Perturbative uncertainties on the p_T Z/W boson spectrum

SM@LHC25 09/04/25
Durham, England

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CLUSTER OF EXCELLENCE
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Outline

- Color singlet production p_T spectrum
- Meaningful theory uncertainty and perturbative uncertainty
  - Correlations
  - Scale variations
  - Theory Nuisance Parameters (TNPs)
- Applications with TNPs
  - $\alpha_s(m_Z)$ from $Z\,p_T$ spectrum
  - W mass determination

Color singlet production p_T spectrum

- Wide-ranging applications, many precise measurements:

  ATLAS '20, ATLAS '24, CMS '17, CMS '19, LHCb '16, ...

  - determination of the strong coupling α_s
  - W mass measurement
  - weak mixing angle
  - determination of PDFs at full N^{3}LO
  - Higgs Yukawa couplings constraining to light quarks

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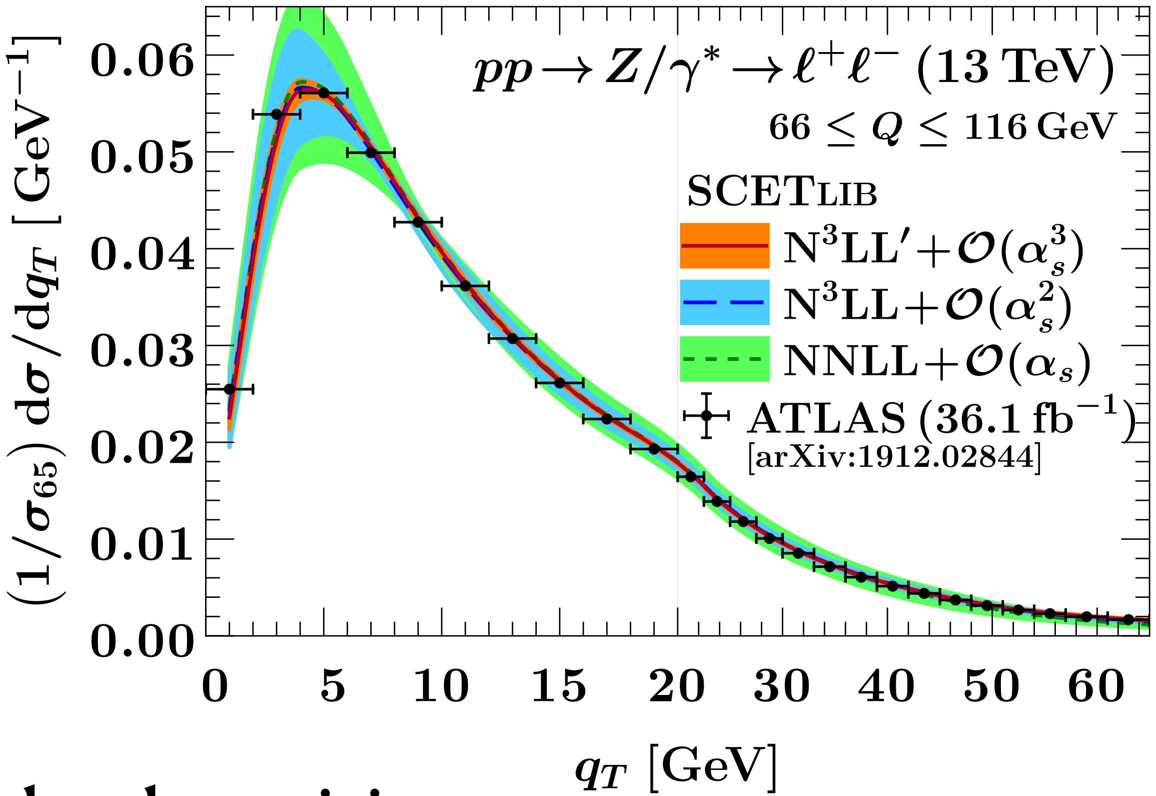

- Many theory requirements to reach $\mathcal{O}(1\%)$ level precision:
  - resummation $\mathcal{O}\left(\log^{2n}(p_T/m_Z)\right) \longrightarrow$ N^{3}LL$'$/ approx N^{4}LL
  - perturbative corrections $\mathcal{O}\left(p_T^2/Q^2\right)$
  - nonperturbative modeling $\mathcal{O}\left(\Lambda_{\mathrm{NP}}^2/p_T^2\right)$
  - finite quark mass corrections $\mathcal{O}\left(m_q^2/p_T^2\right)$
  - electroweak corrections $\left(\alpha_{\mathrm{em}} \sim \alpha_S^2\right)$

Billis, Michel, Tackmann '25,
Moos, Scimeni, Vladimirov, Zurita '24,
Camarda, Cieri, Ferrera '23,
...

Perturbative uncertainty

Consider a series expansion in a small parameter α:

$$f(\alpha) = f_0 + \alpha f_1 + \alpha^2 f_2 + \alpha^3 f_3 + \alpha^4 f_4 + \mathcal{O}(\alpha^5)$$

$$\text{LO}: \; f(\alpha) = \hat{f}_0 \pm \Delta f$$

$$\text{NLO}: \; f(\alpha) = \hat{f}_0 + \alpha \hat{f}_1 \pm \Delta f$$

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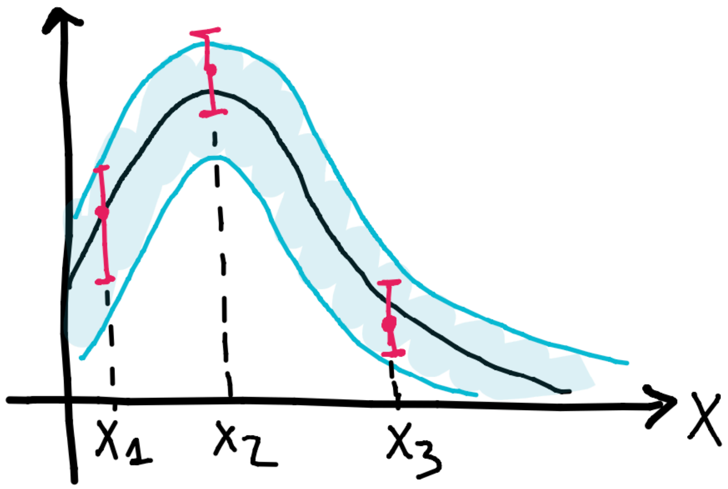
Meaningful theory uncertainty:

- must reflect our degree of knowledge (or ignorance)
- provide correct **correlations** for different predictions
- have a statistical meaning needed for the interpretation of experimental measurements

Correlations and scale variations

Taking a differential spectrum, each bin as separate predictions and separate measurements



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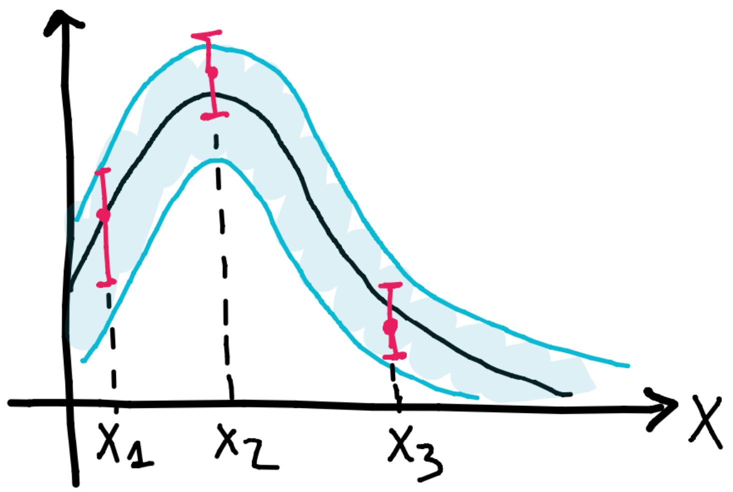

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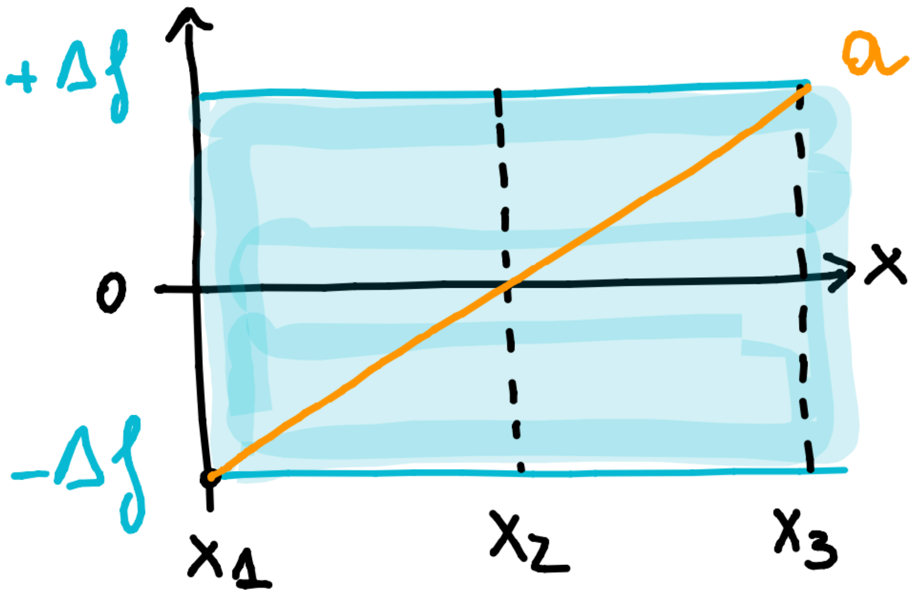
Let's be realistic: uncertainty band given by scale variations. What about its **shape**?



	ρ_{12}	ρ_{13}	ρ_{23}
a	0	−1	0

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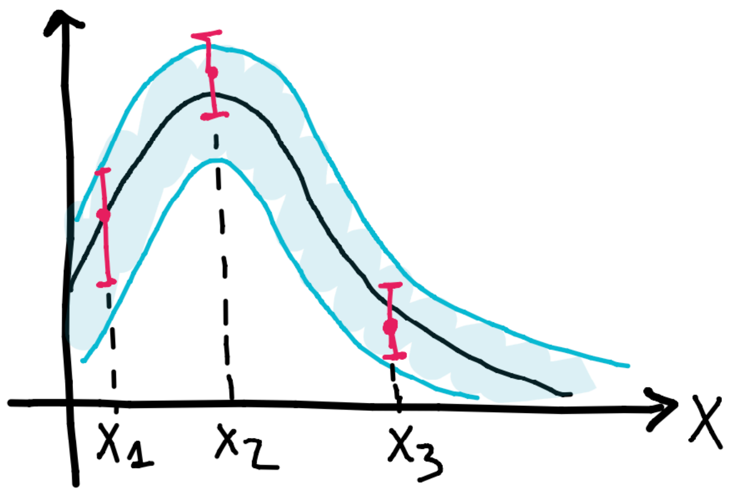

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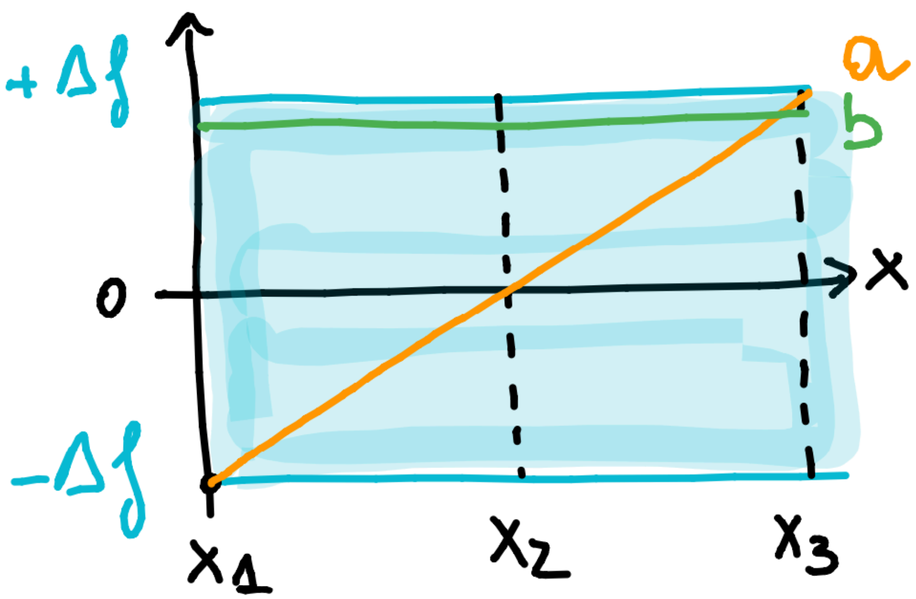

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b	1	1	1

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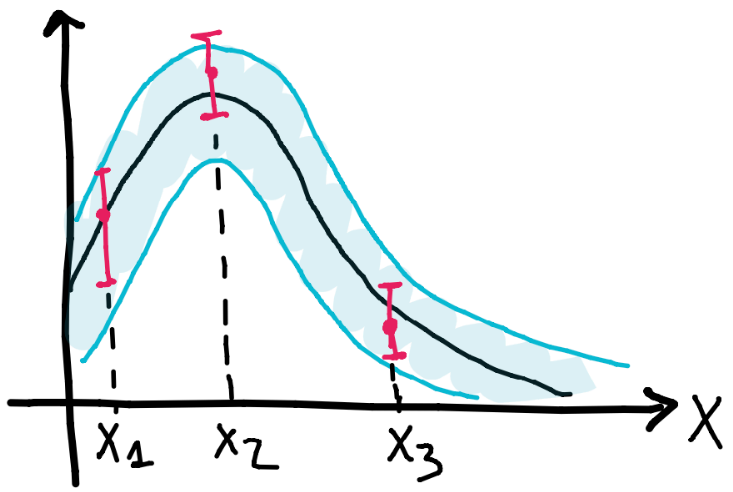

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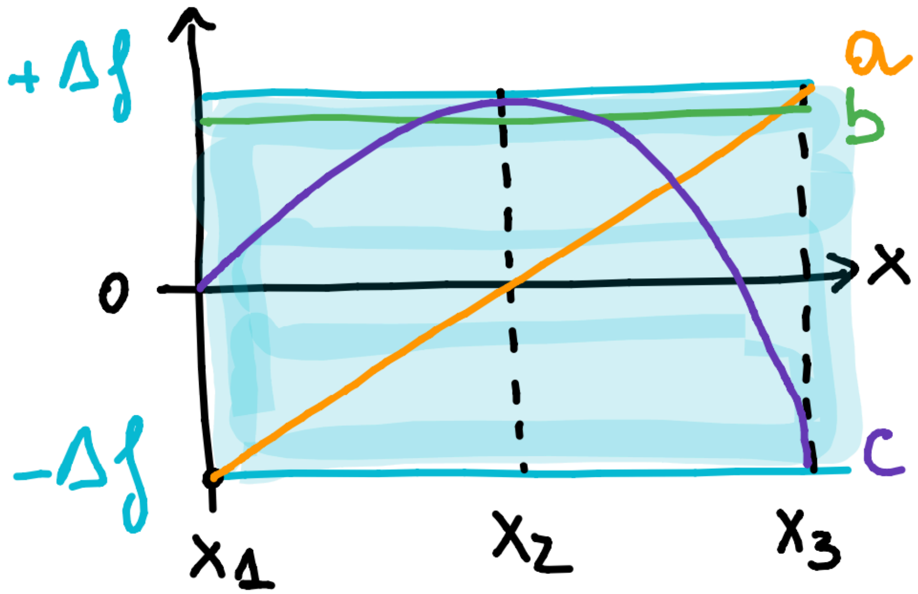

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every line (a, b, c) is a 100 % (anti-) correlated assumption

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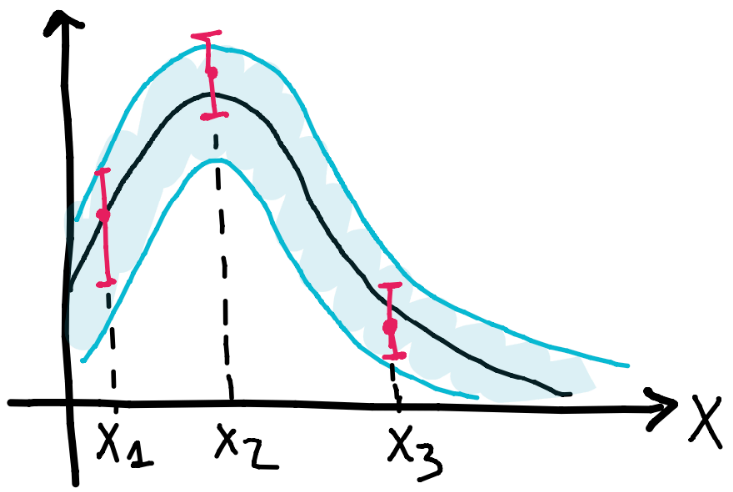

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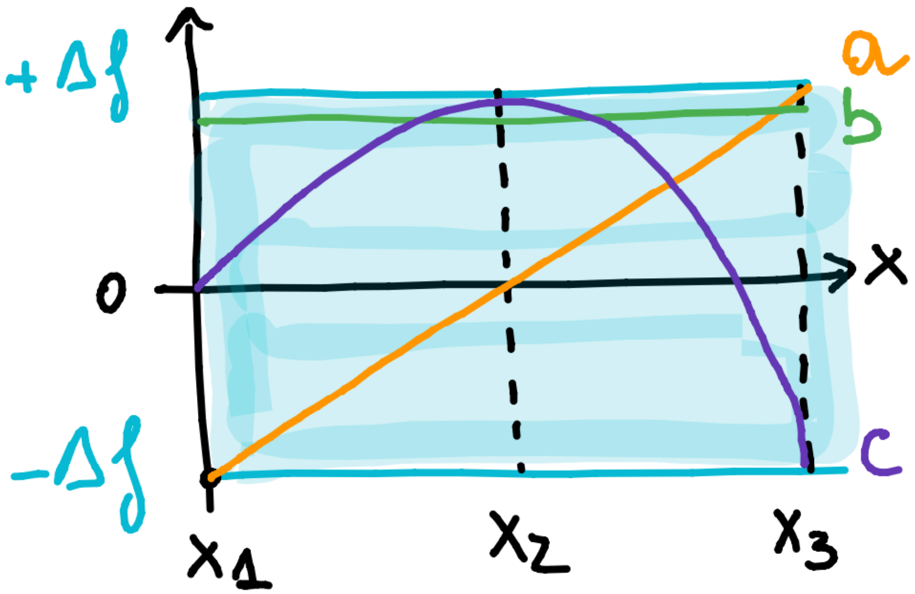

