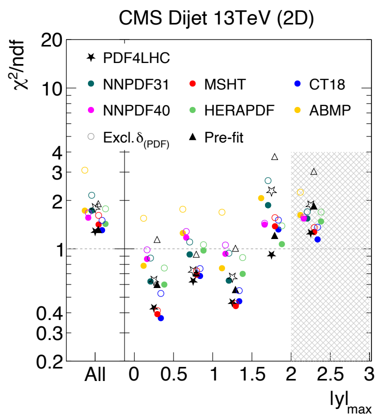
- $\alpha_s$ determined in individual bins of datasets:



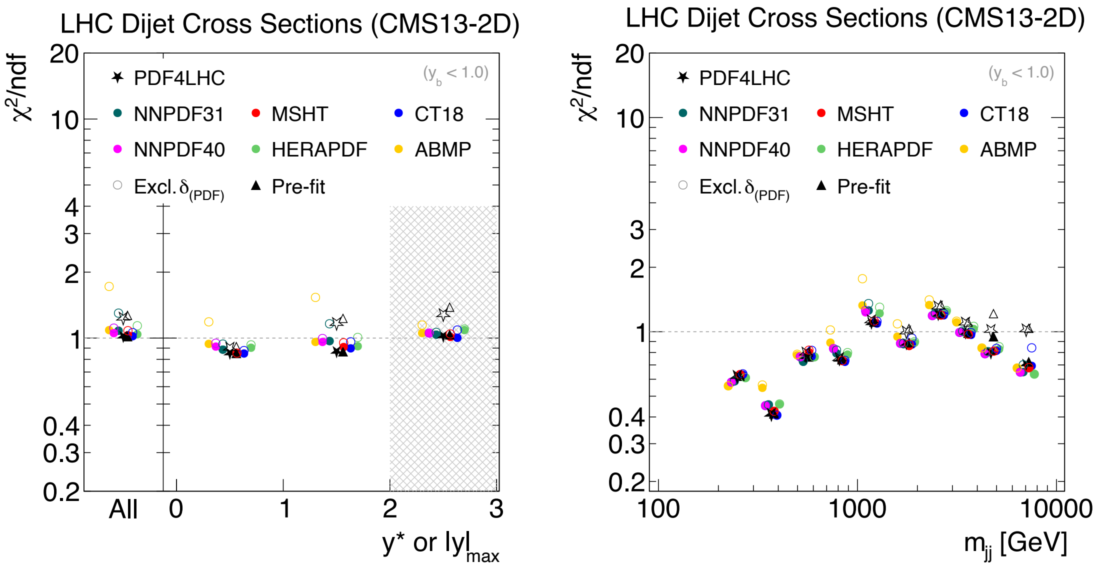




*all LHC datasets:*




- pre- and post-fit $X^2$ reasonable; exceptions only at very large y*, yb or ymax
- $y^{*,(max)} < 2$, yb < 1 in nominal fits
- $X^2$ using different pdfs reasonably consistent

- **overall, the NNLO calculations provide good description**

# $\alpha_s$ fit results

$\alpha_s(m_Z)\,(\mathrm{fit,PDF})\,(\mu_0)\,(\mu_R,\mu_F)$

- **perform fits to:**
- **each** individual LHC dijet data set
- **all** LHC dijet data sets (with *either* CMS 2D *or* 3D)

| Data set | $\chi^2/n_{\mathrm{dof}}$ | $\alpha_s(m_Z)$ |
|---|---|---|
| ATLAS 7 TeV | 74.7/ 77 | 0.1193 (33) (4) ( 6) |
| ATLAS 13 TeV | 87.7/106 | 0.1145 (32) (4) (16) |
| CMS 7 TeV | 50.7/ 45 | 0.1151 (39) (1) ( 9) |
| CMS 8 TeV | 37.0/ 56 | 0.1173 (25) (1) (11) |
| CMS 13 TeV (2D) | 71.6/ 78 | 0.1209 (25) (2) (20) |
| CMS 13 TeV (3D) | 137.7/112 | 0.1181 (20) (1) (15) |
| LHC dijets (CMS13-2D) | 335.3/366 | 0.1178 (14) (1) (17) |
| LHC dijets (CMS13-3D) | 397.9/400 | 0.1172 (14) (1) (14) |

- **$\alpha_s$ from individual data sets:**
- all fits with good $X^2$
- experimental uncert: 0.0020 – 0.0039
- data sets with larger integrated luminosities or higher CM energies yield smaller uncertainties

- **$\alpha_s$ determination using all 5 LHC dijet data sets** yields reduced experimental uncertainties

  (benefits from independent measurements, extended kinematic ranges and multiple CM energies)

- **$\alpha_s$ including HERA+LHC**
- good consistency between HERA and LHC
- further reduced experimental uncertainty
- scale uncertainty dominates as HERA data sit at lower energy scales

| Data set | $\chi^2/n_{\mathrm{dof}}$ | $\alpha_s(m_Z)$ |
|---|---|---|
| HERA | 92.8/118 | 0.1177 (14) (1) (34) |
| LHC+HERA (CMS13-2D) | 428.4/485 | 0.1180 (10) (1) (22) |
| LHC+HERA (CMS13-3D) | 491.0/519 | 0.1177 (10) (1) (24) |

