Axion-less (partial) solution to strong CP problem

—— Presentation for the 5th AEI Workshop

2408.12406, 2505.05142 Q.Liang, R.Okabe, T.T.Yanagida

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Content

- Introduction to strong CP problem and solutions
- Non-invertible symmetry in quark mass matrix
- Discussion

Introduction to strong CP and axion

- Strong CP problem in SM

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \theta\frac{g^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi.$$

- The theory is CP invariant only when $\theta = -\theta'$.
- Physical CP violating angle: $\bar{\theta} = \theta_0 + \mathrm{Arg}[\det(M_d)\det(M_u)]$ Mu and Md are up/down quark mass matrix.
- Neutron electric dipole moment is proportional to physical CP violating angle $d_N = (5.2 \times 10^{-16}\mathrm{e} \cdot \mathrm{cm})\bar{\theta}.$
- Experiments put a tight constraint on $\bar{\theta} < 10^{-10}$

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Axion solution

- QCD axion: prompt the theta angle to a field whose vacuum expectation value is zero. The effective potential in Peccei-Quinn theory is $V_{\text{eff}} \sim \cos\left(\theta + \xi\frac{\langle a\rangle}{f_a}\right)$
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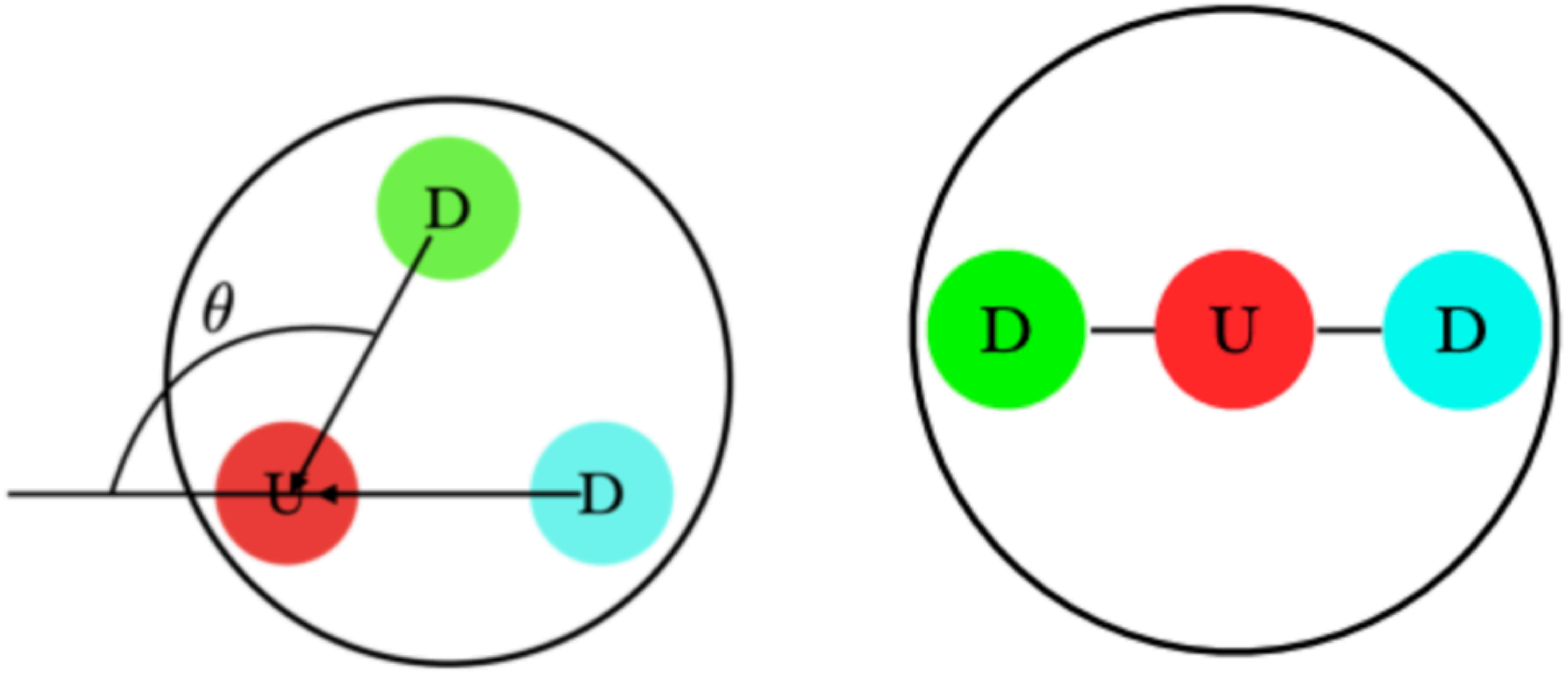