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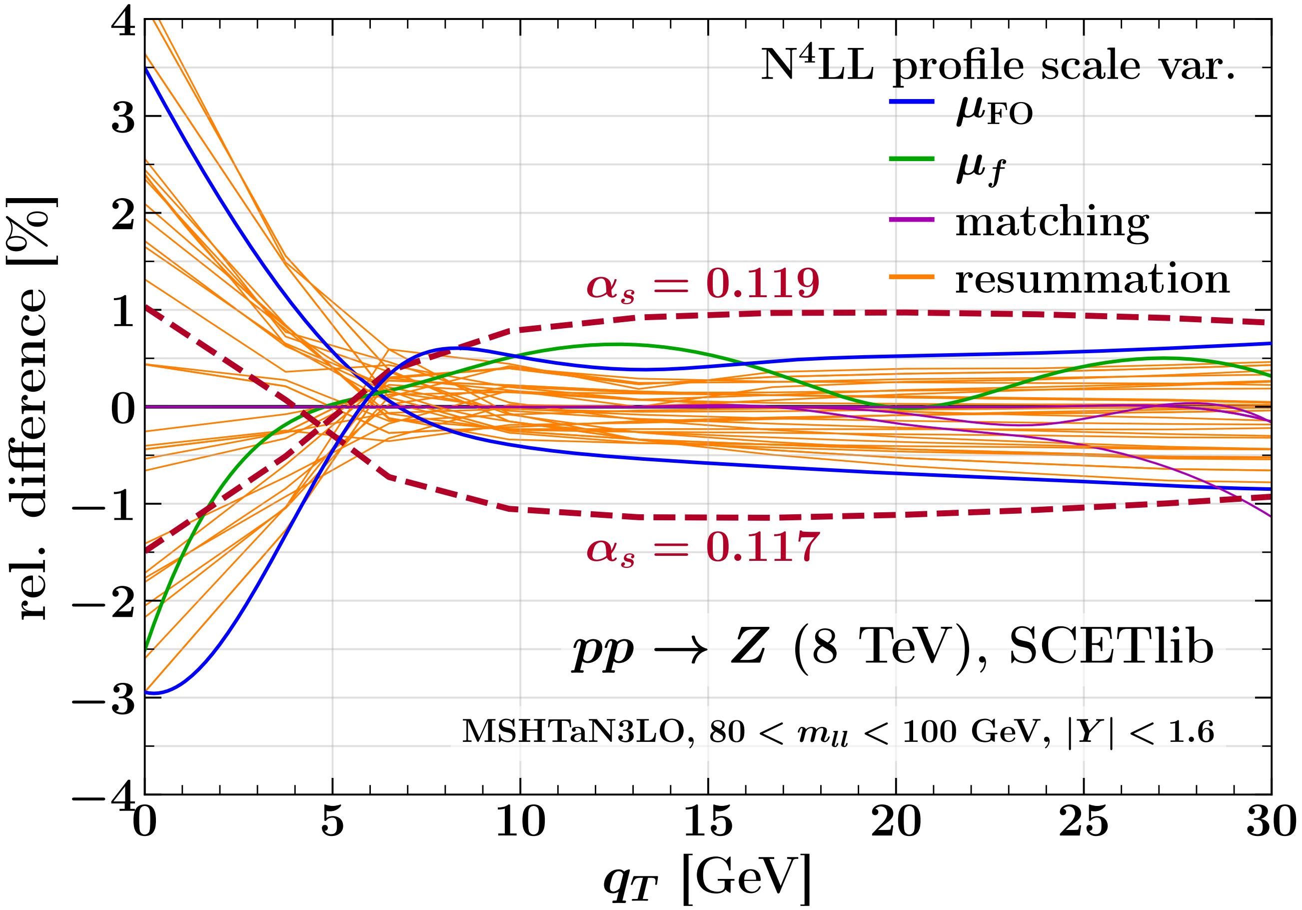
- no idea about the correct shape of scale variations (and therefore correlation): that's why we take envelopes!
- to get correct correlation: breakdown into *independent uncertainty components* required

Extraction of $\Delta\alpha_s$ with scale variations

In the q_T spectrum each bin has its own theory prediction

» point-by-point correlation crucial for the determination of the α_s uncertainty

What are we used to do? Scale Variations!



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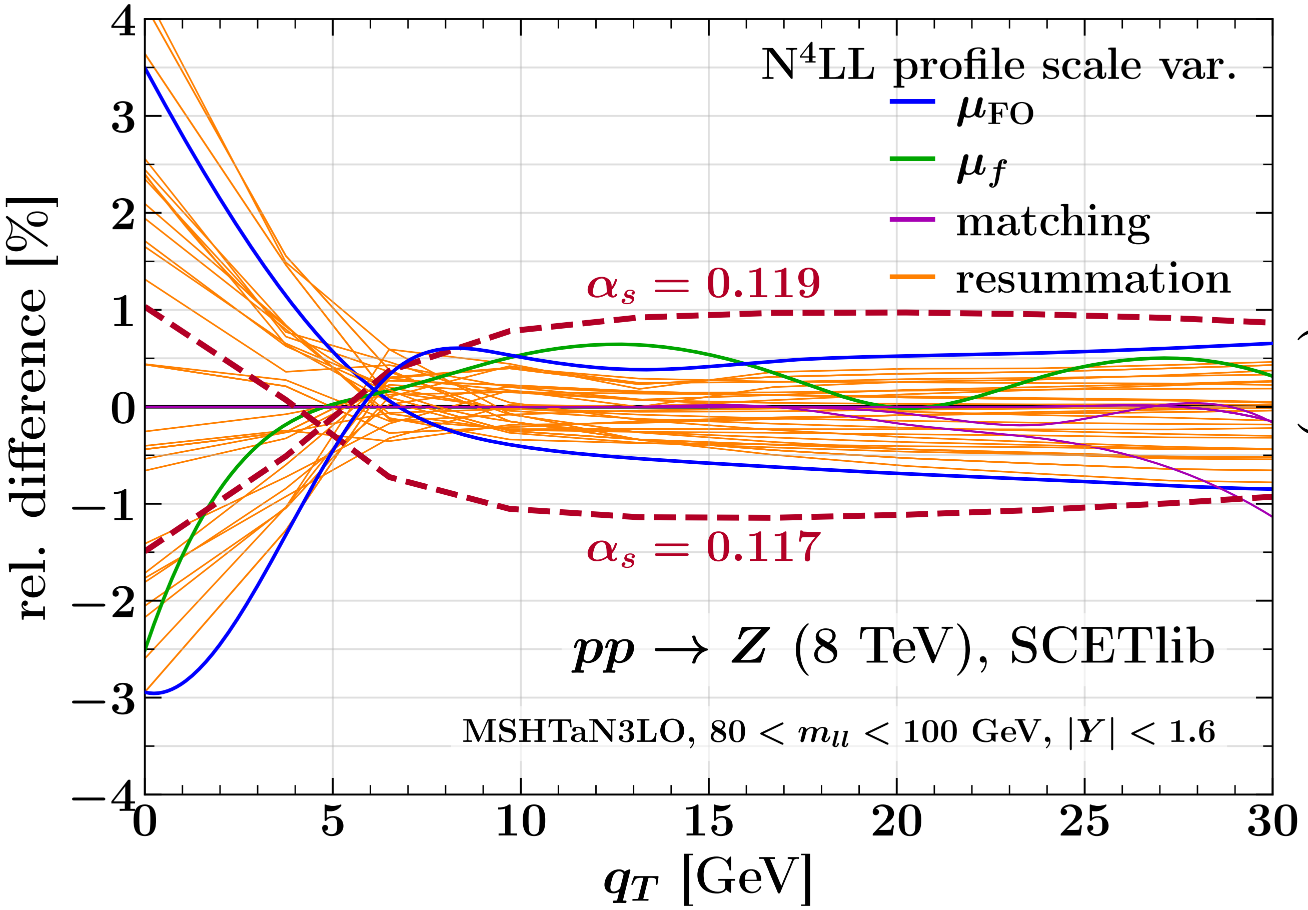
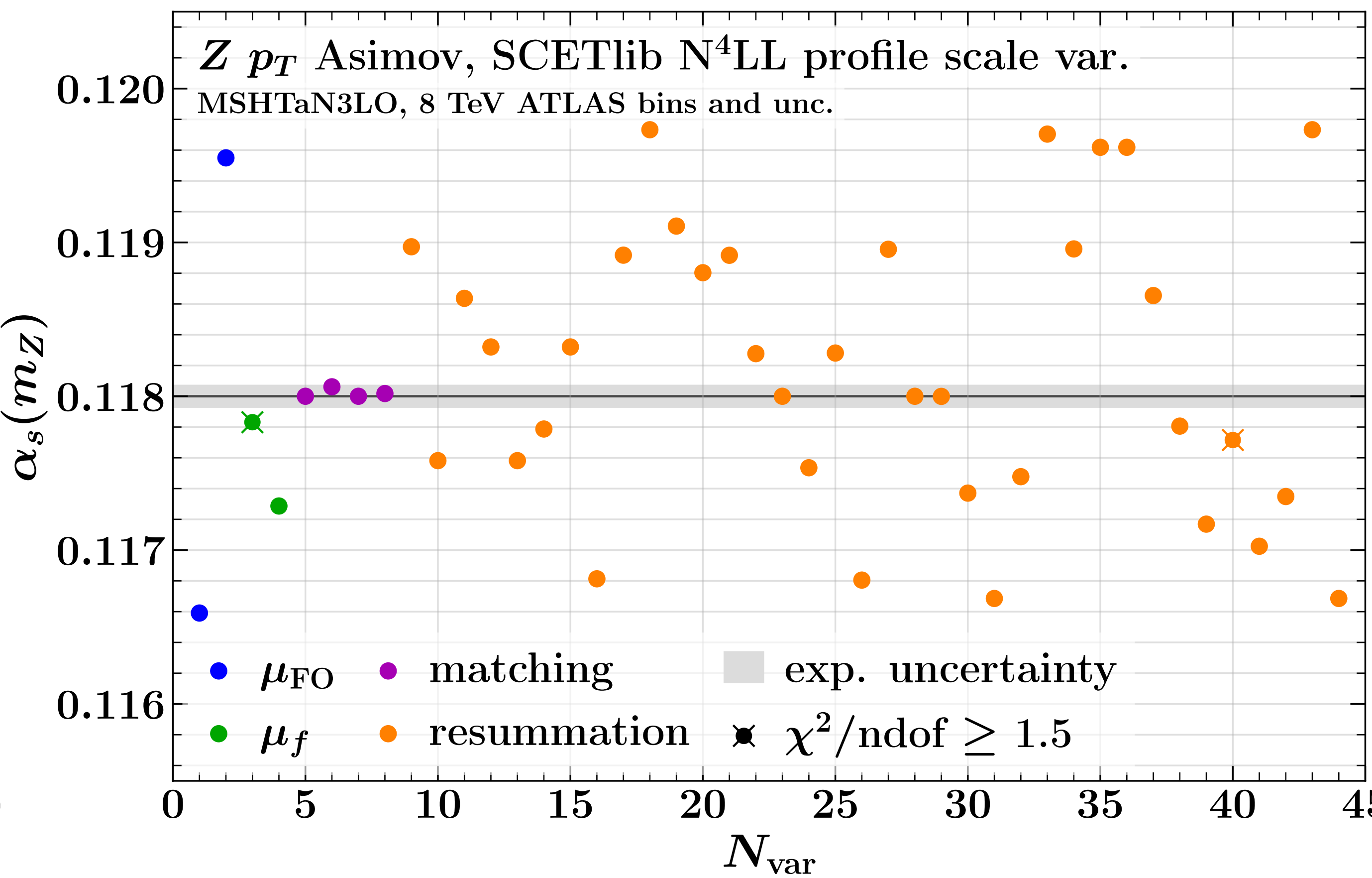


Only fitting α_S



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Sum envelopes of different "sources": $\Delta_{\mathrm{scale}} = 2.3$

Naive envelope: $\Delta_{\mathrm{scale}} = 1.9$

* uncertainties in units of 10^{-3}

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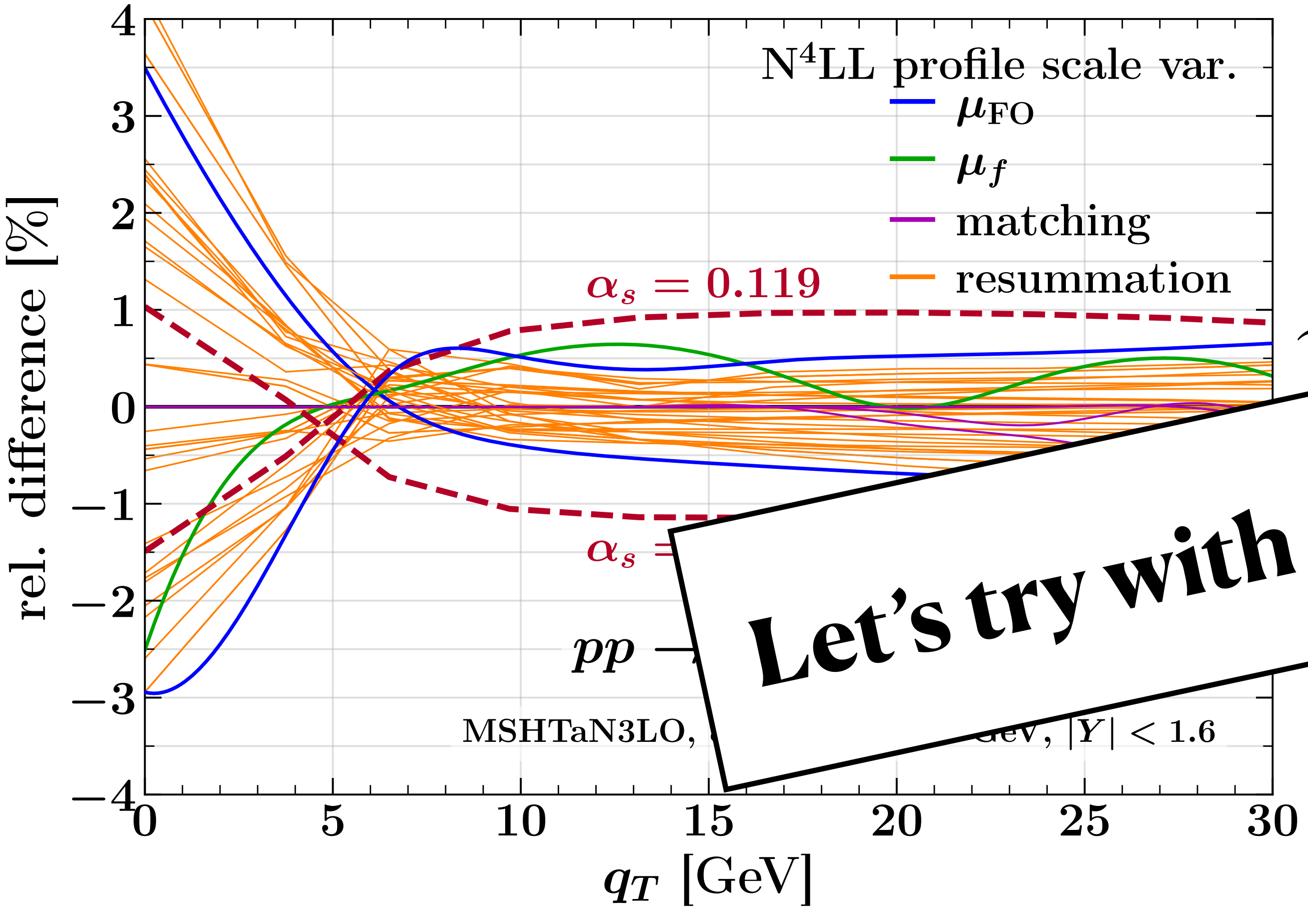

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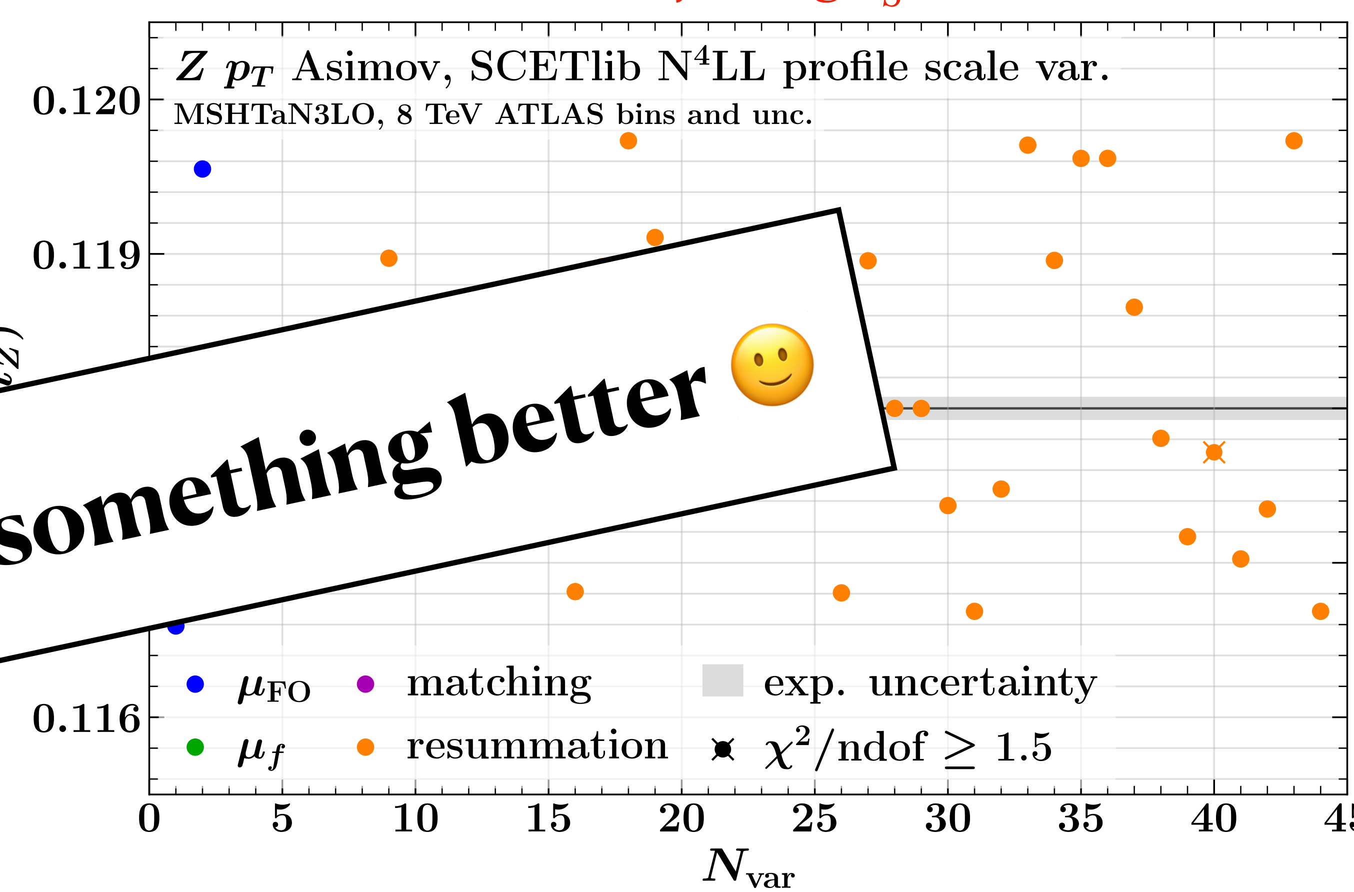

Let's try with something better 🙂

Each variation is a 100 % (anti-) correlated correlation model, strongly impacts the result!

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Theory Nuisance Parameters (TNPs)

all details here Tackmann '24!

Consider the same series expansion:

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2 Parametrize and include the leading source of uncertainty:

$$\text{N}^{2+1}\text{LO}: \ f^{\text{pred}}(\alpha, \theta_3) = \hat{f}_0 + \alpha \hat{f}_1 + \alpha^2 \hat{f}_2 + \alpha^3 f_3\left(\theta_3\right)$$

using theory nuisance parameters θ_n ;

- θ_n have physical true value $\hat{\theta}_n$, such that $\hat{f}_n = f_n(\hat{\theta}_n)$

  ... and therefore encode correct theory correlations

- TNPs well-defined parameters with true but unknown value

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3 How to ***define*** these θ_n?

- simplest case: $f_3(\theta_3) \equiv \theta_3$

- better: account for the internal structure of f_3
  (given the process: partonic channels, color, ...)