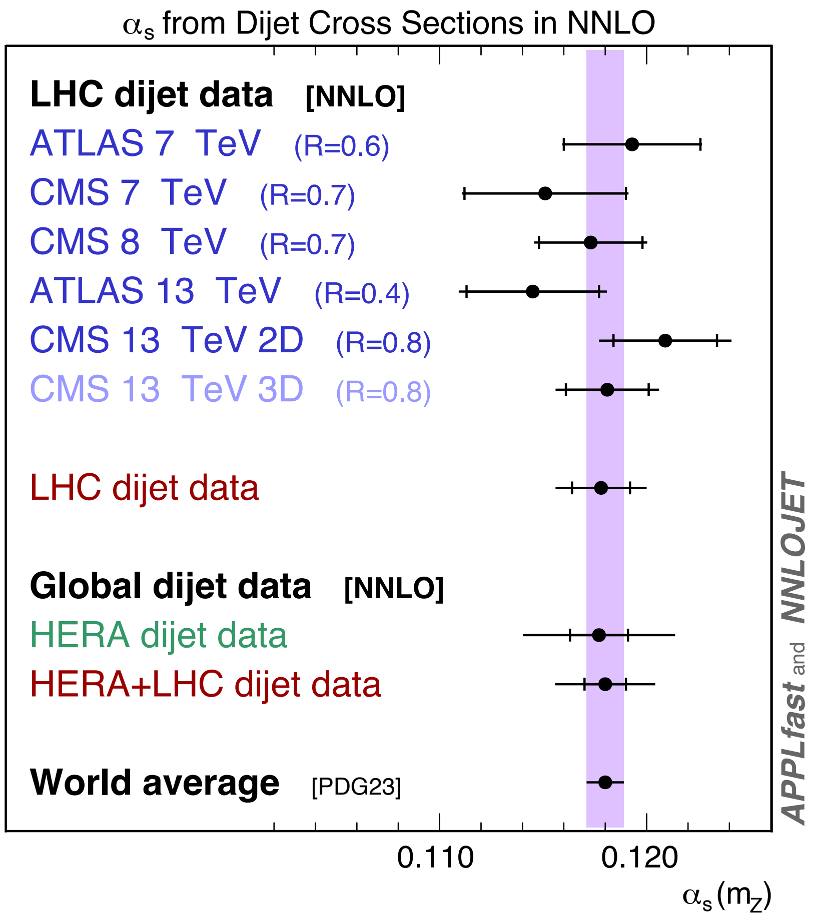
# $\alpha_s$ at the scale of MZ

- **excellent consistency with world average:**
- **$\alpha_s$ main result ("LHC dijet"):**
- uses **all** 5 LHC dijet datasets
- with CMS 2D (smaller sensitivity to pdf parameters, and reaches higher values of mjj c.f. 3D)

$$\alpha_s(m_Z) = 0.1178\ (14)_{(\mathrm{fit,PDF})}\ (1)_{(\mu_0)}\ (17)_{(\mu_R,\mu_F)}$$

- **$\alpha_s$ using HERA dijets** represents first NNLO determination using only DIS dijet production from both H1 and ZEUS
- **$\alpha_s$ determination including HERA+LHC**

$$\alpha_s(m_Z) = 0.1180\,(10)_{(\mathrm{fit,PDF})}\,(1)_{(\mu_0)}\,(22)_{(\mu_R,\mu_F)}$$

