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On-Shell Methods for QCD Matrix Elements

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23th June 2006

Work in collaboration with Nigel Glover and Valya Khoze.

Talk Outline

- Off-Shell Methods.
- On-Shell Methods: Recursion Relations.
- On-Shell Methods: MHV Rules.
- Conclusions and Outlook.

Off-Shell Methods: Feynman Diagrams

- Advantages:
  - Direct link to Lagrangian.
  - Easy to adapt to amplitudes with mass/spin.
  - Easy to automate. [MADEVENT,GRACE]
  - Off-shell recursion. [Berends,Giele]

- Disadvantages:
  - Large number of diagrams.

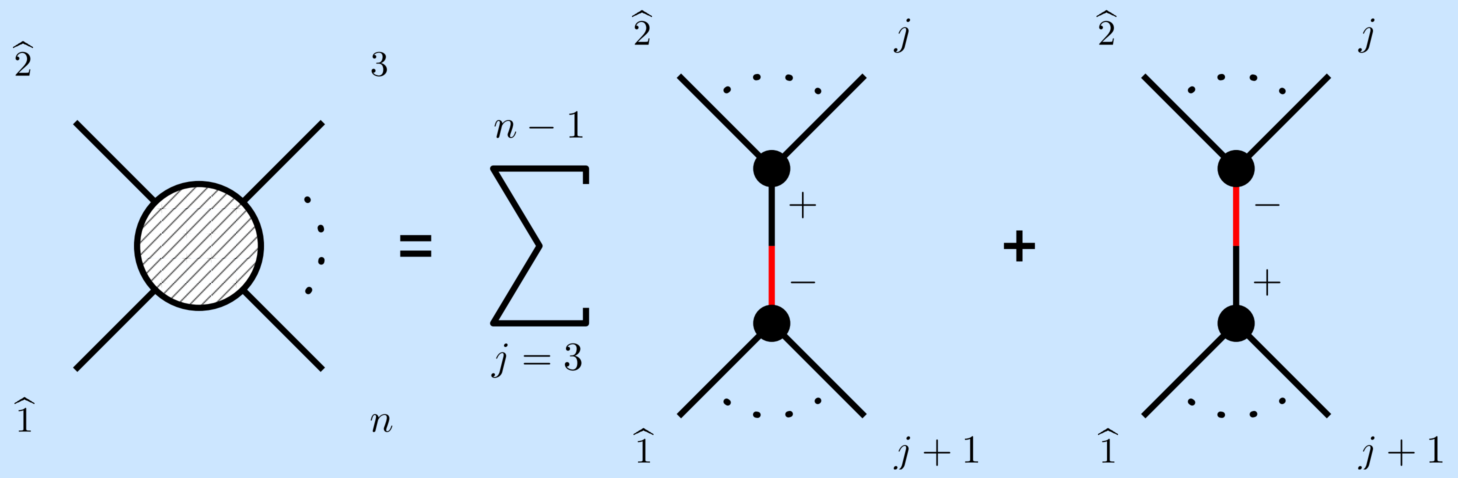
# Number of Particles	4	5	6	7	8	9	10
# Feynman Diagrams	4	25	120	2485	34300	559405	10525900

  - High multiplicity final states will be significant at the LHC.
  - Large cancellations between diagrams.
  - Higher order corrections extremely difficult to evaluate.

On-Shell Recursion: Tree Level Relation

New recursion relations for tree level gluon amplitudes.

[Britto,Cachazo,Feng hep-th/0412308]



All particles **on-shell**, all propagators $\sim 1/P^2$.