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Consider the q_T spectrum, leading power q_T dependence is known to all orders:

$$q_T \frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \left[H \times B_a \otimes B_b \otimes S\right](\alpha_S, L \equiv \ln q_T/m_Z) + \mathcal{O}\left(\frac{q_T^2}{m_Z^2}\right)$$

$F = \{H, B, S\}$ solution to RGE equations

$$F(\alpha_S, L) = \underbrace{F(\alpha_S)}_{\text{boundary conditions}} \exp \int_0^L \mathrm{d}L' \{ \Gamma[\alpha_S(L')] L' + \gamma_F[\alpha_S(L')] \}$$

boundary conditions anomalous dimensions

Theory Nuisance Parameters (TNPs)

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3 Account for dependencies:

- in which we need correlations
- those helping to obtain better theory constraints

$$F(\alpha_S) = 1 + \sum_{n=1} \left(\frac{\alpha_S}{4\pi}\right)^n F_n \qquad \gamma(\alpha_S) = \sum_{n=0} \left(\frac{\alpha_S}{4\pi}\right)^{n+1} \gamma_n$$

C_r leading color factor,

$$F_n(\theta_n^f) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f \qquad \gamma_n(\theta_n^\gamma) = 4C_r(4C_A)^n\,\theta_n^\gamma$$

C_A^{n-1} leading $n-$loop color factor

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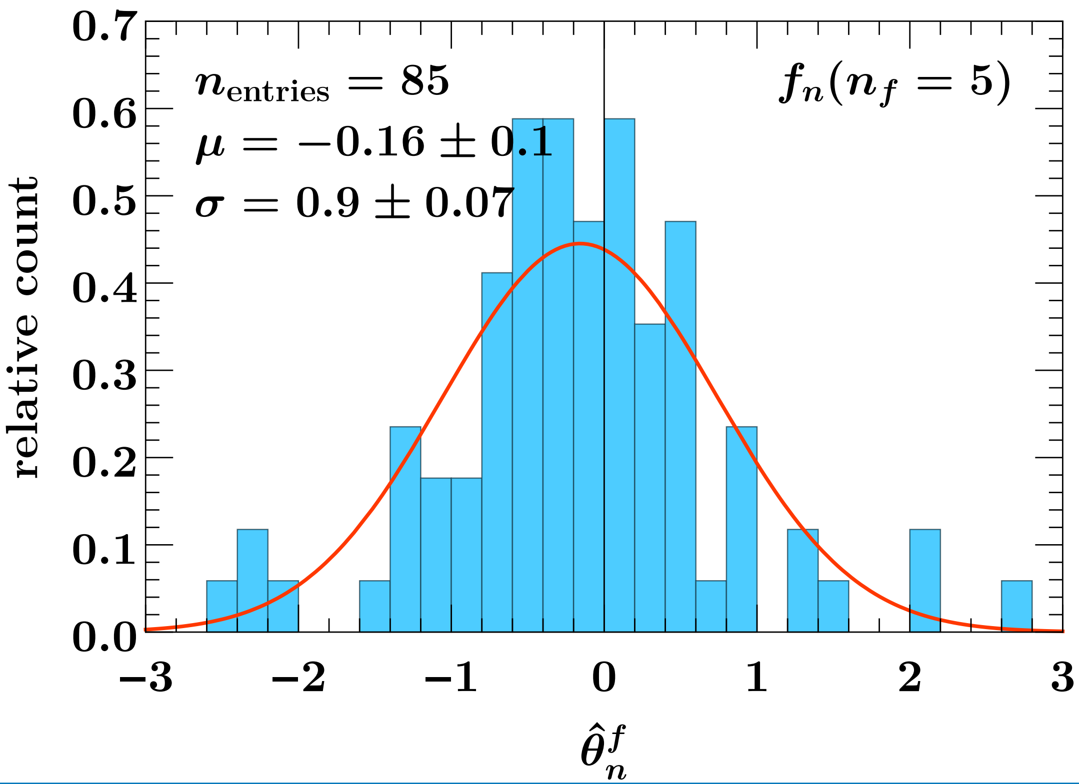
C_r leading color factor,
C_A^{n-1} leading $n-$loop color factor

4 How to ***vary*** θ_n?

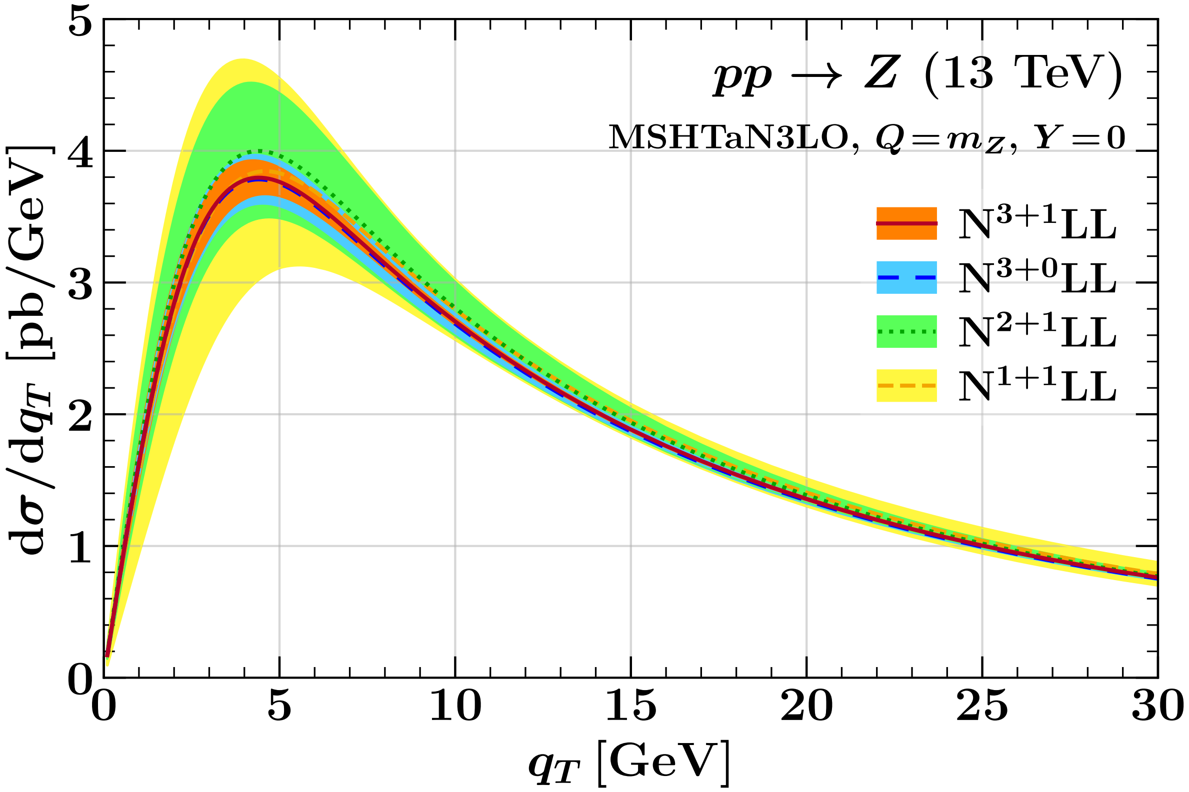


With these normalizations,
expected natural size $|\hat{\theta}_n| \lesssim 1$ $\longrightarrow$ $\boxed{\theta_n = 0 \pm 1}$

look at other known $n-$loop coefficients from population sample here,
validated using known perturbative series

Uncertainty breakdown and correlations

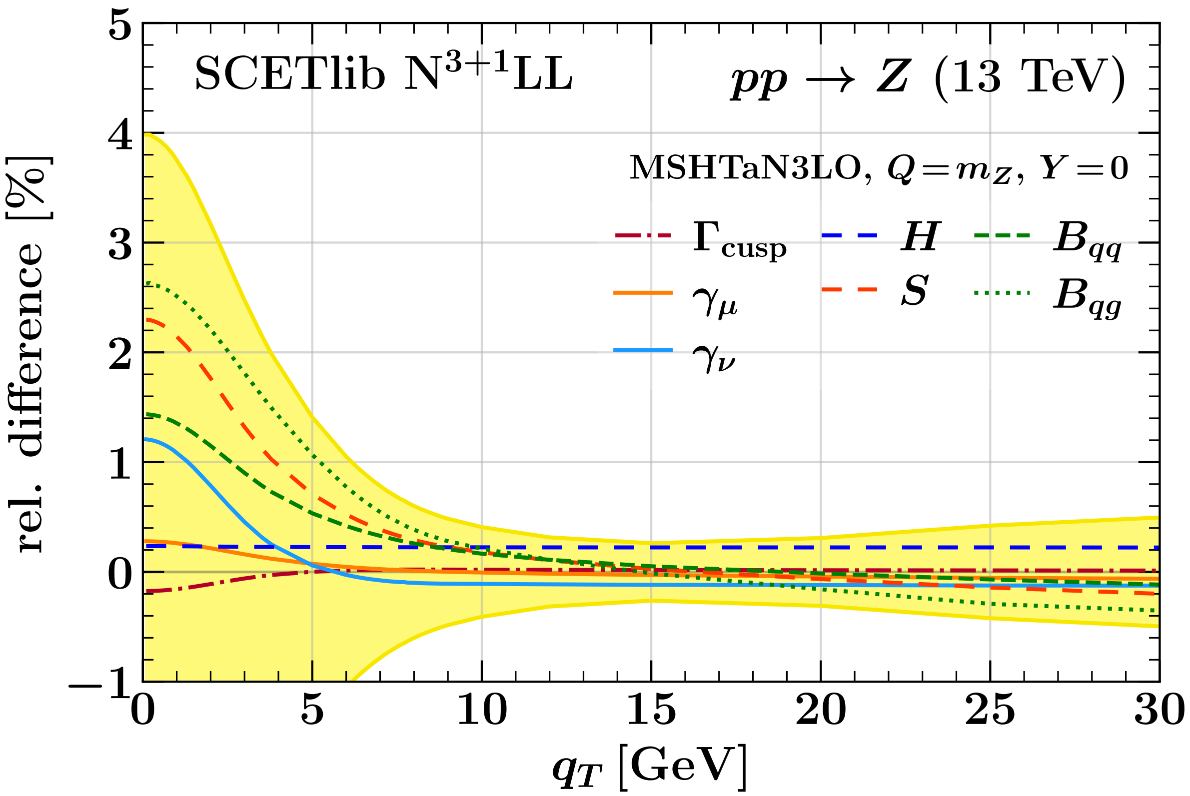
all details here Tackmann '24!



Comparing different orders at 95% theory CL

$(\Delta\theta_n = \pm 2)$

Breakdown of all the TNPs at $N^{3+1}LL$:



- varying each TNP by $\Delta\theta_n = \pm 1$(68% CL)
- providing breakdown into independent sources of uncertainty
- encoding correct point-by-point correlations

*B_{qj} : $F_n(z, \theta_n) = 3/2\,\theta_n\,\hat{F}_n(z)$, DGLAP splitting functions not varied

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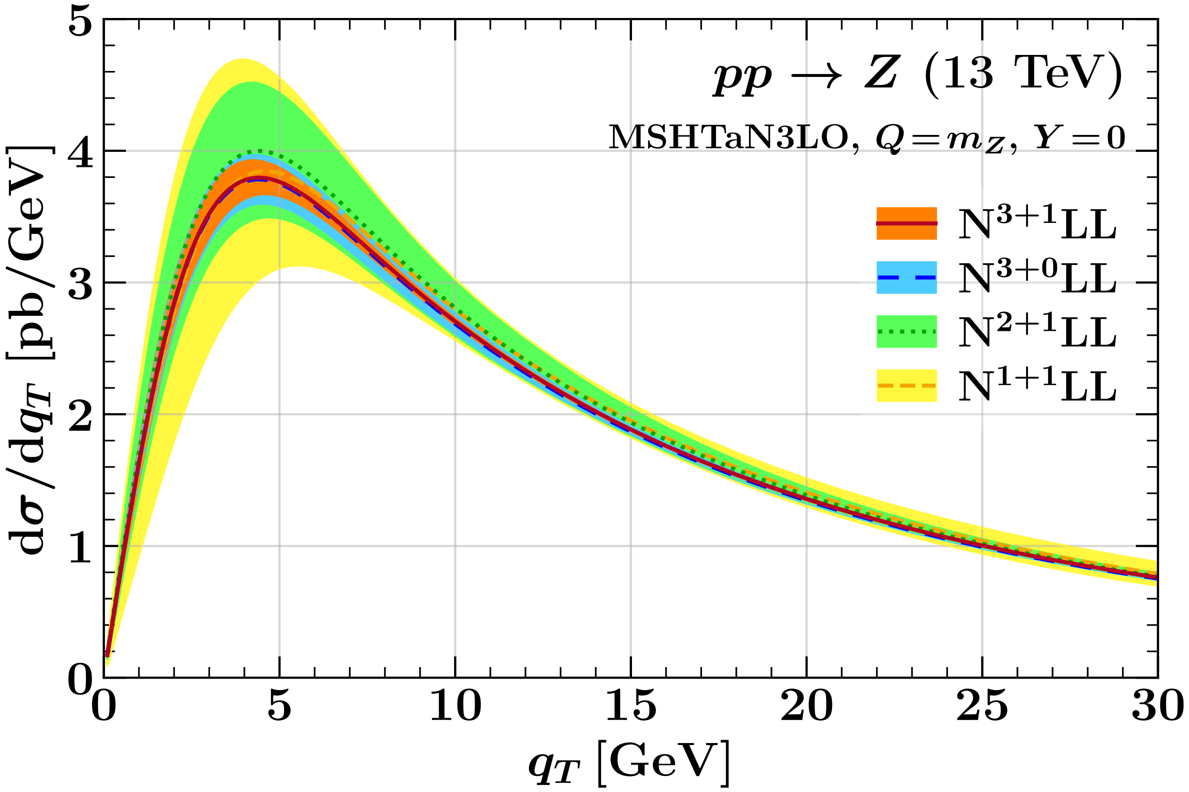

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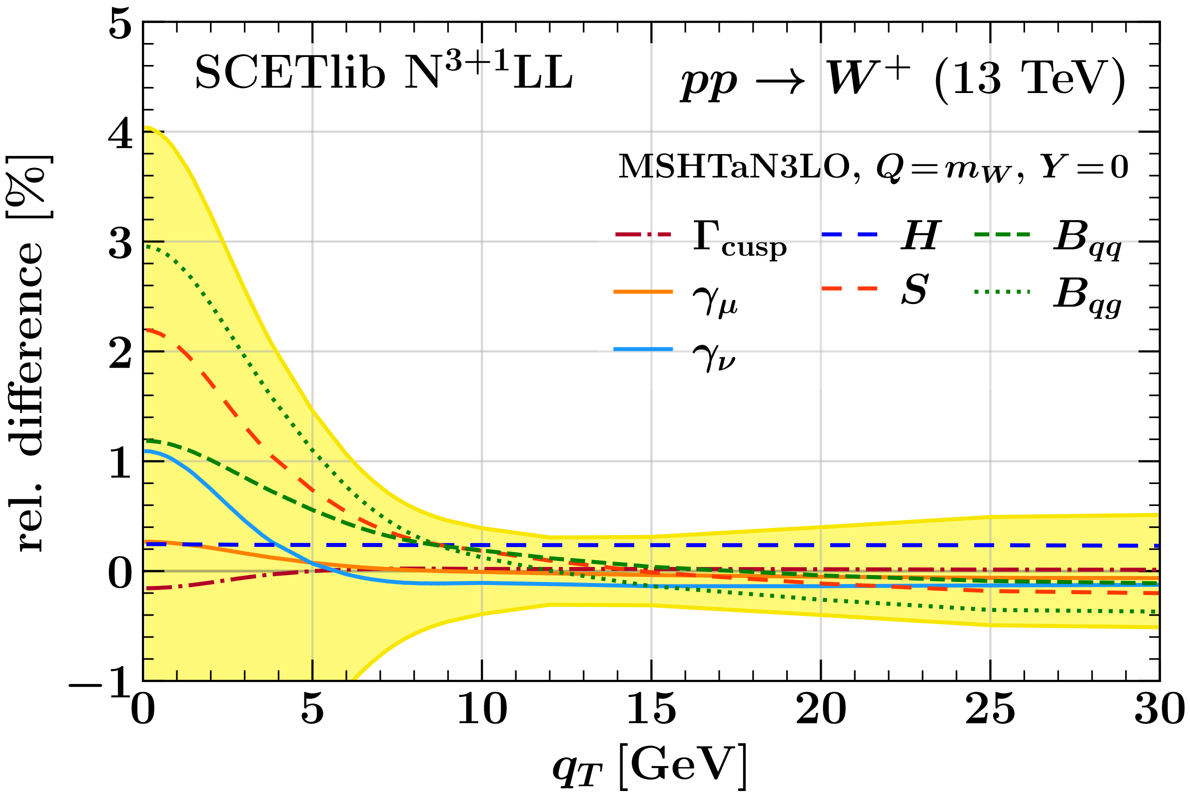

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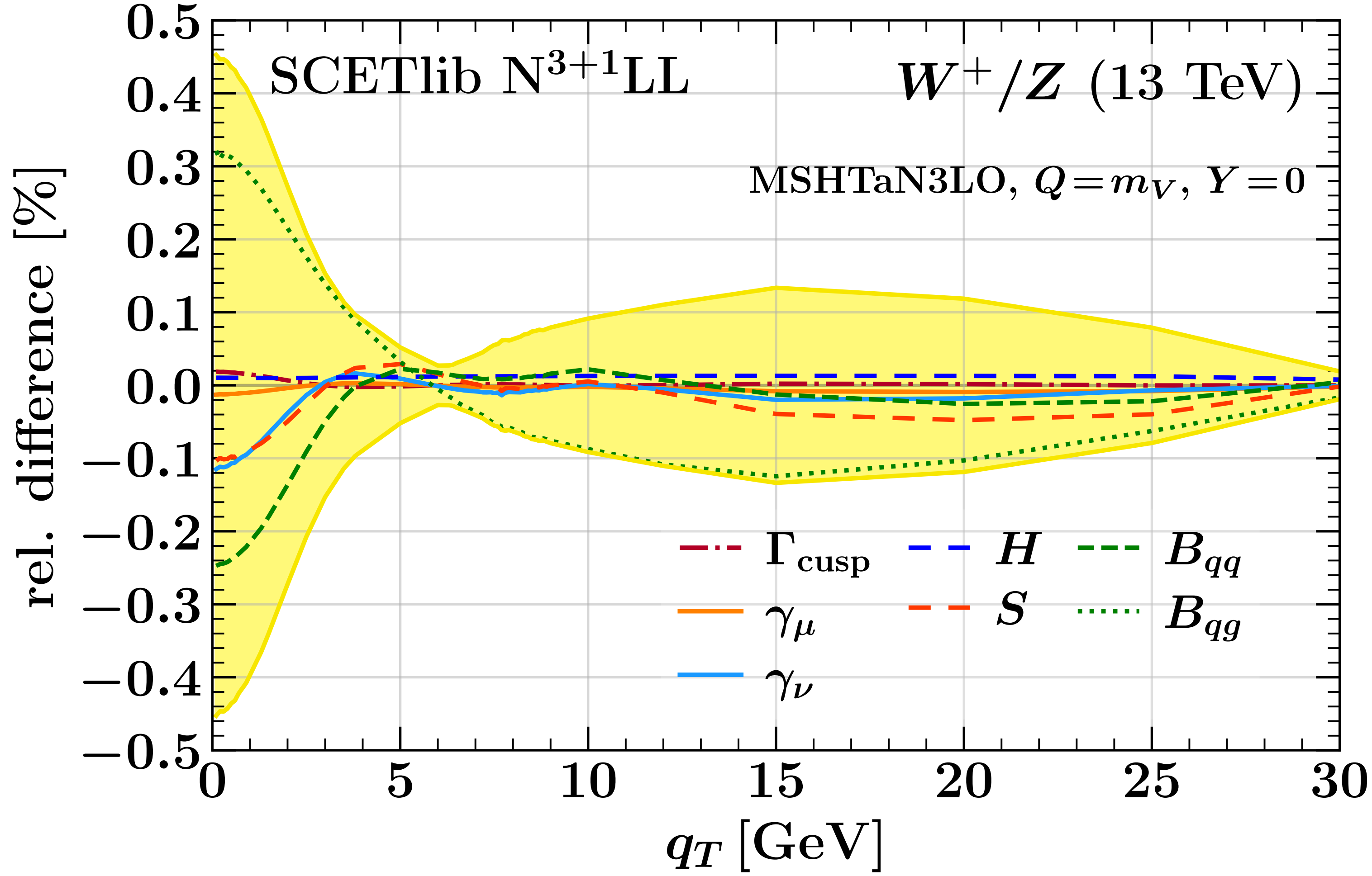
Theory Nuisance Parameters (TNPs)

Relative impacts on W/Z:

all details here Tackmann '24!



