Advances in resummed calculations for $t\bar{t}$ and $t\bar{t}H$ production at the LHC

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Overview of the talk

- 0-jettiness factorization & resummation for $t\bar{t}$ production at the LHC. Soft functions
- q_τ resummation for $t\bar{t}$ production at the LHC [W.L. Ju, M. Schoenherr 2210.09272]
- Soft-gluon resummation for $t\bar{t}H$: comparison of dQCD and SCET methods

Introduction

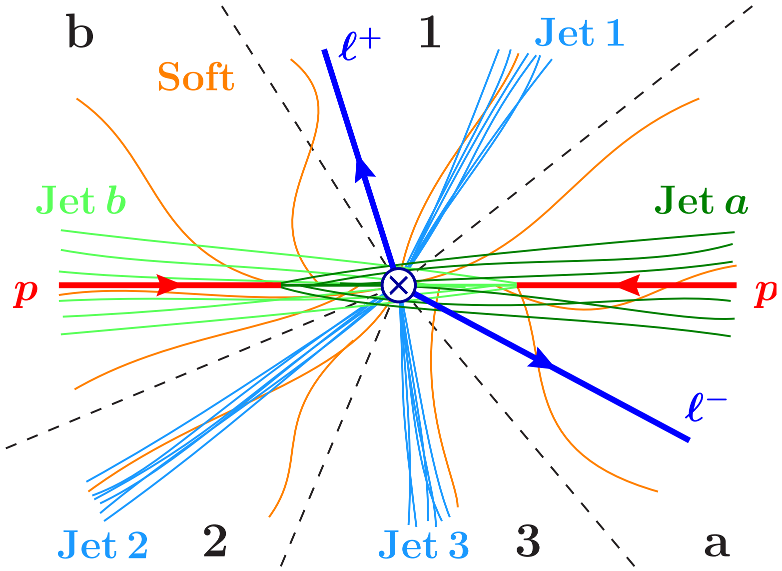
- The availability of highly precise calculations is indispensable to boost the discovery potential of the HL-LHC
- Fixed order calculations alone are not sufficient to describe data in all corners of the phase space due to realistic experimental cuts or kinematical constraints → **resummation**
- Comparison with data at particle level requires matching of partonic calculations with Parton Showers including Hadronization effects
- EFTs provide powerful tools to face current and future precision challenges at colliders and beyond:
    - Resummation → N-jettiness, small q_T, Threshold
    - Subtraction methods for IR divergences → NNLO calculations
    - Systematic study of subleading power corrections

→ Monte Carlo event generator GENEVA

N-Jettiness

- N-jettiness [Stewart, Tackmann,Waalewijn `09,`10] $\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$

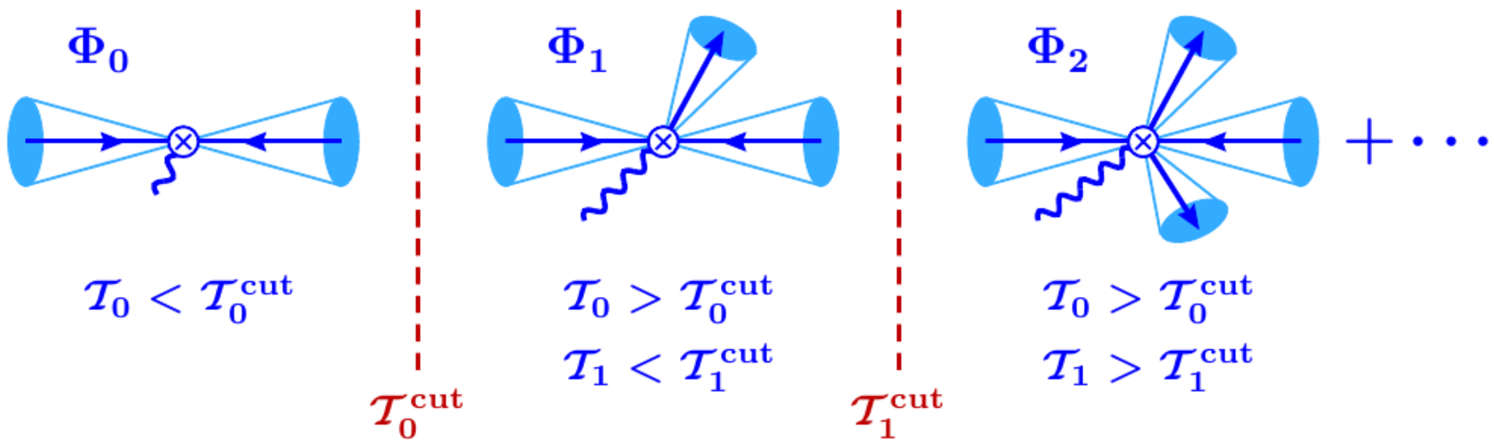
- The limit $\mathcal{T}_N \to 0$ describes a N-jet event where the unresolved emissions can be either soft or collinear to the final state jets or initial state beams



- ***Color-singlet final state***, relevant variable is 0-jettiness aka "beam thrust"

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}| \, e^{-|\eta_k - Y|}$$

- GENEVA MC employs IR-finite definition of events based on a set of N-jettiness resolution variables



- When one takes $\mathcal{T}_N^{\rm cut} \to 0$, large logarithms of $\mathcal{T}_N^{\rm cut}/M_{ll}$, $\mathcal{T}_N/M_{ll}$ appear and need to be resummed

- Implementation of ***colour singlet processes***: DY [Phys. Rev. D. 104 (2021) 9], single [JHEP 05 (2023) 128] and double Higgs production [JHEP 06 (2023) 205], photon pair production [JHEP 04 (2021) 041], ZZ [Phys. Lett. B 818 (2021)], $W^{\pm}\gamma$ [Phys. Lett. B. 826 (2022)], HV [Phys. Rev. D. 100 (2019) 9], Higgs boson decays [JHEP 04 (2021) 254]

0-jettiness resummation for $t\bar{t}$ production

Based on [S. Alioli, AB, M.A. Lim, arXiv:2111.03632]

- NNLO+PS generator for $t\bar{t}$ production available in MINNLOPS
  [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20,`21]
- Including higher-order *resummation* improves the description of observables
- To reach NNLO+PS accuracy in GENEVA
  - NLO calculations for $t\bar{t}$ and $t\bar{t}$+jet
  - Resummed calculation at NNLL` in the resolution variable
  - 0-jettines resummation used for colour-singlet in GENEVA, must be extended for $t\bar{t}$ production. Definition of 0-jettiness has to be adapted with top-quarks in the final state, we choose to treat them like EW particles and exclude them from the sum over radiation
  - Need to develop the resummation framework for $t\bar{t}$ ($t\bar{t}V$)

Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. In case of boosted regime $M_{t\bar{t}} \gg m_t$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

Hard functions (color matrices)

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\,\mathbf{\Gamma}_H^\dagger(M,\beta_t,\theta,\mu)$$

Solution:

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

$$\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{\Gamma}_H(M,\beta_t,\theta,\mu) = \Gamma_{\rm cusp}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$ [Ferroglia, Neubert, Pecjak, Yang,`09]

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in α_s [Buchalla,Buras,Lautenbacher `96]

Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$\mathbf{S}^{(1)}_{\text{bare},\,ij}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = \sum_{\alpha,\,\beta} \boldsymbol{w}_{ij}^{\alpha\beta}\, \hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu)$$

$$\hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k\, v_\beta \cdot k}\, \delta(k^2)\Theta(k^0)$$
$$\times \left[\delta(k_a^+ - k\cdot n_a)\Theta(k\cdot n_b - k\cdot n_a)\,\delta(k_b^+) + \delta(k_b^+ - k\cdot n_b)\Theta(k\cdot n_a - k\cdot n_b)\,\delta(k_a^+)\right]$$

One can average over the two hemisphere momenta, the soft functions satisfies the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s\dagger}\right] \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s}\right]$$

Solution in momentum space, we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\,\mathbf{1}$

$$\mathbf{S}_B(l^+, \beta_t, \theta, \mu) = \exp\left[4S(\mu_s, \mu) + 2a_{\gamma^B}(\mu_s, \mu)\right]$$
$$\times\ \mathbf{u}^\dagger(\beta_t, \theta, \mu, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s}, \beta_t, \theta, \mu_s)\, \mathbf{u}(\beta_t, \theta, \mu, \mu_s)\, \frac{1}{l^+} \left(\frac{l^+}{\mu_s}\right)^{2\eta_s} \frac{e^{-2\gamma_E \eta_s}}{\Gamma(2\eta_s)}$$

