Third-order QCD results on form factors, splitting functions and coefficient functions

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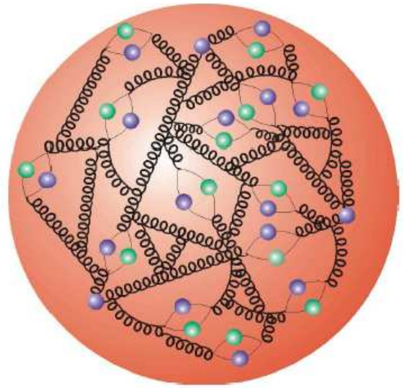
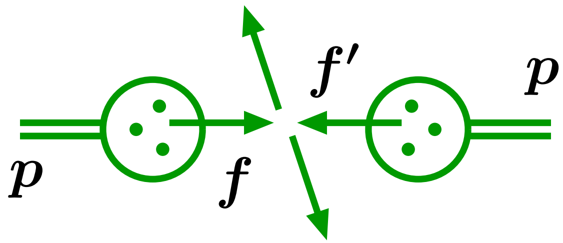
Collaborations with Sven Moch, Jos Vermaseren and Alexander Mitov

Quantitative QCD in collider physics

Proton: very complicated multi-particle bound state

Colliders: wide-band beams of quarks and gluons





$$\sigma^{pp} = \sum f^p * f'^p * \hat{\sigma}^{ff'}$$

Perturbative QCD: factorize non-pert. piece; calculate universal splitting functions P, process-dependent coefficient functions c_a

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \ldots$$

$$c_a = \underbrace{\alpha_s^{n_a} \left[c_a^{(0)} + \alpha_s c_a^{(1)}\right.}_{} \left. + \ldots \right]$$

NLO: first serious cross section estimate

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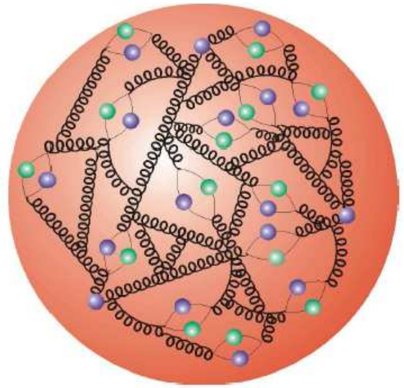

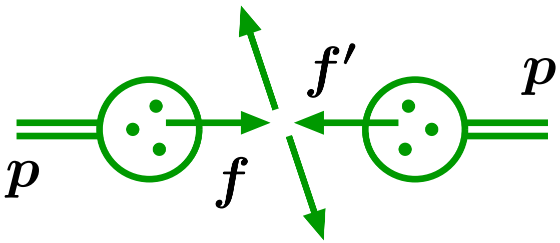

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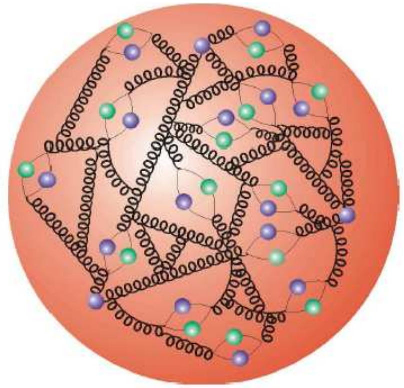

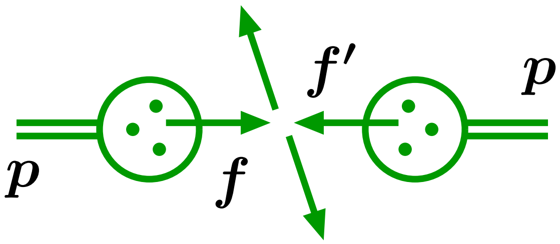

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N^{3}LO: high-accuracy predictions

Special kinematic regions (e.g., thresholds): all-order resummations

Research topics and references

- **Coefficient functions for deep-inelastic scattering at $\mathcal{O}(\alpha_s^3)$**

  hep-ph/0411112 = PLB 606 (2005) 123, hep-ph/0504242 = NPB 724 (2005) 3, . . .

- **Soft-gluon resummation beyond next-to-next-leading logs**

  hep-ph/0506288 = NPB 726 (2005) 317

- **On-shell quark and gluon form factors up to three loops**

  hep-ph/0507039 = JHEP 08 (2005) 049, hep-ph/0508055 = PLB 625 (2005) 245

- **Approximate N^{3}LO for Higgs production at hadron colliders**

  hep-ph/0508265 = PLB 631 (2005) 48

- **NNLO photon-quark and photon-gluon splitting functions**

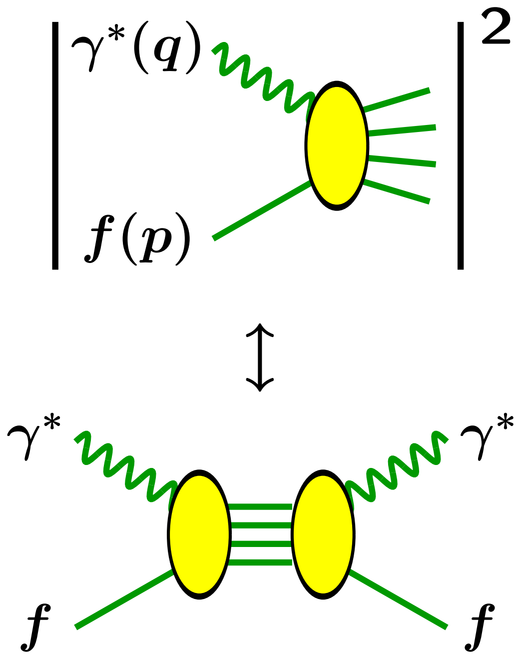
  hep-ph/0511112 = APP B37 (2006) 638, . . .

