Theory predictions for "toponium" at the LHC: present status and open issues

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partly on the basis of
M.V.G., G. Limatola, S.-O. Moch, M. Steinhauser, O. Zenaiev,
[arXiv:2412.16685]

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Standard Model at the LHC 2025
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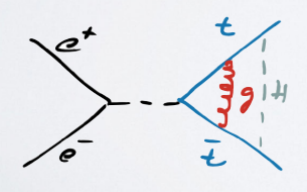
First investigations of “toponium”

- in e^+e-:
  - I. Bigi, Yu. Dockshitzer, V. Khoze, J. Kuehn, P. Zerwas, PLB 181 (1986) 157
  - V. Fadin, V. Khoze, JETP Lett. 46 (1987) 417. Yad Fiz. 88 (1988) 487.
- in *pp*:
  Y. Fadin, V. Khoze and T. Sjostrand, “On the threshold behaviour of heavy top production” CERN-TH 5687/90
  - well before the discovery of top quark
  - analogy with γ exchange in DY production
  - multiple exchange of virtual gluons between t and $\bar{t}$ at threshold dominated by pQCD effects because of m_t large (difference with charmonia and bottomonia) and described in terms of a Coulomb-like potential
    - attractive for color-singlet states
    - repulsive for color-octet states
  - $\Gamma_t \neq 0$ and bound-state effects accounted for through Green’s functions.
  - This might lead to color-singlet “quasi” bound states
  - Effect more pronounced for low m_t.
  - “not distinguishable from continuum....in hadron collider it would be impossible to study the details of the threshold behaviour, even for $m_t \sim 100$ GeV. An e^+e^- collider would in principle do better.”

$e^+e^- \to t\bar{t}$ close to threshold at lepton colliders



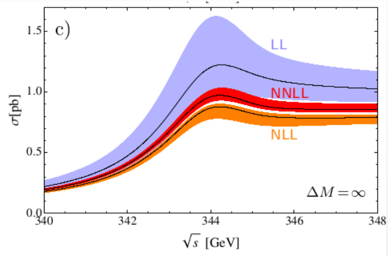
$*$ $S = \hat{s}$, LO $t\bar{t}$ pairs always in color-singlet states.

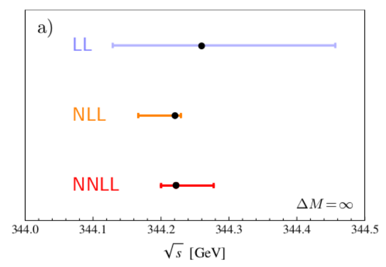
$*$ Top quark non relativistic close to threshold ($v_t/c \sim \alpha_s <<< 1$) in $t\bar{t}$ rest frame.
Regime for NRQCD and its variants (VNRQCD, PNRQCD, etc...). Plenty of studies, due to sensitivity to m_t.

$*$ Simultaneous sensitivity to Γ_t, α_s, $y_{t\bar{t}h}$, to be considered when the process is used for m_t extraction via top-threshold scan in short-distance m_t renormalization scheme ($\Delta m_t \sim$ some tens MeV vs. $\Delta m_t^{\mathrm{MC}} \sim 300$ MeV in the experimental extraction of m_t^{MC} at the LHC).

$*$ Even $e^+e^- \to l^+\nu_l b l^- \bar{\nu}_l \bar{b}$ predictions available (although at lower accuracy).

Predictions for m_t extraction at linear colliders





from A. Hoang and M. Stalhofen, [arXiv:1309.6323]

$*$ VNRQCD (RGI NRQCD variant) allows for separation of hard (m_t), soft ($m_t v$), ultrasoft scales ($m_t v^2$).
Resummation of both α_s^n / v^n contributions and $(\alpha_s log v)^n$ ones.