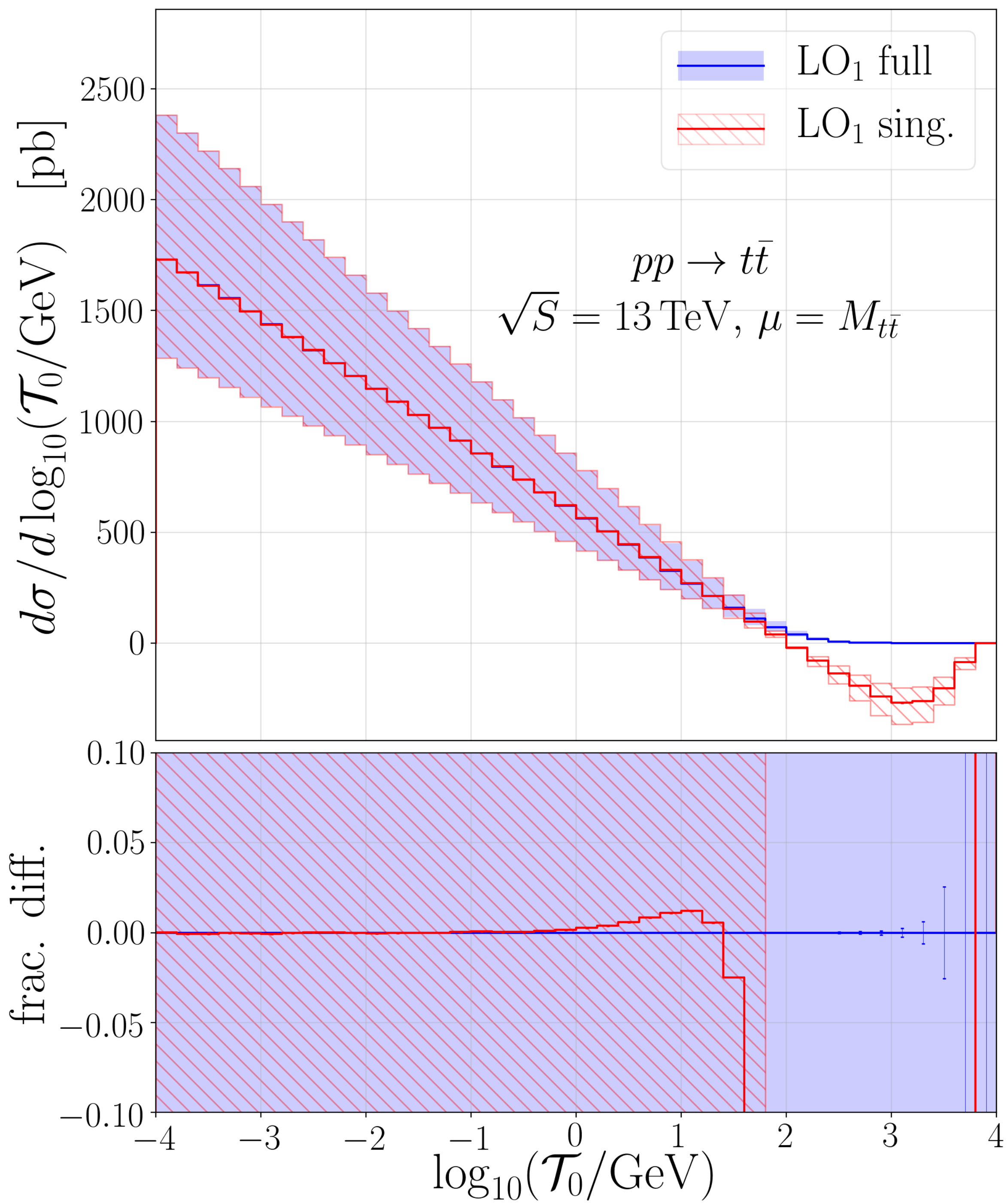
$$\eta_s \equiv -2a_\Gamma(\mu_s, \mu)$$

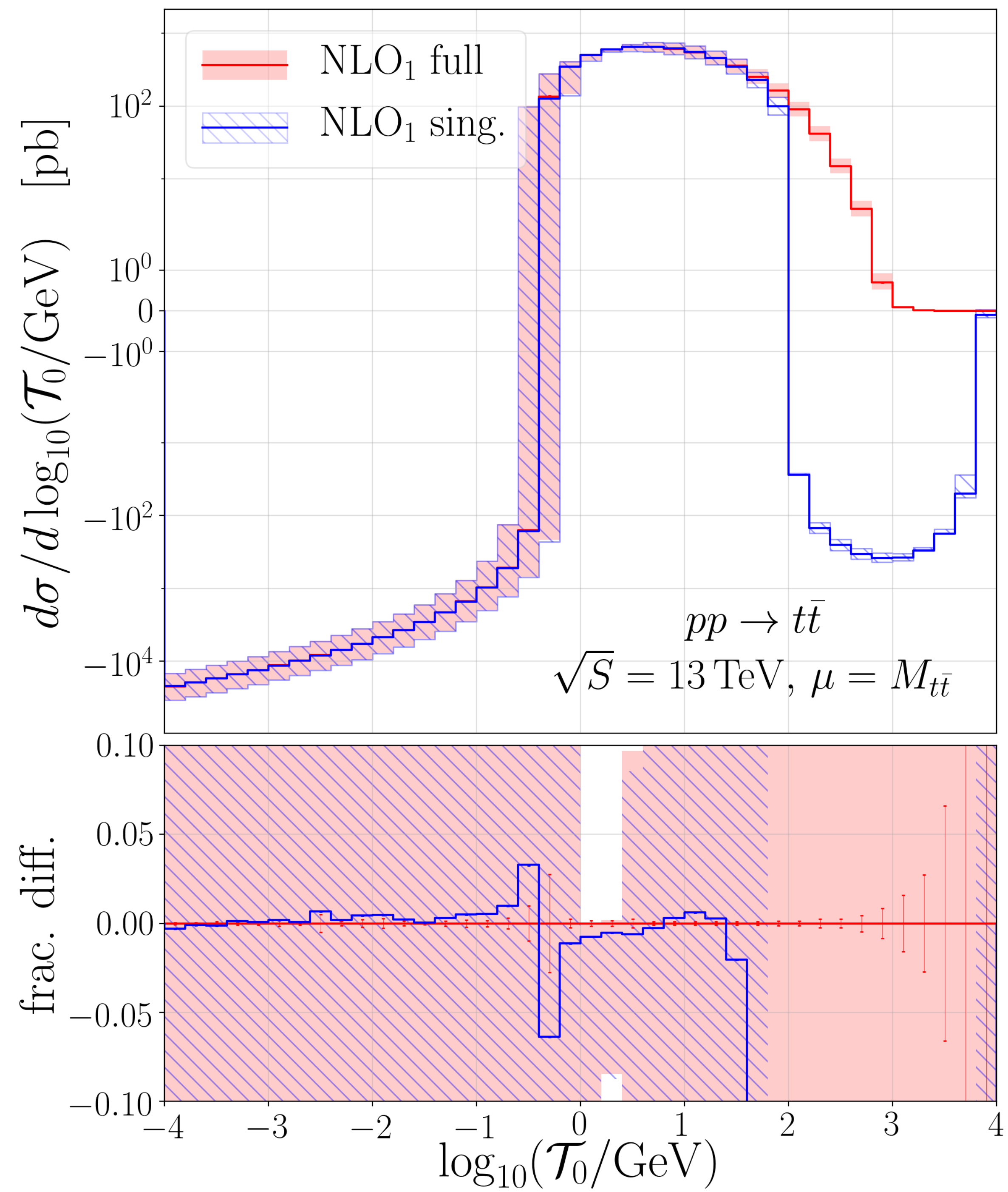
Resummed result for the cross section

- We have
  - hard functions at NLO
  - soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we only miss $\delta(\mathcal{T}_0)$ terms at NNLO
  - beam functions at NNLO (for initial states with quarks and gluons)
  - two-loop anomalous dimensions
- We can resum to NNLL. We are missing $\delta(\mathcal{T}_0)$ terms (from NNLO hard and soft functions). If we include everything else we know we obtain a $\mathrm{NNLL}'_{\mathrm{a}}$ result
- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_0 > 0$) *(also needed for nonlocal N-jettiness subtractions)*

Singular vs fixed order

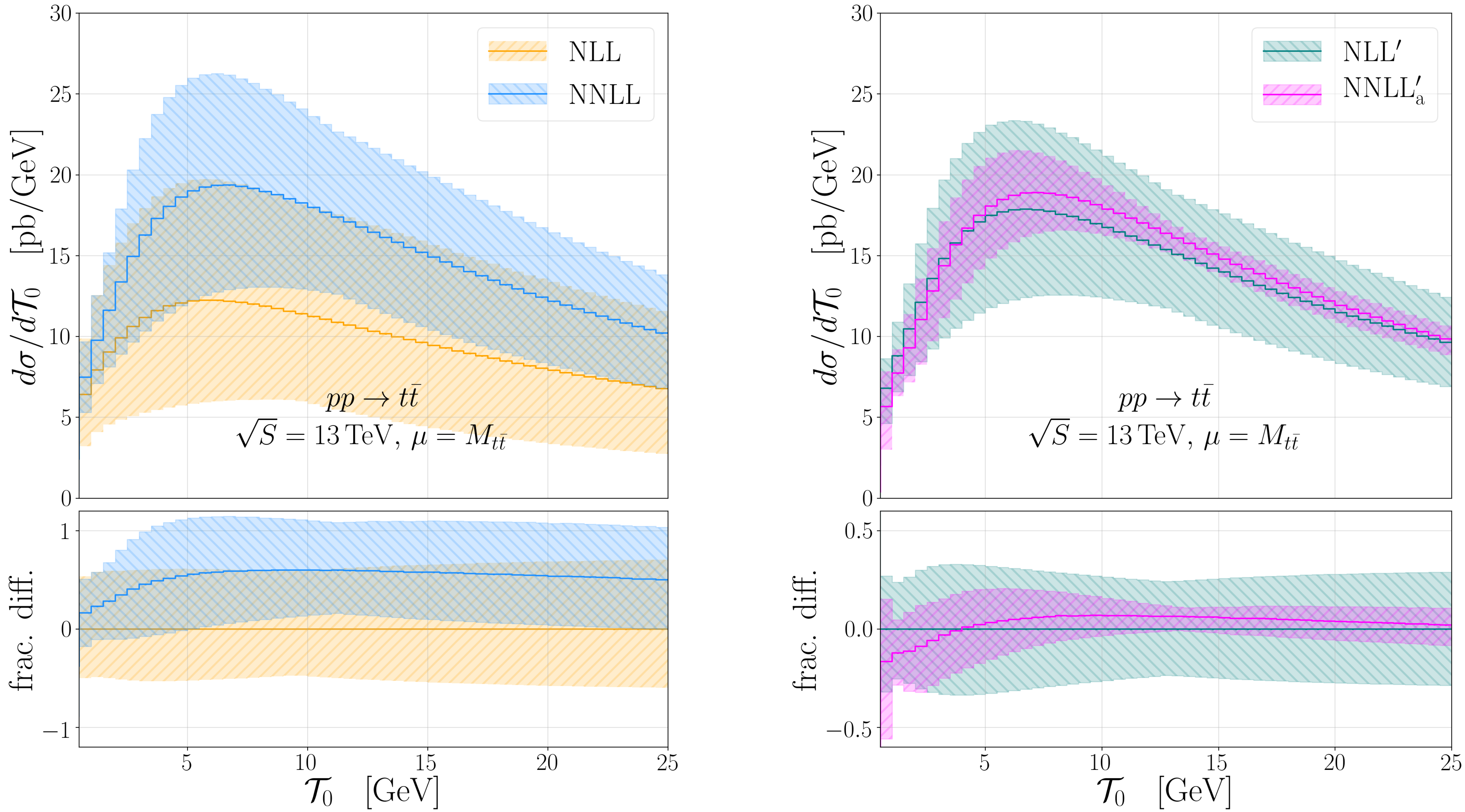
Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1