- **Non-singlet time-like splitting functions to the third order in α_s**

  hep-ph/0604053 = PLB 638 (2006) 61, . . .

Our three-loop calculation of inclusive DIS

Optical theorem: $\gamma^* f$ total cross sections $\leftrightarrow$ forward amplitudes



	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
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hϕ		1	33	1184
sum	3	18	350	9607

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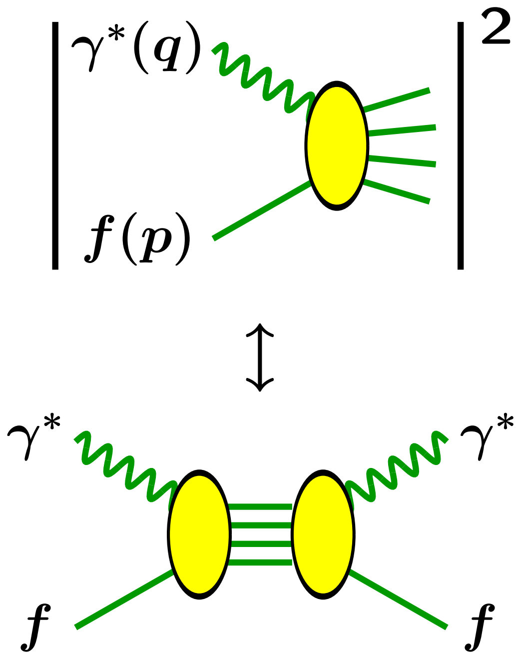

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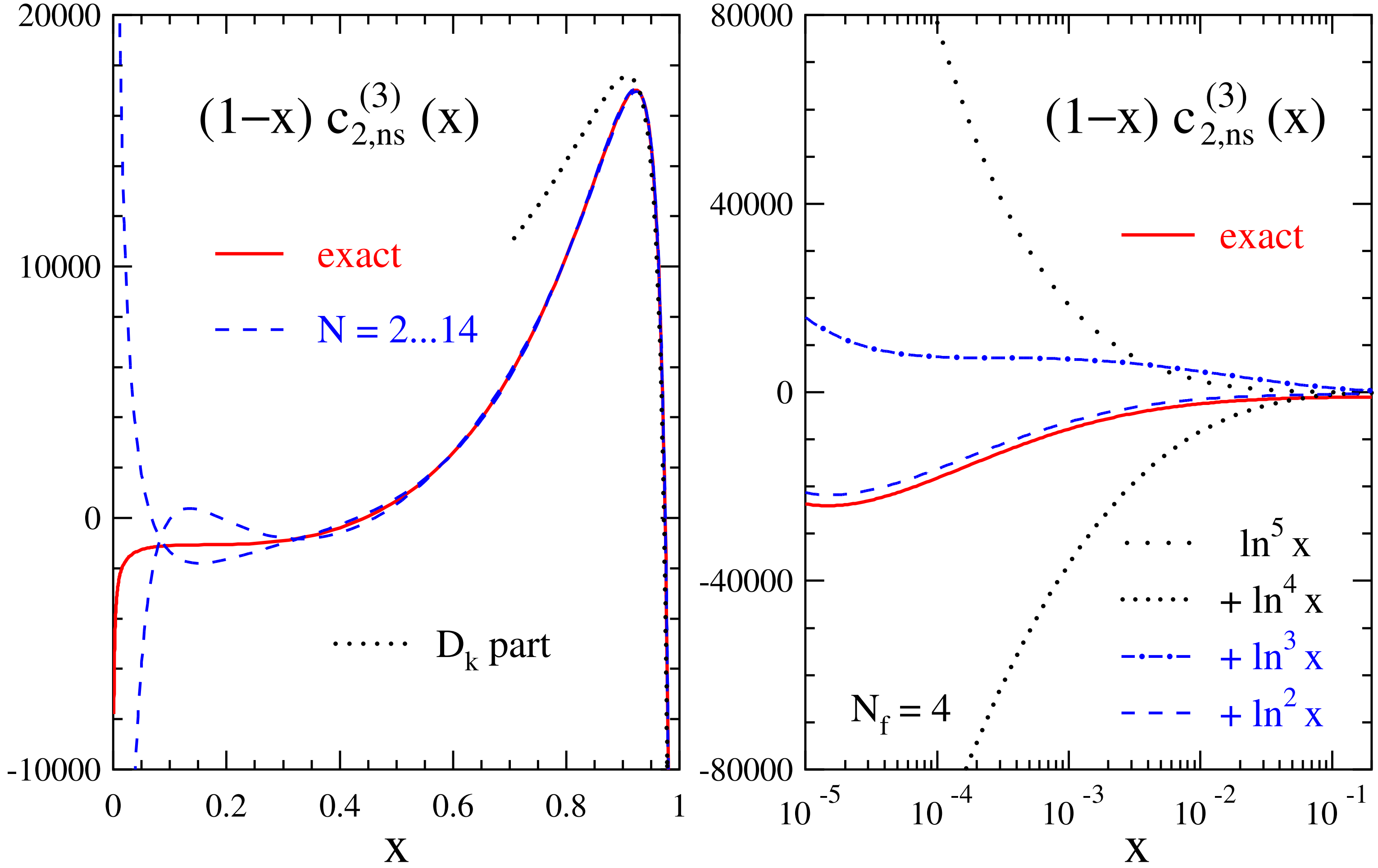
Coefficient of $(2p\cdot q)^N$ $\leftrightarrow$ N-th moment $A^N = \int_0^1 dx\, x^{N-1} A(x)$

