CHARGE QUANTISATION

AEI 2025

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9/29/25

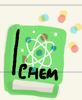
Outline

I. Groups locally & globally

II. The Standard Model Group & Spectrum

III. Embeddings

VI. SU2Y model



ArXiv: 2507.01777
RA, Dimakou, Ha & Khoze

ArXiv: 2404.03438
RA, Dimakou & West

Charge Quantisation

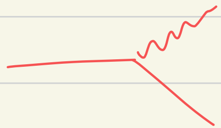
$$\Updownarrow$$

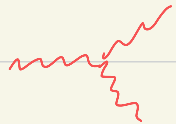
Compact group

I Local vs Global properties of groups.

[Insert appraisal of Gauge Theory here]

Local properties of groups tell us a great deal





$$[T_i, T_j] = i f_{ijk} T^k$$

In particular, given a few inputs, they give the perturbative **S**-matrix, correlators etc

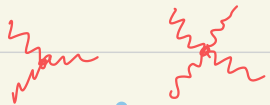
SU(2) versus SO(3)

SU(2): σ_i Pauli Matrices

$$\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\epsilon_{ijk}\frac{\sigma_k}{2}$$

as they act on a doublet ψ

SAME "GLUEDYNAMICS"



SO(3): $T_i = -T_i^T = T_i^\dagger$ 3×3 Matrices

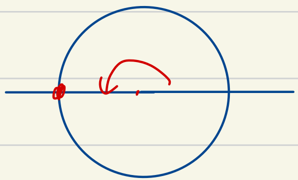
$$[T_i, T_j] = i\epsilon_{ijk} T_k$$

as they act on 3-vector $\vec{v}$

Would need some non-pert. effect to tell them apart

SU(2) versus SO(3)

SU(2): σ_i Pauli Matrices $\left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$



as they act on a doublet ψ

$e^{2\pi i \frac{\sigma_3}{2}}\psi = -\psi$

Rauch et al. '75 Phys. Lett. A

SO(3): $T_i = -T_i^T = T_i^\dagger$ 3 x 3 Matrices

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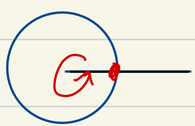
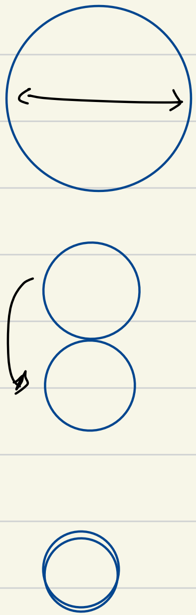


as they act on 3-vector $\vec{V}$

$e^{2\pi i T_3}\vec{V} = \vec{V}$

SU(2) versus SO(3)



σ_i Pauli Matrices $\quad \left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2}\right] = i\varepsilon_{ijk}\frac{\sigma_k}{2}$

irreps

		ψ		
SU(2):	1	2	3	...
$\sim$				
SO(3):	1	$\cancel{2}$	3	
			$\vec{V}$	

$$\boxed{SO(3) = \frac{SU(2)}{\mathbb{Z}_2}}$$

$\mathbb{Z}_2 = \{1, -1\}$
$\equiv \xi$

T_i 3×3 Matrices $\quad [T_i, T_j] = i\varepsilon_{ijk} T_k$

SU(2) double covers SO(3)

$\xi R = R$

in SO(3) reps don't see ξ

Remarks on U(1)

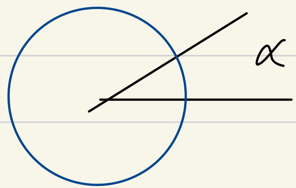
If one declares the group is $U(1) \Rightarrow$ Charge is quantised

$$\psi_Q \to e^{iQ\alpha} \psi_Q \qquad \alpha + t \qquad Q\alpha_1 = 2\pi\mathbb{Z}$$

$$\Rightarrow Q/Q_{min} = \mathbb{Z}$$

$Q_{min}^{exp} = 1$

$\alpha_1 = 2\pi$



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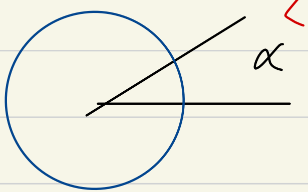
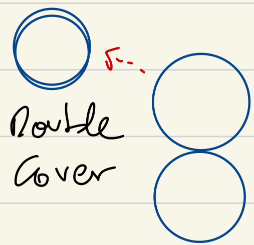
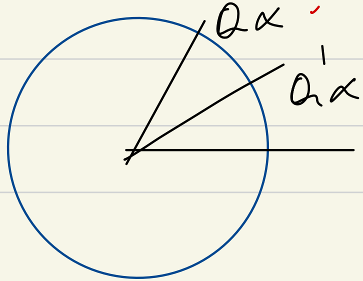