Resummed results

NNLL'_a is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



Zero-jettiness soft functions at NNLO

We are computing the correlated emissions of NNLO soft functions [Bell,AB,Dehnadi,Edelmann,Lim,Rahn, in progress]. Soft amplitudes from [Angeles-Martinez,Czakon,Sapeta 18'], for example double real (RR) C_A part

$$\mathcal{M}^{*(0)}_{g,g,a_1,\ldots}(k,l,p_1,\ldots)\,\mathcal{M}^{(0)}_{g,g,a_1,\ldots}(k,l,p_1,\ldots)$$
$$\simeq \frac{1}{2}\sum_{ijkl}\mathcal{S}_{ij}(k)\mathcal{S}_{kl}(l)\,\langle\mathcal{M}^{(0)}_{a_1,\ldots}(p_1,\ldots)|\,\{\mathbf{T}_i\cdot\mathbf{T}_j,\mathbf{T}_k\cdot\mathbf{T}_l\}\,|\mathcal{M}^{(0)}_{a_1,\ldots}(p_1,\ldots)\rangle$$
$$-\,C_A\sum_{ij}\mathcal{S}_{ij}(k,l)\,\langle\mathcal{M}^{(0)}_{a_1,\ldots}(p_1,\ldots)|\mathbf{T}_i\cdot\mathbf{T}_j|\mathcal{M}^{(0)}_{a_1,\ldots}(p_1,\ldots)\rangle$$

Using colour conservation and symmetries it can be rewritten as

$$\sum_{i\neq j}\mathbf{T_i}\cdot\mathbf{T_j}\big(S_{ij}(k,l)-S_{ii}(k,l)/2-S_{jj}(k,l)/2\big)$$

$$I_{RR}(\epsilon)=\frac{(4\pi e^{\gamma_E}\tau^2)^{-2\epsilon}}{(2\pi)^{2d-2}}\int d^dk\,\delta(k^2)\,\theta(k^0)\int d^dl\,\delta(l^2)\,\theta(l^0)\,|M_{RR}(k,l)|^2\,\mathscr{M}(\tau;k,l)$$

Change of variables: from light-cone variables to the variables defined in [Bell, Rahn, Talbert, 1812.08690]

$$k_-=\frac{a\,b\,p_T}{(1+ab)\sqrt{y}},\quad k_+=\frac{b\sqrt{y}\,p_T}{(a+b)}\qquad\qquad l_-=\frac{p_T}{(1+ab)\sqrt{y}},\quad l_+=\frac{a\sqrt{y}\,p_T}{(a+b)}$$

Zero-jettiness soft functions at NNLO

UV renormalization: start from hard anomalous dimension (only dipoles) [Ferroglia, Neubert, Peck, Yang, 09]

$$\boldsymbol{\Gamma}\left(\{\underline{p}\},\{\underline{m}\},\mu\right)\Big|_{\text{2-parton}} = \sum_{(i,j)} \frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma_i(\alpha_s)$$

$$- \sum_{(I,J)} \frac{\mathbf{T}_I\cdot\mathbf{T}_J}{2}\gamma_{\text{cusp}}(\beta_{IJ},\alpha_s) + \sum_I \gamma_I(\alpha_s)$$

$$+ \sum_{(I,j)} \frac{\mathbf{T}_I\cdot\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\ln\frac{m_I\mu}{-s_{Ij}}.$$

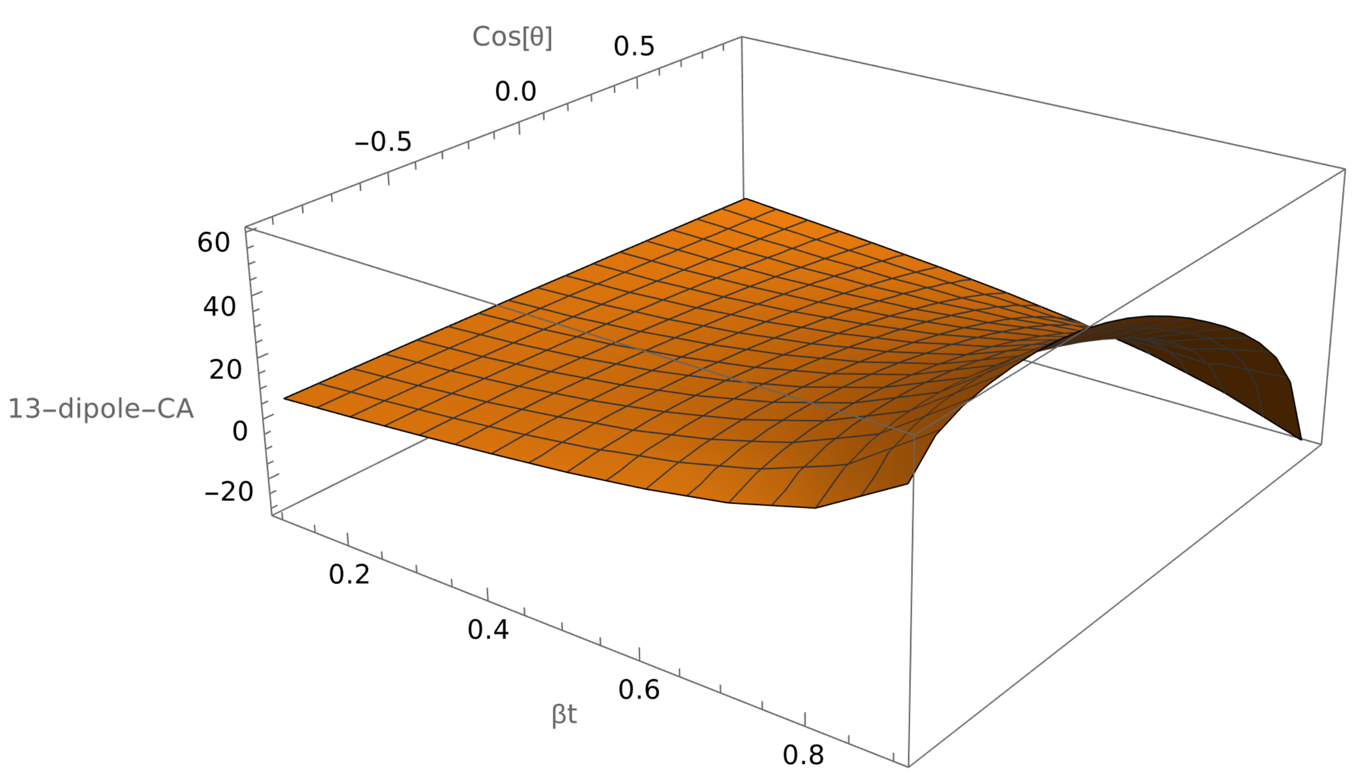
Combining the hard anomalous dimensions with beam anomalous dimensions to obtain the soft anomalous dimensions $\boldsymbol{\Gamma}_s = -\boldsymbol{\Gamma}_h - 2\,\Gamma_B$. We can renormalize additively (the logarithm of) the soft function dipole by dipole with

$$\ln \boldsymbol{Z} = \frac{\alpha_s}{4\pi}\left(\frac{\Gamma_0'}{4\varepsilon^2}+\frac{\boldsymbol{\Gamma}_0}{2\varepsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left[-\frac{3\beta_0\Gamma_0'}{16\varepsilon^3}+\frac{\Gamma_1'-4\beta_0\boldsymbol{\Gamma}_0}{16\varepsilon^2}+\frac{\boldsymbol{\Gamma}_1}{4\varepsilon}\right] + \mathcal{O}(\alpha_s^3)$$

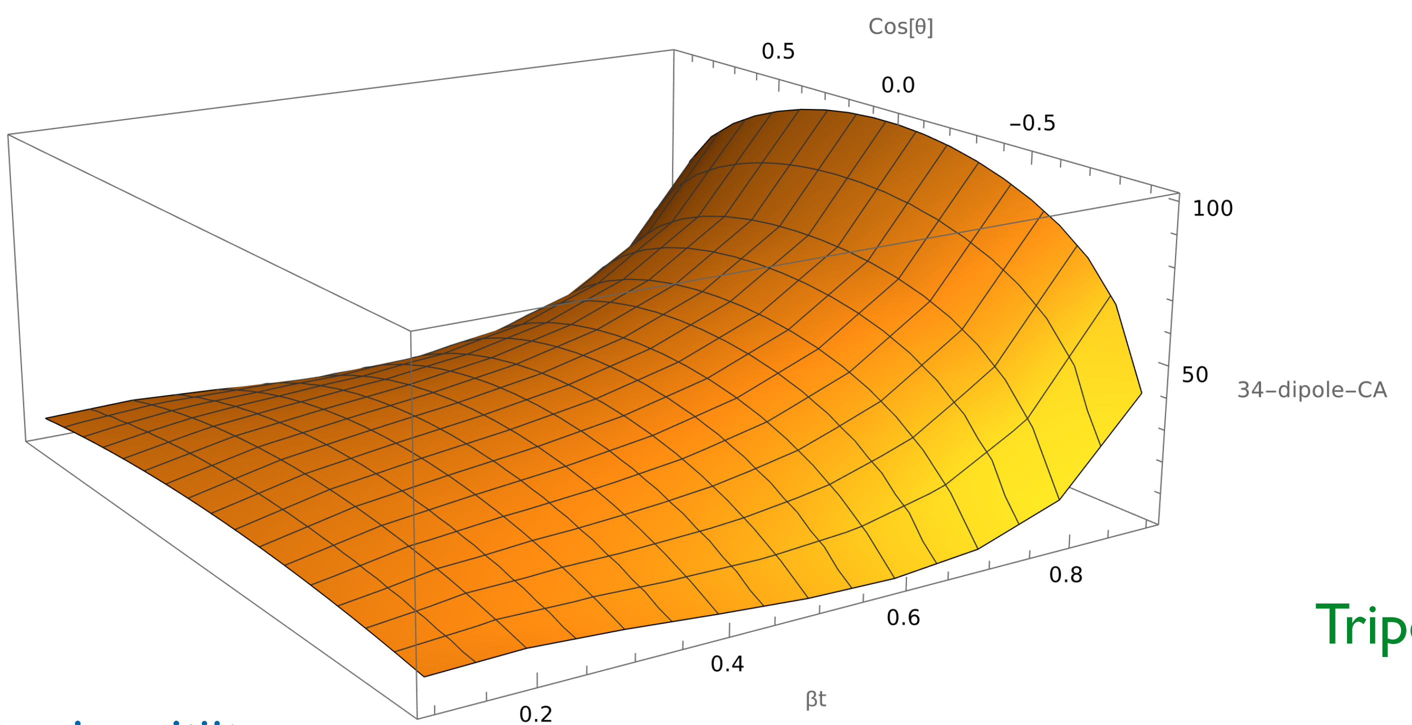
Casimirs of diagonal terms can always be rewritten in terms of colour dipoles using colour conservation