UV and mass singularities : dimensional regularization, $D = 4 - 2\epsilon$

$1/\epsilon$ poles : splitting functions, ϵ^0 part : coefficient functions

The third-order coefficient function for $F_{2,\mathrm{ns}}$

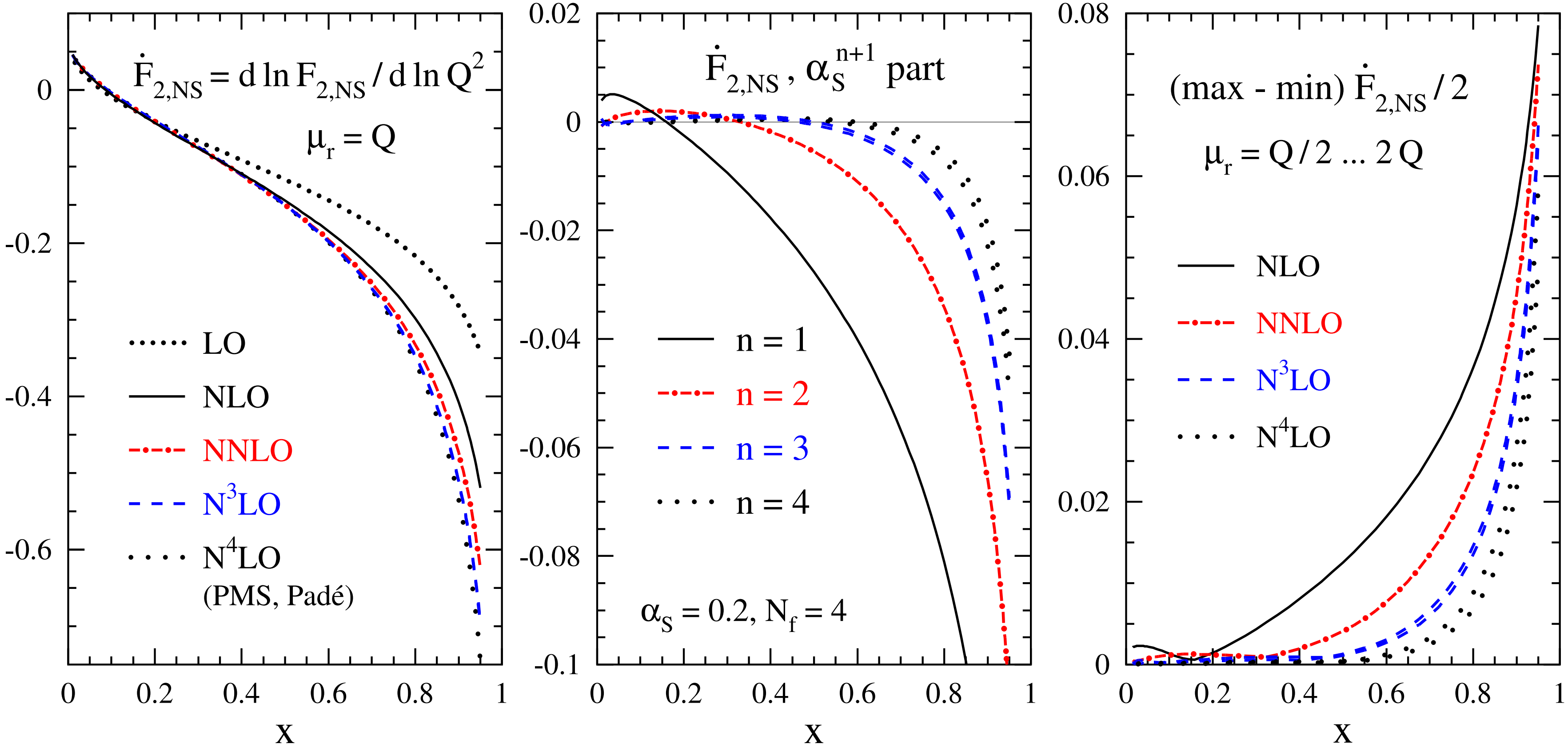


Large x: region of soft dominance shrinks with perturbative order

Small x: no dominance of (sub-)leading $\ln^k x$ terms : do not resum

Physical kernel for non-singlet evolution

Large-x convergence of P series: approx. N^{3}LO structure functions



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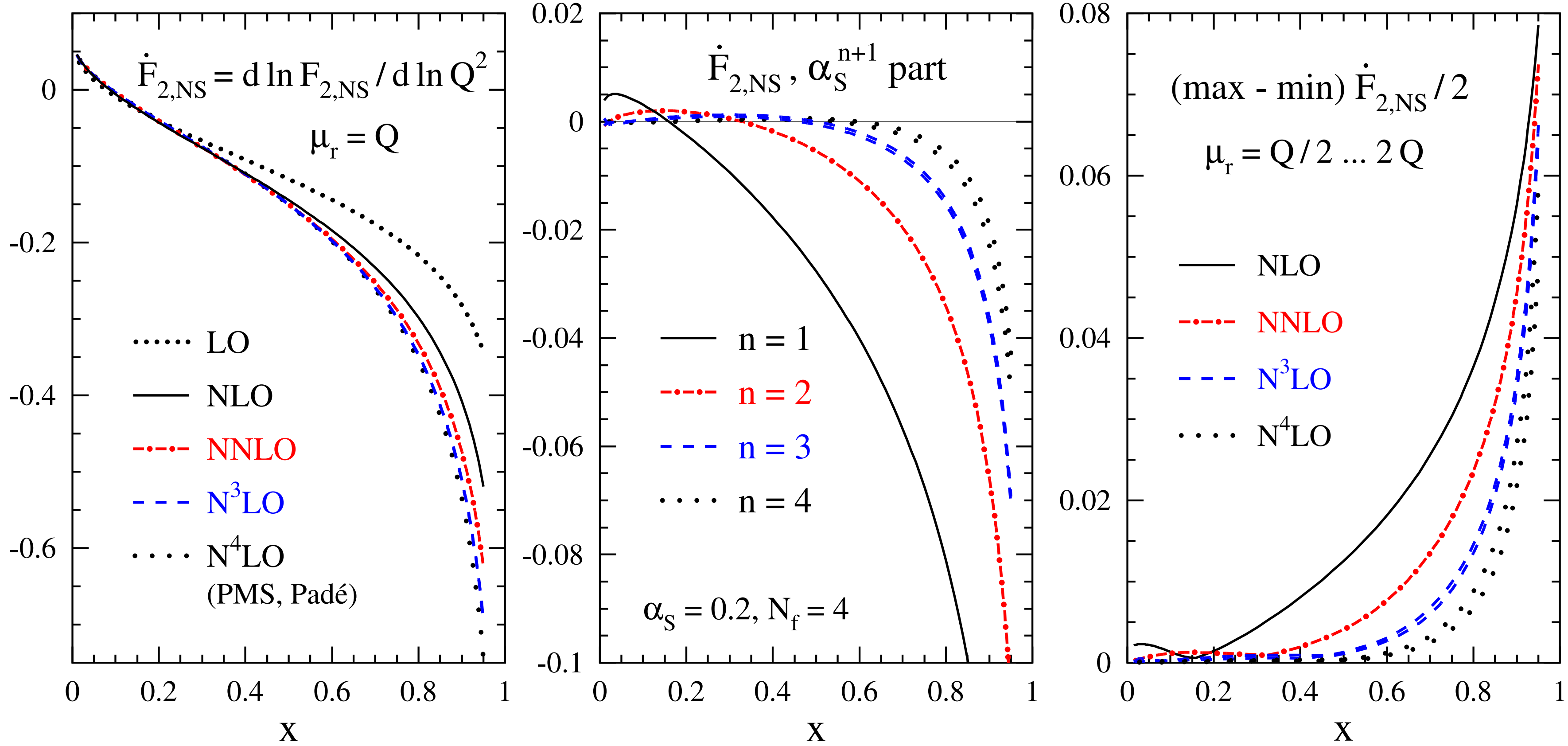
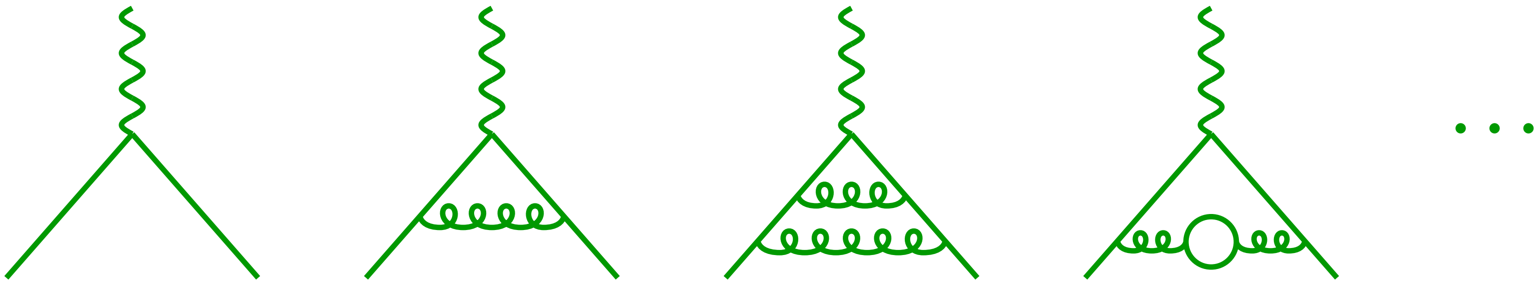


Potential for ‘gold-plated’ α_s determination: $\Delta_{\rm pert.}\alpha_s(M_Z) < 1\%$