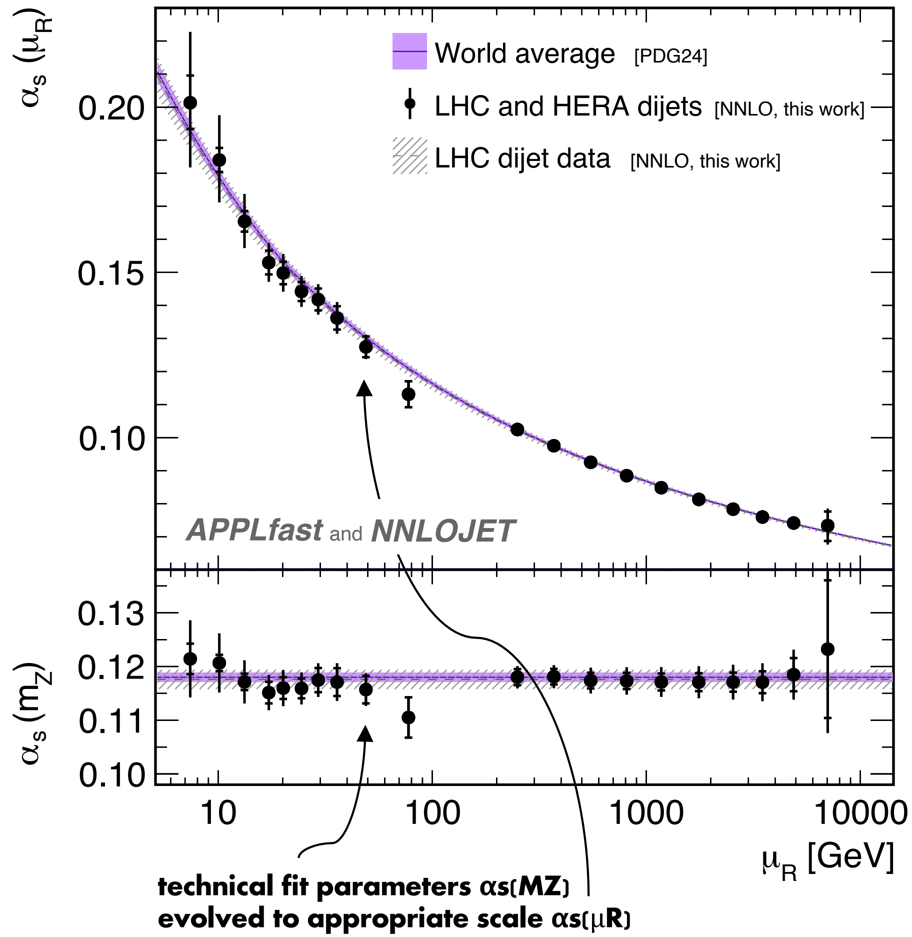


# $\alpha_s$ running

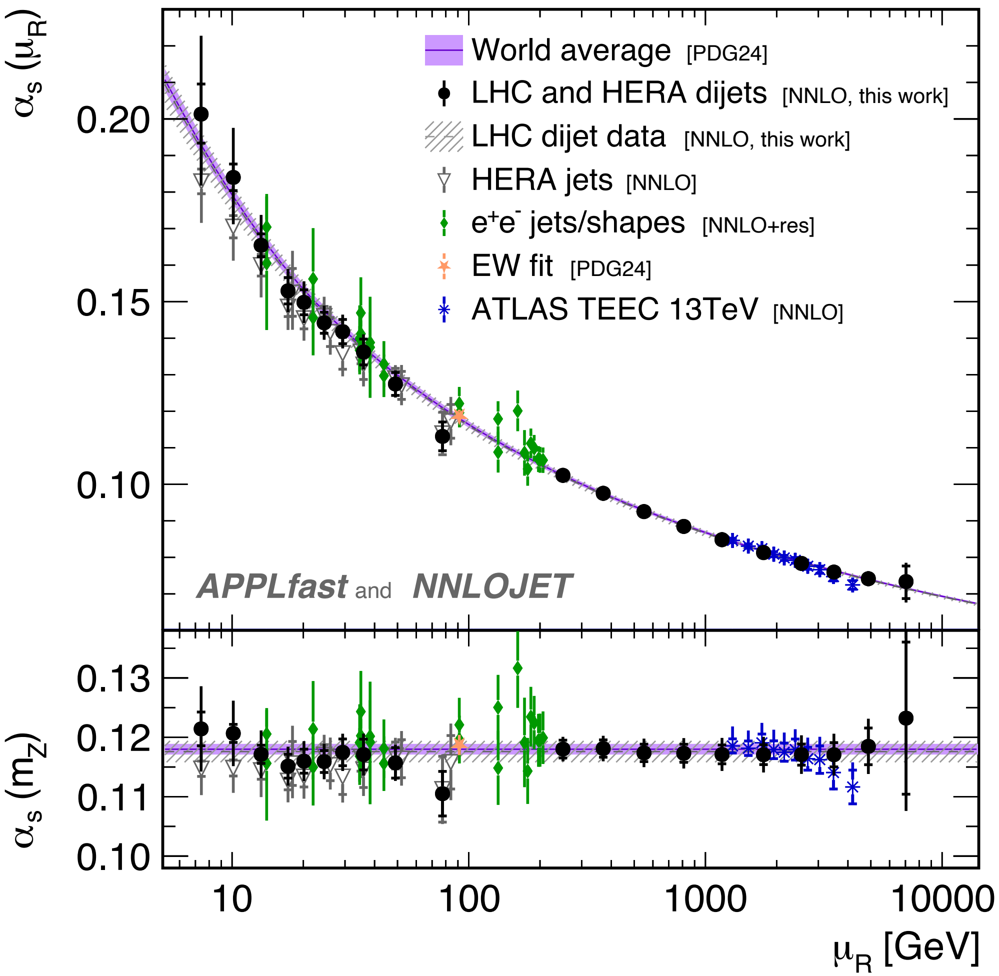
- **test running of strong coupling**:
- perform **$\alpha_s$** fits to subsets of data points at similar scale
- assumes running to be valid within limited range covered by interval
- all fits have good $X^2$

- **results**:
- consistent with expectation at all scales (7 GeV $\lesssim \mu \lesssim$ 7 TeV )
- at scales of a few hundred GeV, experimental and theory uncertainties of similar size (±0.0015)
- scale uncertainty dominates at low μ

- **scales up to 7 TeV probed for the first time**



# $\alpha_s$ in a global context




- smaller uncertainties and cover significantly larger range in scale than any previous determination