Zero-jettiness soft functions at NNLO

$(\beta_t, \cos\theta)$-plane, non-logarithmic term



Renormalized result of massless-massive (CA part) dipole after combining RR+VR and α_s renormalization. UV poles removed via Z factor in EFT

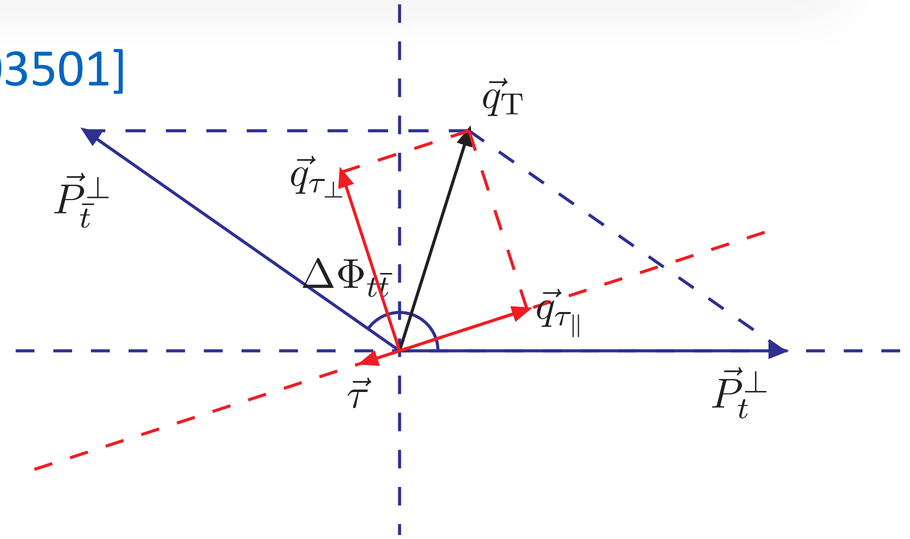


Renormalized result of massive-massive (CA part) dipole after combining RR+VR and α_s renormalization. UV poles removed via Z factor in EFT

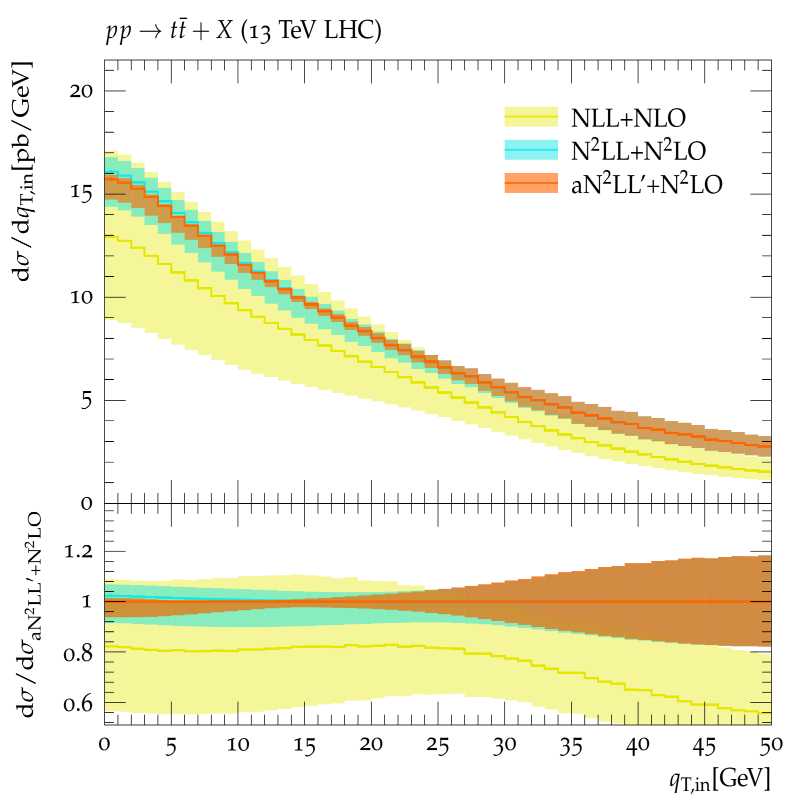
Tripole contributions on the way, only contribute to VR (RR tripole diagrams sum up to zero)

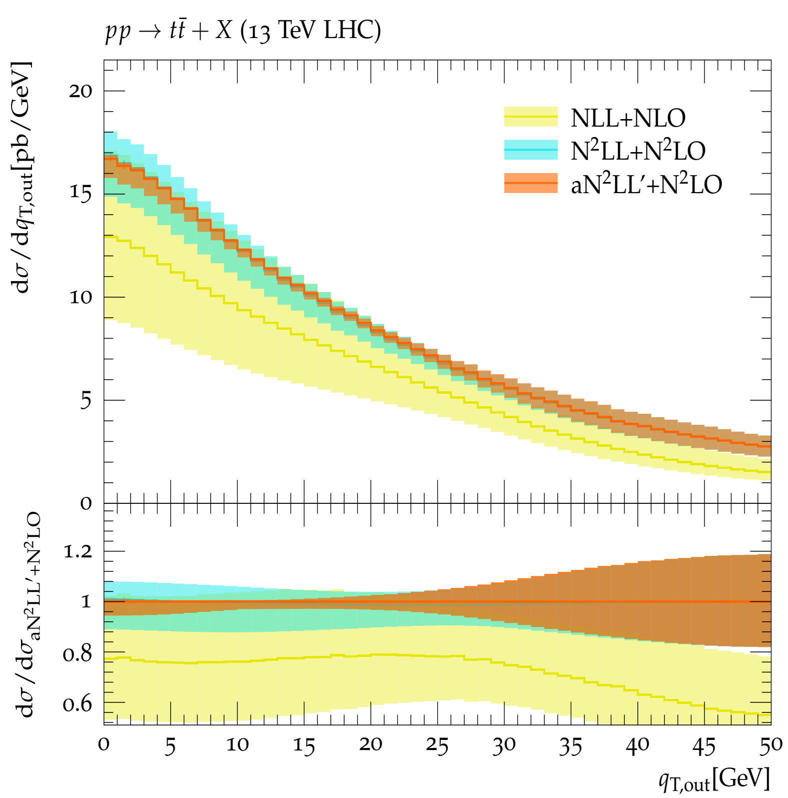
Projected transverse momentum resummation

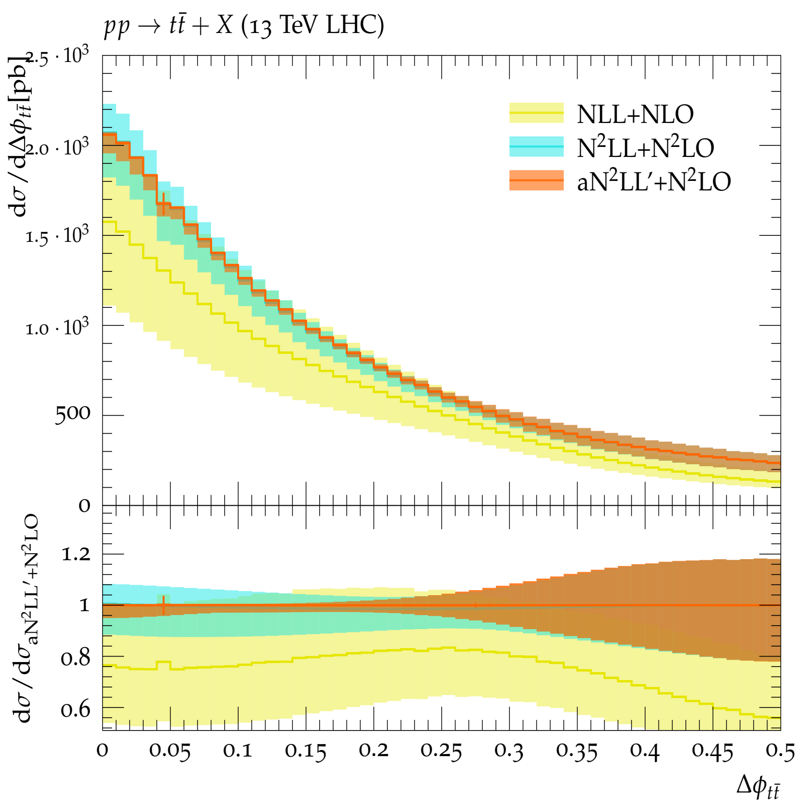
[W.L. Ju, M. Schoenherr, 2210.09272 and 2407.03501]



- Decomposition: $\vec{q}_\mathrm{T} = \vec{q}_{\tau_\perp} + \vec{q}_{\tau_\parallel} = q_{\tau_\perp}\vec{\tau}\times\vec{n} + q_{\tau_\parallel}\vec{\tau} \qquad q_\tau \equiv |\vec{q}_{\tau_\parallel}|$
- Projected transverse momentum spectrum on $\vec{\tau}$, $q_{\tau_\perp}$ component is integrated. Factorization of the cross section is shown.
- Removes the azimuthal correlation divergences (in addition to the azimuthally averaged case $|\vec{q_T}|$)
- Three observables are defined depending on the choice of $\vec{\tau}$ parallel ($q_{T,\mathrm{in}}$) or perpendicular ($q_{T,\mathrm{out}}$) to the top-quark transverse momentum and $\Delta\phi_{t\bar{t}} \equiv (\pi - \Delta\Phi_{t\bar{t}}) \sim q_{T,\mathrm{out}}/|\vec{p}_t^{\perp}|$







Soft gluon resummation for $t\bar{t}H$ in EFT

[AB,Ferroglia,Pecjak,Signer, Yang, JHEP 03 (2016) 124],[AB,Ferroglia,Pecjak,Yang, JHEP 02 (2017) 126],

[AB,Ferroglia,Frederix, Pagani,Pecjak,Tsinikos, JHEP 08 (2019) 039]

Threshold limit

When real radiation is present in the final state $\longrightarrow$ $\hat{s} \neq Q^2$, $z = Q^2/\hat{s} \to 1$