Introduction to strong CP and axion

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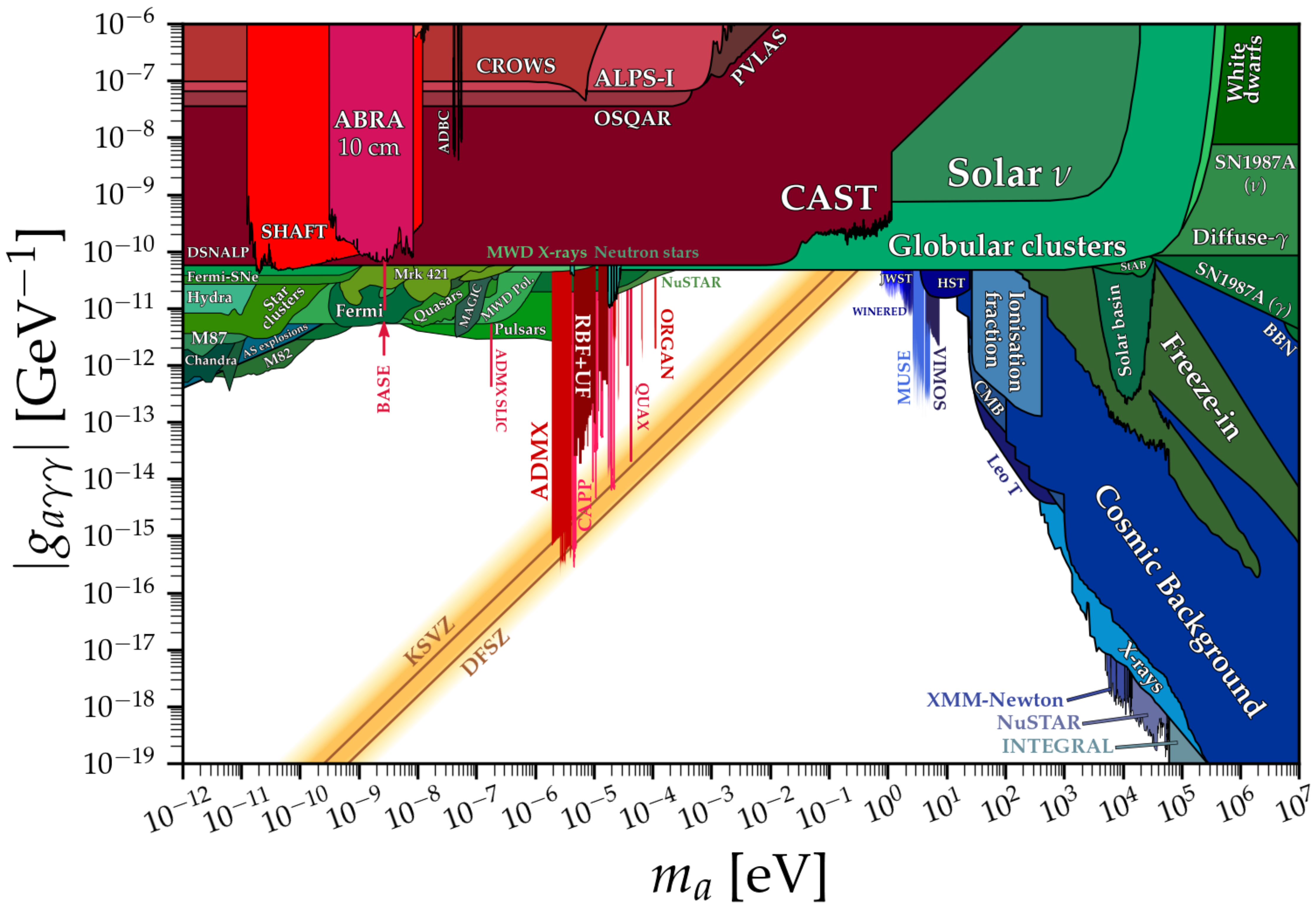
- QCD axion is also a popular dark matter candidate through misalignment. Constraints through their anomalous coupling to electromagnetic：
- Can arise from string theory and other UV completion. hep-th/0605206.