Tomorrow discover

$Q' = \frac{1}{2} \Rightarrow \alpha_1 = 4\pi$



Again

$$U(1) = \frac{\mathbb{R}}{\mathbb{Z}}$$

Math vs Phy

SU(N)xU(1) versus U(N)

$$\mathbb{1} \to e^{iQ'\theta}\,\mathbb{1}$$

SU(N) x U(1) Unrelated

$$\square \to e^{iQ_F\theta}\, e^{iT_a\theta^a}\,\square$$

other than $\frac{Q'}{Q_F}, \frac{Q_S}{Q_F} \in \mathbb{Z}$

$$\square\square \to e^{iQ_S\theta}\, e^{iT_{\square\square}\theta}\,\square\square$$

U(N) elementary unit:

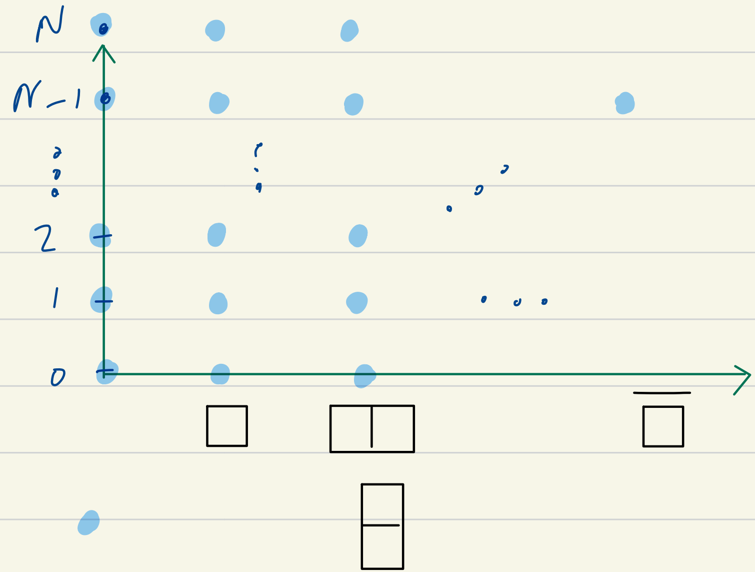
$$\square \to e^{i(\theta Q_F + \theta^a T^a_F)}\,\square$$

$$\square \otimes \square \to e^{2Q_F i\theta + i\theta^a\left(T^a_{\square\square} + T^a_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\right)}\,\square\square \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}$$

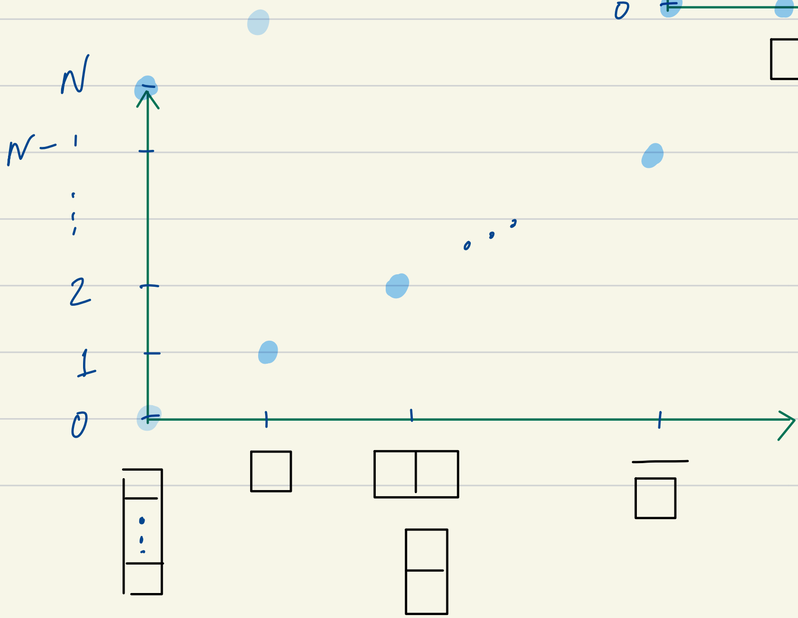
$$\left.\begin{matrix}\square\\\vdots\\\square\end{matrix}\right\} N \longrightarrow e^{NQ_F i\theta}\,\begin{matrix}\square\\\vdots\\\square\end{matrix}$$