- Mellin space factorization derived in SCET+HQET ($N \to \infty$)

$$d\widetilde{\hat{\sigma}}_{ij}(N,\mu) = \mathrm{Tr}\left[\mathbf{H}_{ij}(\{p\},\mu)\,\widetilde{\mathbf{s}}_{ij}\left(\ln\frac{Q^2}{\bar{N}^2\mu^2},\mu\right)\right] \qquad \bar{N} = Ne^{\gamma_E}$$

- Resummation performed by deriving and solving RG equations

$$d\widetilde{\hat{\sigma}}_{ij}(\mu_f) = \exp\left[\frac{4\pi}{\alpha_s(\mu_h)}g_1(\lambda,\lambda_f) + g_2(\lambda,\lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi}g_3(\lambda,\lambda_f) + \ldots\right]$$
$$\times\,\mathrm{Tr}\left[\widetilde{\mathbf{u}}_{ij}(\mu_h,\mu_s)\mathbf{H}_{ij}(\{p\},\mu_h)\widetilde{\mathbf{u}}^{\dagger}_{ij}(\mu_h,\mu_s)\widetilde{\mathbf{s}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},\{p\},\mu_s\right)\right]$$

$$\lambda \equiv \beta_0\frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_s), \qquad \lambda_f \equiv \beta_0\frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_f)$$

- In the original papers predictions depend on μ_f (with $\mu_r = \mu_f$) and the matching scales μ_h, μ_s

Introducing the scale μ_r

- When matching to NNLO it is reasonable to vary independently $\mu_f \neq \mu_r$ (standard procedure). To make contact to FO, we have introduced μ_r in the resummation formula

  - Eliminate $\alpha_s(\mu_h)$ in favor of $\alpha_s(\mu_r)$ using

$$\alpha_s(\mu_h) = \frac{\alpha_s(\mu_R)}{X}\left[1 - \frac{\alpha_s(\mu_R)}{4\pi}\frac{\beta_1}{\beta_0}\frac{\ln X}{X} + \left(\frac{\alpha_s^2(\mu_R)}{4\pi}\right)^2 \left(\frac{\beta_1^2}{\beta_0^2}\frac{\ln^2 X - \ln X - 1 + X}{X^2} + \frac{\beta_2}{\beta_0}\frac{1-X}{X}\right) + \cdots\right], \qquad X = 1 - \frac{\alpha_s(\mu_R)}{2\pi}\beta_0 \ln\frac{\mu_R}{\mu_h}$$

- ***SCET resummation formula (and resummed expanded) updated to introduce the scale μ_r ,*** necessary to identify the cause of the different shape of scale variations between SCET and dQCD predictions [Kulesza,Motyka,Stebel,Theeuwes,1704.03363]

- Current implementation:

  - We vary μ_f and μ_r with the standard 7 point method

  - Soft scale ***always*** fixed to $\mu_s = M_{t\bar{t}H}/\bar{N}$ even if the other scales are related to H_T or to $(m_t + m_H/2)$

  - We have freedom on what to do with μ_h. We provide predictions for the case $\mu_h = \mu_f$ and $\mu_h = \mu_r$. In order to incorporate some hard scale variations we finally *take the envelope of 11 independent scale choices as the measure of uncertainty*

Differences between SCET and dQCD formulas

- However, the two methods make different choices in evaluating the NNLL
  - SCET: expand to NNLL using $\alpha_s(\mu_r)\,\ln \mu_i/\mu_j = \mathcal{O}(1)$
  - dQCD: expand to NNLL using only $\alpha_s(\mu_r)\,\ln N = \mathcal{O}(1)$
  - dQCD: keep some partial N^{3}LL terms ***in diagonal part*** of the evolution that are dropped in SCET formula (but could be kept). $\tilde{\mathbf{u}}_{ij}(\mu_h, \mu_s)$ terms are in both approaches expanded in α_s

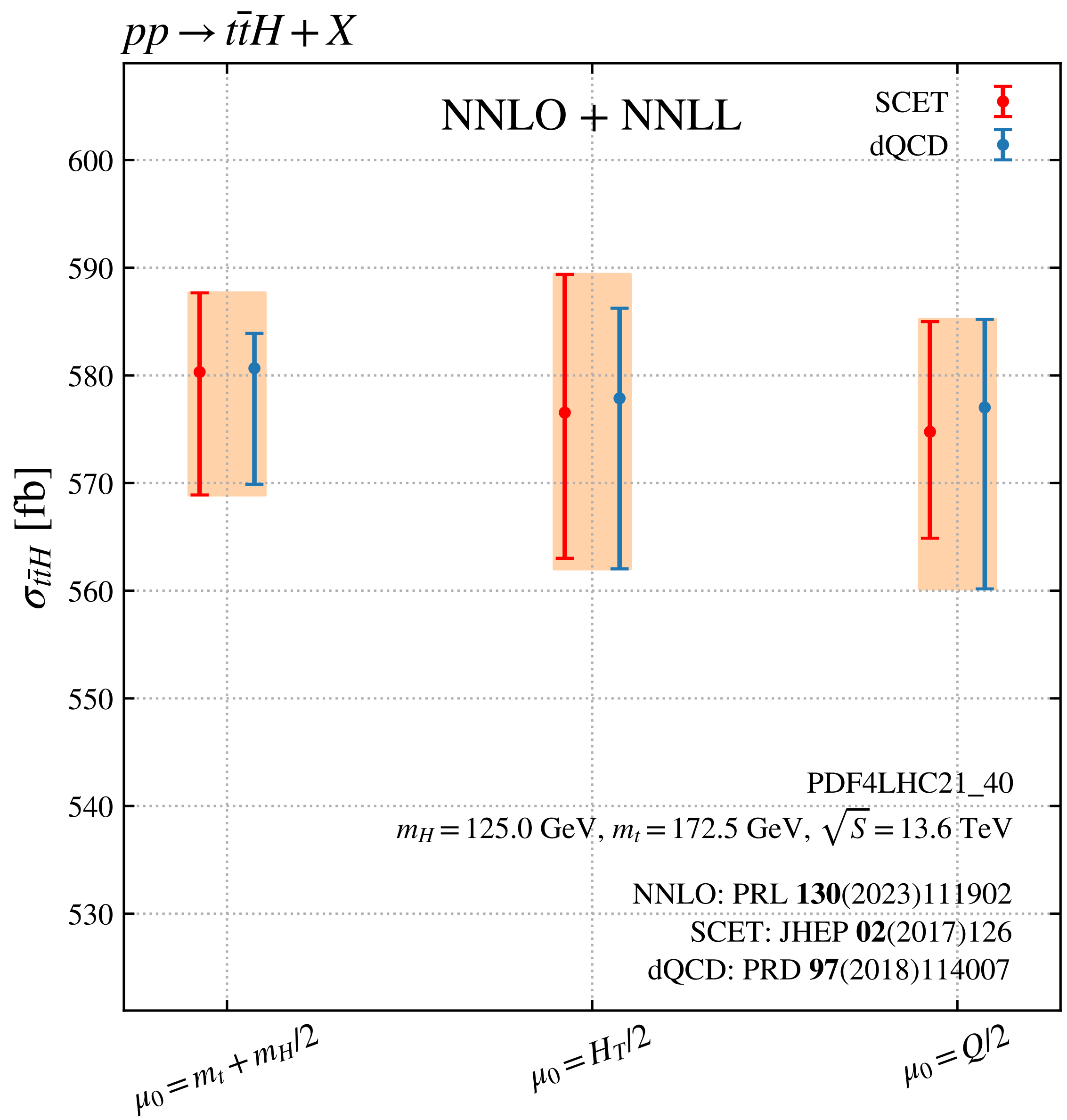
$$\exp[\alpha_s(\mu_r)\, g_3(\lambda)] \to 1 + \alpha_s(\mu_r)\, g_3(\lambda)$$

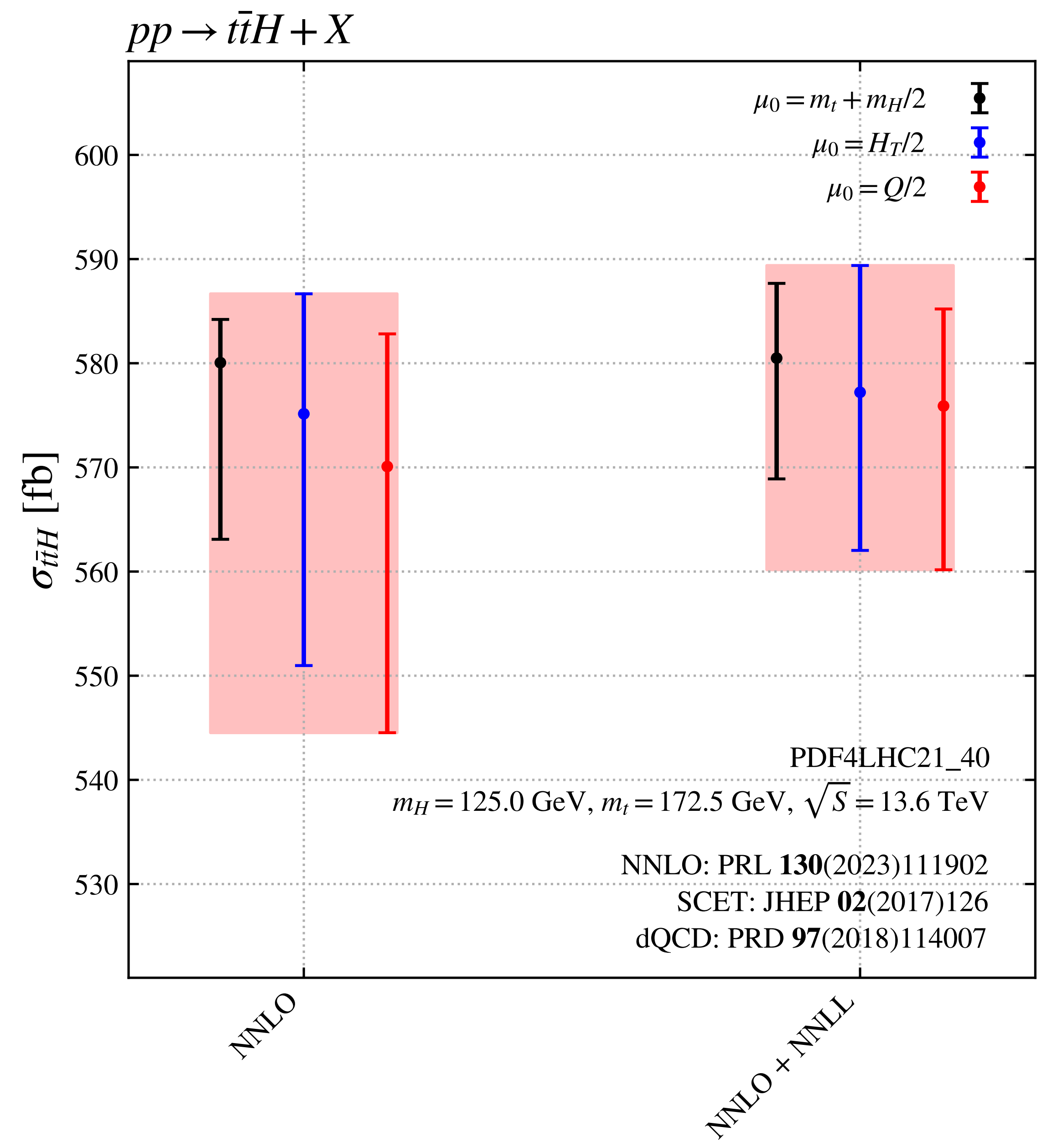
dQCD SCET

These different choices are the cause of the ***small numerical differences*** remaining between the SCET and dQCD formulas at NNLO+NNLL