https://cajohare.github.io/AxionLimits/docs/ap.html

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Axion

- However, axion has not been identified by experiments.
- Moreover, it has been claimed that axion generally has the quality problem. (Quantum correction will generally lead to a non-zero vev)
- 1. From Effective filed theory point of view, without symmetry, axion should not have the desired coupling: $\mathcal{L} \supset \left(\frac{a}{f_a} + \theta\right) \frac{1}{32\pi^2} G\tilde{G}.$
- 2. Quantum gravity should break all symmetries that are not gauged. Gravitational effects will induce additional mass terms that are not centered at zero.
- This also seems incompatible with cosmology:

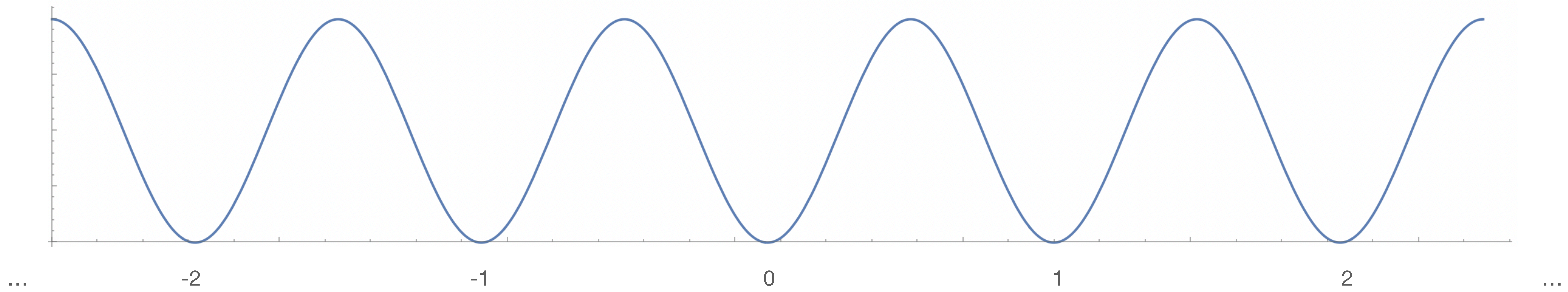
2301.00549 [hep-th]
2312.07650 [hep-ph].
Phys. Rev. D 32 (1985) 3178.

Axion-less solutions to strong CP problem

- Massless quark: chiral rotation to eliminate θ angle, but ruled out by experiments.
- Parity-based solutions: parity requires θ to vanish, and quark mass matrix are hermitian to have real determinant. arXiv:hep-ph/9511376
- Nelson-Barr: CP symmetry is fundamentally exact and breaks at low energy to give rise to CP-violation phase in CKM matrix. A. E. Nelson, Phys. Lett. B 143, 165 (1984) 10.1103/PhysRevLett.53.329

Axion-less solutions to strong CP problem

- Quantum corrections are hard to control.
- Symmetry at the Lagrangian level does not lead to symmetry at the state level.



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Nelson-Barr solutions to strong CP problem

- Turn back to the original question about the strong CP.

$$\bar{\theta} = \theta_0 + \mathrm{Arg}[\det(M_d)\det(M_u)] \qquad \bar{\theta} < 10^{-10}$$

- Spontaneous CP violation as an alternative: $\theta_0 = 0$
- For this type of solutions, one need to have real mass matrix determinant: $\mathrm{Arg}[\det(M_d)\det(M_u)] = 0$
- This is not easy to construct since quark mass matrix contains complex phases, and harder to protect them from loop correction.

Nelson-Barr solutions to strong CP problem

- The 4-th quark pair A. E. Nelson, Phys. Lett. B 143, 165 (1984) 10.1103/PhysRevLett.53.329
- Extra Higgs L. Hall, C. A. Manzari, and B. Noether, arXiv:2407.14585 [hep-ph].
- Modular invariance F. Feruglio, M. Parriciatu, A. Strumia, and A. Titov, arXiv:2406.01689 [hep-ph].