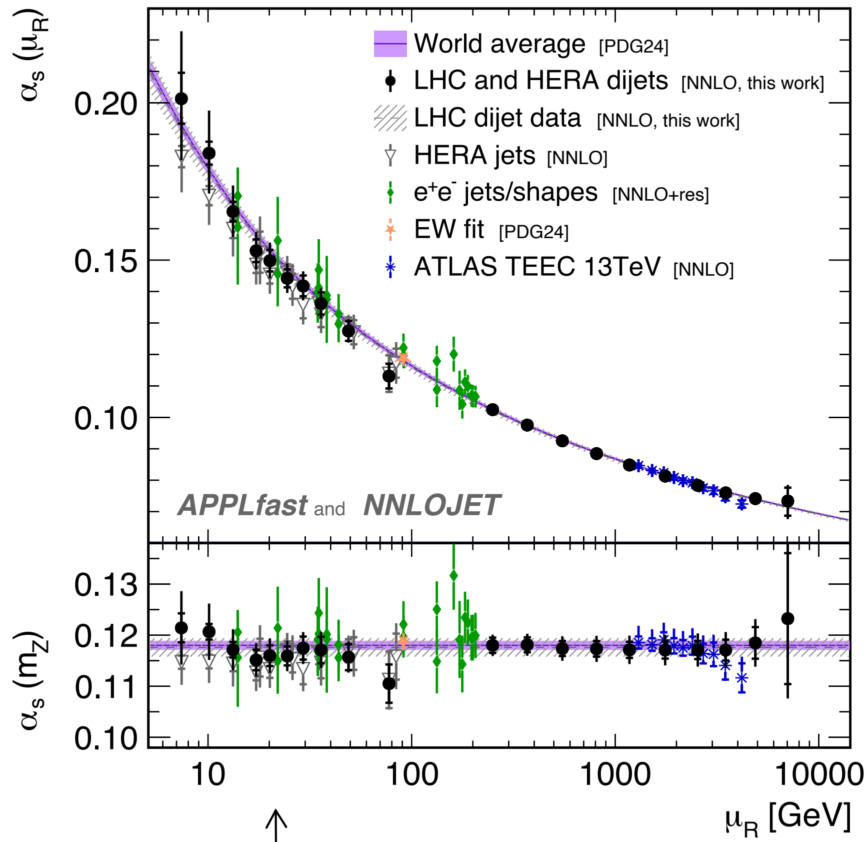
# summary

- **strong coupling $\alpha_s$** determined from dijet data for the first time based on **complete NNLO QCD calculations** (with sub-leading colour contributions)

- using multiple selected **LHC dijet** data sets from ATLAS and CMS at 7, 8 and 13 TeV, $\alpha_s(M_Z)$ is determined:

$$\boxed{\alpha_s(m_Z) = 0.1178\,(22)_{(\mathrm{tot})}}$$

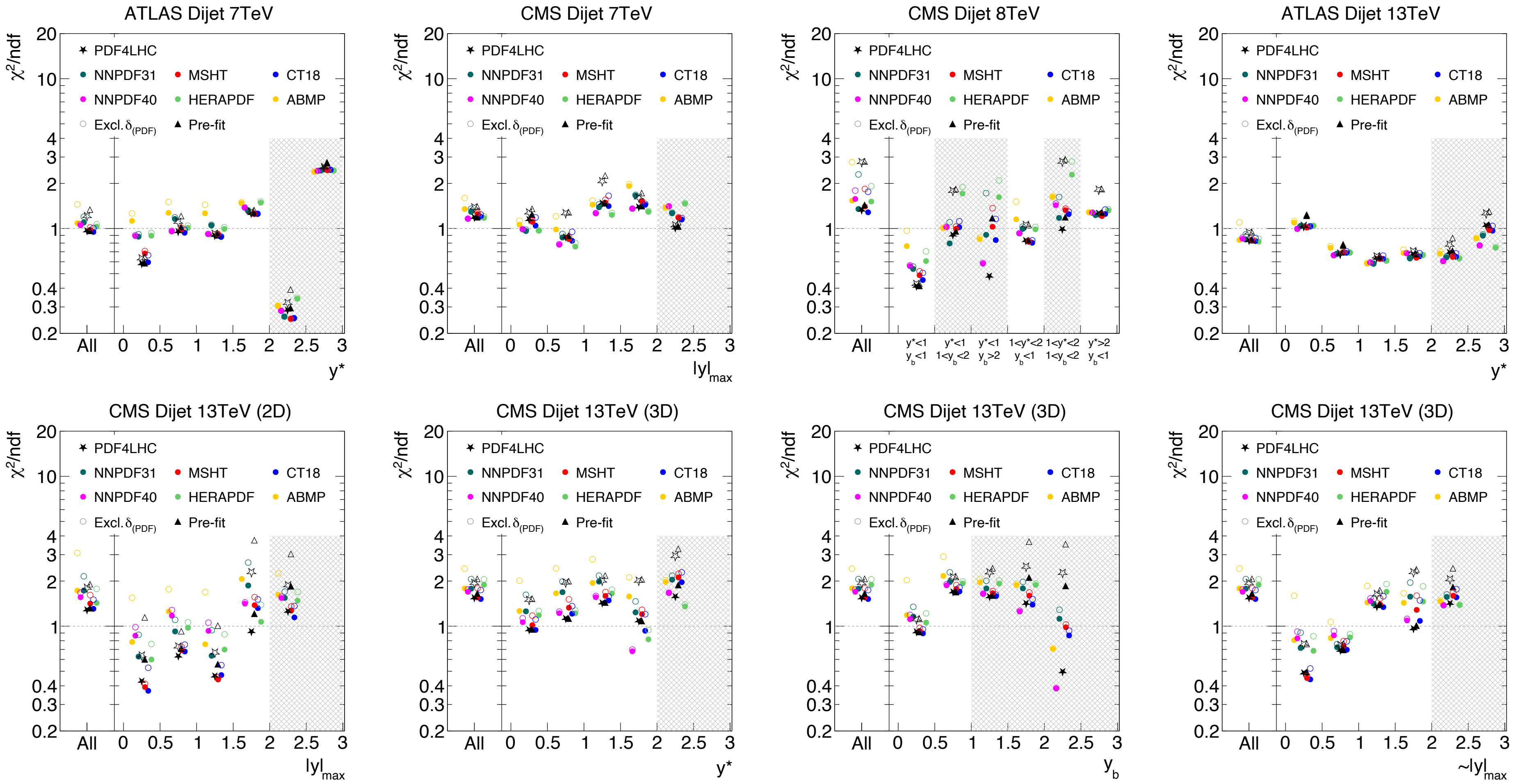
- including also dijet cross sections from **HERA** makes this **one of the most comprehensive and precise tests of the QCD renormalisation group running of $\alpha_s(\mu)$ to date**, probing unprecedented range of scales from about 7 GeV to 7 TeV