Comparisons SCET vs dQCD vs NNLO

$$d\sigma_{ttH}^{\mathrm{NNLO+NNLL}} = d\sigma_{ttH}^{\mathrm{NNLL}} + \left(d\sigma_{ttH}^{\mathrm{NNLO}} - d\sigma_{ttH}^{\mathrm{NNLL}} \Big|_{\substack{\mathrm{NNLO}\\ \mathrm{expansion}}} \right)$$





Comparison between dQCD and SCET

[2503.15043]

NNLO vs NNLO+NNLL

[Devoto,Grazzini,Kallweit,Mazzitelli,Savoini,2411.15340]

See Chiara's talk!

Outlook

- Calculate and extract all the missing ingredients to reach NNLL$'$ accuracy for the top-quark pair production process (hard and soft functions)
- Event Generator implementation
- Extend to associated production of a top-pair and a heavy boson $t\bar{t}V$ ($V = H, W^{\pm}, Z$)

Thank you!

Backup slides

Beam functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_i(x,\mu)$

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t, z/\xi, \mu) f_j(\xi,\mu)$$

I_{ij} kernels are known up to N^{3}LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \Big[-2\,\Gamma_{\rm cusp}(\alpha_s)\,L_c + \gamma_i^B(\alpha_s)\Big]\tilde{B}_i(L_c,z,\mu)$$

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln\left(M\kappa/\mu^2\right)$

Resummed result for the cross section

We can combine the solutions for the hard, soft and beam functions to obtain

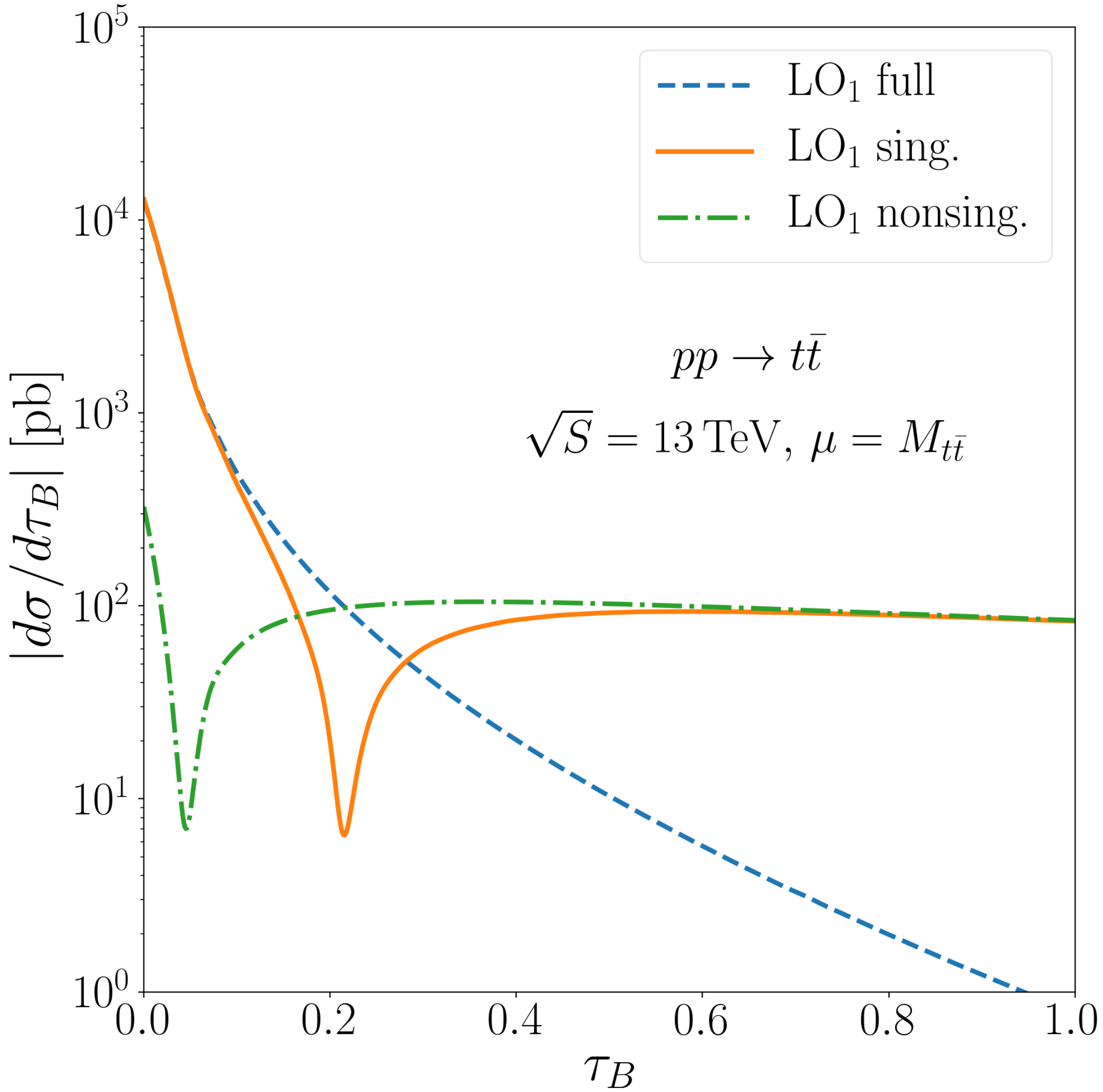
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} &= U(\mu_h, \mu_B, \mu_s, L_h, L_s) \\
&\times \mathrm{Tr}\left\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \right\} \\
&\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\mathrm{tot}}}} \frac{e^{-\gamma_E \eta_{\mathrm{tot}}}}{\Gamma(\eta_{\mathrm{tot}})}
\end{aligned}
$$

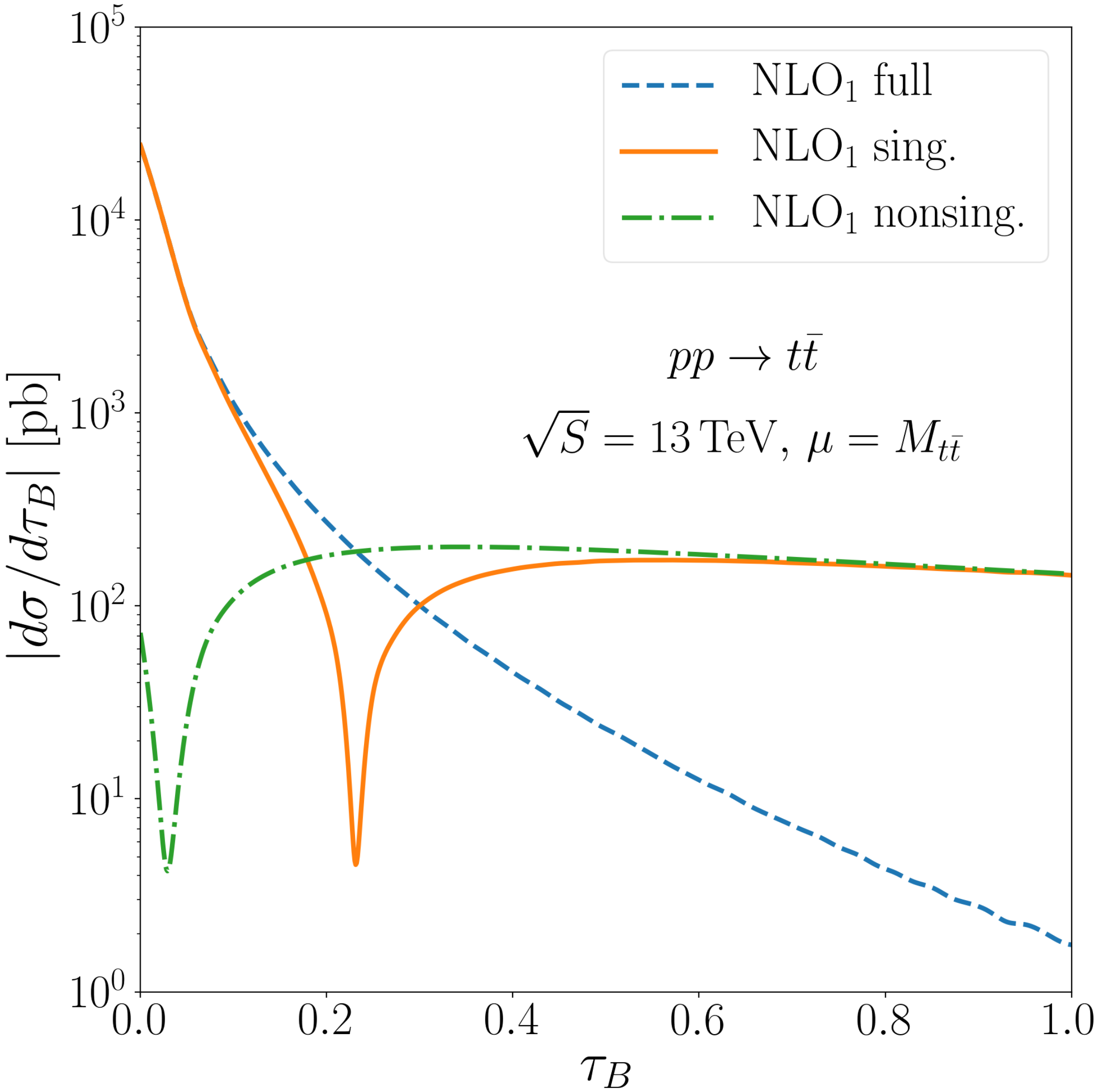
where

$$
U(\mu_h, \mu_B, \mu_s, L_h, L_s) = \exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \right]
$$

and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\mathrm{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