  S. T. Petcov and M. Tanimoto, arXiv:2404.00858 [hep-ph].

Three-zero texture of quark mass matrix

- It has been proposed that 7 three-zero textures of quark mass matrix can fit data well. One such example of the down quark mass matrix is

$$M_d = \begin{pmatrix} 0 & a & 0 \\ a' & be^{-i\phi} & c \\ 0 & c' & d \end{pmatrix}$$

a [MeV]	a' [MeV]	b [MeV]	c [MeV]	c' [GeV]	d [GeV]	ϕ [$\circ$]
16 - 17.5	10 - 15	92 - 104	78 - 95	1.65 - 2.0	2.0 - 2.3	37 - 48

- M. Tanimoto and T. T. Yanagida, “Occam’s Razor in Quark Mass Matrices,” PTEP 2016 no. 4, (2016) 043B03, arXiv:1601.04459 [hep-ph].

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- More interestingly, the determinant of this matrix is real even it contains complex elements!

- This looks like a good way to solve strong CP. But why these elements have to be exact zero? Is there any symmetry protecting them?

Three-zero texture of quark mass matrix for strong CP

- We first want to show that 4D ordinary symmetry does not work
-

$\mathbf{10}_1 H \bar{\mathbf{5}}_2$ Neutral

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$\mathbf{10}_2 H \bar{\mathbf{5}}_1$ Neutral

$$\mathbf{10}_i = (q_L, \bar{u}_R)_i$$
$$\bar{\mathbf{5}}_i = (\bar{d}_R)_i$$

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Md11 should have opposite charge of Md 22, no reason to be zero!

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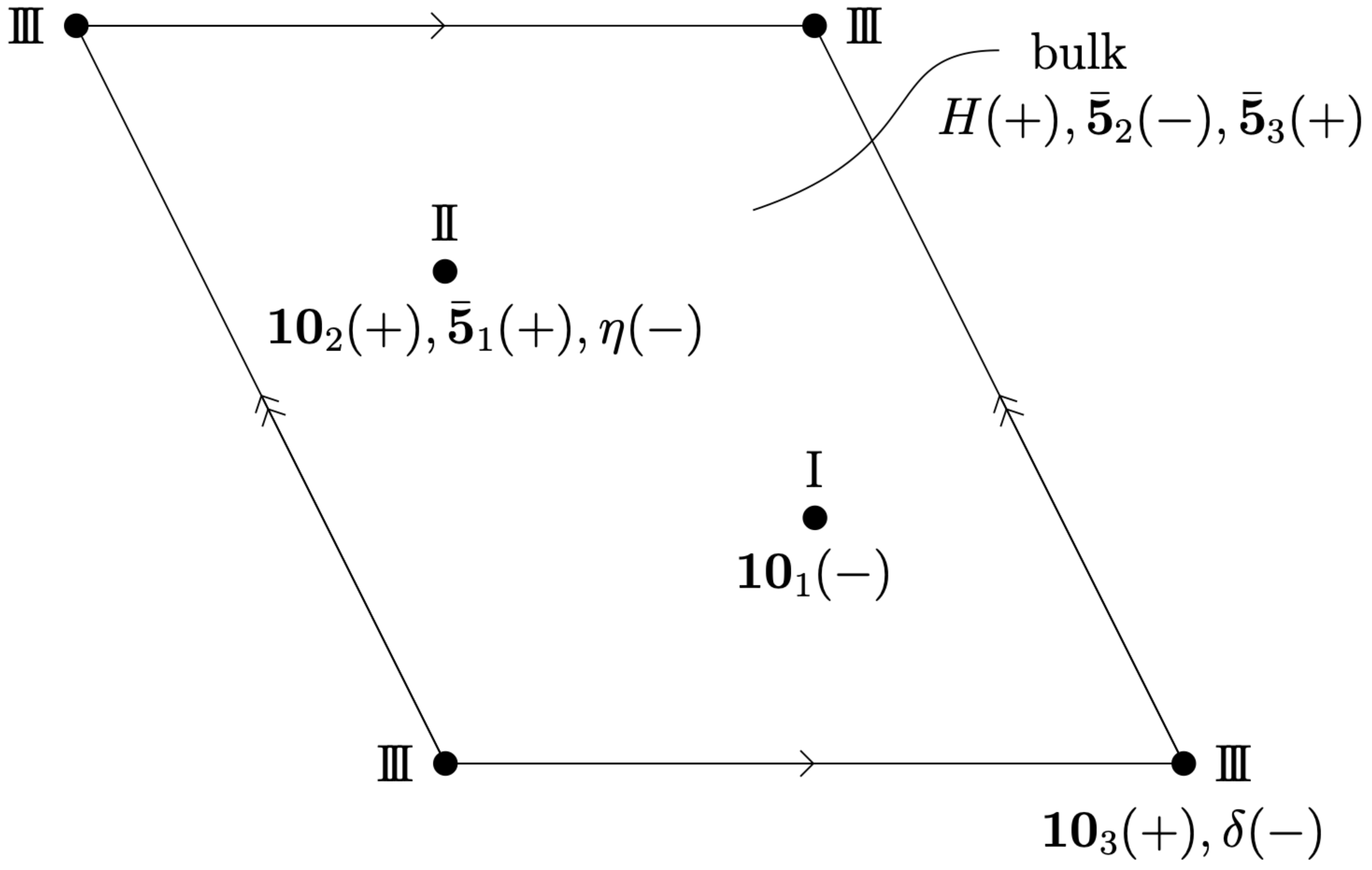
- Maybe generalized symmetry? Maybe extra dimension?