$*$ Uncertainties from μ_h, $\mu_s = \nu \mu_h$, $\mu_U = \nu^2 \mu_h$ variation

$*$ $\delta m_t \sim \pm 20$ MeV (+ uncertainties on $e^+ e^-$ luminosity)

$t\bar{t} + X$ production close to threshold at hadron colliders

$*$ Far less studies at hadron coliders w.r.t. lepton colliders

$*$ Interest suddenly enhanced by the threshold excesses seen in the results of recent CMS analyses searching for BSM $H \to t\bar{t}$ decays (currently under cross-check by ATLAS).

$*$ NRQCD predictions, including singlet and octet contributions for the $m_{t\bar{t}}$ distribution $\Rightarrow$ *Revival of the theory chain in Kiyo, Kühn, Moch, Steinhauser and Uwer, [arXiv:0812.0919]*

- attractive (repulsive) effects in the color-singlet (octet) channels due to terms depending on α_s^n/v^n associated to the exchanges of Coulomb-like virtual gluons between the t and $\bar{t}$ quarks (process analogous to γ exchange between ℓ^+ and ℓ^- in DY).
- bound-state effects in the color-singlet channels
- $\Gamma_t \neq 0$ effects (as opposite to the NWA, $\Gamma_t = 0$)
- plus resummation of large logarithms due to real soft-gluon emissions close to threshold (at NLL).

$M_{t\bar{t}}$ differential distribution

$$M_{t\bar{t}} \frac{\mathrm{d}\sigma_{P_1 P_2 \to T}}{\mathrm{d}M_{t\bar{t}}}(S, M_{t\bar{t}}^2) = \sum_{i,j} \int_\rho^1 \mathrm{d}\tau \left[\frac{\mathrm{d}\mathcal{L}_{ij}}{\mathrm{d}\tau}\right](\tau, \mu_f^2)\; M_{t\bar{t}} \frac{\mathrm{d}\hat{\sigma}_{ij\to T}}{\mathrm{d}M_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2)$$

with $\tau = \hat{s}/S$, $\rho = M_{t\bar{t}}^2/S$,

$$\left[\frac{d\mathcal{L}_{ij}}{d\tau}\right](\tau, \mu_f^2) \;=\; \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f_{i/P_1}(x_1, \mu_f^2) f_{j/P_2}(x_2, \mu_f^2)\, \delta(\tau - x_1 x_2)$$

and

$$M_{t\bar{t}} \frac{\mathrm{d}\hat{\sigma}_{ij\to T}}{\mathrm{d}M_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) \;=\; F_{ij\to T}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2)\, \frac{1}{m_t^2}\, \mathrm{Im}\, G^{[1,8]}(M_{t\bar{t}} + i\Gamma_t)$$

Last formula is valid in NRQCD for the considered distribution at NLO:

- still to be generalized at higher-orders
- still to be generalized for other distributions

Factorization of the partonic cross-section

$$M_{t\bar{t}} \frac{d\hat{\sigma}_{ij \to T}}{dM_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) = F_{ij \to T}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) \frac{1}{m_t^2} \operatorname{Im} G^{[1,8]}(M_{t\bar{t}} + i\Gamma_t)$$

valid in NRQCD for the considered distribution at NLO

- $F_{ij \to T}$: hard-scattering function for producing the $T = {}^{2S+1}L_j^{[1,8]}$ state, containing threshold logarithms becoming large close to the partonic threshold (i.e. for τ reaching its minimum value ρ, equivalent to $z = M_{t\bar{t}}^2/\hat{s} \to 1$).
- $G^{[1,8]}$: attractive and repulsive non-relativistic Green's functions, solution of Schroedinger eq. accounting for exchange of potential gluons between t and $\bar{t}$. They depend on Γ_t and m_t.
  - $G^{[1]}$ from attractive Coulomb-like QCD potential,
    opening the possibility for the formation of "toponium" quasi-bound states
    $\Gamma^t >> \Lambda_{QCD}$: the top quark decays before a proper bound state can be formed.
  - $G^{[8]}$ from repulsive Coulomb-like QCD potential.