Very large x: soft-gluon resummation **A.V. (01), MVV (05)**

Form factors of massless quarks and gluons



On-shell $m = 0$ quark form factor $\mathcal{F}_q$: QCD corr's to $\gamma^* qq$ vertex

$$\Gamma_\mu \;=\; \mathrm{i} e_q \, (\bar{u} \gamma_\mu u) \, \mathcal{F}_q(\alpha_s, Q^2)$$

Gauge invariant, divergent: dimensional regularization, $D = 4 - 2\varepsilon$

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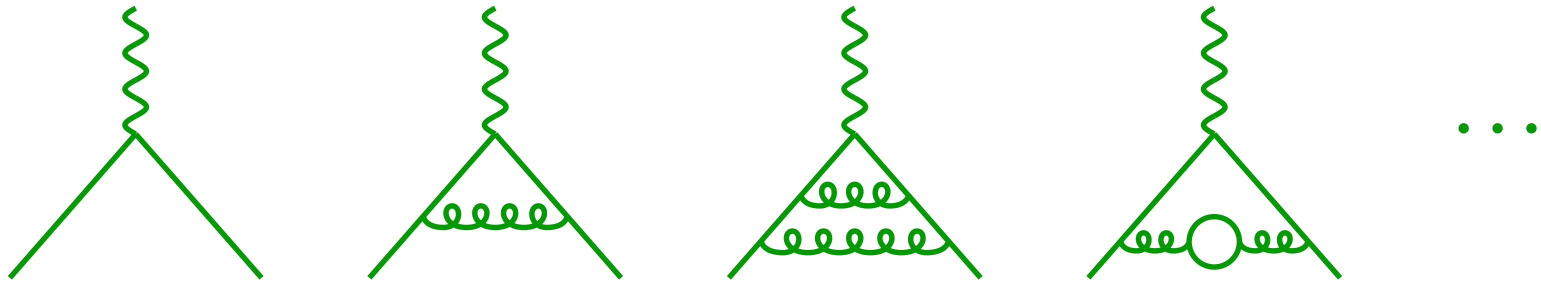

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Gluon form factor $\mathcal{F}_g$: effective Hgg vertex in heavy top-quark limit

$$\mathcal{L}_{\text{eff}} \;=\; -\frac{1}{4} \, C_H \, H G^a_{\mu\nu} G^{a,\mu\nu}$$