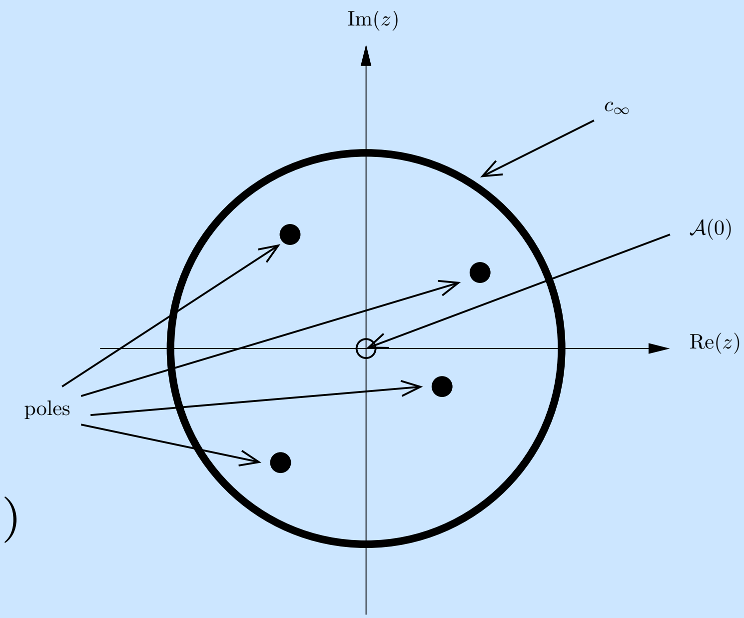
Proved using only simple complex analysis. [Britto,Cachazo,Feng,Witten hep-th/0501052]

On-Shell Recursion: Proof

Consider a generic scattering amplitude:

$$\mathcal{A}(p_1, \ldots, p_n) \to \mathcal{A}(p_1(z), p_2(z), p_3, \ldots, p_n) = \mathcal{A}(z)$$

where $p_1(z) + p_2(z) = p_1 + p_2$ and
$p_1(z)^2 = p_2(z)^2 = 0$.



$$0 = \frac{1}{2\pi i}\int_\gamma dz \frac{\mathcal{A}(z)}{z} = \mathcal{A}(0) + \sum_{i=3}^{n-1}\sum_{h=\pm}\mathcal{A}_L(z_i)\frac{1}{P_{1,i}^2}\mathcal{A}_R(z_i)$$

- Condition for complex momenta can be solved for massless particles:

$$\widehat{p}_1^{\mu}(z) = p_1^{\mu} + \tfrac{1}{2}\langle p_1|\sigma^{\mu}|p_2] \qquad \widehat{p}_2^{\mu}(z) = p_2^{\mu} - \tfrac{1}{2}\langle p_1|\sigma^{\mu}|p_2]$$

- Holds for more general shifts into complex momenta.

- Relies on vanishing of $\mathcal{A}(z)$ at the $z \to \infty$ boundary.

On-Shell Recursion: Applications

Generates extremely compact expressions for many amplitudes:

- Works for amplitudes of gluons and quarks. [Luo,Wen hep-th/0501121,hep-th/0502009]

- Applies to massive propagating particles: [SB,Glover,Khoze,Svrček hep-th/0504159] [Forde,Kosower hep-ph/0507292]
  - Propagator simply changes from $P_{1,i}^2 \to P_{1,i}^2 - m_P^2$
  - Solutions to constraints for shifting one external massive particle.

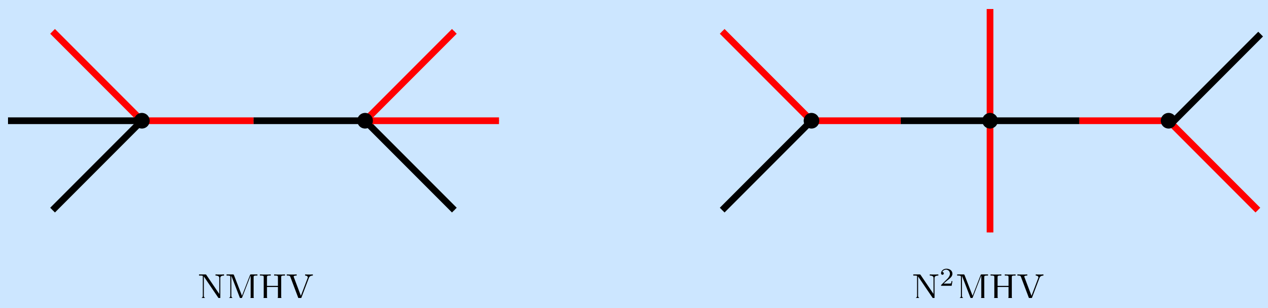
- Multi-Vector Bosons Currents and Massive Fermions [SB,Glover,Khoze hep-th/0507173] [Ozeren,Stirling hep-ph/0603071],[Rodrigo hep-ph/0508138],[Schwinn,Weinzierl hep-th/0601012]