Resummation of threshold logarithms

$*$ We resum threshold logarithms up to NLL for the most relevant T channels ($gg \to^1 S_0^{[1]},^1 S_0^{[8]}$ and $q\bar{q} \to^3 S_1^{[8]}$), working in N-space (N conjugate to z).

$*$ We consider the Mellin transform of the hard function

$$F^N_{ij\to T}(M^2_{t\bar{t}}, \mu_f^2) = \int_0^1 dz\, z^{N-1}\, F_{ij\to T}(\hat{s}, M^2_{t\bar{t}}, \mu_f^2)$$

and the Mellin transform $\mathcal{L}_{ij}^N$ of the luminosity function $\mathcal{L}_{ij}(\tau)$.

$*$ We then recover predictions in z-space from the inverse Mellin transform of $(\mathcal{L} \otimes \mathcal{F})^{\mathcal{N}} = \mathcal{L}^{\mathcal{N}} \mathcal{F}^{\mathcal{N}}$, performed numerically, using the minimal prescription.

$*$ This includes matching of the resummed results to the NLO ones, avoiding double counting, leading to NLO + NLL predictions.

$*$ Uncertainties related to different prescriptions for avoiding the Landau pole in the inverse Mellin transform and matching not yet included.

Theory predictions: input

* Present setup:
  - $\sqrt{S} = 13$ TeV
  - NNPDF3.1 NNLO (PDFs + $\alpha_s(M_Z)$),
  - $m_t = 172.5$ GeV,
  - $\Gamma_t = 1.36$ GeV,
  - $\mu_R = \mu_F = [m_t, 4m_t]$, with $\mu_{R,0} = \mu_{F,0} = 2m_t$.

* Ongoing study of parametric uncertainties

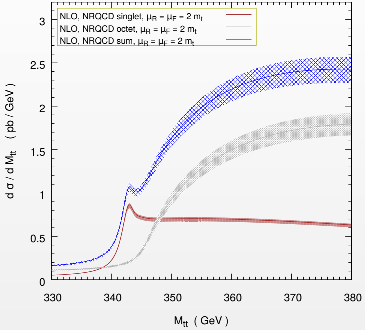
* Possibility to provide predictions with different inputs, according to the experimental requests

Predictions for the convolution $\mathcal{L} \otimes \mathcal{F}$

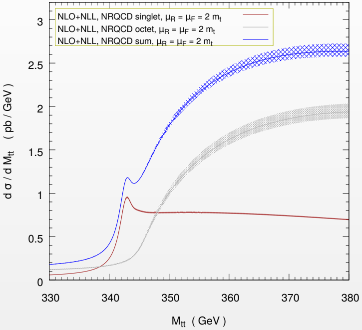
	NLO			resummed		
$gg \to {}^1S_0^{[1]}$	18.2	18.7	18.3	19.4	20.5	21.1
$gg \to {}^1S_0^{[8]}$	55.8	55.2	52.8	60.0	61.5	62.0
$q\bar{q} \to {}^3S_1^{[8]}$	21.7	22.3	22.0	22.4	22.4	22.0

TABLE I. Comparison of the NLO and NLO+NLL resummed result of the convolution $\mathcal{L} \otimes F$ (in units 10^{-6} GeV^{-2}) for the LHC configuration $\sqrt{S} = 13$ TeV with NNPDF3.1 PDFs at the reference point $M_{t\bar{t}} = 2m_t$. The three columns correspond to the scale choices $\mu_r = \mu_f \in \{m_t, 2m_t, 4m_t\}$.