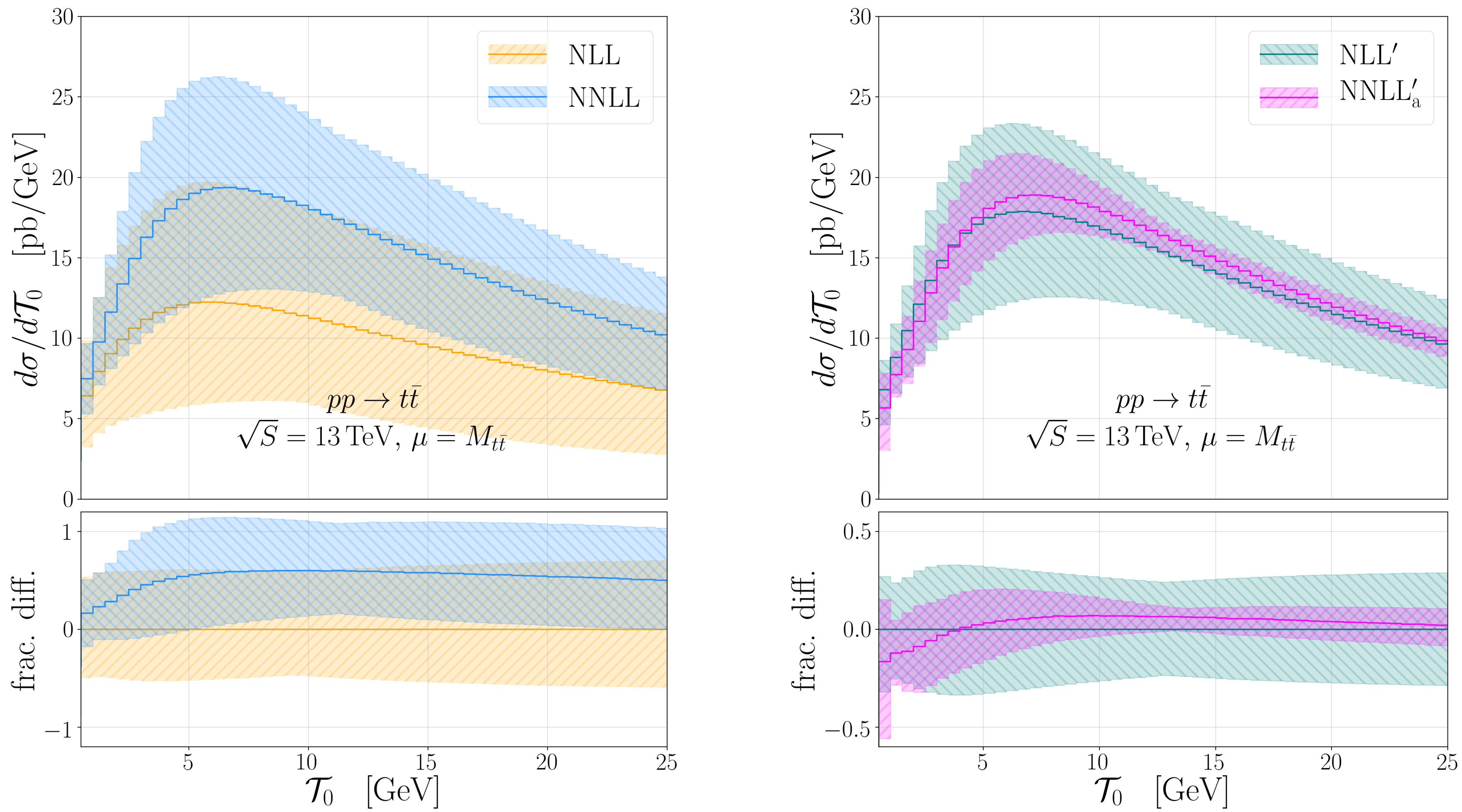
Singular vs Nonsingular contributions





Resummed results

NNLL′ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



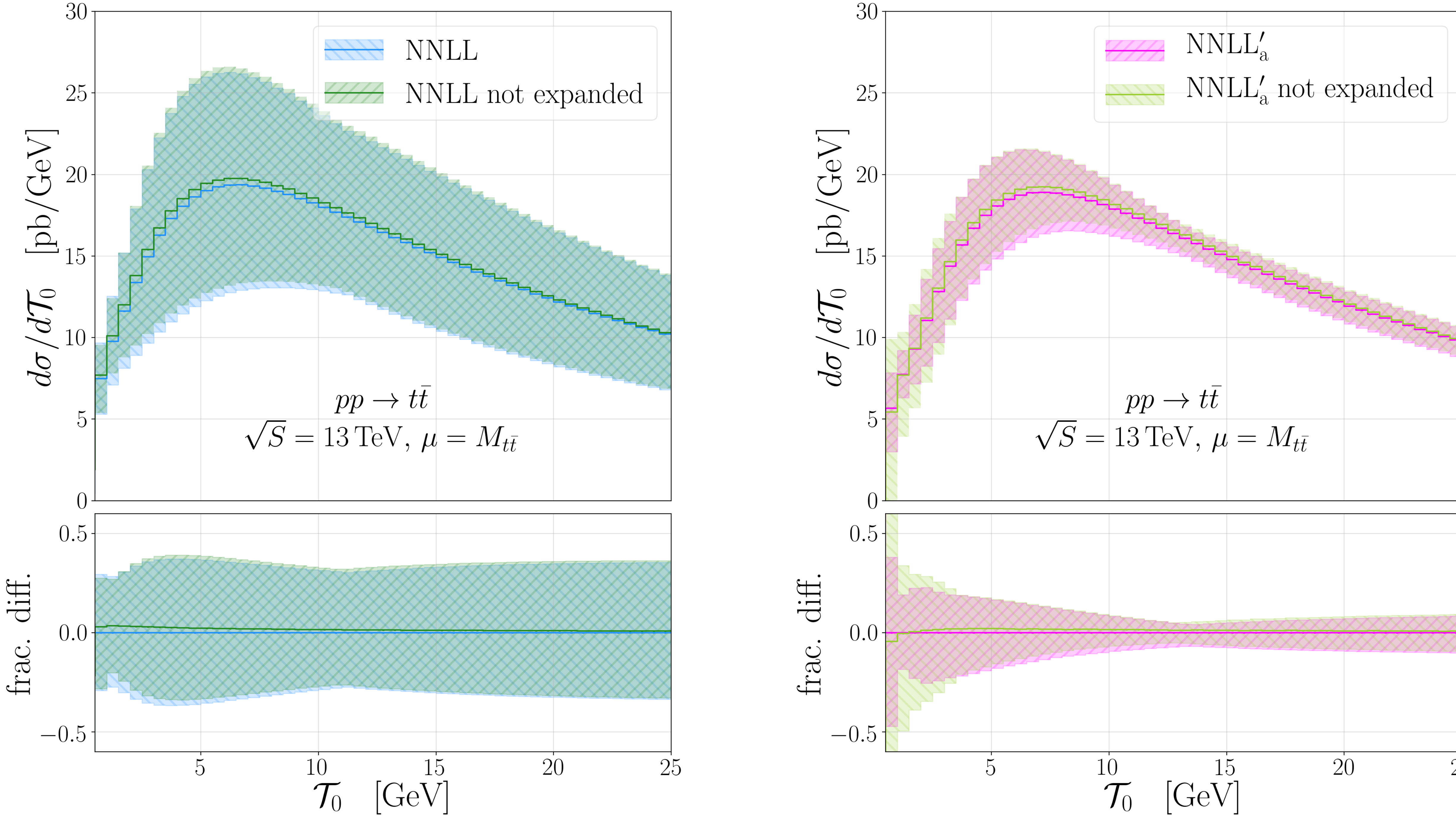
$$\mu_H = \mu_{\rm NS}\,,$$
$$\mu_S(\mathcal{T}_0) = \mu_{\rm NS}\ f_{\rm run}(\mathcal{T}_0/M)\,,$$
$$\mu_B(\mathcal{T}_0) = \mu_{\rm NS}\ \sqrt{f_{\rm run}(\mathcal{T}_0/M)}$$

$$f_{\rm run}(y) = \begin{cases} y_0\big[1+(y/y_0)^2/4\big] & y \le 2y_0\,, \\ y & 2y_0 \le y \le y_1\,, \\ y + \frac{(2-y_2-y_3)(y-y_1)^2}{2(y_2-y_1)(y_3-y_1)} & y_1 \le y \le y_2\,, \\ 1 - \frac{(2-y_1-y_2)(y-y_3)^2}{2(y_3-y_1)(y_3-y_2)} & y_2 \le y \le y_3\,, \\ 1 & y_3 \le y\,. \end{cases}$$