arXiv:2412.21165

# extras

# consistency study



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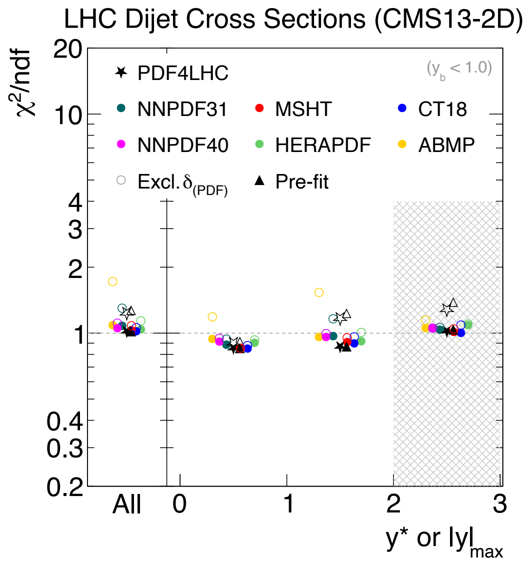
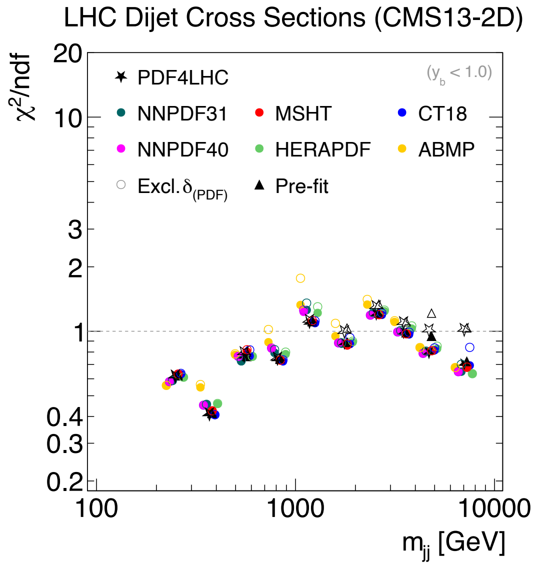
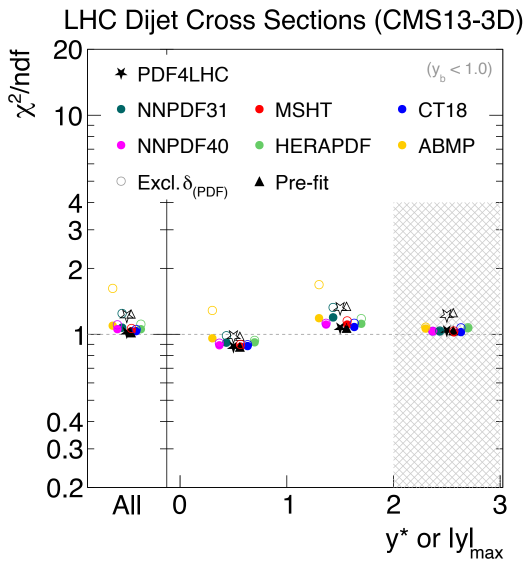
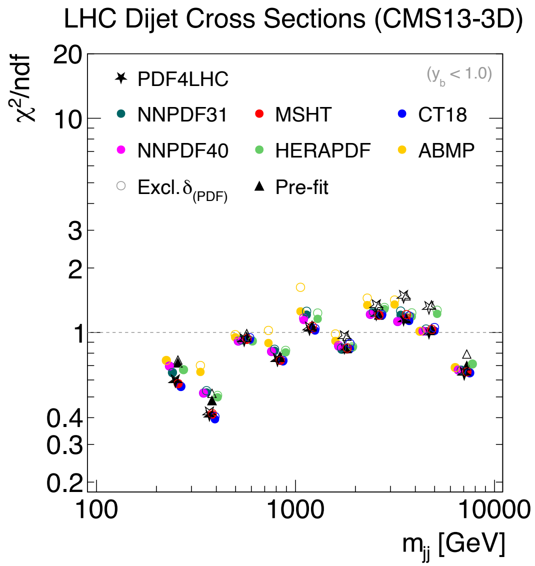
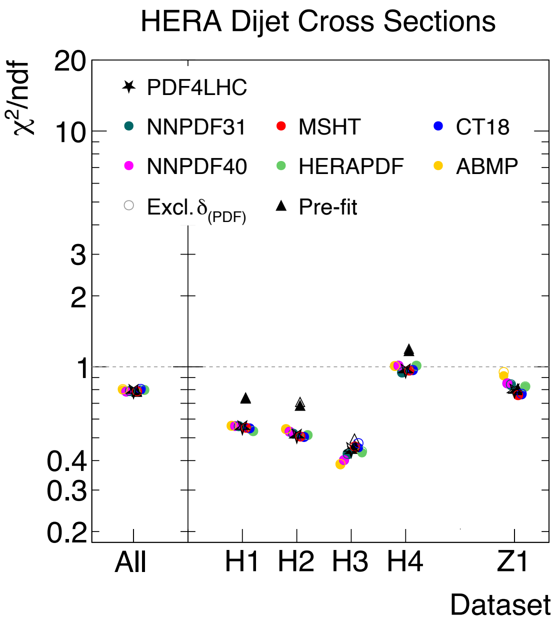