Coefficient C_H known to N^{3}LO — Chetyrkin, Kniehl, Steinhauser (97)

Renormalization of $G^a_{\mu\nu} G^{a,\mu\nu}$: $\qquad Z_{G^2} \;=\; \left[1 - \beta(a_s)/(a_s \varepsilon)\right]^{-1}$

Extraction of $\mathcal{F}_3$ from (ϕ)DIS at third order

a_s expansion of the bare structure functions at large Bjorken-x

$$
\begin{aligned}
F_0^{\rm b} &= \delta(1-x) \\
F_1^{\rm b} &= 2\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1 \\
F_2^{\rm b} &= (2\,\mathcal{F}_2 + \mathcal{F}_1^2)\delta(1-x) + 2\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2 \\
F_3^{\rm b} &= (2\,\mathcal{F}_3 + 2\,\mathcal{F}_1\mathcal{F}_2)\delta(1-x) + (2\,\mathcal{F}_2 + \mathcal{F}_1^2)\mathcal{S}_1 + 2\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3
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$\mathcal{F}_l$: bare l-loop space-like q or g form factor. $\mathcal{S}_l$: soft real emissions

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\mathcal{S}_k &= \mathsf{S}_k(\varepsilon)\cdot\varepsilon[\,(1-x)^{-1-k\varepsilon}\,]_+ \\
&= \mathsf{S}_k(\varepsilon)\left\{-\frac{1}{k}\,\delta(1-x) + \sum_{i=0}\frac{(-k\varepsilon)^i}{i\,!}\,\varepsilon\,\mathcal{D}_i\right\}\,, \quad \mathcal{D}_i \equiv \left[\frac{\ln^i(1-x)}{(1-x)}\right]_+
\end{aligned}
$$

Calculation of $F_3^{\rm b}$ to order ε^m $\Rightarrow$ $\mathcal{F}_3$ and $\mathcal{S}_3$ to order ε^{m-1}

MVV(04/05): coefficient fct's for (ϕ)DIS + dedicated n_f calc. to $\mathcal{O}(\varepsilon)$

Pole structure of $q\bar{q} \longrightarrow \gamma^*$ and $gg \longrightarrow H$

α_s^n expansion coefficients of bare partonic cross sections to $n = 3$

cf. Matsuura, van Neerven (88)

$$W_0^{\rm b} = \delta(1-x)$$

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$$

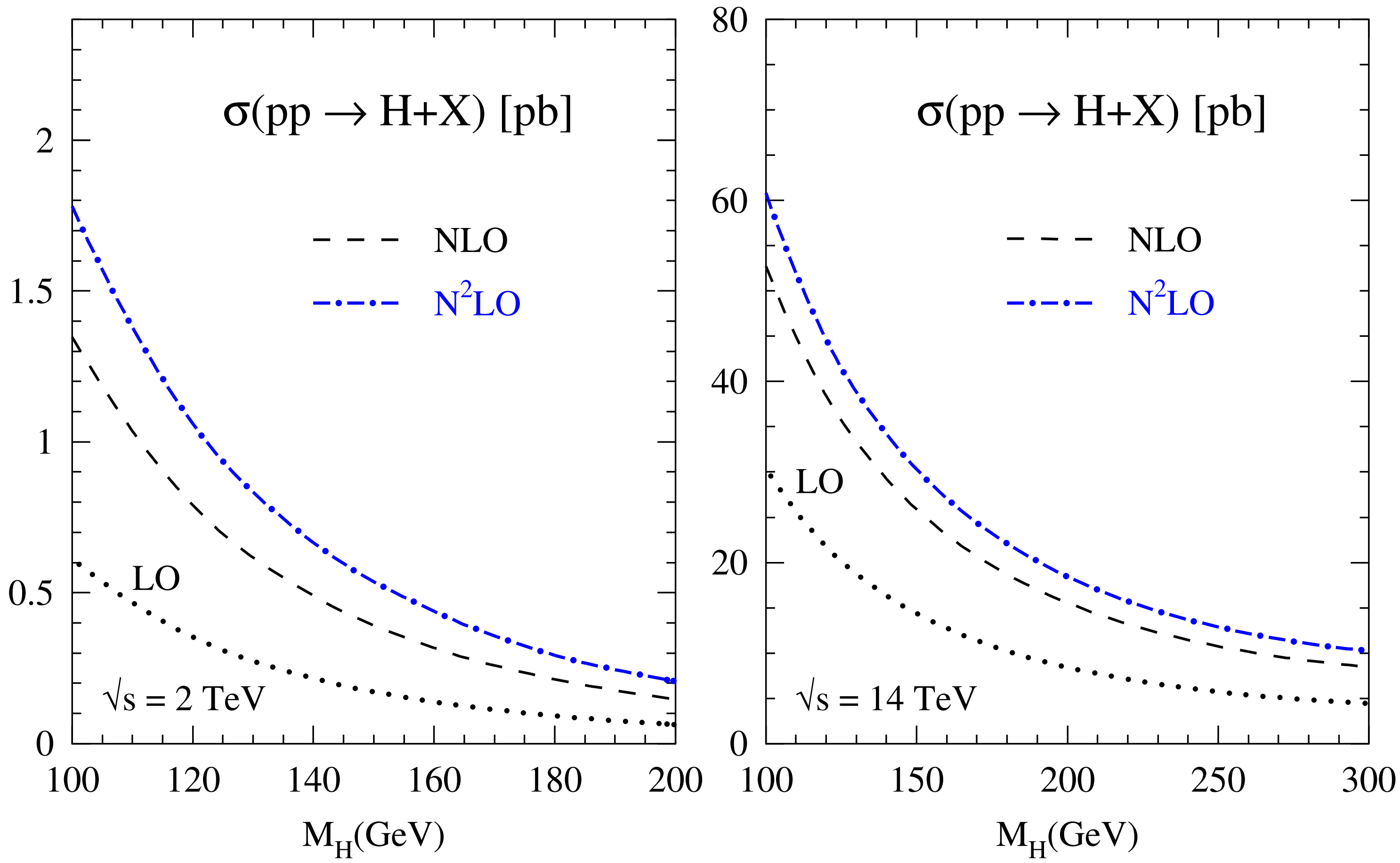
Poles in $\varepsilon = 2 - D/2$: KLN, renormalization, mass factorization