- uncertainties very similar for Z and W processes: same TNPs for both

  $\longrightarrow$ each TNP impacts are 100% correlated between the processes:

  nice cancellation in the ratio!

*just for illustration: only leading **massless** contribution

Some applications

Asimov fits for $\alpha_S(m_Z)$ from $Z\, p_T$ spectrum

WIP Cridge, Marinelli, Tackmann

Asimov fits: standard procedure to estimate expected uncertainties in a fully controlled setting

- study the *dominant* sources of uncertainty and their impact on the extracted α_s
  not concerned with subleading effects: neglected both in pseudodata and theory model

  $\longrightarrow$ still necessary for fitting real data

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  → still necessary for fitting real data

Our theory inputs:

- SCETlib N^{3+1}LL and N^4LL only resummed contribution

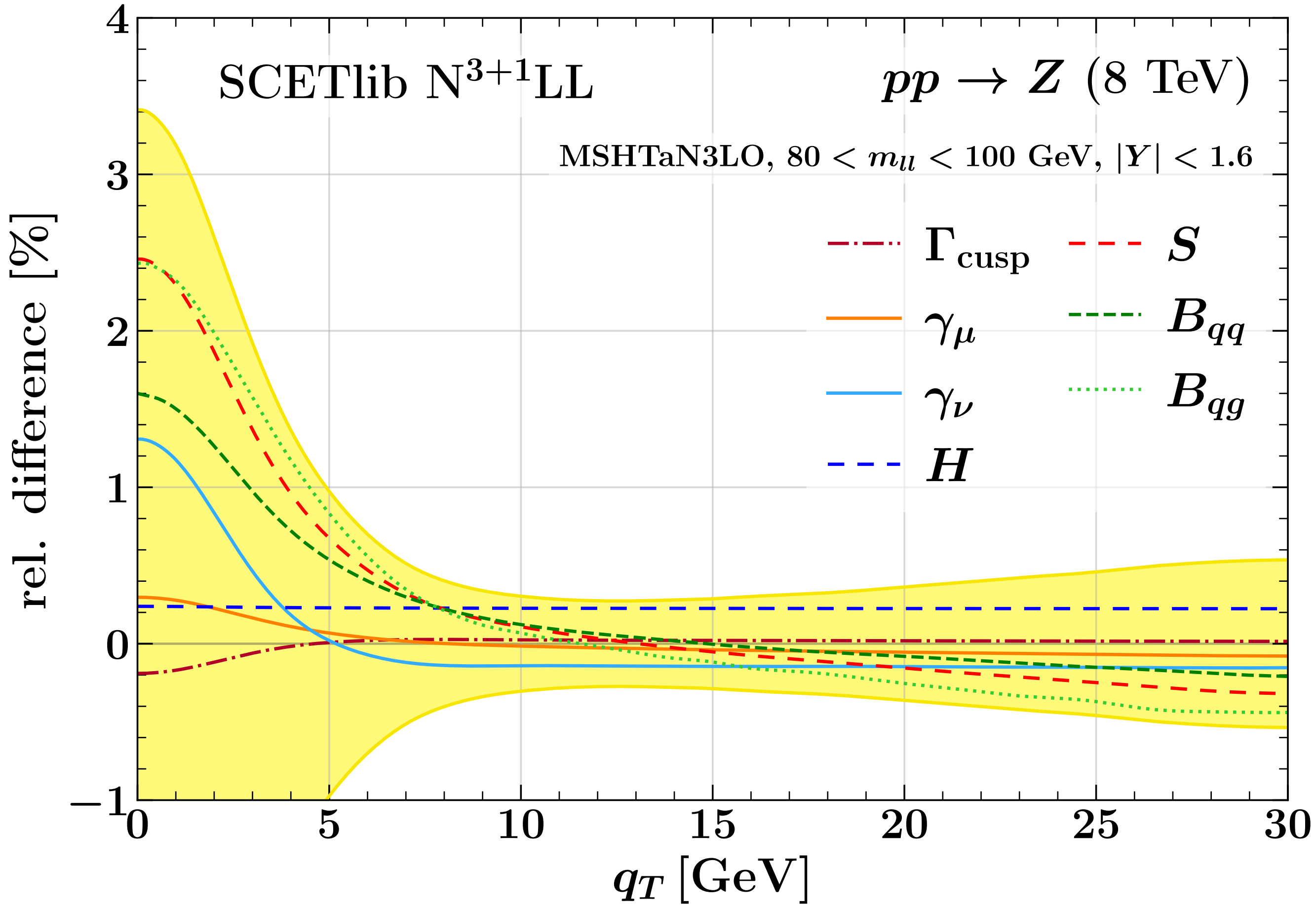
Our toy data:

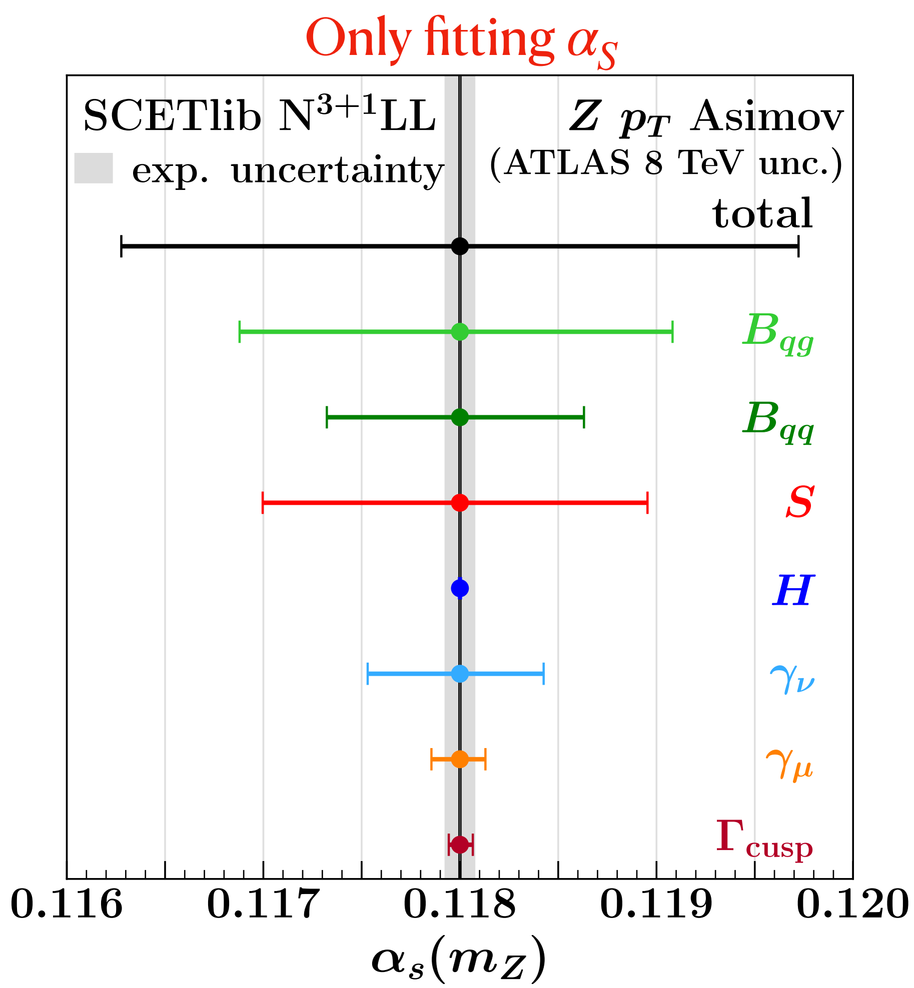
- Data defined as central theory prediction $[\alpha_S = 0.118]$
  [fixed nonp. params, MSHT20aN3LO PDF set]

- 72 data points in ATLAS binning,
  9 q_T bins in [0,29] GeV for each 8 Y bin in [0.0,3.6]
  [integrated in q_T, Y and Q]

- Using ATLAS exp. uncertainties and complete correlations for all 72 bins arXiv:2309.12986

Scanning TNPs





Repeat fit separately varying each TNP by $\Delta\theta_n = \pm 1$

- providing breakdown into independent sources of uncertainty
- encoding correct point-by-point correlations
- can now sum in quadrature $\Delta_{\text{total}} = 1.7$

still does not let the fit decide between moving α_S or theory

rather profiling

* uncertainties in units of 10^{-3}

Perturbative uncertainty with profiling TNPs

Profiling: fitting α_S *together* with all TNPs (allows the fit to decide what to do)

- TNPs are proper parameters, included in the fit with Gaussian constraint $\theta_n = 0 \pm 1$
- allows data to constrain TNPs and thereby reduce theory uncertainty

Pseudodata: central [$\alpha_s = 0.118$] N^{4}LL prediction

- simulates the fit to real data, which contains the all-order result

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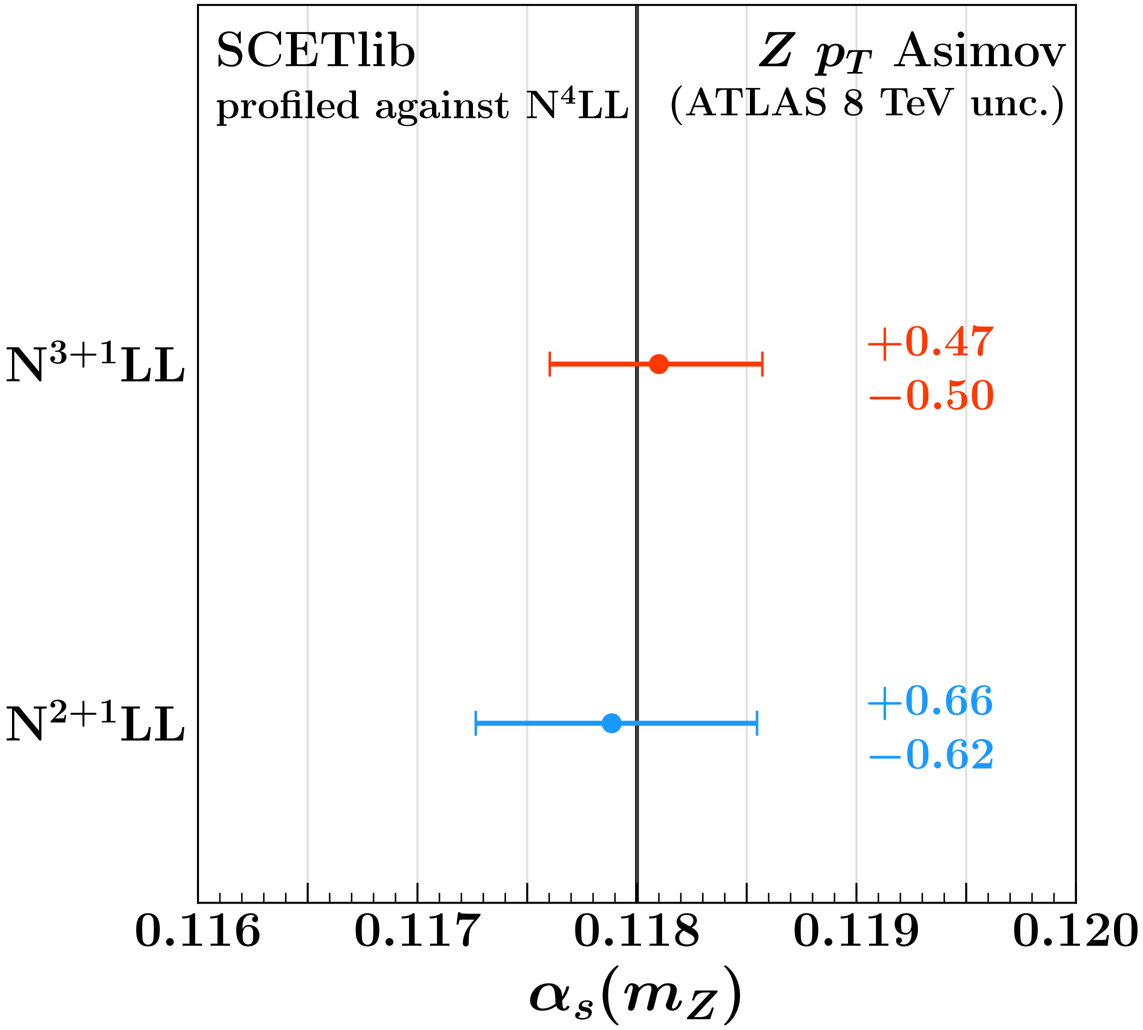
1 N^{2+1}LL theory model

2 N^{3+1}LL theory model

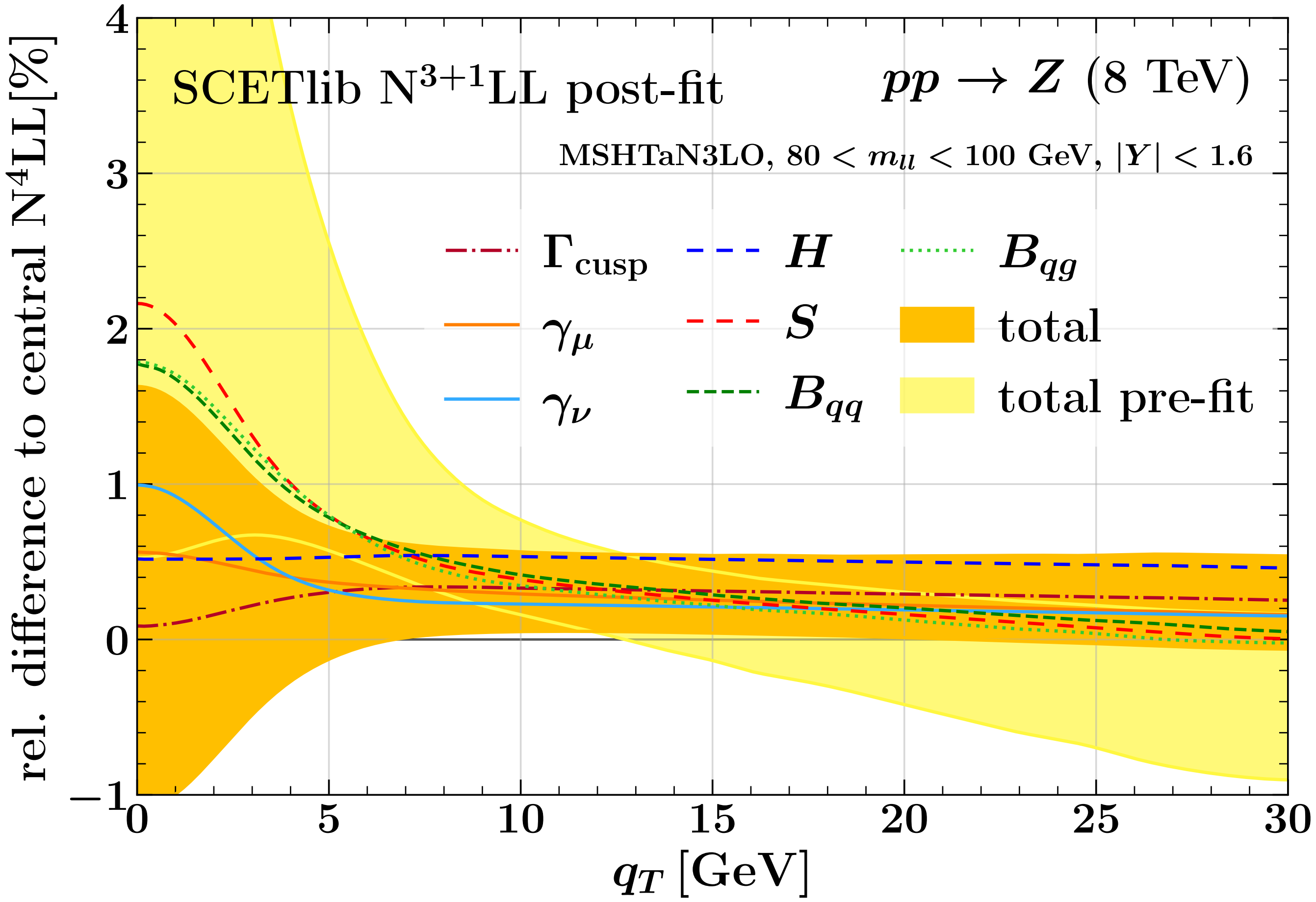


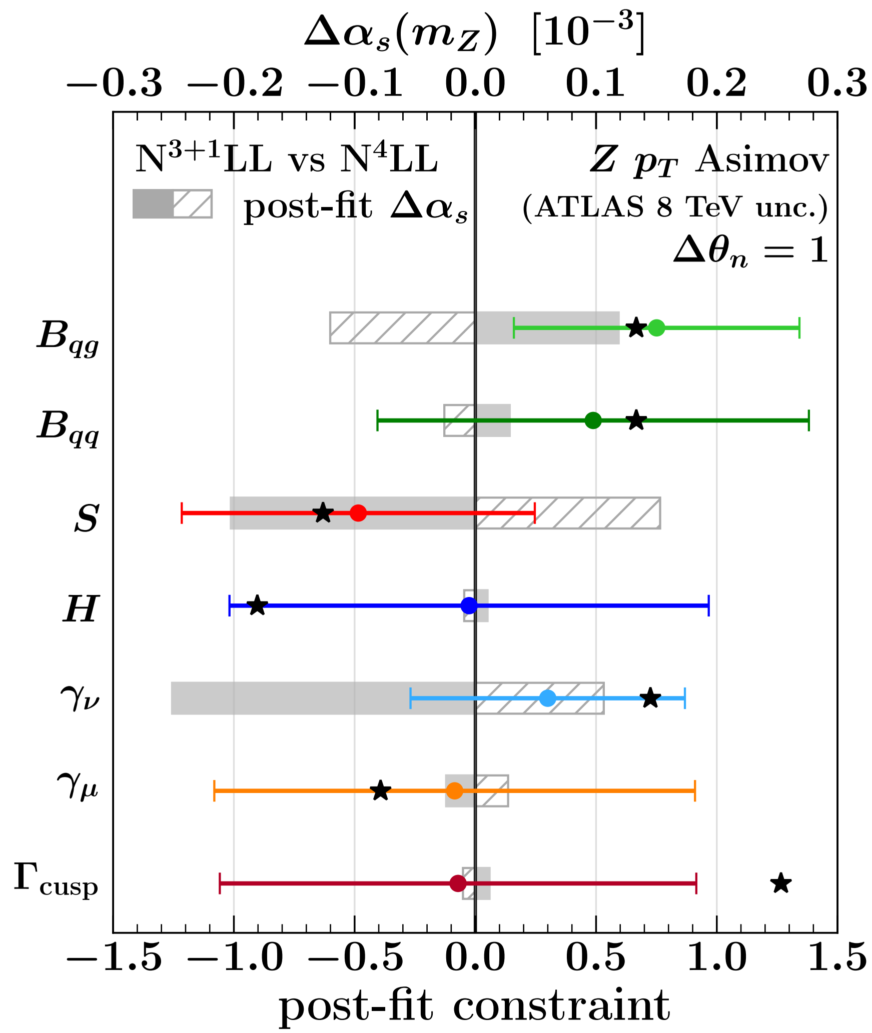
→ look at the post-fit constraints on TNPs