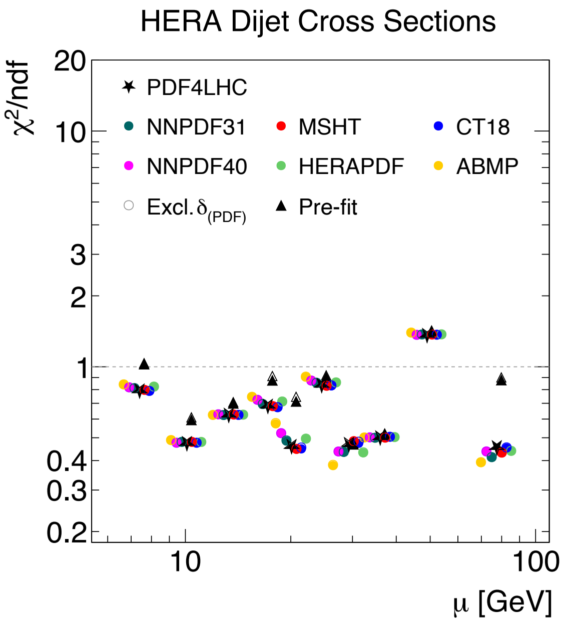



# consistency study

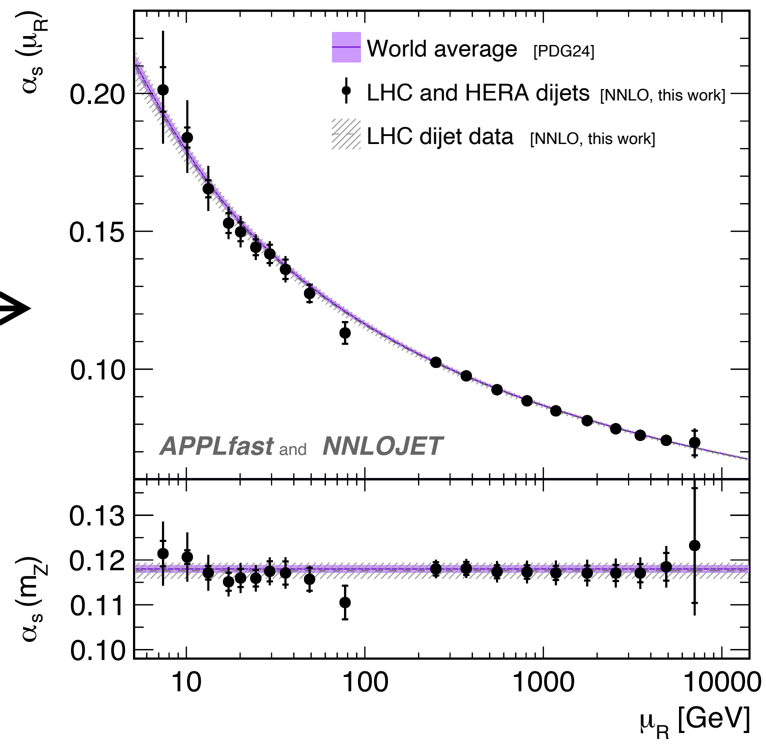


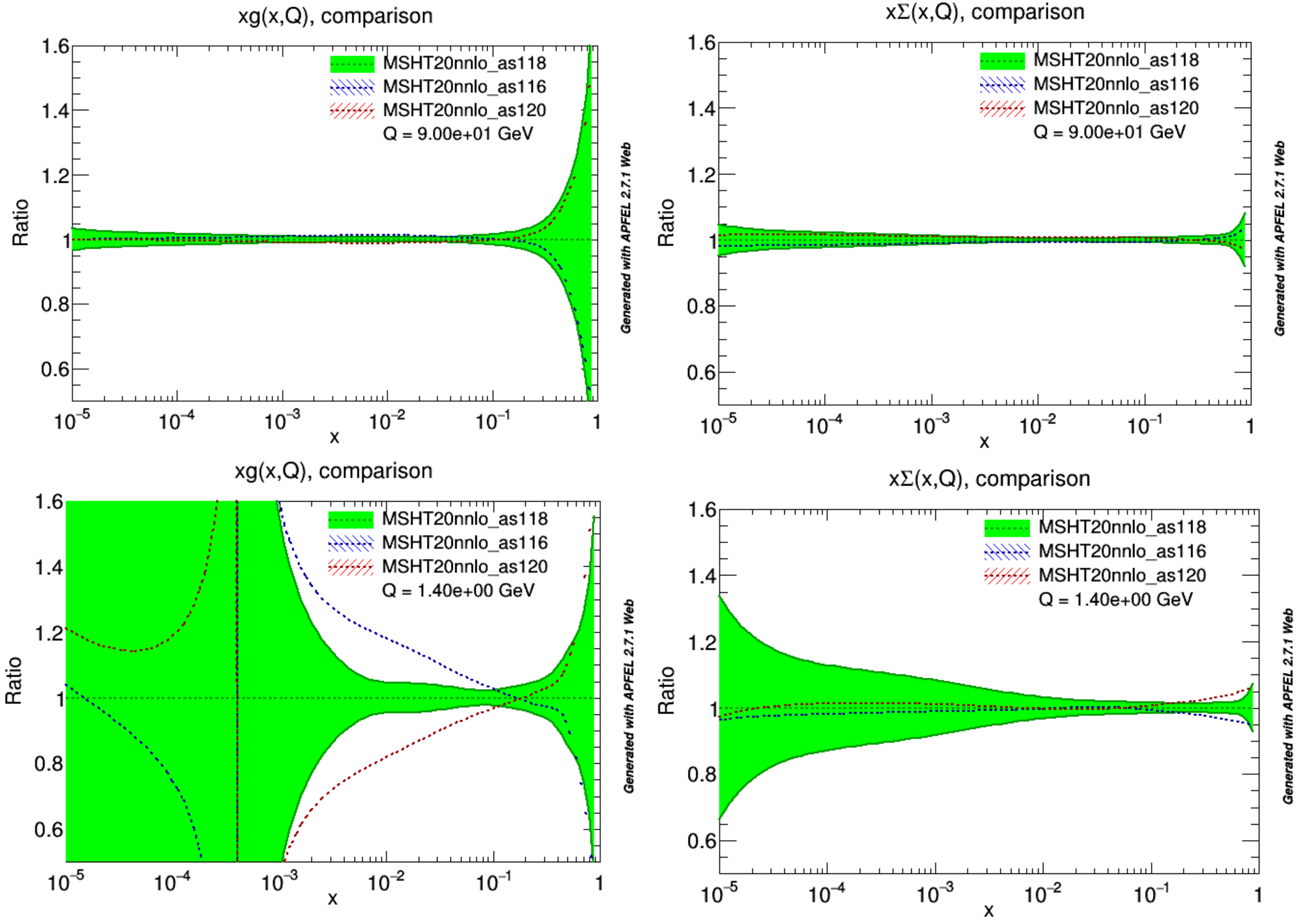
# Running of $\alpha_s$

| $\mu_R^{avg}$ [GeV] | $\alpha_s(m_Z)$ | $\alpha_s(\mu_R)$ |
|---|---|---|
| 7.4 | 0.1214 (28) (1) (69) | 0.2013 (82) (4) (196) |
| 10.1 | 0.1207 (15) (1) (56) | 0.1840 (37) (2) (130) |
| 13.3 | 0.1171 (15) (0) (41) | 0.1654 (31) (0) (77) |
| 17.2 | 0.1151 (20) (0) (29) | 0.1530 (36) (1) (47) |
| 20.1 | 0.1160 (20) (1) (31) | 0.1498 (34) (1) (46) |
| 24.5 | 0.1159 (18) (0) (27) | 0.1442 (29) (1) (37) |
| 29.3 | 0.1175 (23) (0) (24) | 0.1418 (33) (0) (32) |
| 36.0 | 0.1171 (26) (0) (25) | 0.1362 (35) (1) (33) |
| 49.0 | 0.1157 (26) (1) (17) | 0.1275 (31) (1) (20) |
| 77.5 | 0.1105 (37) (3) ( 8) | 0.1131 (39) (3) (12) |
| 250 | 0.1180 (15) (1) (14) | 0.1025 (11) (1) (11) |
| 370 | 0.1181 (15) (1) (16) | 0.0975 (10) (1) (11) |