$M_{t\bar{t}}$ distribution for $pp \to t\bar{t} + X$ close to threshold



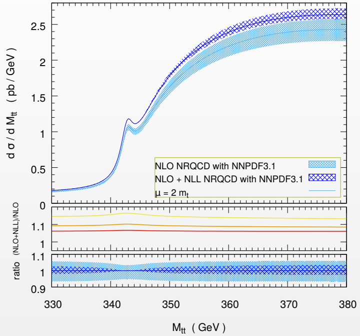
NLO



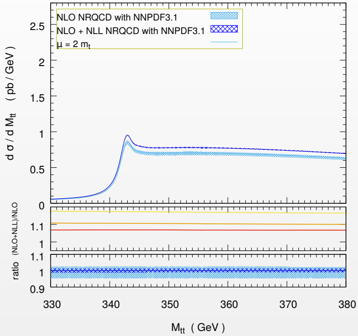
NLO + NLL

* Multiple singlet and octet contributions included at fixed order: all S-wave channels, i.e. the $gg, gq, q\bar{q} \to {}^1S_0^{[1,8]}$ channels, the $gg, gq, q\bar{q} \to {}^3S_1^{[8]}$ channels, as well as the $gg \to {}^3S_1^{[1]}$ one.
* NLL effects (so far applied only on the $gg \to {}^1S_0^{[1]}, {}^1S_0^{[8]}$ and $q\bar{q} \to {}^3S_1^{[8]}$ channels) enhance predictions and reduce considerably uncertainty bands w.r.t. NLO.

$M_{t\bar{t}}$: NLO vs. NLO + NLL predictions



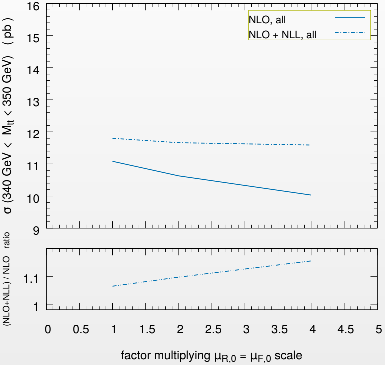
including octets

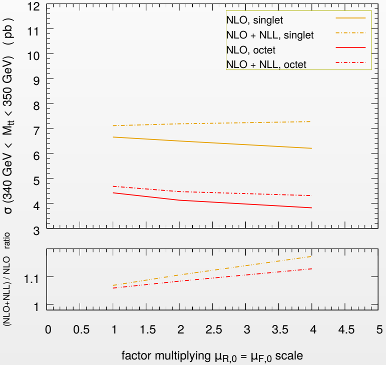


only singlet(s)

* For the present scale choice, NLO + NLL and NLO uncertainty bands do not overlap in most of the $M_{t\bar{t}}$ interesting region.
* K-factors almost flat $\sim 8 - 14\%$, depending on the scale: the effect of resummation is enhanced at large scales
* Uncertainties reduced in NLO + NLL w.r.t. NLO, especially in the peak region.

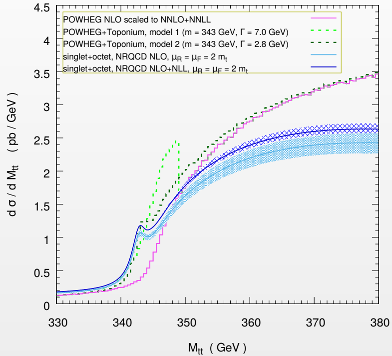
Integrated cross-section in the region of validity of NRQCD vs. scale





* (NLO + NLL/NLO) K-factors $\sim$ 10% for both singlet and octets
* NLL effects increase with the value of the scale $\mu_{R,0} = \mu_{F,0}$
* other scale choices under investigation

Comparison with predictions by the experimentalists using the 2021 model of toponium by Fuks et al.

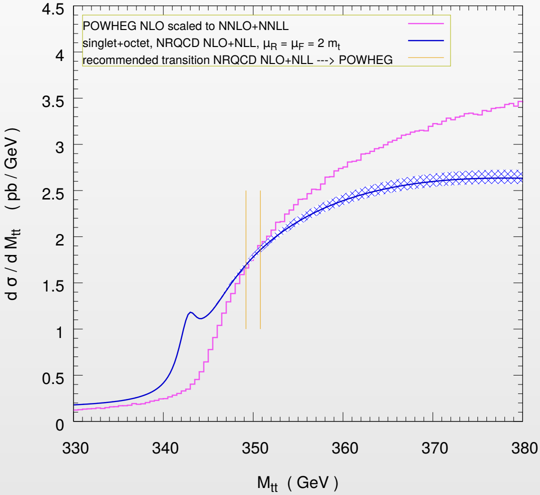


∗ POWHEG and Fuks et al. model input produced by the CMS experimentalists.