$1/\varepsilon$ pieces of $\mathcal{F}_n$ + n-loop splitting fct's $\longrightarrow$ $1/\varepsilon$ coefficients of S_n

$\longrightarrow$ all soft-enhanced $\mathcal{D}_{2n-1,\ldots,0}$ terms of N^{n}LO coefficient fct's c_n

Higgs production at Tevatron and LHC

Parameters ($m_{\text{top}} = 173.4$ GeV etc): Ravindran, Smith, van Neerven (03)



NNLO: Harlander, Kilgore; Anastasiou, Melnikov (02); RSvN (03)

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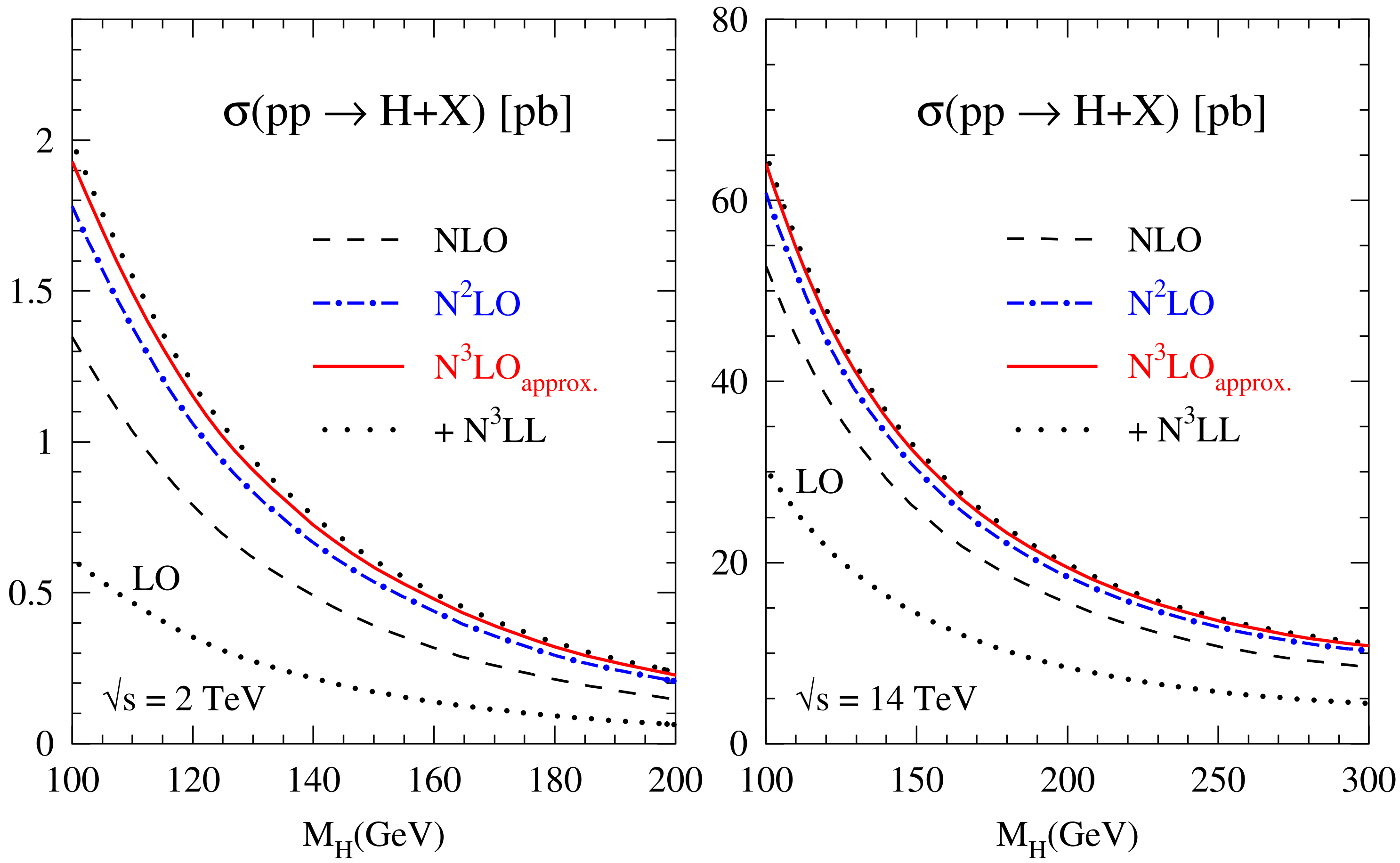
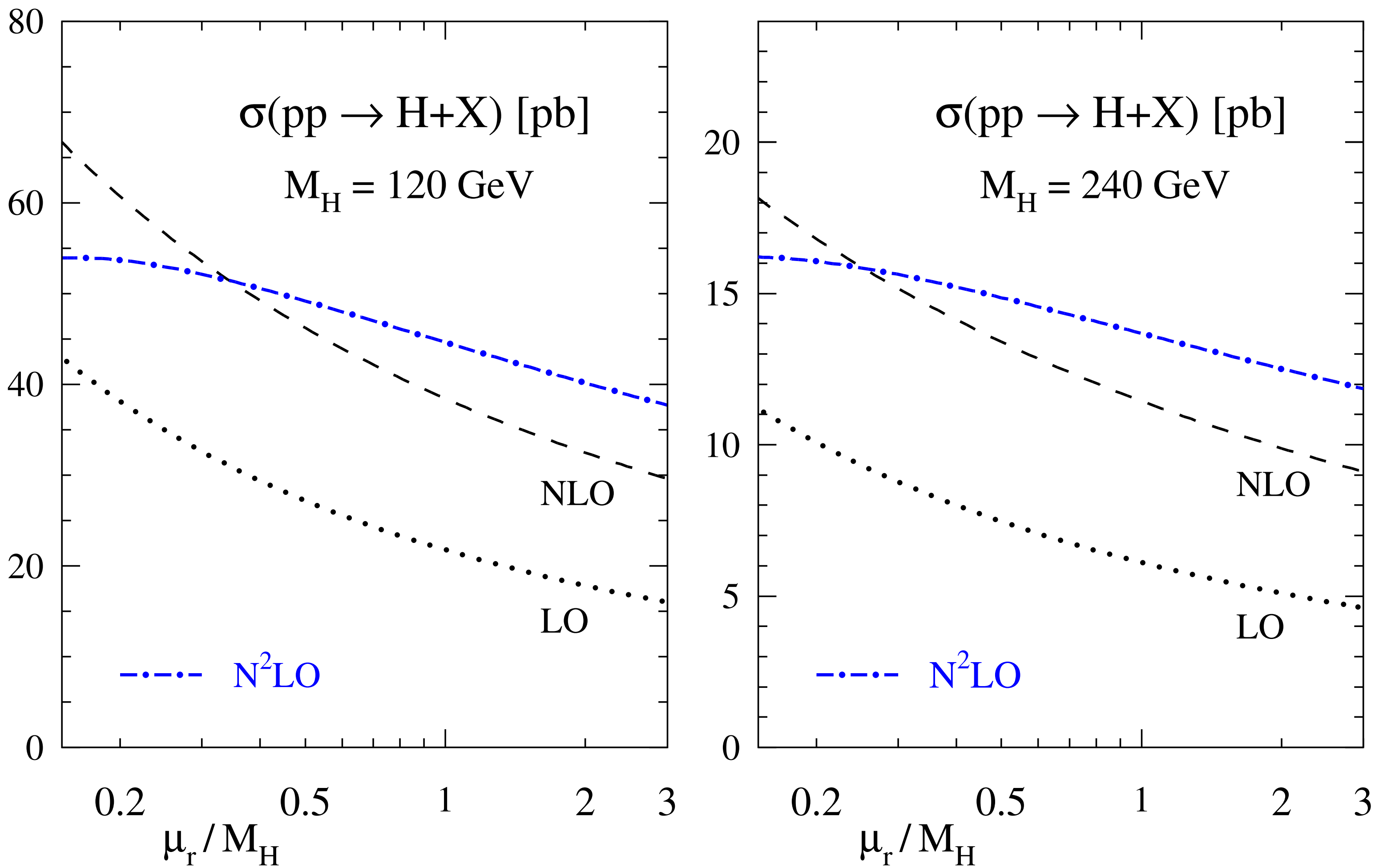