$$y_0 = 1.0\,\mathrm{GeV}/M\,, \quad \{y_1, y_2, y_3\} = \{0.1, 0.175, 0.25\}$$

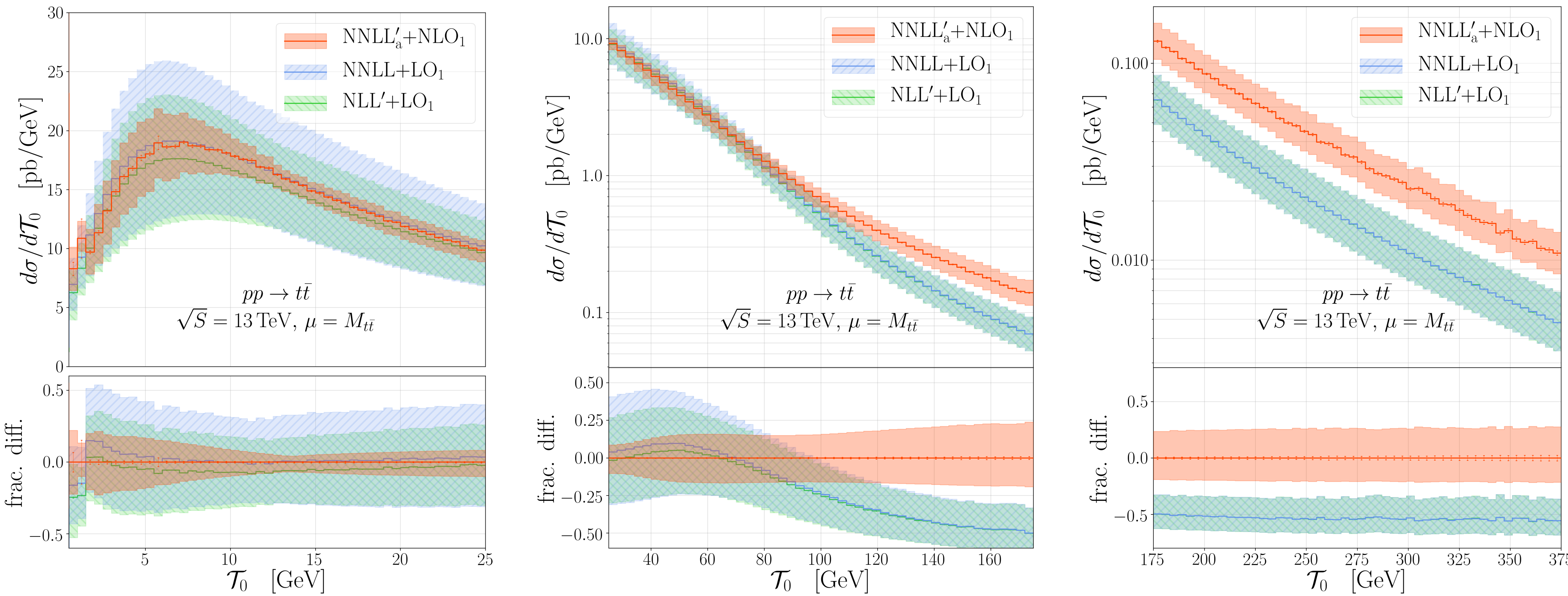
Resummed results

The evolution matrix **u** is evaluated in α_s expansion, we can choose to expand or not expand U, the difference is quite small



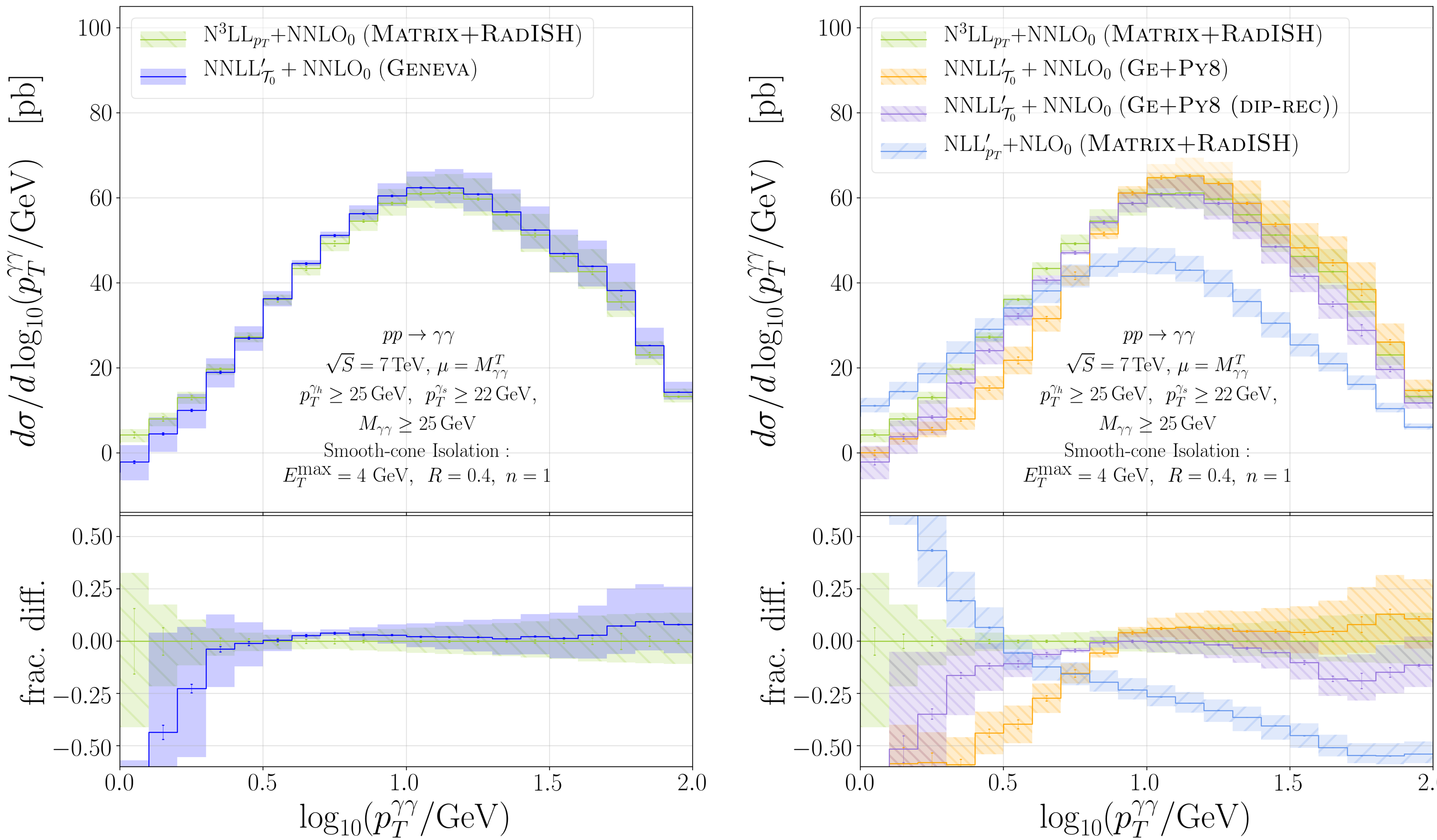
Matched results to fixed-order

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



GENEVA vs q_T resummation

- Inclusive quantities are not modified, changes are expected in exclusive observables
- Shower recoil schemes large impact in predictions of colour singlet p_T



Introducing the scale μ_r

- When matching to NNLO it is reasonable to vary independently $\mu_f \neq \mu_r$ (standard procedure). To make contact to FO, we have introduced μ_r in the resummation formula
  - Eliminate $\alpha_s(\mu_h)$ in favor of $\alpha_s(\mu_r)$ using

$$\alpha_s(\mu_h) = \frac{\alpha_s(\mu_R)}{X}\left[1 - \frac{\alpha_s(\mu_R)}{4\pi}\frac{\beta_1}{\beta_0}\frac{\ln X}{X} + \left(\frac{\alpha_s^2(\mu_R)}{4\pi}\right)^2\left(\frac{\beta_1^2}{\beta_0^2}\frac{\ln^2 X - \ln X - 1 + X}{X^2} + \frac{\beta_2}{\beta_0}\frac{1-X}{X}\right) + \cdots\right], \qquad X = 1 - \frac{\alpha_s(\mu_R)}{2\pi}\beta_0 \ln\frac{\mu_R}{\mu_h}$$

  - Re-expand perturbatively treating $\ln \mu_i/\mu_j = \mathcal{O}(1)$ with $\mu_{i,j} \in \{\mu_f, \mu_r, \mu_h, \mu_s\}$. With this counting, NNLO expansion of NNLL takes a simple form (for case $\mu_f = \mu_r$)

$$d\widetilde{\hat{\sigma}}^{(2)}_{ij}(N,\mu_F) = \mathrm{Tr}\left[\mathbf{H}^{(2)}_{ij}(\mu_F)\,\tilde{\mathbf{s}}^{(0)}_{ij}(\mu_F) + \mathbf{H}^{(1)}_{ij}(\mu_F)\,\tilde{\mathbf{s}}^{(1)}_{ij}(\mu_F) + \mathbf{H}^{(0)}_{ij}(\mu_F)\,\tilde{\mathbf{s}}^{(2)}_{ij}(\mu_F)\right] - \mathrm{Tr}\left[\mathbf{H}^{(2)}_{ij}(\mu_h)\,\tilde{\mathbf{s}}^{(0)}_{ij}(\mu_s) + \mathbf{H}^{(1)}_{ij}(\mu_h)\,\tilde{\mathbf{s}}^{(1)}_{ij}(\mu_s) + \mathbf{H}^{(0)}_{ij}(\mu_h)\,\tilde{\mathbf{s}}^{(2)}_{ij}(\mu_s)\right]$$

- ***SCET resummation formula (and resummed expanded) updated to introduce the scale μ_r ,*** necessary to identify the cause of the different shape of the scale variations between SCET and dQCD predictions [Kulesza,Motyka,Stebel,Theeuwes,1704.03363]

Scale choices and uncertainties in the EFT framework

- Original NNLL SCET+HQET implementation:

  - H_T scales: $\mu_f = \mu_r = \mu_h = H_T/2, \mu_s = H_T/\bar{N}$
    Invariant mass scales: $\mu_f = \mu_r = M_{t\bar{t}H}/2, \mu_h = M_{t\bar{t}H}, \mu_s = M_{t\bar{t}H}/\bar{N}$
    Fixed scales: $\mu_f = \mu_r = \mu_h = (m_t + m_H/2), \mu_s = (2m_t + m_H)/\bar{N}$

  - *Varied μ_f, μ_h, μ_s independently by factors of 2, added upper and lower variations in quadrature (as customary in SCET), total of 7 scales varied* ***but matching to FO only for $\mu_f = \mu_r$ scales***

- Current implementation:

  - We vary μ_f and μ_r with the standard 7 point method

  - Soft scale ***always*** fixed to $\mu_s = M_{t\bar{t}H}/\bar{N}$ even if the other scales are related to H_T or to $(m_t + m_H/2)$

  - We have freedom on what to do with μ_h. We provide predictions for the case $\mu_h = \mu_f$ and $\mu_h = \mu_r$. In order to incorporate some hard scale variations we finally *take the envelope of 11 independent scale choices as the measure of uncertainty*

New vs Old SCET results at NNLO