∗ Height, position and width of the Fuks-modelled singlet provided by the experimentalists and added to their POWHEG predictions do not coincide with our NLO (+ NLL) QCD singlet and singlet + octet predictions.

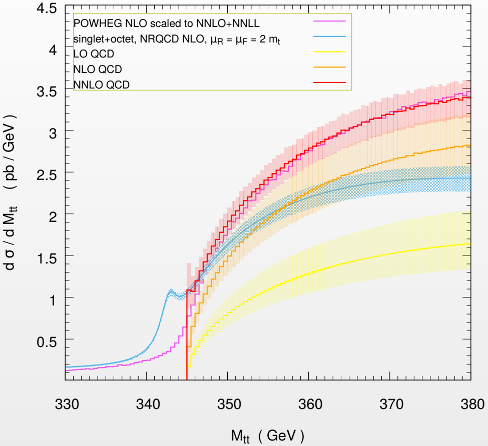
∗ Big uncertainties on the Green's functions used in our NLO QCD predictions, not shown in our plots. Missing higher orders would enhance these functions.

Recommended transition from NLO + NLL NRQCD to standard QCD POWHEG predictions scaled to NNLO + NNLL QCD



Warning: the POWHEG prediction shown here corresponds to our scale. The experimental analyses, however, use other scales in their simulations, corresponding to POWHEG predictions rising much more steeply.

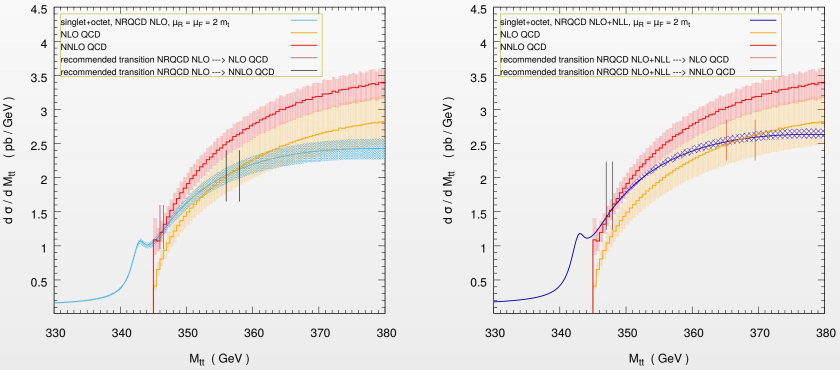
Scaled Powheg vs. fixed-order standard QCD predictions vs. NRQCD



Limitations of our fixed-order predictions in standard QCD: the NWA, as opposite to NRQCD predictions, including the effects of Γ_t in the Green's functions.

Warning: with our scale choice, the perturbative convergence is slow.

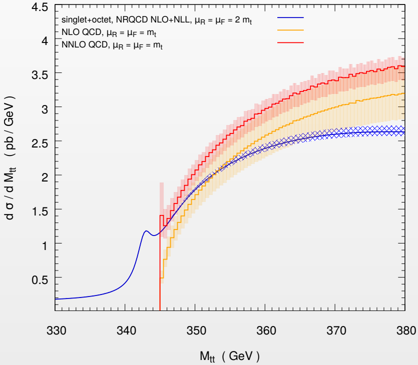
Recommended transitions from NRQCD to QCD fixed-order predictions (for the considered scale choice)

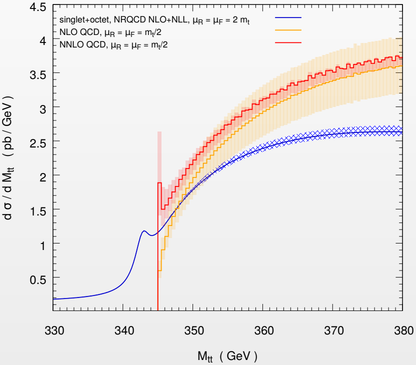


∗ fixed-order QCD predictions with MATRIX at LO, NLO, NNLO, with their scale uncertainty bands.