NNLO: Harlander, Kilgore; Anastasiou, Melnikov (02); RSvN (03); **N^3LO:** Moch, A.V. (05)

$N^3LO_{\text{approx.}}$: trf. $\mathcal{D}_k$ to N, drop $1/N$ terms (10% error for $N^{(2)}$LO corr.)

LHC Higgs production: renormalization scale



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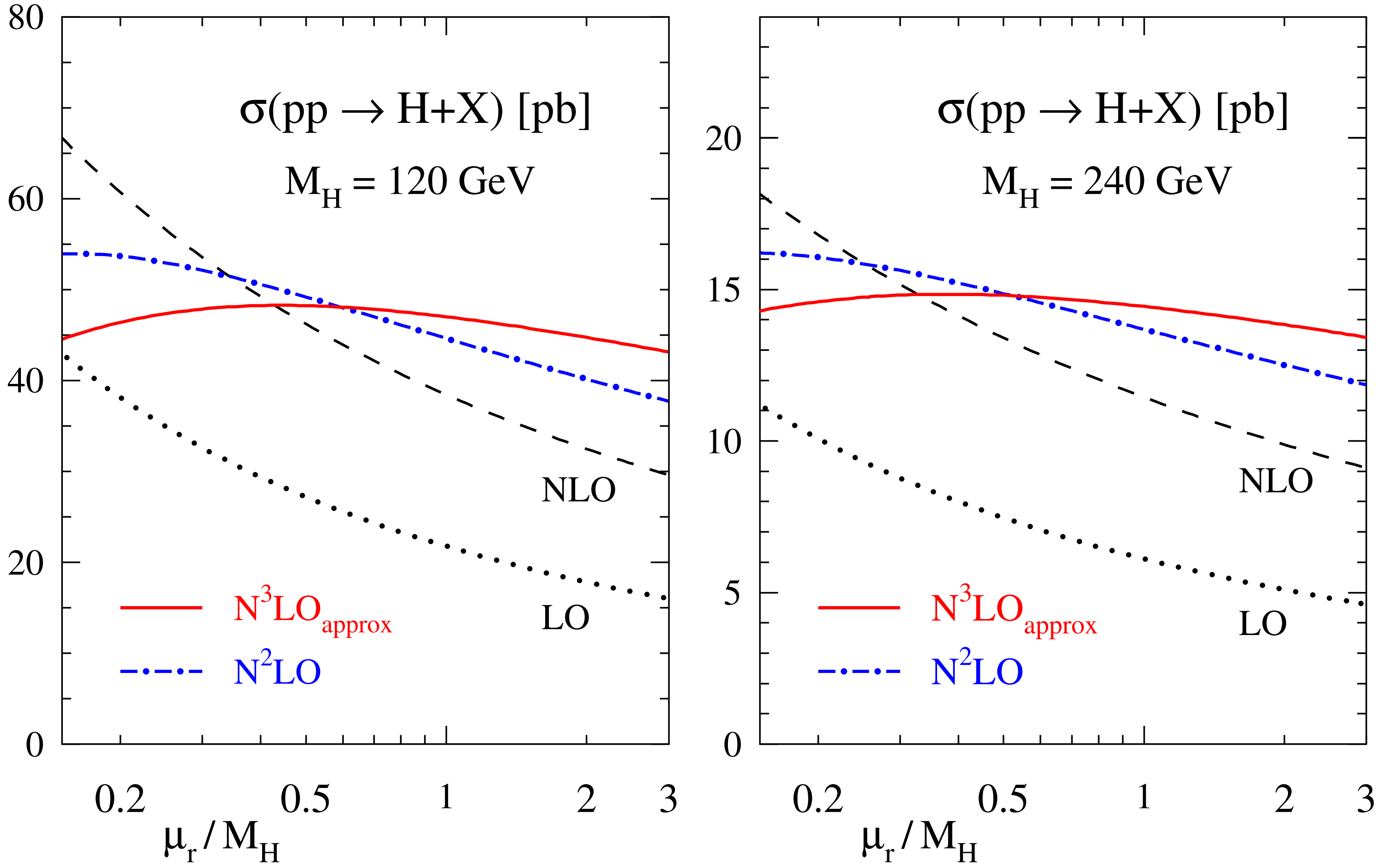