SU(N)xU(1) versus U(N)

$SU(N) \times U(1)$



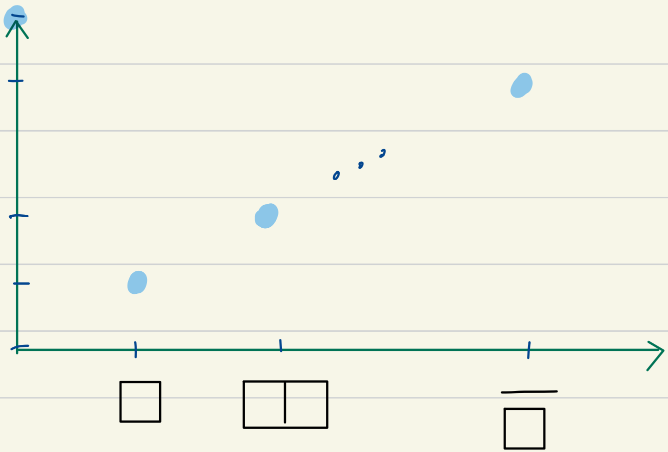
$U(N)$



A way to tell them apart

SU(N)xU(1) versus U(N)

One can again find centre; $U(N)$ reps satisfy



$$\mathrm{Exp}\left(\frac{2\pi i n_\psi}{N} - \frac{2\pi i Q}{N Q_F}\right)\psi = \psi$$

$$\equiv \xi \psi$$

ξ spans a $\mathbb{Z}_N$ group

$$\mathbb{Z}_N = \{1, \xi, \xi^2, \cdots \xi^{N-1}\}$$

$$\boxed{U(N) = \frac{SU(N)\times U(1)}{\mathbb{Z}_N}}$$

The centre/zentrum

We can identify all possible isomorphic groups

Given a Lie Algebra, obtain the centre

Z : Centre $\equiv$ subset of elements which commute with everyone else

The "largest" group is the universal cover $\tilde{G}$ while all others are of the form

$$\boxed{G_p = \frac{\tilde{G}}{Z_p}} \; ; \quad Z_p \text{ subgroup of } Z$$

$\boxed{\mathrm{II}}$

The gauge group of the SM

$\tilde{G} \equiv$ \ irreps	q_L	u_R	d_R	l_L	e_R	H
$SU(3)$	3	3	3			
$SU(2)$	2			2		2
$U(1)$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$

Given the spectrum is the a centre ?

$\mathbb{Z}_3 \times \mathbb{Z}_2 \times U(1) \longrightarrow \mathbb{Z}_6$

triality

"duality"

Yes: $\xi \equiv \mathrm{Exp}\left[\frac{2\pi i}{6}\left(6\,Q_Y + 2n_c + 3n_L\right)\right] \equiv \mathrm{Exp}\left[\frac{2\pi i}{6} n_6\right] \equiv e^{\frac{2\pi i Q_6}{6}}$

Hexality

The gauge group of the SM

$$\mathbb{Z}_6 = \{ \overset{\xi^6}{1}, \ \xi, \ \xi^2, \ \xi^3, \ \xi^4, \ \xi^5 \}$$

$$\frac{\tilde{G}}{\mathbb{Z}_6} = S[U(3) \times U(2)]$$

$$\frac{\tilde{G}}{\mathbb{Z}_3} = U(3) \times SU(2)$$

$$\frac{\tilde{G}}{\mathbb{Z}_2} = SU(3) \times U(2)$$

$$\frac{\tilde{G}}{\mathbb{Z}_1} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_1}$$