* uncertainties in units of 10^{-3}

Post-fit constraints on TNPs

Profiling lower order against higher order: N^{3+1}LL





- N^{3+1}LL pulled, toward correct true values [★]
- post-fit prediction for q_T spectrum driven by constraints from data
- grey → TNP down variation, dashed grey → TNP up variation

  γ_ν, S and B_{qg} have the largest remaining impact on $\alpha_s(m_Z)$ after profiling

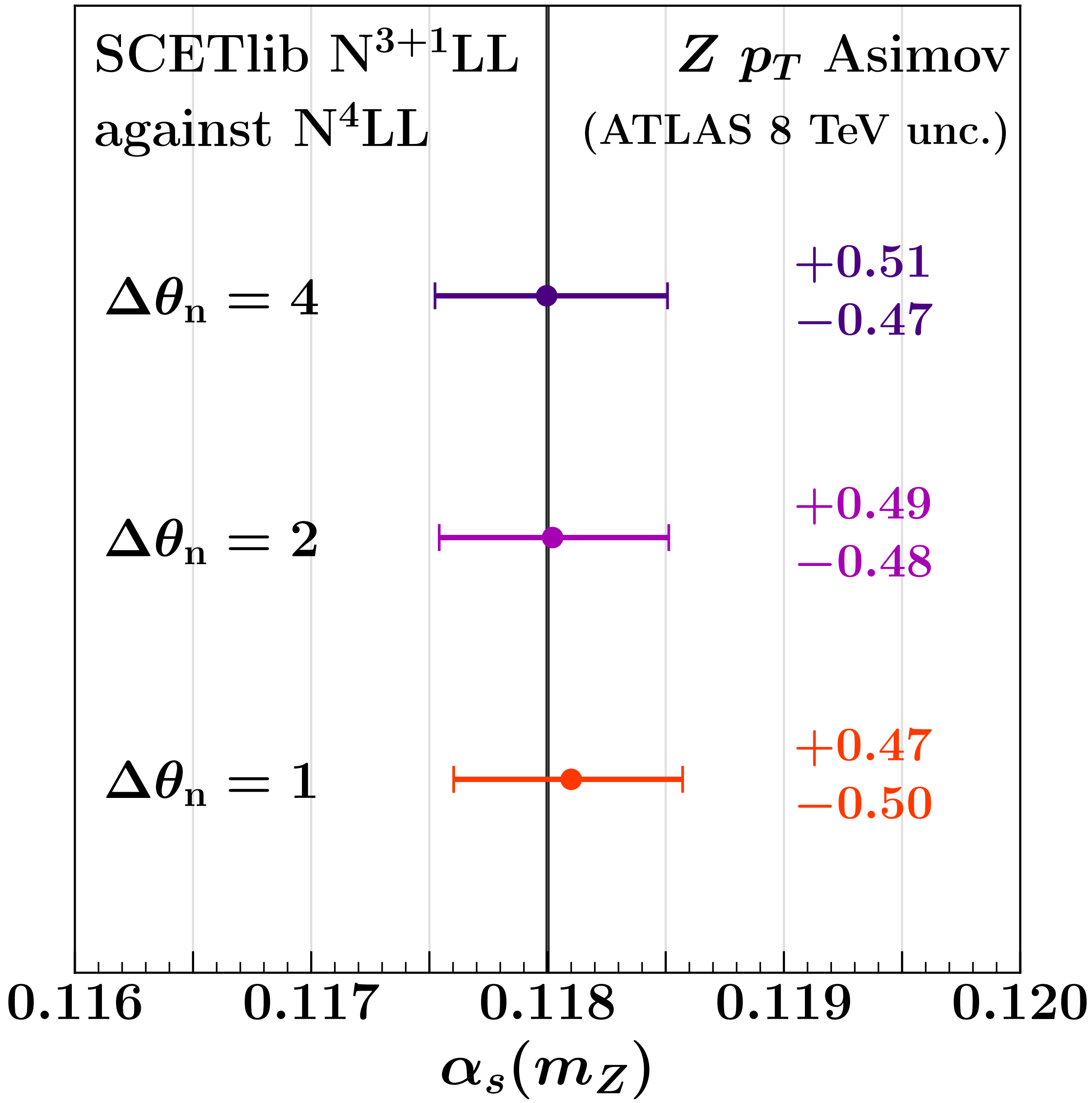
→ for the exact correlation between parameters, look at the post-fit covariance matrix!

Different theory constraints on TNPs

What happens by changing the prior theory constraint?

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 1, 2, 4$

Fit N^{3+1}LL against N^{4}LL data



- profiling substantially reduces the dependence on theory constraint (with scanning, α_s unc. directly depends on choice of $\Delta\theta_n$)
- the effect relative to the theory constraint strongly depends on the power of the experimental constraint
- only slight difference in the uncertainties when relaxing the TNP constraint

* uncertainties in units of 10^{-3}

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 1$

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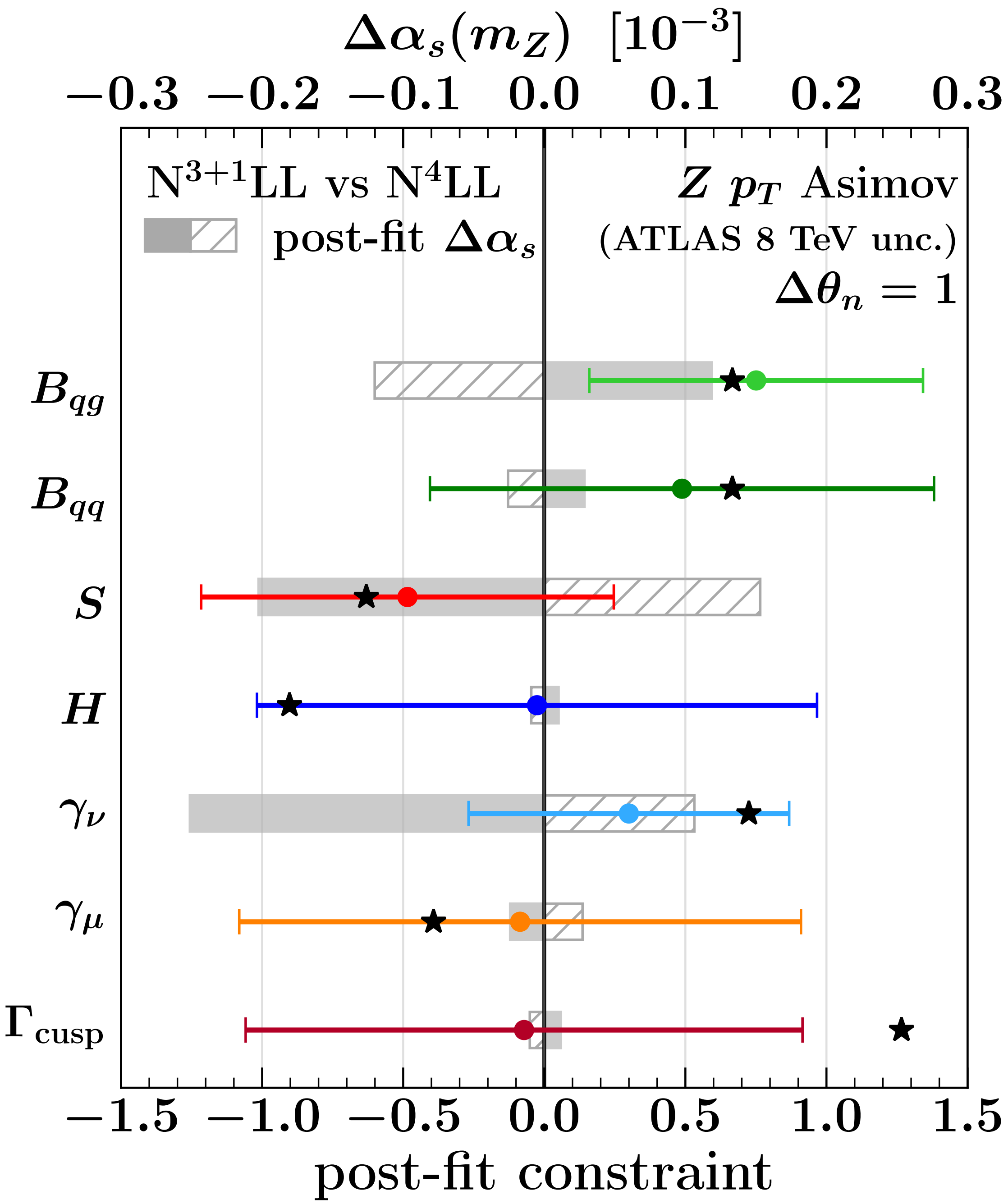
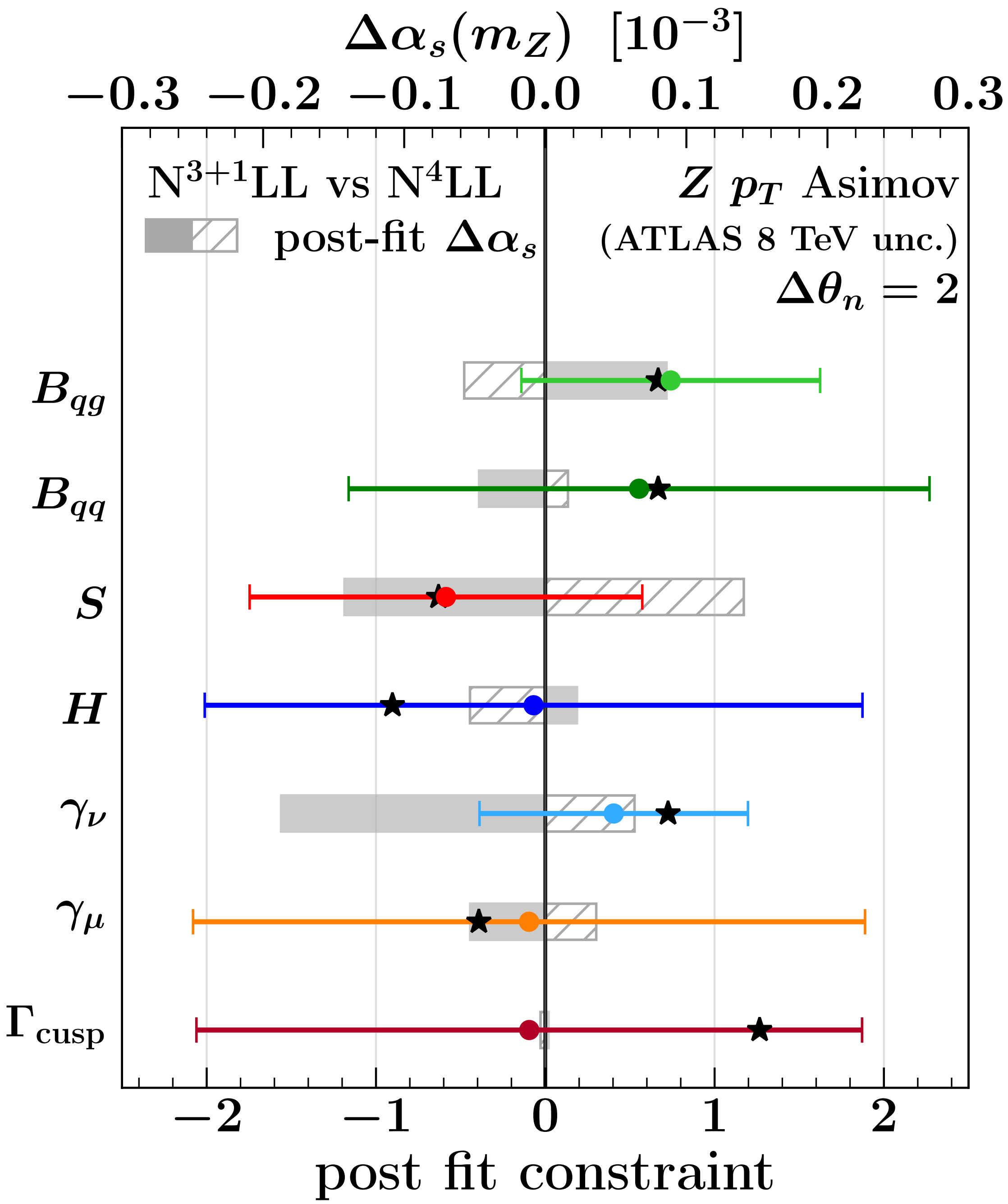


[don't be fooled by the different x-range!]

1 $\Delta\theta_n = 1$ start seeing the exp. constraint

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 2$



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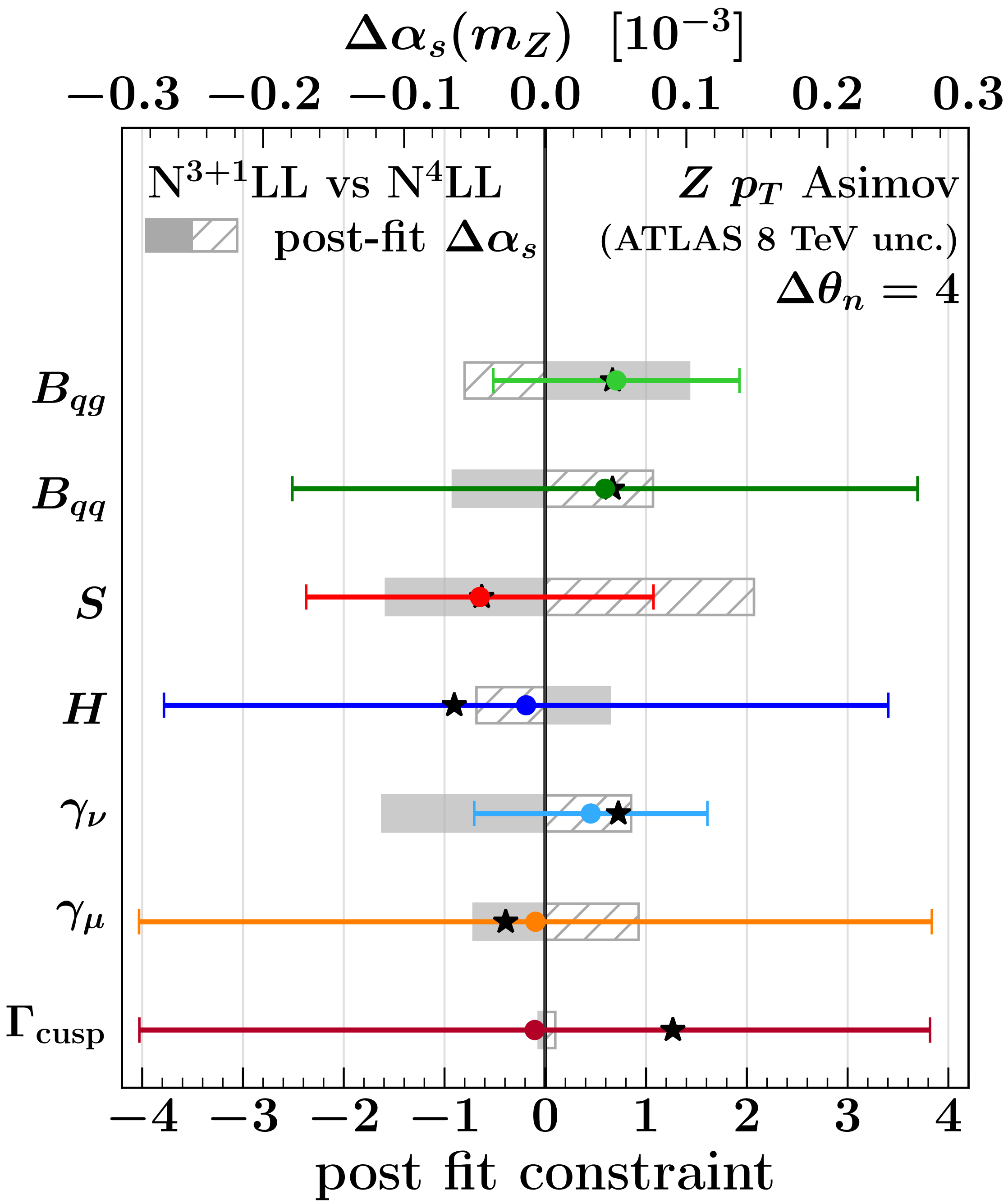
Fit N^{3+1}LL against N^{4}LL data

1. $\Delta\theta_n = 1$ start seeing the exp. constraint

2. $\Delta\theta_n = 2$ it's basically a factor 2 w.r.t $\Delta\theta_n = 1$

Different theory constraints on TNPs

Using now $\theta_n = 0 \pm \Delta\theta_n$ with $\Delta\theta_n = 4$



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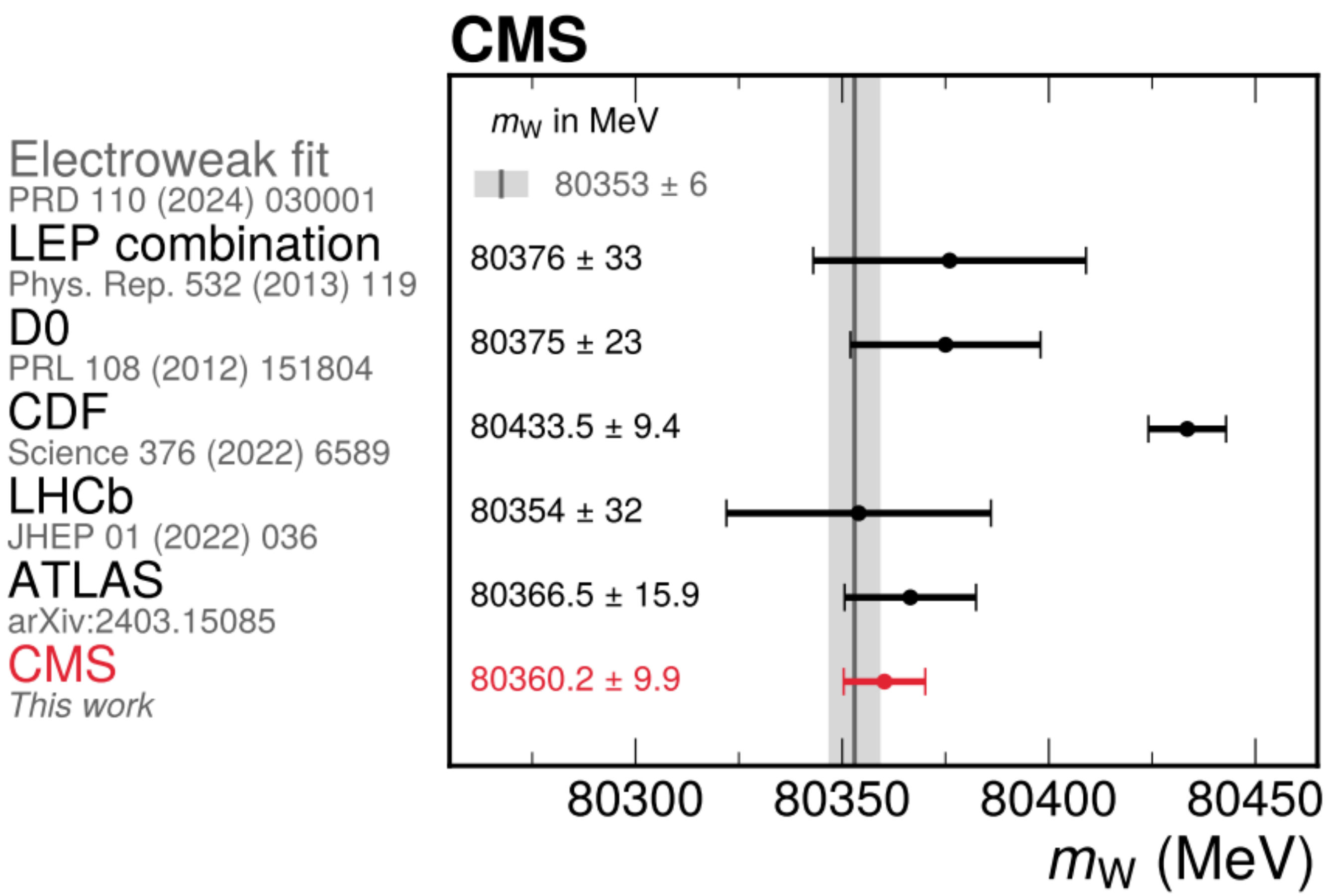
Fit N^{3+1}LL against N^{4}LL data