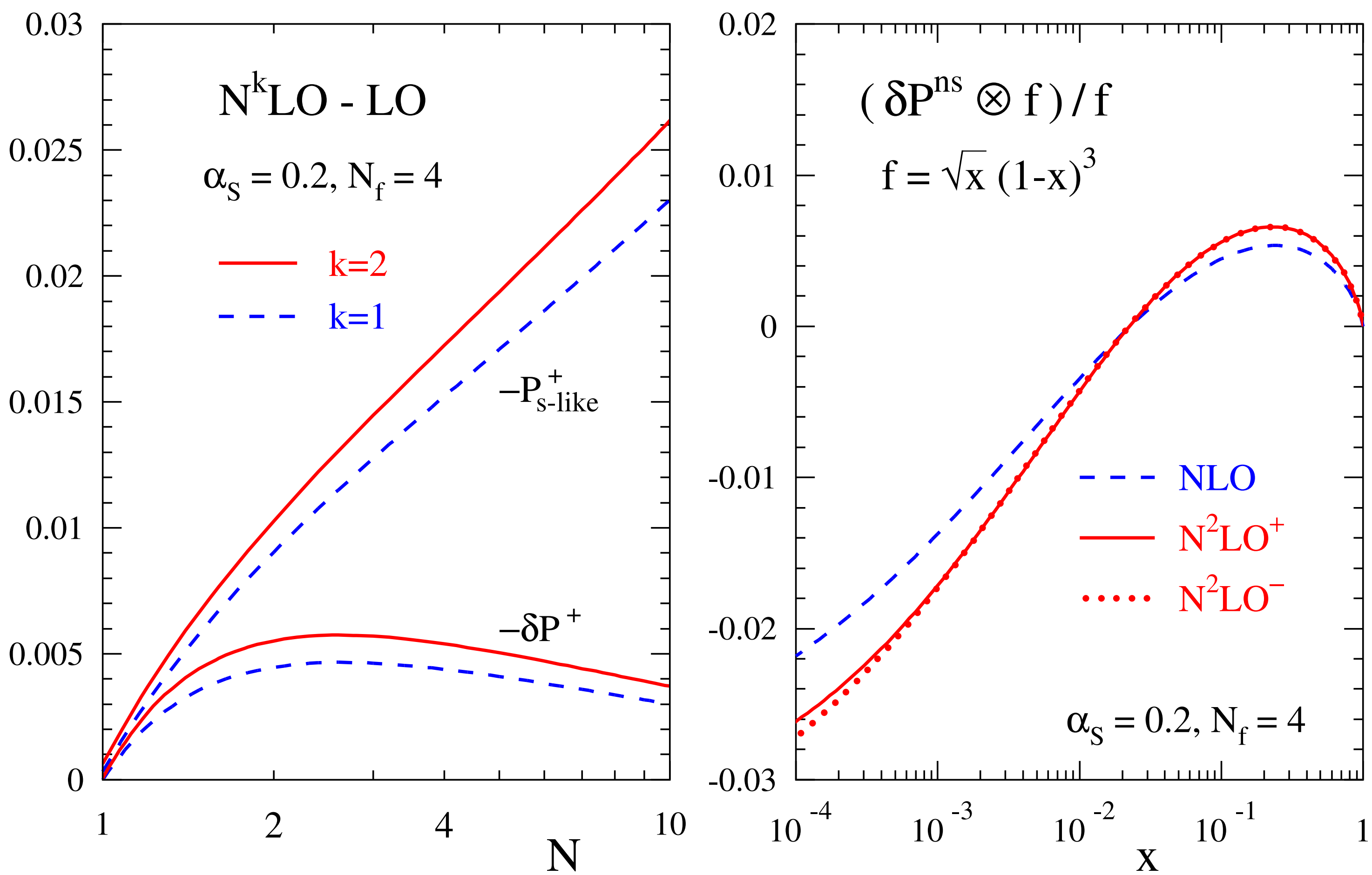


N^{3}LO increase at $\mu_r = M_H$: 5% (NNLO pdf's). μ_r variation: 4%

Estimated higher-order uncertainty: 5% for LHC, 7% for Tevatron

Non-singlet time-like splitting functions

From space-like case by analytic continuation and DMS conjecture



Time-like evolution slightly (mildly) faster (slower) at large (small) x