Hucks 1991

Tong 2017

The group and its shortcuts

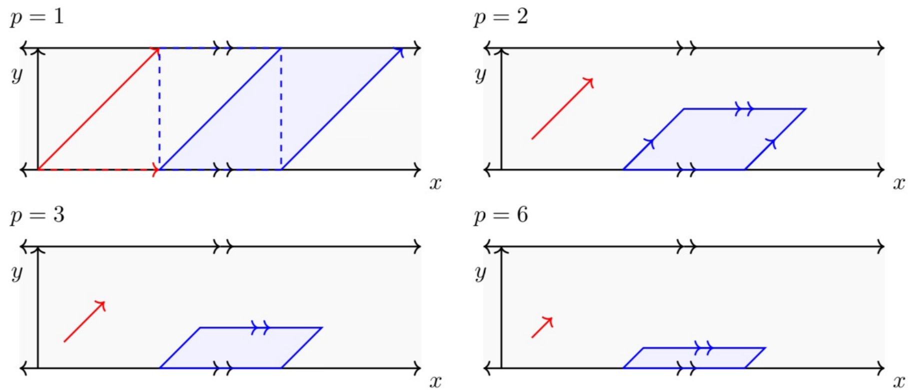
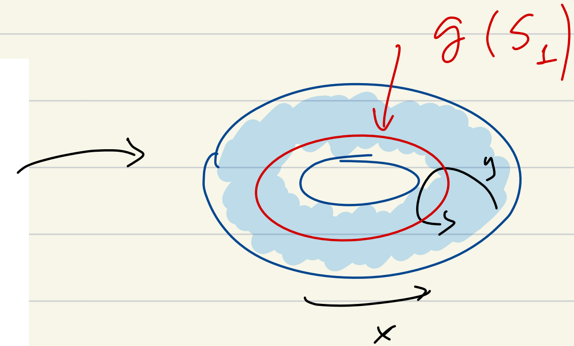
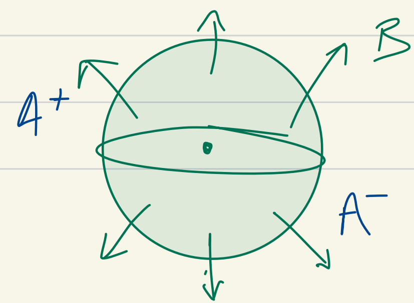


Figure 2. The action of the quotient group, i.e. the equivalence relation we impose on the group, in the plane (x, y) where $X = 2\pi x(6Q_Y) + 2\pi y(2\tilde{\lambda}_8 + 3\tilde{T}_{3L})$ and a group element is $g = \exp(iX)$. We have $x \in \mathbb{R}_Y$ while y is periodic (by definition) with period 1, the red arrow represents the action of the generating element of K_p so any two points differing by an integer times this vector are identified. The blue-filled rhomboid is the minimum domain all points in x, y can be mapped to, one can think of it as a torus with its interior (a non-Abelian off-diagonal direction) filled in.



Monopole! A lá Wu-Yang

$$A^+ - A^- = g^{-1}(S_1)\, d\, g(S_1)$$



The spectrum

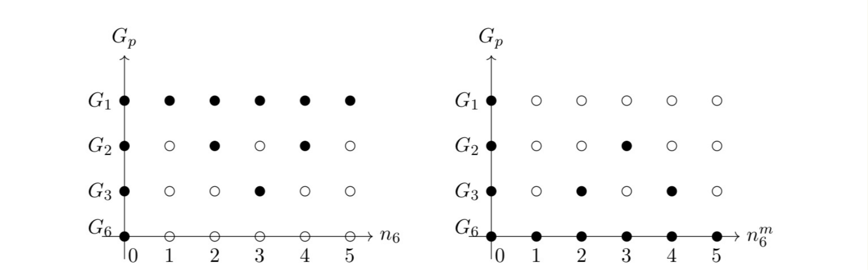
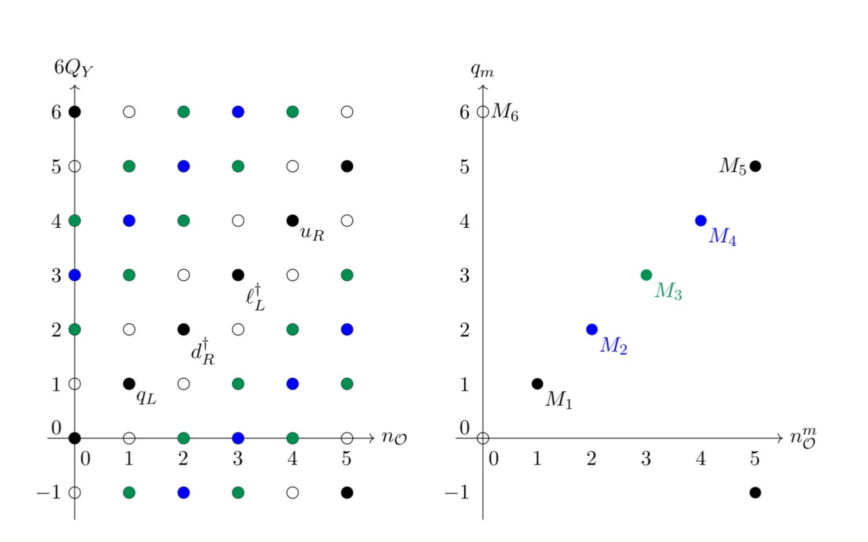