Three-zero texture of quark mass matrix for strong CP

QL, R. Okabe, T.T.Tsutomu 2408.12406

- We go to higher dimension $\mathbf{T}^2/\mathbb{Z}_3$ orbifold with fixed points.



	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	H	η	δ
$\mathbb{Z}_2$	$-$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$-$

TABLE II. $\mathbb{Z}_2$ charge for each particle.

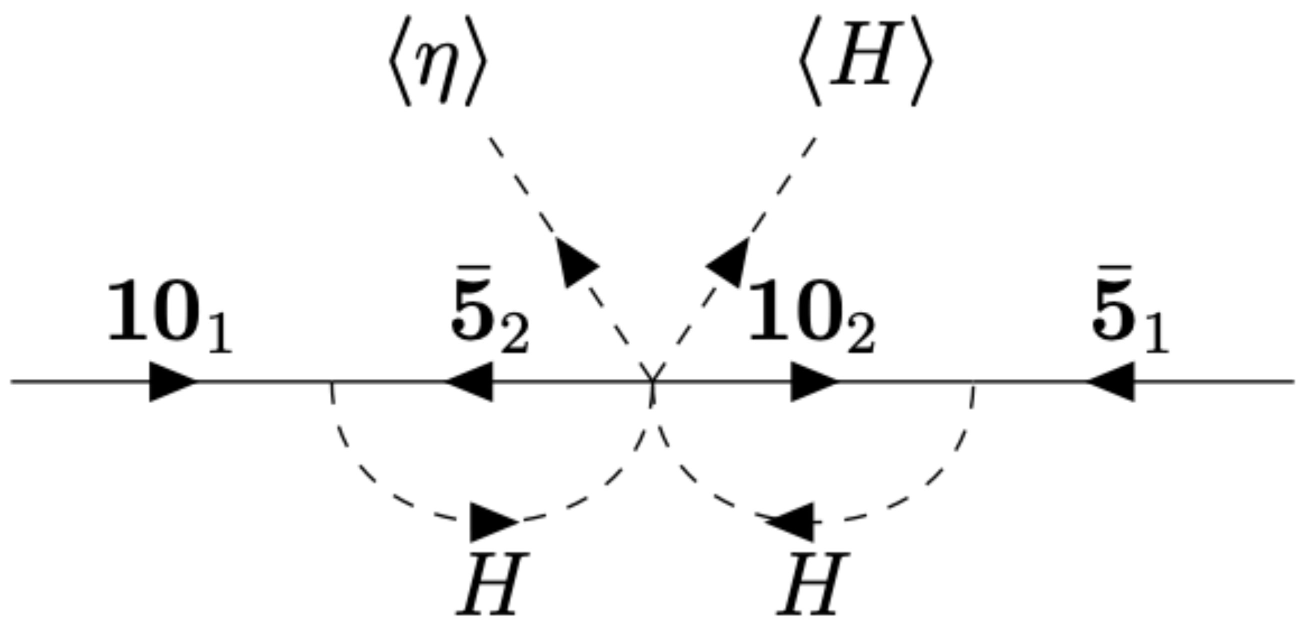
- Complex phase are realized through eta condensation.