| 550 | 0.1174 (15) (1) (19) | 0.0925 ( 9) (1) (12) |
| 810 | 0.1173 (15) (2) (20) | 0.0885 ( 9) (1) (11) |
| 1175 | 0.1171 (16) (2) (22) | 0.0848 ( 8) (1) (12) |
| 1760 | 0.1171 (17) (2) (24) | 0.0813 ( 8) (1) (12) |
| 2545 | 0.1171 (18) (2) (26) | 0.0783 ( 8) (1) (12) |
| 3490 | 0.1171 (20) (2) (28) | 0.0760 ( 8) (1) (12) |
| 4880 | 0.1185 (31) (3) (34) | 0.0742 (12) (1) (13) |
| 7040 | 0.1232 (128) (12) (37) | 0.0734 (43) (4) (30) |

$\longrightarrow$



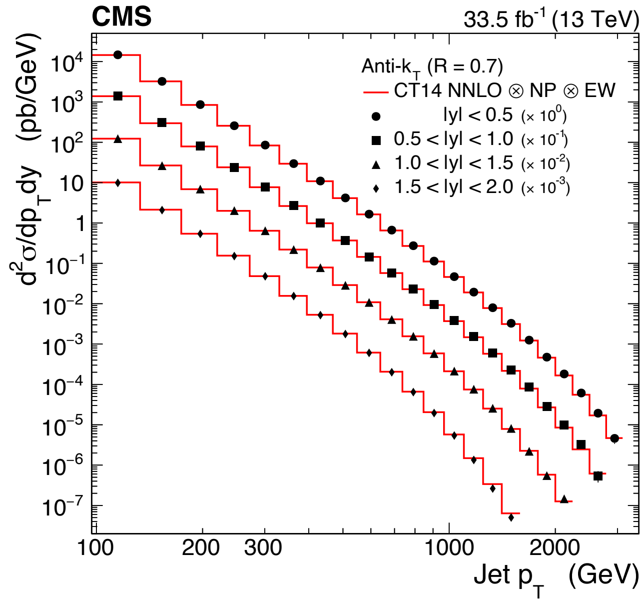
# impact on the pdfs of varying αs at the start scale



# $\alpha_s$ extraction using CMS inclusive jet data

| $\sqrt{s}$ [TeV] | $\mathcal{L}$ [fb$^{-1}$] | $N_p$ | $p_T$ [GeV] | $\lvert y \rvert$ |
|---|---|---|---|---|
| 2.76 | 0.0054 | 80 | 74–592 | 0.0–3.0 |
| 7 | 5.0 | 130 | 114–2116 | 0.0–2.5 |
| 8 | 20 | 165 | 74–1784 | 0.0–3.0 |
| 13 | 33.5 | 78 | 97–3103 | 0.0–2.0 |