Figure 1. Left panel shows the allowed values of n_6 characterising all possible electric representations for each G_p choice of the SM group as black filled-in nodes. On the right we show the corresponding magnetic spectrum n_6^m for each G_p theory.



$$n_6 = 6Q_Y + 2n_c + 3n_2 \mod 6$$

$$n_{\mathcal{O}} = -2n_c + 3n_2 \mod 6$$

$$g(s_1) = e^{\frac{2\pi i}{6} n_6^m Q_6 \phi}$$

The Electric Spectrum

$$Q_{em} = \frac{n_6}{6} - \frac{n_c}{3} + \mathbb{Z}$$

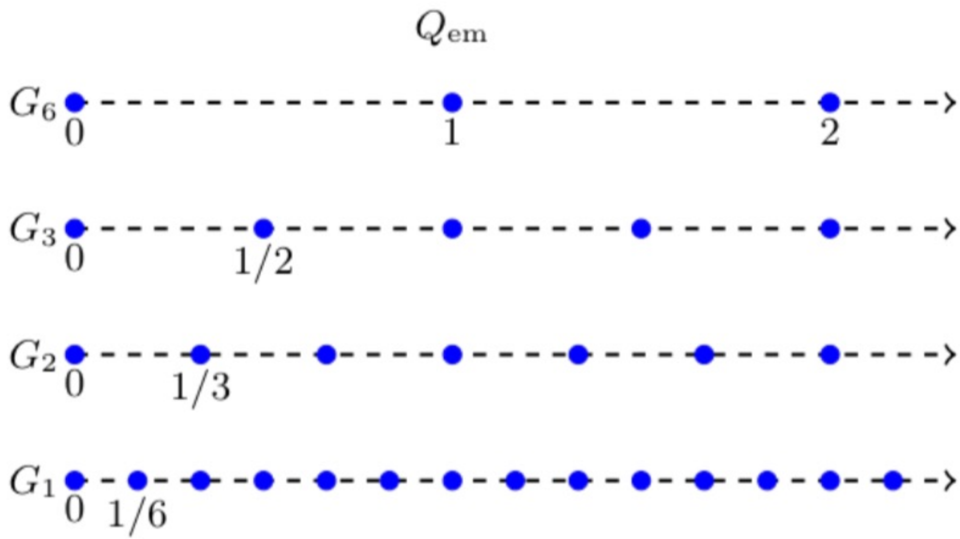


FIG. 4. Electric charge spectrum for hadrons and leptons in the case $k = 0$.

Embeddings

If the SM group is contained in G semi-simple prediction for G_p

For exple **SU(5)**

$$Q_Y = \frac{1}{6}\begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix} \qquad \tilde{\lambda}_8 = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \qquad \tilde{T}_{32} = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}$$

$$\xi = \mathrm{Exp}\left(\frac{2\pi i}{6}\left(2\tilde{\lambda}_8 + 3\tilde{T}_{32} + 6\,Q_Y\right)\right) = \mathrm{Exp}\left[2\pi i \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & -1 & \\ & & & & 0 \end{pmatrix}\right] = \mathbb{1}$$

$$\Rightarrow G_6 = \frac{\tilde{G}}{\mathbb{Z}_6}$$

The map of simplest embeddings

$$
\begin{tikzcd}
 & E_6 \arrow[d] \arrow[ddr] & \\
 & \mathrm{Spin}(10) \arrow[dl] \arrow[d] & \\
\frac{\mathrm{SU}(4)\times\mathrm{SU}(2)^2}{\mathbb{Z}_2} \arrow[dr] & \mathrm{SU}(5) \arrow[d] & \frac{\mathrm{SU}(3)^3}{\mathbb{Z}_3} \arrow[dl] \\
 & G_6 &
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathrm{SU}(4)\times\mathrm{SU}(2)^2 \arrow[d] \\
G_3
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathrm{SU}(3)^3 \arrow[dr] & & \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)^2}{\mathbb{Z}_2} \arrow[dl] \\
 & G_2 &
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathrm{SU}(3)\times\mathrm{SU}(2)^2 \arrow[d] \\
G_1
\end{tikzcd}
$$