∗ A proper matching between NRQCD and QCD predictions still to be devised...

What happens when using smaller $\mu_{R,0} = \mu_{F,0}$ scales ?

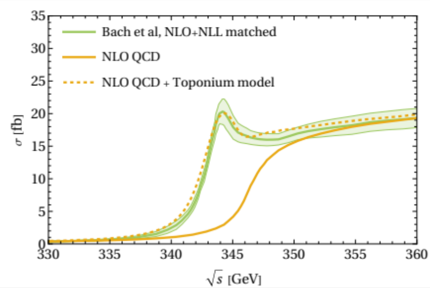


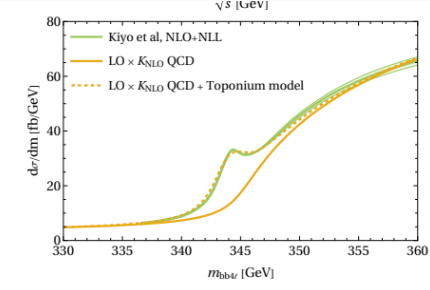


∗ Better perturbative convergence, but NNLO and NLO uncertainty bands do overlap only for large enough $M_{t\bar{t}}$, not at threshold.

⇒ importance of porting NRQCD predictions to NNLO:

ψ and η_t model





from F. Maltoni et al. [arXiv:2404.08049]

Varying a single parameter (coupling of η_t, ψ to $t\bar{t}$) one can reproduce NRQCD predictions at lepton and hadron colliders

Working with the MSR mass

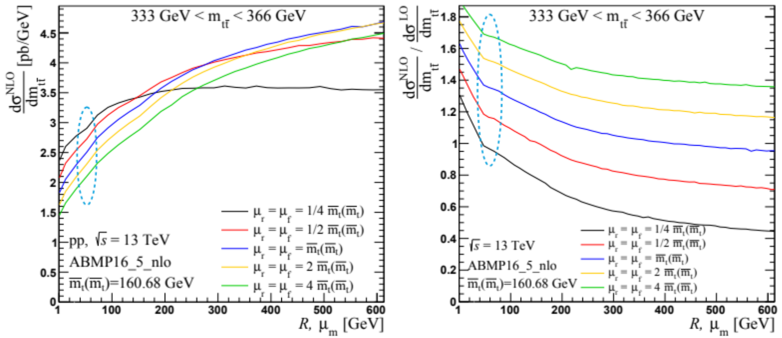


Figure 2: The NLO cross section (left) and the ratio of the LO and NLO cross sections (right) for $m_{t\bar{t}} \in [333, 366]$ GeV. The transition from a region suffering from the missing Coulomb corrections to a more stable region where the threshold effects become less important is seen at $R \gtrsim 60\,\text{GeV}$ (dashed blue). Further, predictions obtained using small values of μ_r, μ_f are observed to stabilize the prediction quickly as a function of R or μ_m.

from T. Makela et al., [arXiv:2301.03546]

Conclusions

∗ Further studies ongoing, especially focused on uncertainties.

∗ So far, we are limited to inclusive predictions, cuts still to be applied.

∗ Investigation of how to best use our predictions:
bin-by-bin reweighting ? Alternatives ?

∗ How safe/robust is extrapolating to other distributions (i.e. the distributions currently measured by the experimentalists)?

∗ Future interesting studies: compare shape of enhancements at threshold from models with BSM Higgs boson decaying into a $t\bar{t}$ pair to our results, and understand up to which extent it is possible to discriminate between the effects of BSM Higgs and toponium.

∗ Please note that our threshold enhancement, corresponding to the color-singlet config., occurs mostly **below** threshold, in contrast with many BSM Higgs models.

∗ **Our current predictions: lower limit to the true ones**.

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