CMS 13 TeV inclusive jet



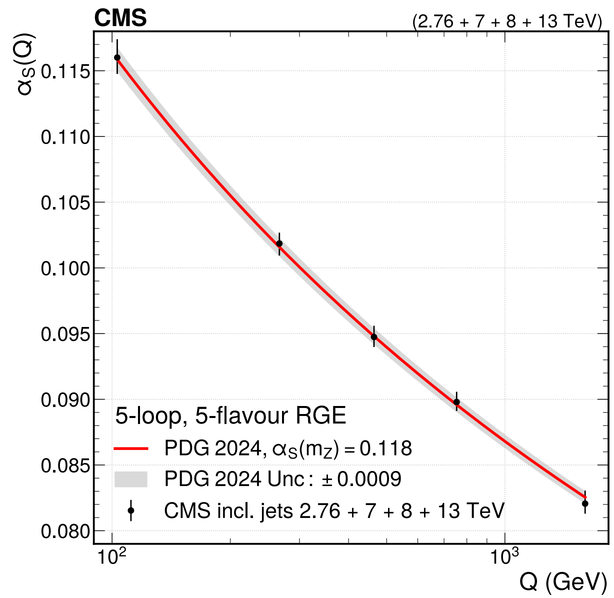

- CMS **NNLO** $\alpha_s$ extraction in **LC** approximation:

- **$\alpha_s(M_Z)$ :**

$$\boxed{\alpha_S(m_Z) = 0.11759\,^{+0.0009}_{-0.0009}\,(\text{fit})\,^{+0.0006}_{-0.0004}\,(\text{model})\,^{+0.0009}_{-0.0012}\,(\text{scale})\,^{+0.}_{-0.00004}\,(\text{param.})}$$

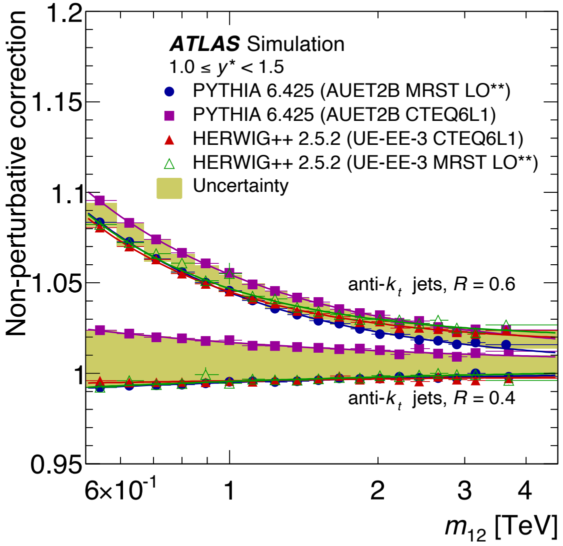
- **running of $\alpha_s$ :**
- covering energy scales between 100 and 1600 GeV

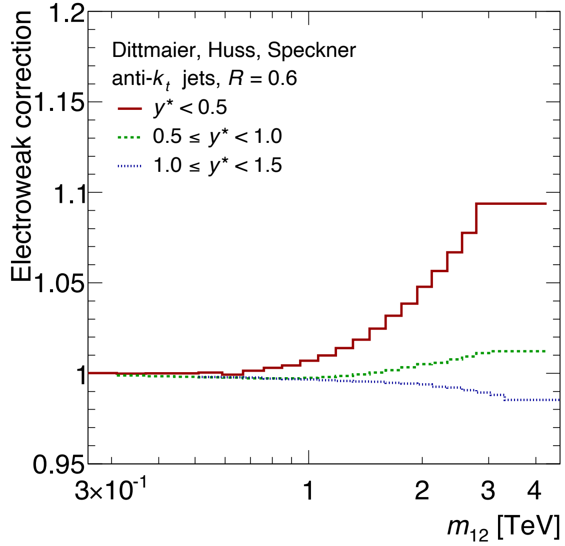
| $p_T$ (GeV) | $\langle Q \rangle$ | $\alpha_S(m_Z)$ (tot) | $\alpha_S(Q)$ | (fit) | (scale) | (model) | (param.) | (tot) |
|---|---|---|---|---|---|---|---|---|
| 74–220 | 103.06 | $0.1182\,^{+0.0013}_{-0.0012}$ | 0.1160 | $^{+0.0011}_{-0.0011}$ | $^{+0.0007}_{-0.0005}$ | $^{+0.0006}_{-0.0004}$ | $^{+0.0}_{-0.0}$ | $^{+0.0014}_{-0.0012}$ |
| 220–395 | 266.63 | $0.1184\,^{+0.0011}_{-0.0012}$ | 0.1019 | $^{+0.0007}_{-0.0007}$ | $^{+0.000021}_{-0.0004}$ | $^{+0.0004}_{-0.00029}$ | $^{+0.0}_{-0.000033}$ | $^{+0.0008}_{-0.0009}$ |
| 395–638 | 464.31 | $0.1179\,^{+0.0012}_{-0.0012}$ | 0.0947 | $^{+0.0007}_{-0.0007}$ | $^{+0.00033}_{-0.000032}$ | $^{+0.0004}_{-0.00027}$ | $^{+0.0}_{-0.000008}$ | $^{+0.0009}_{-0.0008}$ |
| 638–1410 | 753.66 | $0.1184\,^{+0.0013}_{-0.0012}$ | 0.0898 | $^{+0.0006}_{-0.0006}$ | $^{+0.00027}_{-0.00004}$ | $^{+0.0004}_{-0.00026}$ | $^{+0.000005}_{-0.0}$ | $^{+0.0008}_{-0.0007}$ |
| 1410–3103 | 1600.5 | $0.1170\,^{+0.0020}_{-0.0016}$ | 0.0821 | $^{+0.0007}_{-0.0007}$ | $^{+0.0004}_{-0.0}$ | $^{+0.0005}_{-0.00034}$ | $^{+0.00004}_{-0.0}$ | $^{+0.0010}_{-0.0008}$ |



# EG, NP and EW corrections

7 TeV




13 TeV