Figure 20. Atlas of UV embeddings of $G_p = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{Z_p}$ comprising four distinct maps for different values of $p = 1, 2, 3, 6$.

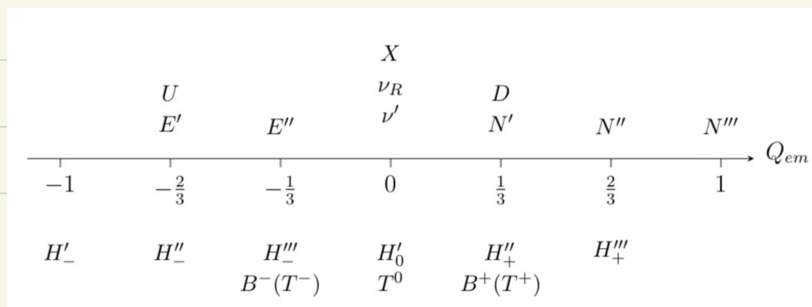
SU2Y: A model for G_1

$U(1)_Y \subset SU(2)_Y$

$SU(3)_c \times SU(2)_L \times SU(2)_Y$	$(\mathbf{1},\mathbf{2},\mathbf{4})$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$(\mathbf{1},\mathbf{1},\mathbf{3})$
SM fields	H		B^0_μ
n_6	0		0
BSM fields	H', H'', H'''	T^+, T^0, T^-	B^+_μ, B^-_μ
n_6	0, 2, 4	2, 0, 4	2, 4

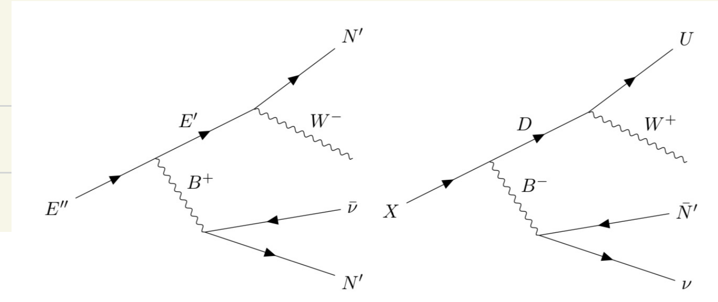
$SU(3)_c \times SU(2)_L \times SU(2)_Y$	$(\mathbf{3},\mathbf{2},\mathbf{2})$	$(\mathbf{1},\mathbf{2},\mathbf{4})$	$(\mathbf{3},\mathbf{1},\mathbf{5})$	$(\mathbf{1},\mathbf{1},\mathbf{7})$
SM fields	q_L	ℓ_L	u_R, d_R	e_R
n_6	0	0	0	0
BSM fields	Q_L	L'_L, L''_L, L'''_L	U_R, X_R, D_R	$E'_R, E''_R, \nu'_R, N'_R, N''_R, N'''_R$
n_6	4	2, 4, 0	4, 2, 4	2, 4, 0, 2, 4, 0