MHV Rules

Maximal Helicity Violating (MHV) amplitudes take a remarkably simple form:

$$\mathcal{A}_n^{(0)}(\ldots, i^-, \ldots, j^-, \ldots) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

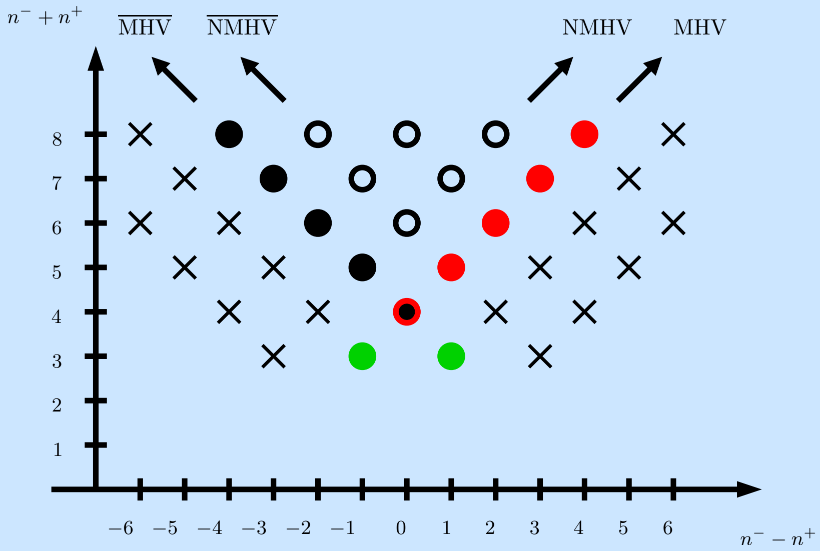
Following from Witten's twistor string/$\mathcal{N} = 4$ duality, Cachazo, Svrček and Witten proposed a scalar perturbation theory with the MHV amplitudes as vertices. [Cachazo,Svrček,Witten hep-th/0403047]



Lines in red(black) indicate **negative(positive)** helicity gluons.

MHV Rules

Rules generate simple n-point amplitudes for specific helicity configurations.



- Has been applied to tree level amplitudes with gluons, quarks, Higgs and Vector Bosons [Georgiou,Khoze hep-th/0404072], [Georgiou,Glover,Khoze hep-th/0407027] [Dixon,Glover,Khoze hep-th/0411092], [SB,Glover,Khoze hep-th/0412275], [Bern,Forde,Kosower,Mastrolia hep-th/0412275]

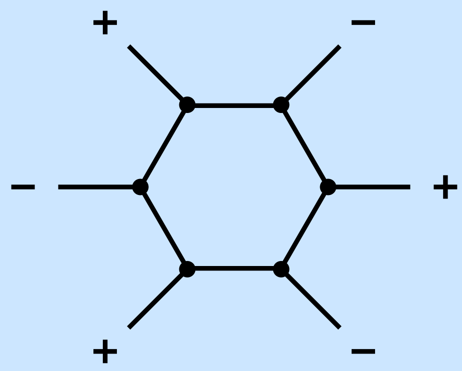
- Proved by showing equivalence between MHV method and recursion relation with special shift to complex momenta:
  Shift all negative helicity particles: $|\widehat{m}_i] \rightarrow |\widehat{m}_i] + zr_i|\eta]$
  This ensures no $\overline{\text{MHV}}$ 3-vertices appear. [Risager hep-th/0508206]

Outlook

- On-shell methods have provided efficient new ways to calculate tree level matrix elements.

- Methods extremely general and easy to apply to many different gauge theories at tree level.

Progress at NLO - Combining on-shell recursion and unitarity methods to find QCD matrix elements:



- Gluon and Quark "finite" amplitudes. [Bern,Dixon,Kosower hep-ph/0501240+hep-ph/0505055]
- Gluon MHV amplitudes. [Bern,Dixon,Kosower hep-ph/0507005, Forde,Kosower hep-ph/0509358]
- MHV rules at 1-loop. [Bedford,Brandhuber,Spence,Travaglini hep-th/0412108]
- Finite parts for all 6 and 7 gluon matrix elements now available. [Berger,Bern,Dixon,Forde,Kosower hep-ph/0605195]