1. $\Delta\theta_n = 1$ start seeing the exp. constraint
2. $\Delta\theta_n = 2$ it's basically a factor 2 w.r.t $\Delta\theta_n = 1$
3. $\Delta\theta_n = 4$ data can constrain TNPs more
4. remaining impact on $\alpha_s(m_Z)$ after profiling proportional to $\Delta\theta_n$

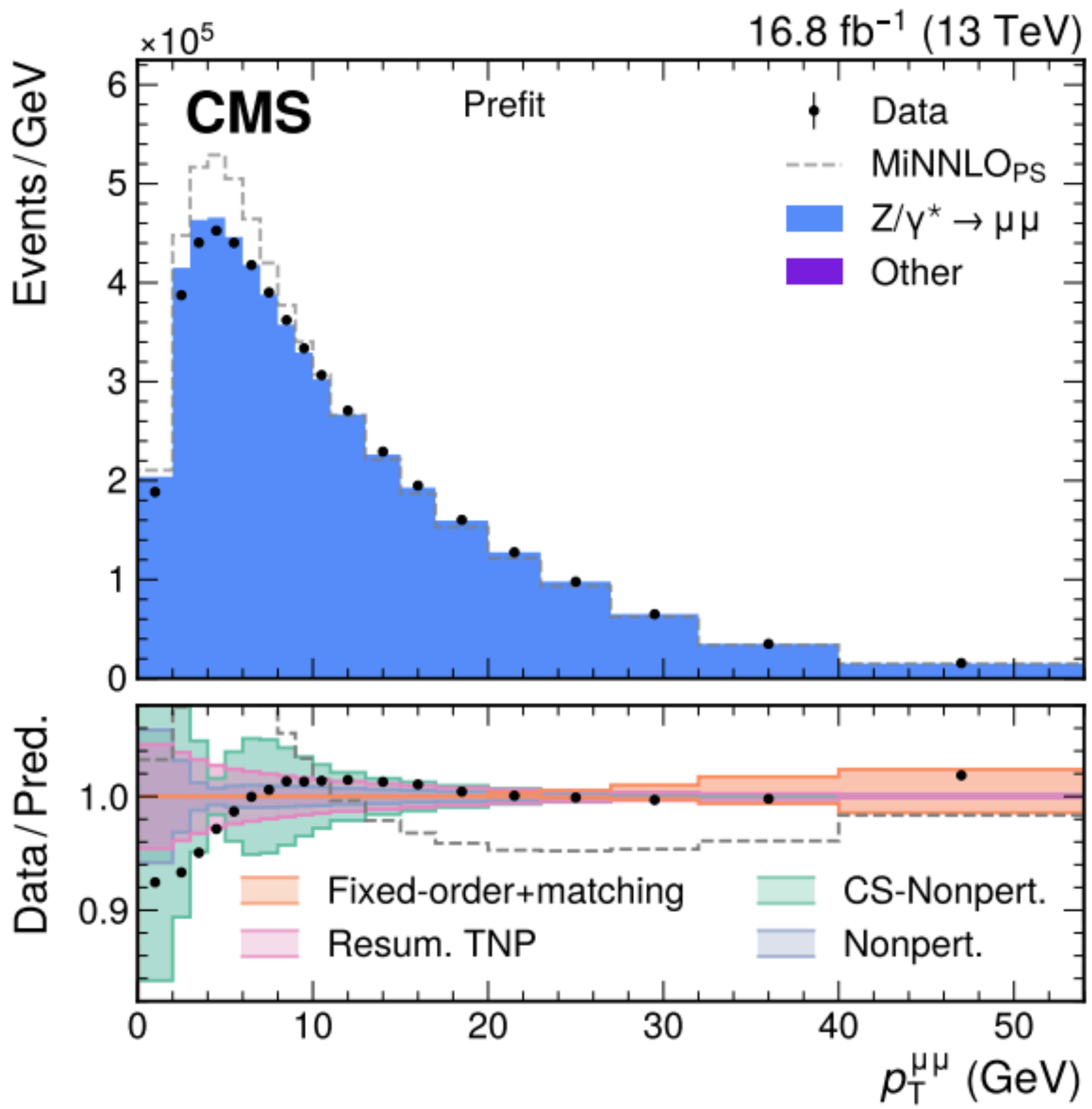
CMS W mass measurement

See Chiara's talk!

Recent CMS W mass measurement arXiv:2412.13872



<u>Theory input</u>: N^{3+0}LL+NNLO (SCETlib and DYturbo)



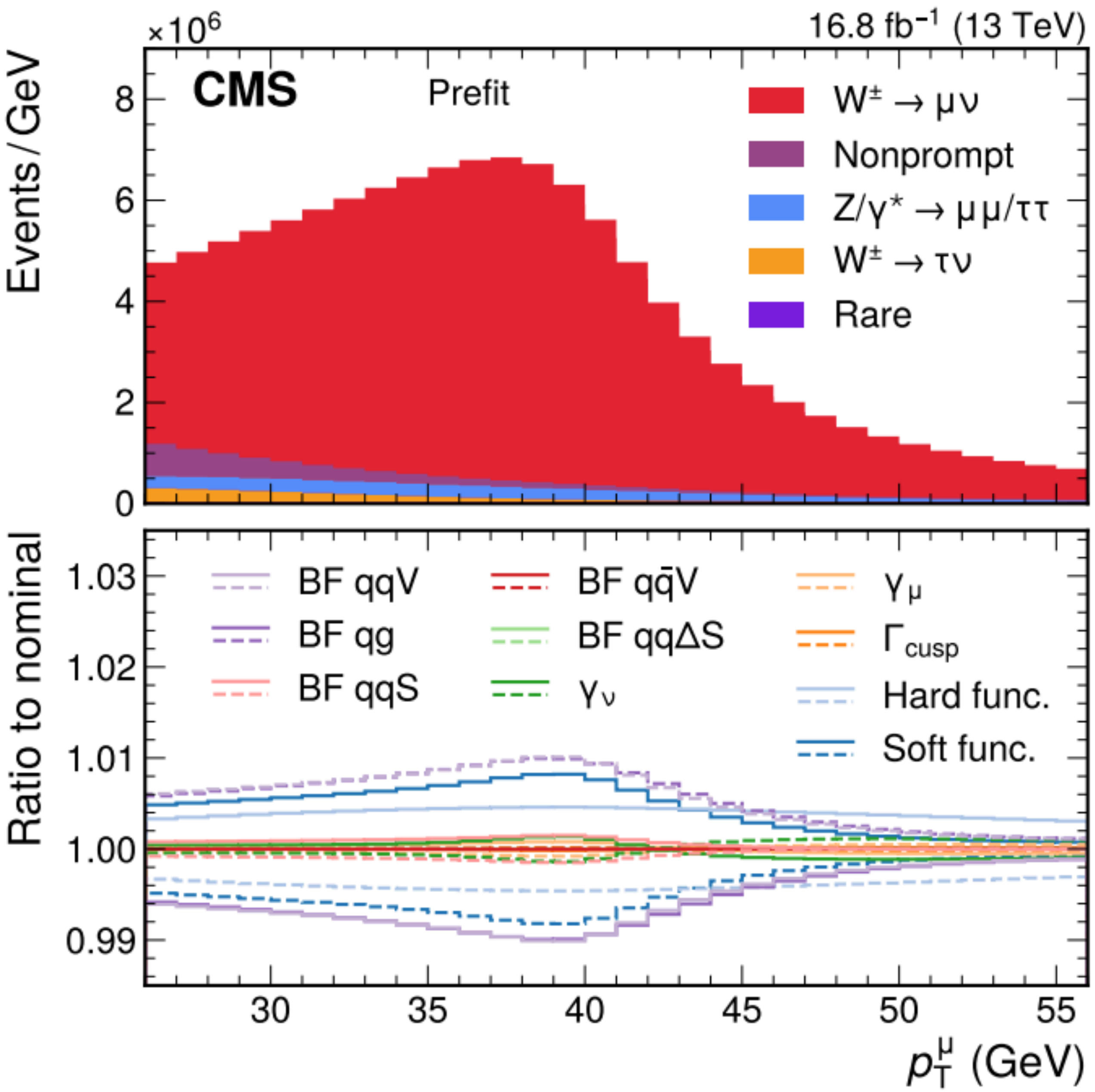
- p_T^W modeling fundamental: uncertainties in the low p_T region affect the shape as m_W variation
- theory correlations are crucial: uncertainty propagated from p_T^W to p_T^μ to m_W!

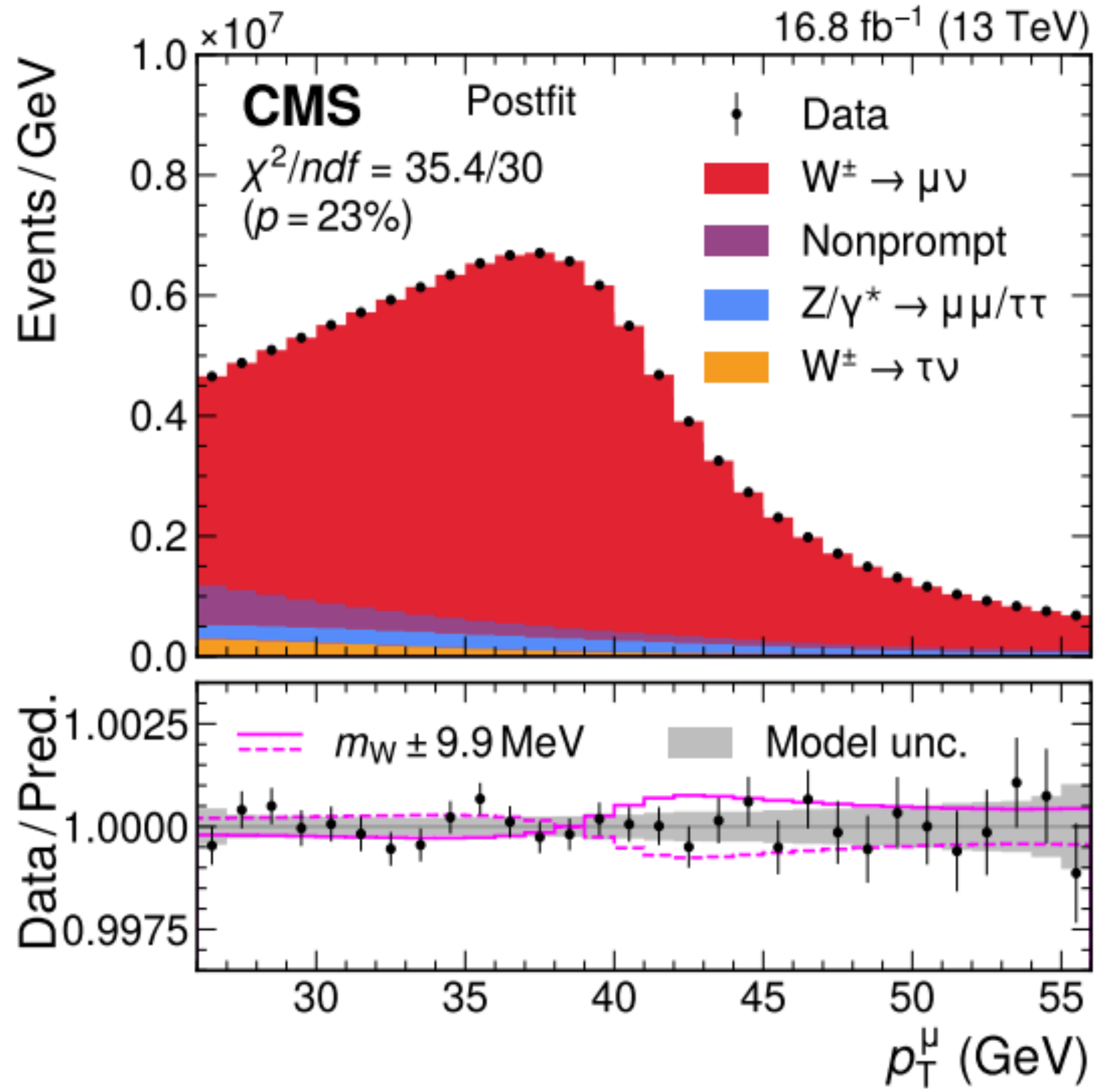
CMS W mass measurement

See Chiara's talk!

Perturbative uncertainties in the resummed prediction: N^{3+0}LL SCETlib*

- contribution of all theoretical and experimental uncert. before and after profiling





* for details about this order look here

Summary

p_T Z/W crucial benchmark observables for LHC precision physics program

$\longrightarrow$ that's why we need meaningful theory uncertainties!

Correlations are fundamental for interpretation of precision measurements:

having meaningful theory uncert. is as important as meaningful exp. uncert.!

1 Theory Nuisance Parameters perfect candidate

- include correct point-by-point correlations across the q_T spectrum, different processes ,...
- can be constrained by data reducing theory uncertainty
- first applications work as advertised

2 Perturbative uncertainty with TNPs

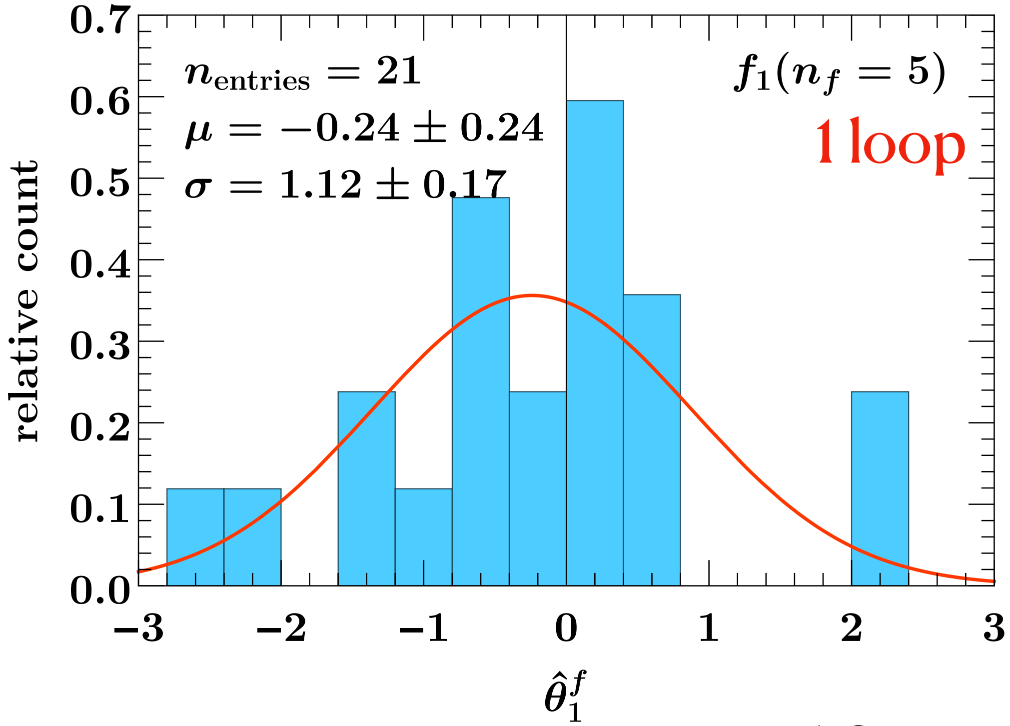
- perturbative uncertainty can be correctly profiled
- TNPs not "easy and cheap" as scale variation, but worth it!

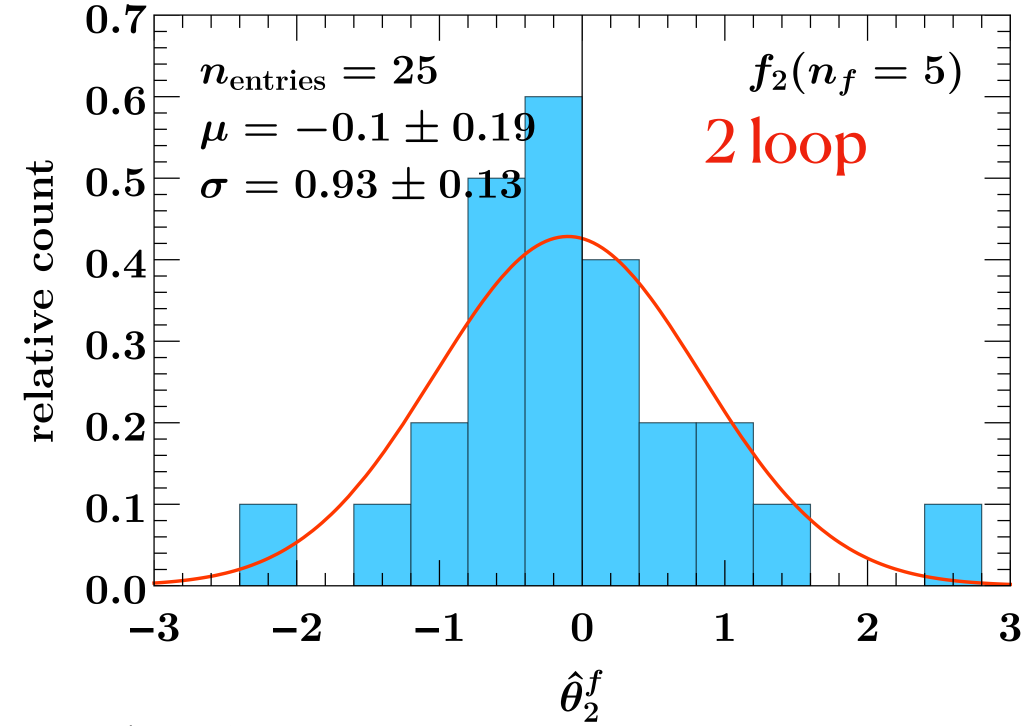
THANK YOU!