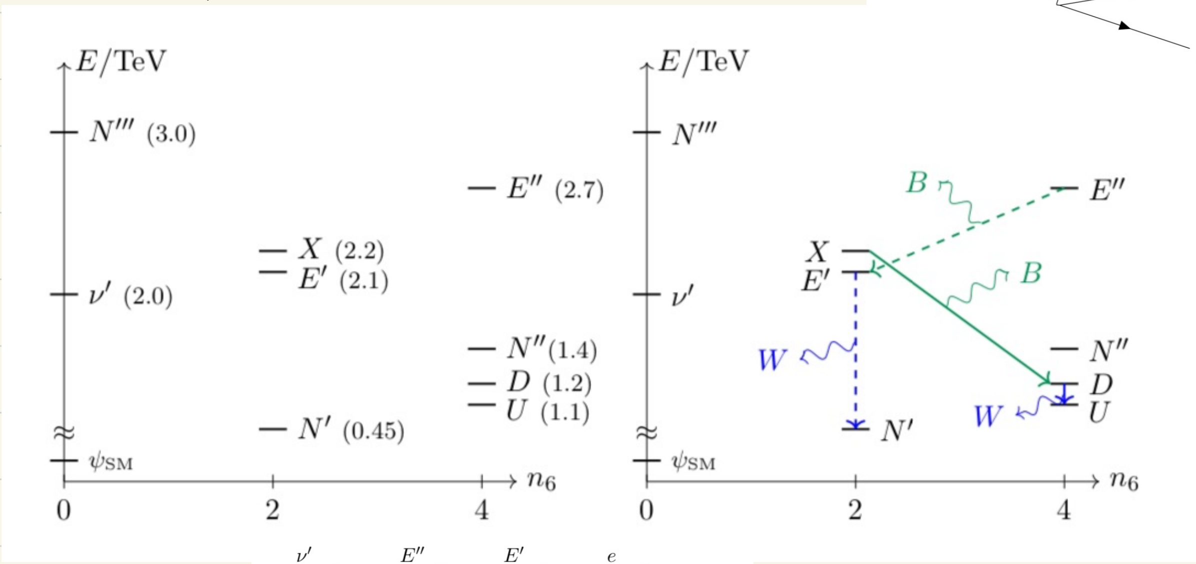
$SU(3)_c \times SU(2)_L \times SU(2)_Y$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{1})$
BSM fields	X_L	ν_R
n_6	4	0

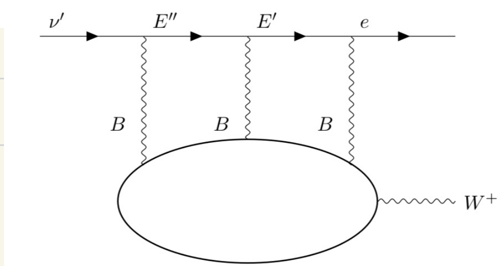


SU2Y: A model for G_1

Hexality as stabiliser







- Anomaly free
  - Renormalisable
    - SM mass spectrum
      - Non-decoupling

Summary

Hopefully you learned something about charge quantisation you didn't know.

Integer $\qquad G_p = 1, 2, 3, 6$

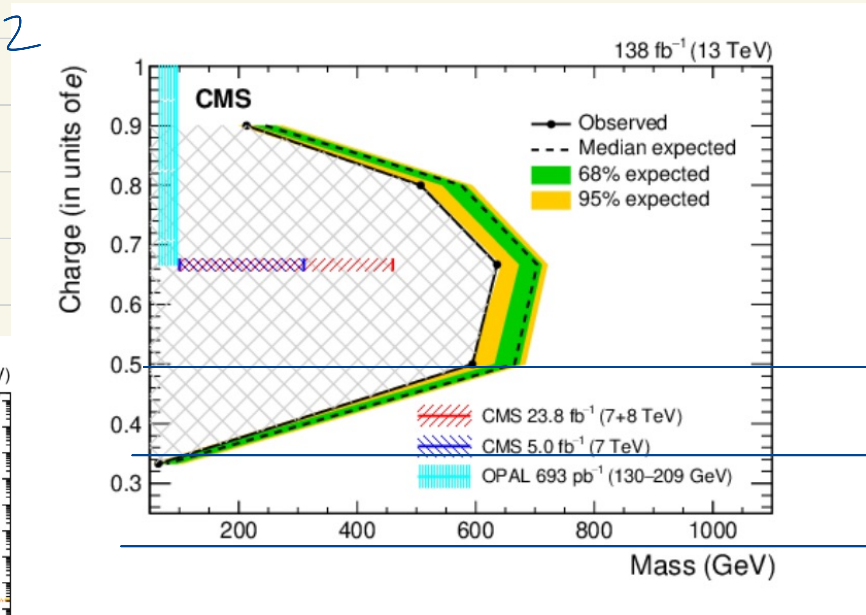
$$Q_{em} = \frac{\mathbb{Z}\ p}{6(1 + pK)}$$