Backup slides

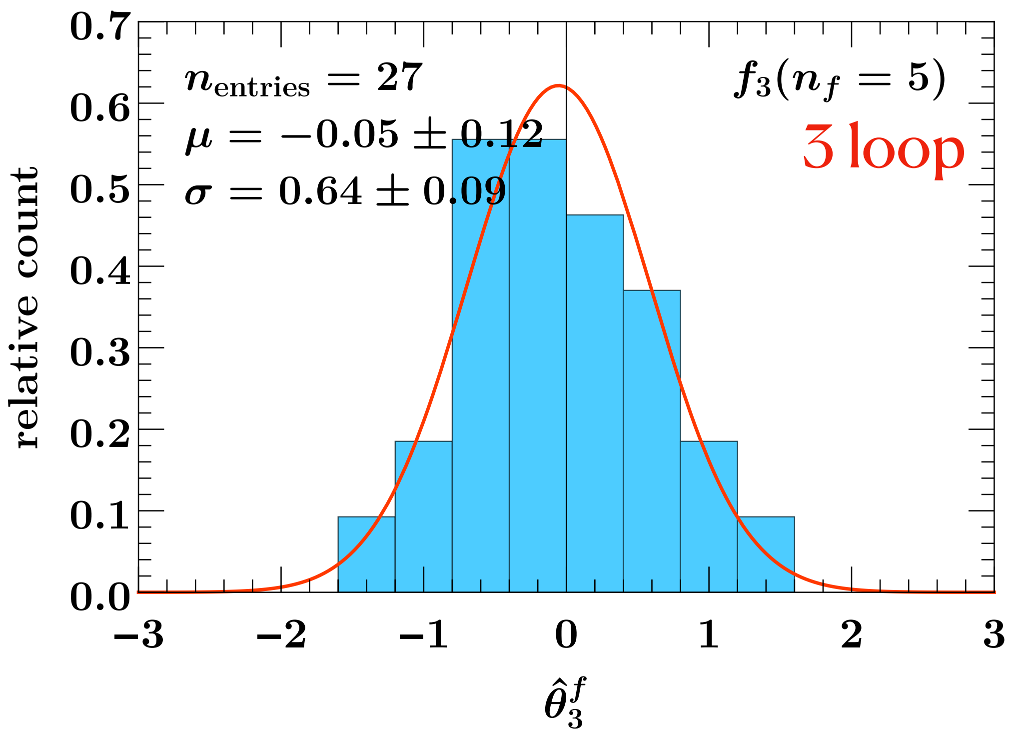
TNPs for Boundary Conditions

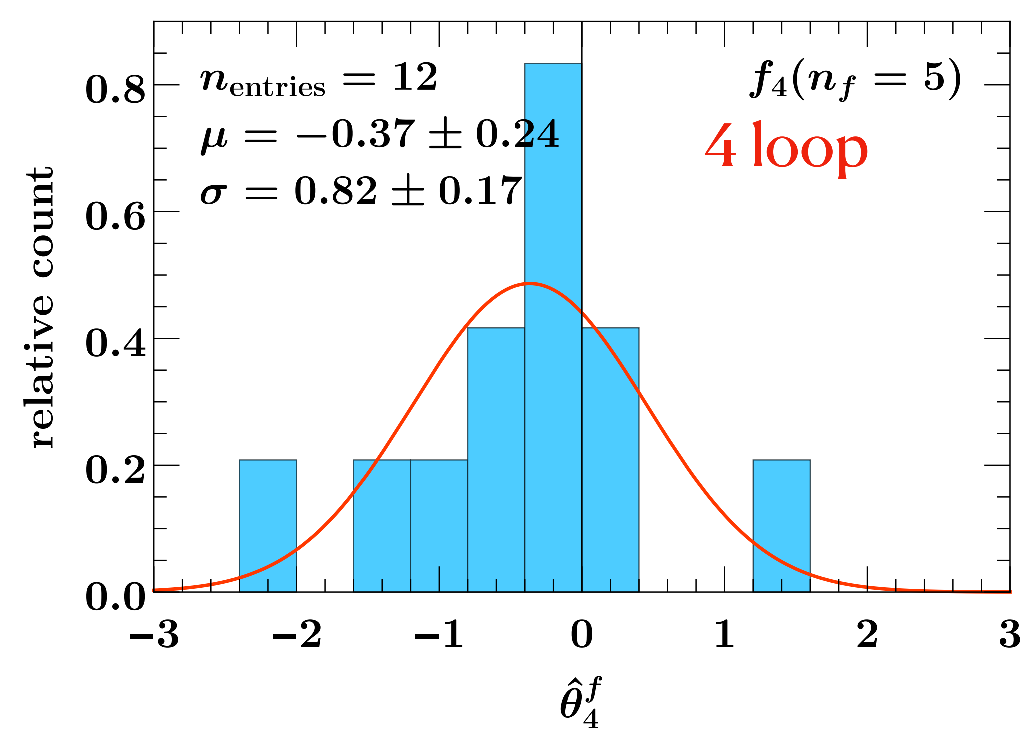
$$F_n(\theta_n) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f$$





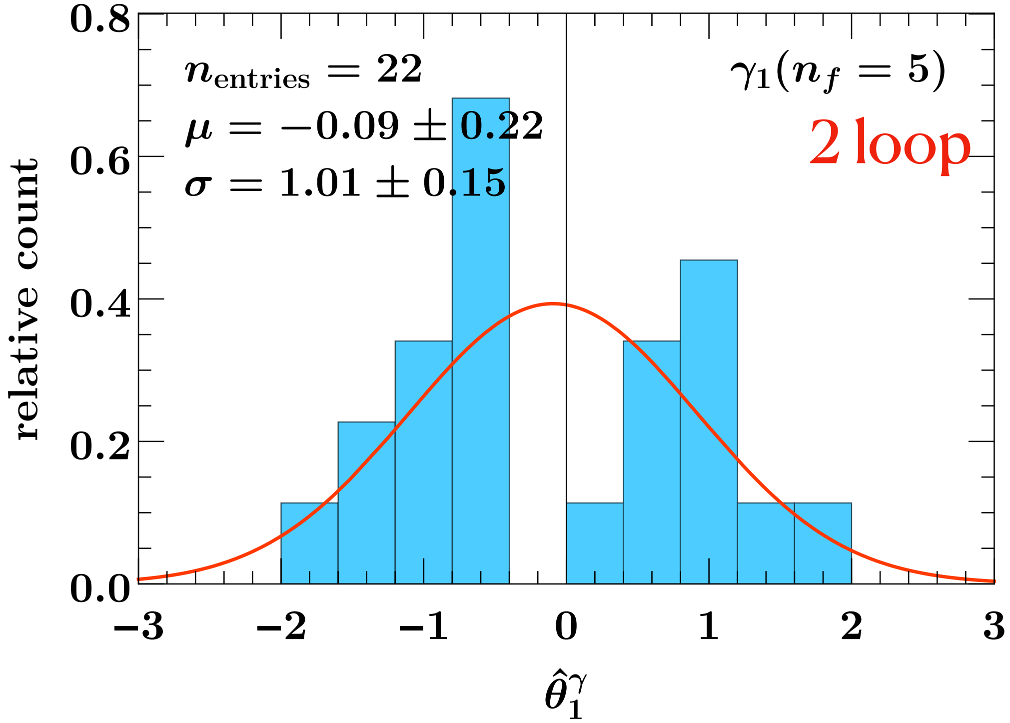
Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$

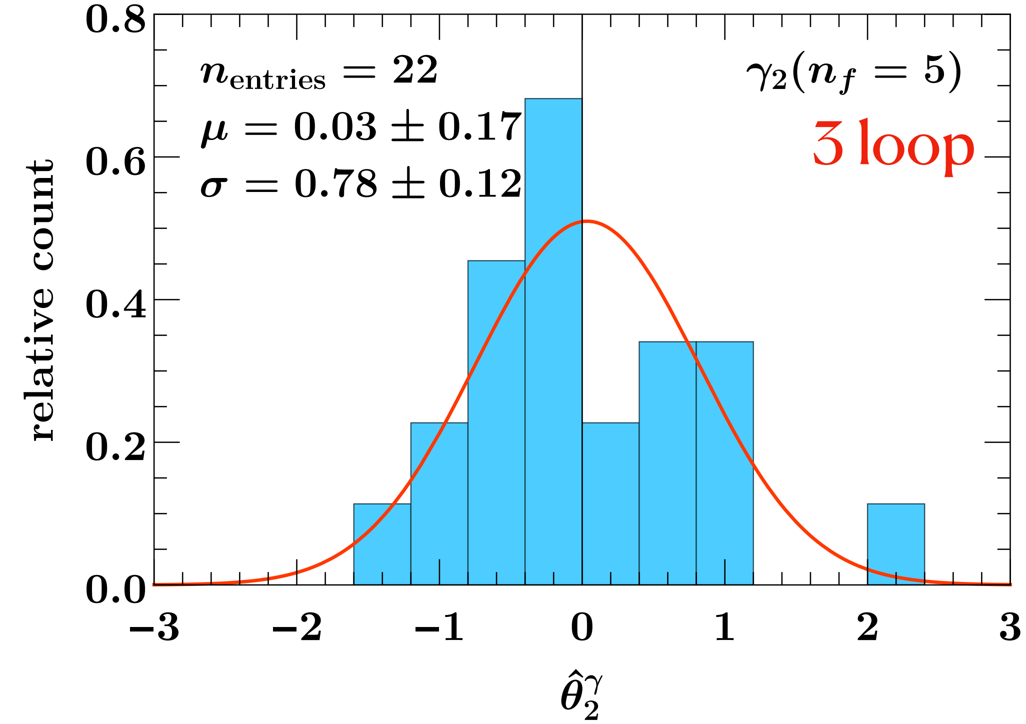




TNPs for Anomalous Dimensions

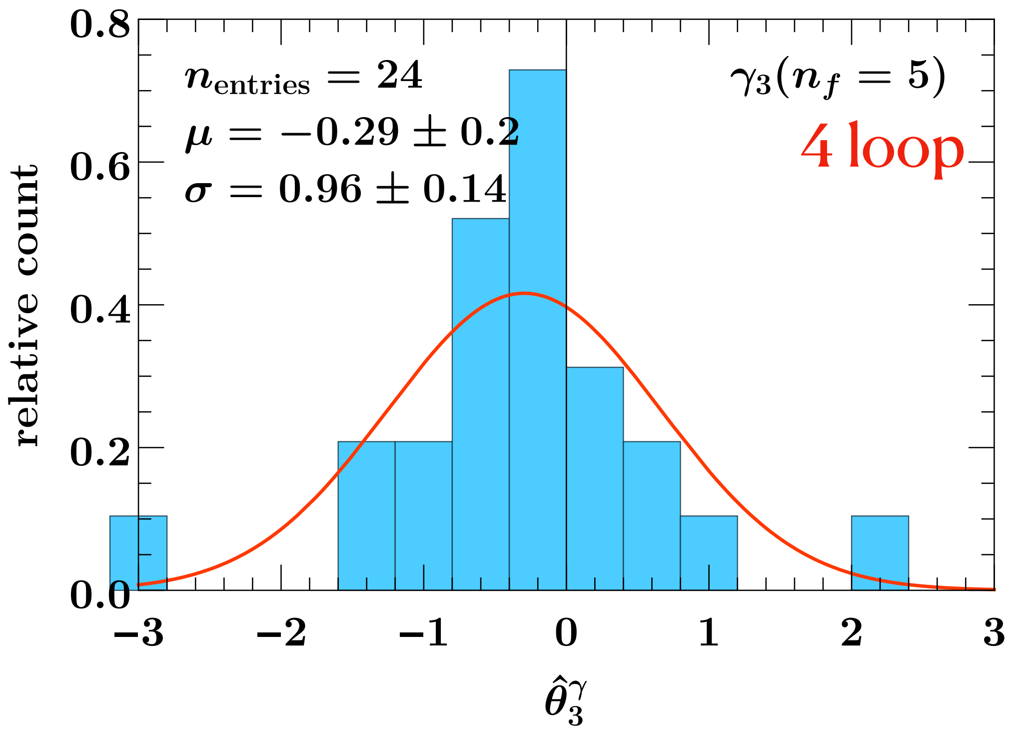
$$\gamma_n(\theta_n) = 4C_r(4C_A)^n\,\theta_n^\gamma$$

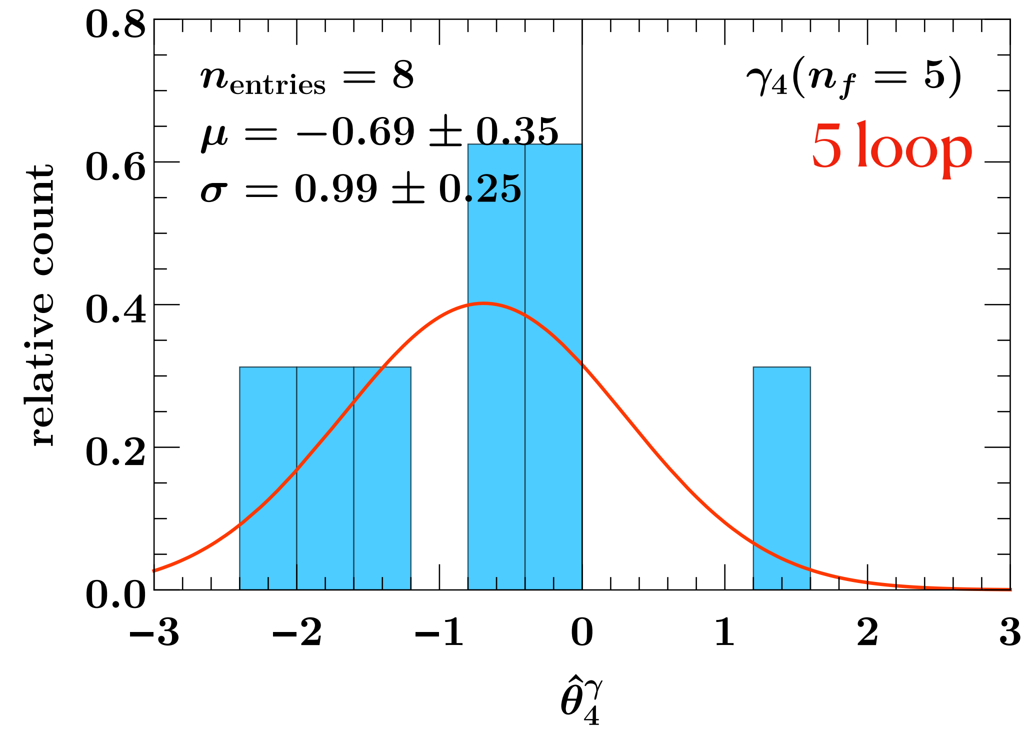




Good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$

✓



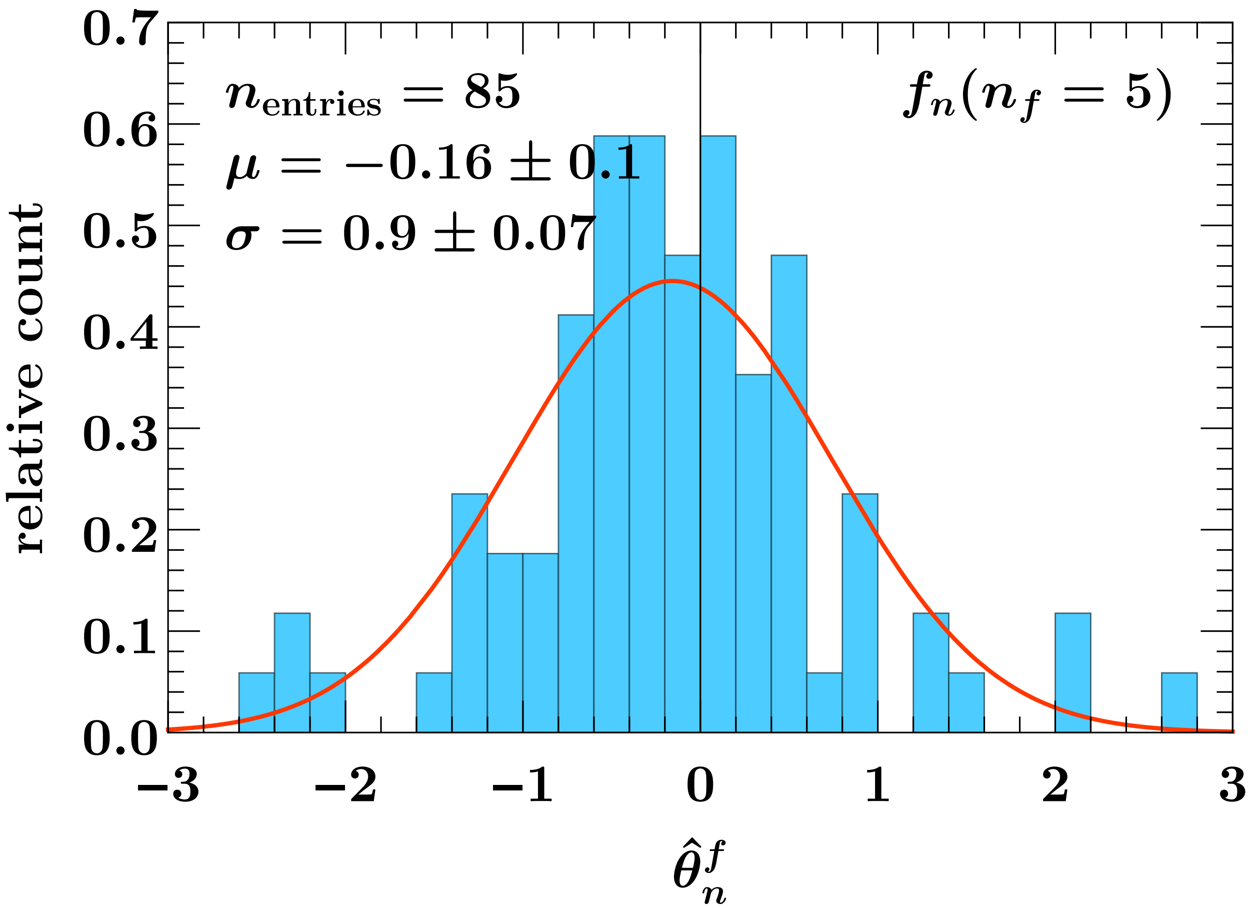


TNPs for BC and ADm

Considering all together from 1 to 4 loop:

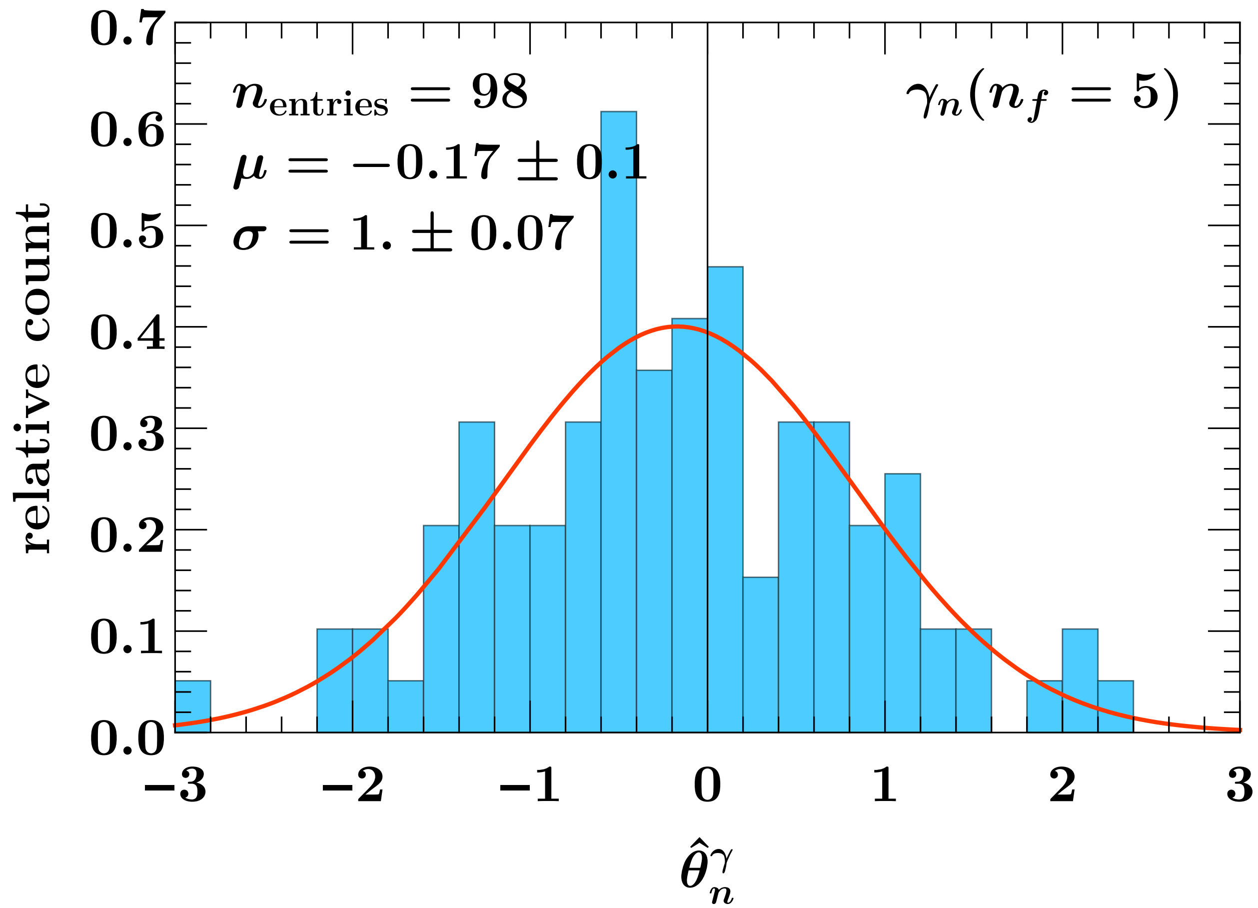
Boundary Conditions

$$F_n(\theta_n) = 4C_r(4C_A)^{n-1}(n-1)!\,\theta_n^f$$



Anomalous Dimensions

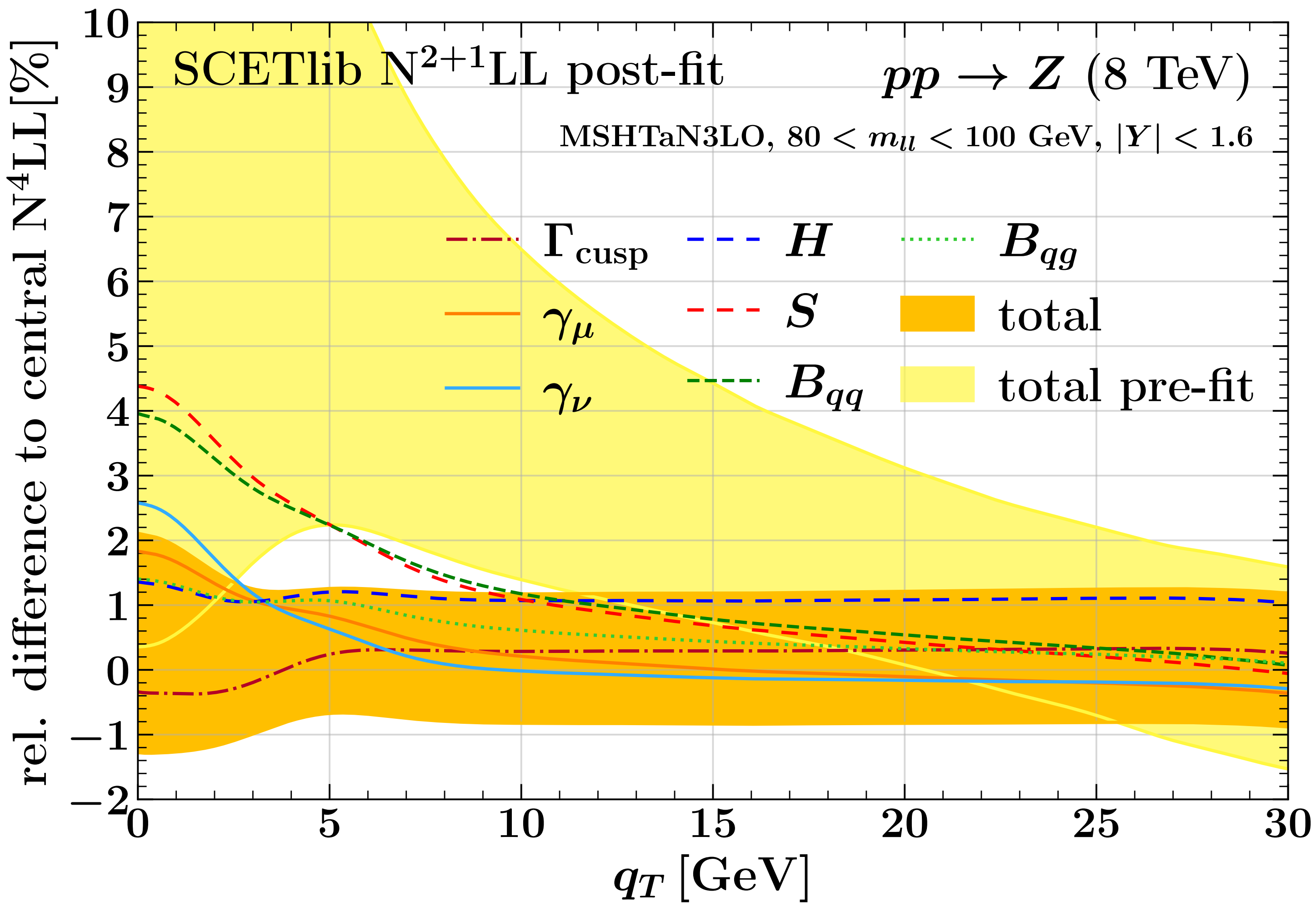
$$\gamma_n(\theta_n) = 4C_r(4C_A)^n\,\theta_n^\gamma$$

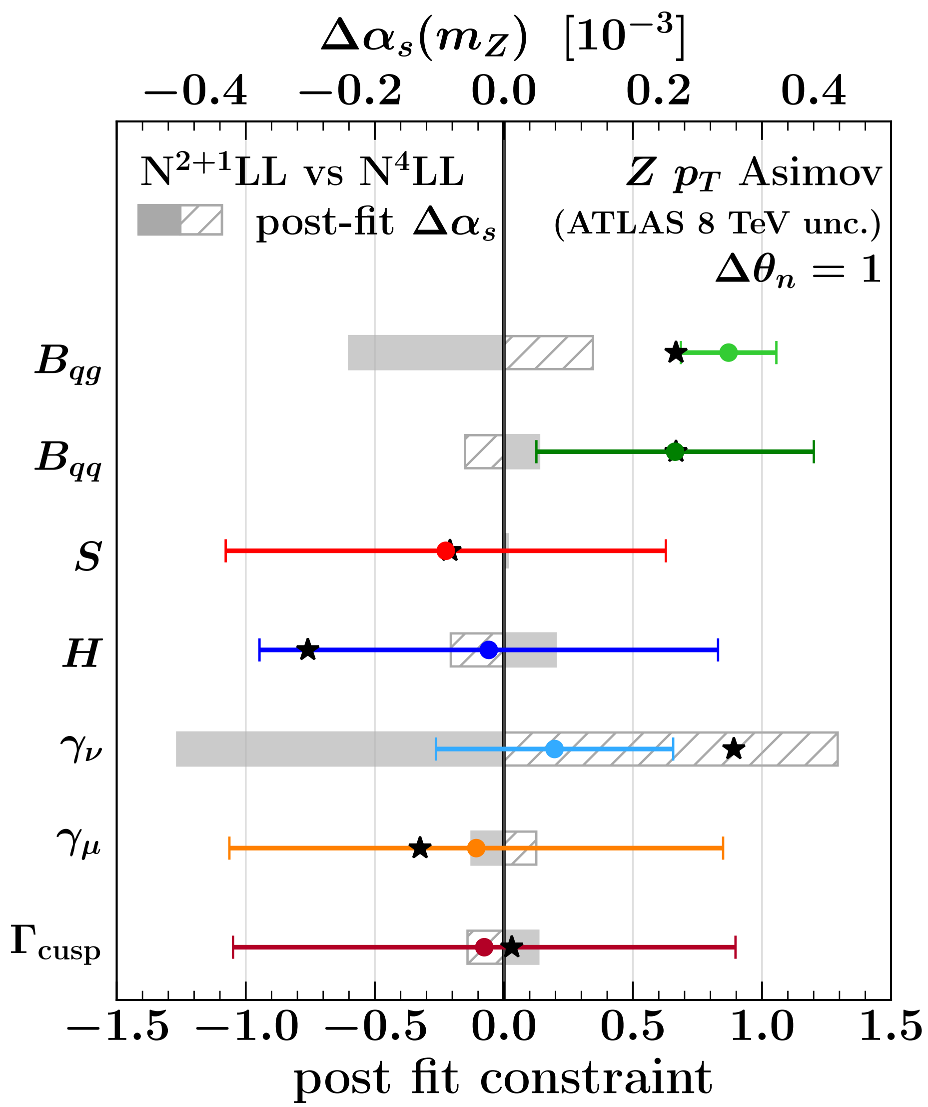


Very good fit to a Gaussian with $\theta_n \approx 0$ and $\Delta\theta_n \approx 1$ ✓

Post-fit constraints on $N^{2+1}LL$

Profiling lower order against higher order: $N^{2+1}LL$



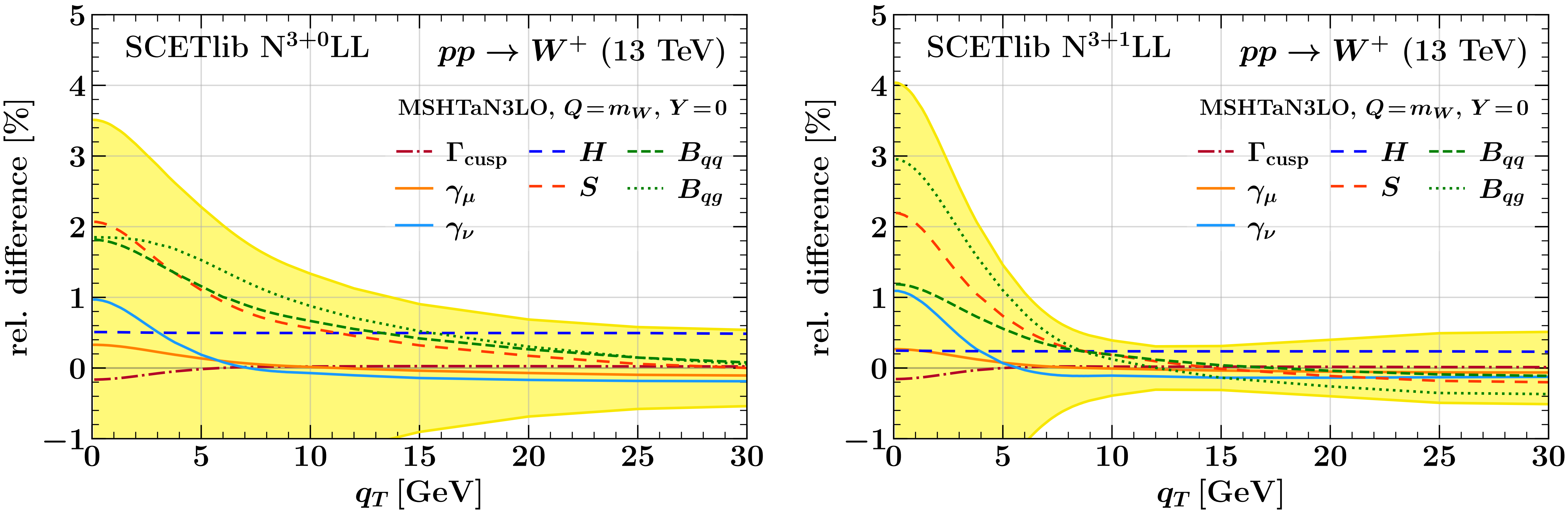


- $N^{2+1}LL$ strongly pulled, toward correct true values [★]
- post-fit prediction for q_T spectrum driven by constraints from data

TNPs at N^{3+0}LL

N^{3+0}LL is an approximation of N^{3+1}LL:

consider the N^{3}LL structure but absorb the N^{3+1}LL TNPs uncert. term into the N^{3}LL structure



- The impact of the parameters is only approximately correct!
- Limited effect on the overall size of theory uncert., but may have bigger effect on theory correlations

$\longrightarrow$ if possible prefer the N^{m+1}LL prescription!

SCETlib nonperturbative models

- Collins-Soper (CS) kernel

$$\tilde{\gamma}_\nu(b_T) = \tilde{\gamma}_\nu^{\text{pert}}\Big(b_6^*(b_T)\Big) + \tilde{\gamma}_\nu^{\text{nonp}}(b_T)$$

$$\tilde{\gamma}_\nu^{\text{nonp}}(b_T) = -\lambda_\infty\, f_\nu\left(\frac{\lambda_2}{\lambda_\infty} b_T^2 + \frac{\lambda_4}{\lambda_\infty} b_T^4\right)$$

- Transverse Momentum Distributions (TMDs)

$$\tilde{f}(b_T) = \tilde{f}_{\text{pert}}(b_T)\, \tilde{f}_{\text{nonp}}(b_T)$$

$$\ln\Big(\tilde{f}_{\text{nonp}}(b_T)\Big) = -\Lambda_\infty\, b_T\, f\left(\frac{\Lambda_2}{\Lambda_\infty} b_T + \frac{\Lambda_4}{\Lambda_\infty} b_T^3\right)$$

λ_2, λ_4 and Λ_2, Λ_4 quadratic/quartic small b_T coefficients

$\lambda_\infty, \Lambda_\infty$ determine $b_T \to \infty$ behavior

What about $f_\nu(x)$ and $f(x)$? [Collins and Rogers '14]

$$\tilde{\gamma}_\nu^{\text{nonp}}(b_T \to 0) \sim b_T^2, \quad \tilde{\gamma}_\nu^{\text{nonp}}(b_T \to \infty) \sim const \quad f_\nu\left(\frac{\lambda_2}{\lambda_\infty} b_T^2 + \frac{\lambda_4}{\lambda_\infty} b_T^4\right) = \tanh\left(\frac{\lambda_2}{\lambda_\infty} b_T^2 + \frac{\lambda_4}{\lambda_\infty} b_T^4\right)$$

$$\log\Big(\tilde{f}_{\text{nonp}}(b_T \to 0)\Big) \sim b_T^2, \quad \log\Big(\tilde{f}_{\text{nonp}}(b_T \to \infty)\Big) \sim b_T \quad f\left(\frac{\Lambda_2}{\Lambda_\infty} b_T + \frac{\Lambda_4}{\Lambda_\infty} b_T^3\right) = 2\tanh\left(\frac{\Lambda_2}{\Lambda_\infty} b_T + \frac{\Lambda_4}{\Lambda_\infty} b_T^3\right)$$

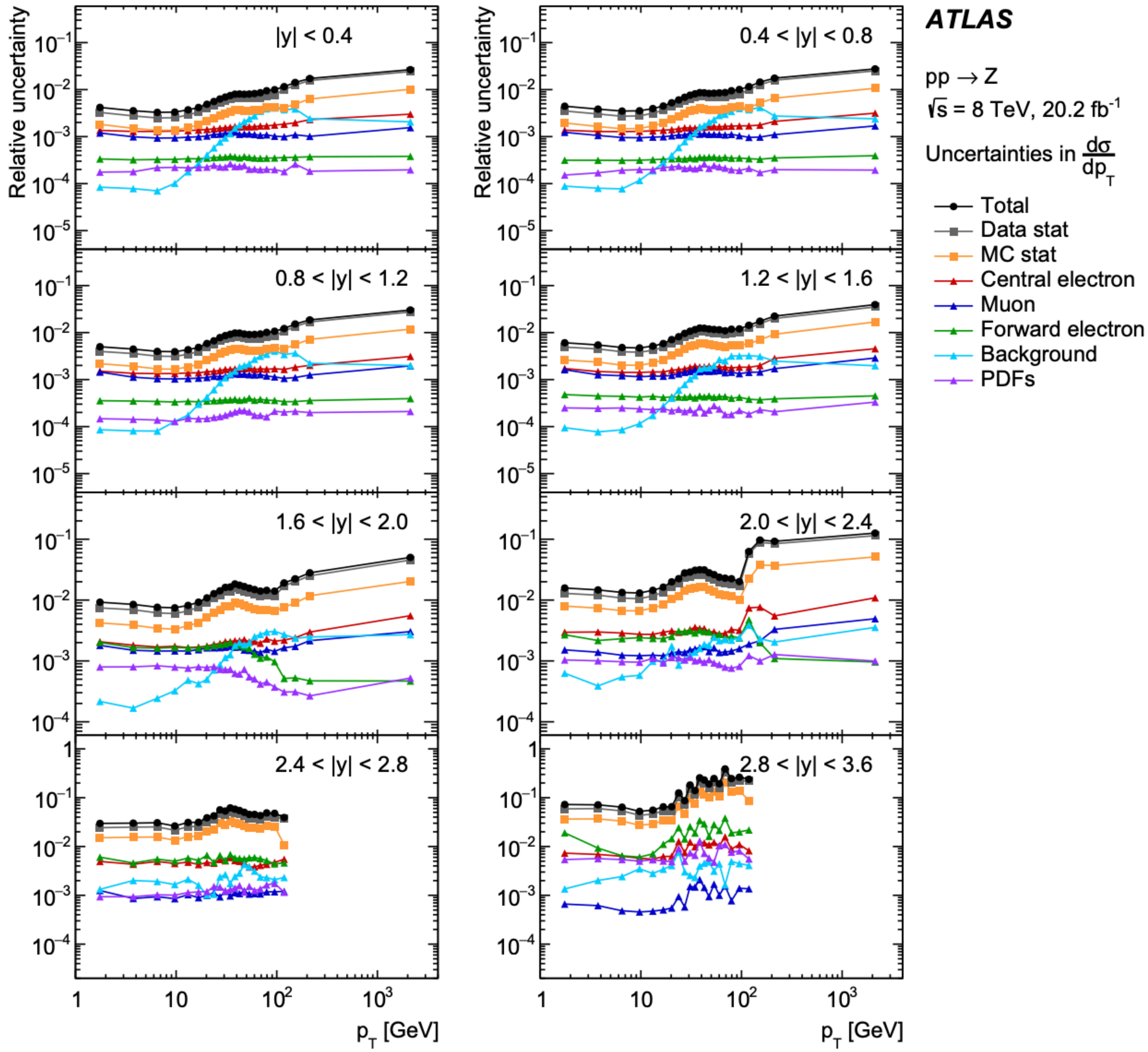
$\alpha_s(m_Z)$ determination from ATLAS [arXiv:2309.12986]

Fit of strong coupling constant using p_T in [0,29] GeV based on N^{3}LO+N^{4}LLa from DYTurbo

$\alpha_S(m_Z) = 0.1183 \pm 0.0009$

Breakdown of uncertainties on α_s in units of 10^{-3}:

Experimental uncertainty	± 0.44	
PDF uncertainty	± 0.51	
Scale variation uncertainties	± 0.42	
Matching to fixed order	0	-0.08
Non-perturbative model	$+0.12$	-0.20
Flavour model	$+0.40$	-0.29
QED ISR	± 0.14	
N^{4}LL approximation	± 0.04	
Total	$+0.91$	-0.88



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