$K = 0, \pm 1, \pm 2, \ldots$

Searches at the LHC

CMS 2402.09932

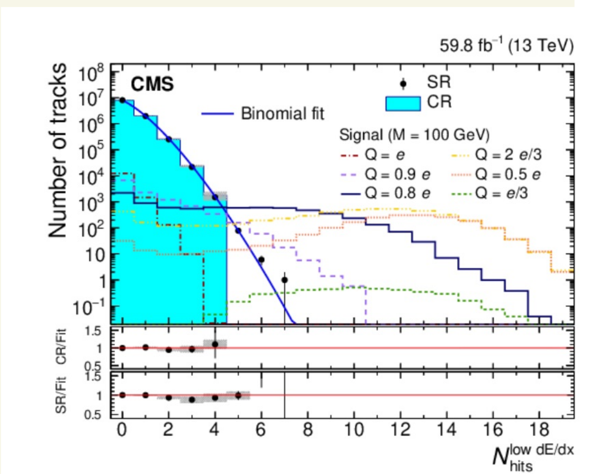
$(1,1)_{Q_4}$



G_3

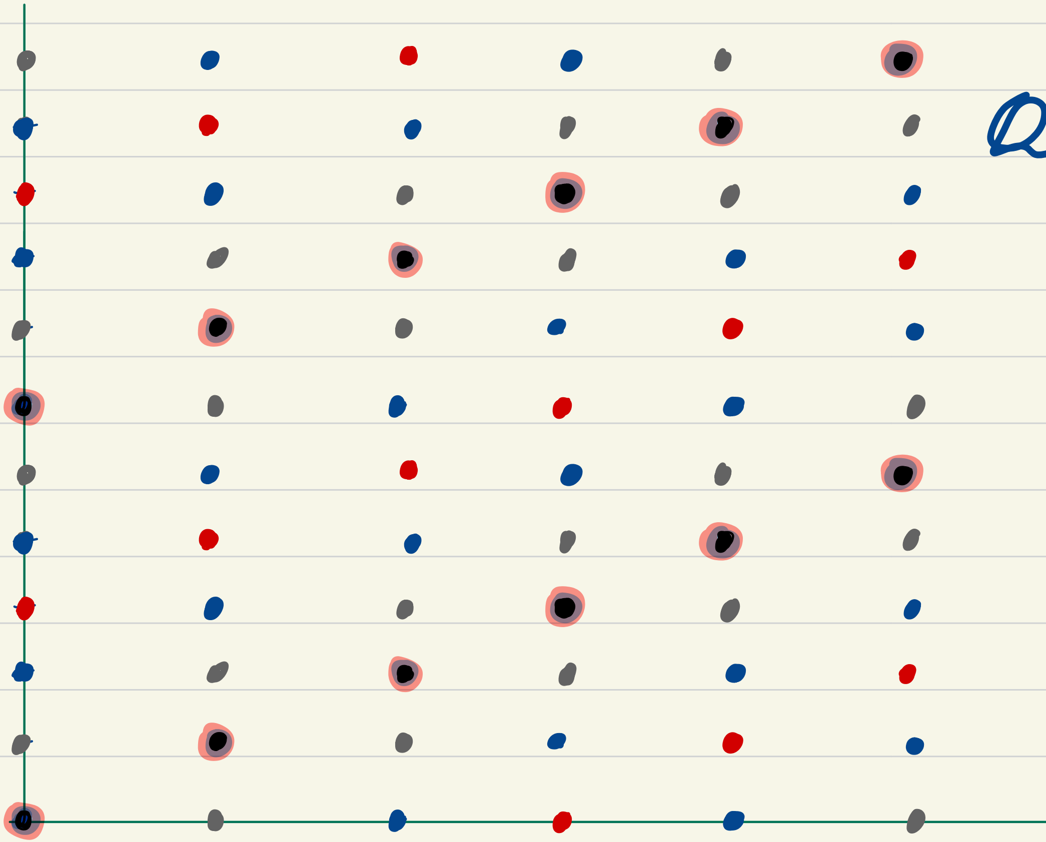
G_2

G_1



Martin & Koren 2406.17850

The caveat/presumption



Integer $\qquad G_{p=1,2,3,6}$

$$Q_{em} = \frac{\mathbb{Z}\; p}{6\,(1 + pK)}$$

$K = 0, \pm 1, \pm 2, \ldots$

e.g. $\quad G_6 \qquad p = 6$

$$\frac{Q^{e^+}}{Q_{min}} = \begin{cases} 1 & K = 0 \\ -5 & K = -1 \\ 7 & K = 1 \end{cases}$$