Three-zero texture of quark mass matrix for strong CP

QL, R. Okabe, T.T.Tsutomu 2408.124066

- Loop corrections are manageable. $\bar{\theta} = \mathrm{Arg}\det M_d \sim \frac{\mathrm{Im}[\Delta_{11}(be^{-i\phi}d - cc')]}{aa'd} \sim g \times 10^{-9}$
- Quark mass hierarchy can be approached

$$m_u : m_c : m_t \simeq \epsilon^2 : \epsilon : 1 \text{ with the } \epsilon \simeq 1/300$$



through the ratio of the Higgs wave function at the fixed points to satisfy $|\Psi(H)|_{\mathrm{I}} : |\Psi(H)|_{\mathrm{II}} : |\Psi(H)|_{\mathrm{III}} \simeq \epsilon^2 : \epsilon : 1.$

Exotic symmetry in 4D to realize the three-zero texture?

QL, T.T.Tsutomu, 2505.05142

Introduction to generalized symmetries

- The concept of symmetry has been generalized in the following ways:
- Higher form symmetry — the charged objects are not local fields but extended objects
- Non-invertible symmetry — the operation is not described by group structure anymore
- Subsystem symmetry — symmetries that are not enjoyed by the whole system, fractonic symmetry⋯
- See reviews in 2306.00912, 2504.05960, etc..

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  → Strong CP problem 2505.05142
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Introduction to generalized symmetries

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Non-invertible symmetries

- Symmetries that are not describe by the group structure, usually with category theories (See lecture notes in SH Shao , S.Schafer-Namek). Non-invertible Peccei-Quinn symmetry to solve strong CP problem has been discussed in C.Córdova, S.Hong,c S. Koren 2402.12453.
- The one that I will talk about is a zero-form non-invertible symmetry, obtained through gauging Z2 from Zn, that has been discussed in T Kobayashi, Y Nishioka, H Otsuka, and M Tanimoto 2409.05270, 2305.18296, M Suzuki and L-X Xu 2503.19964
- I will focus on the selection rule implied by this specific non-invertible symmetry, there exists other non-invertible symmetry from higher group structure Y Tachikawa 1712. 09542 , J. Kaidi, Y. Tachikawa, H. Y. Zhang 2402.00105, J.J.Heckman, etal, 2402.00118

Non-invertible symmetries

- Add comparison between group $\mathbb{Z}_M$ and non-invertible $\widetilde{\mathbb{Z}}_M$

	$\mathbb{Z}_M$	$\widetilde{\mathbb{Z}}_M$
Generator	$g^k\ ,\ k = 0, 1, \cdots M-1 \bmod M$	$[g^k] = \{hg^kh^{-1} \vert h \in \mathbb{Z}_2\}\ ,\ k = 0, 1, \cdots \left\lfloor \frac{M}{2} \right\rfloor$ **Gauging operation identify charge k with -k !**
Operation	$g^{k_1}g^{k_2}\cdots g^{k_n} = g^{k_1+k_2+\cdots k_n}$	$[g^{k_1}][g^{k_2}]\cdots[g^{k_n}] = [g^{k_1+k_2+\cdots k_n}] + [g^{M-k_1+k_2+\cdots k_n}] + [g^{k_1+M-k_2+\cdots k_n}] + \cdots + [g^{k_1+k_2+\cdots M-k_n}]$
Selection rule	$\sum k_i = 0$	$\sum \pm k_i = 0$ **Fusion rule!**

Non-invertible symmetries

QL, T. Yanagida :2505.05142

- Talk about M=3

	$\mathbb{Z}_3$				$\widetilde{\mathbb{Z}}_3$		
Generator	g^0 , g^1 , g^2				$[g^0]$, $[g^1]$		
3 point coupling	(000)	(111)	(222)	(012)	$([0][0][0])$	$([1][1][1])$	$([0][1][1])$

- Winberg mass matrix

$$\bar{q}_L H d_R : \begin{pmatrix} 0 & \delta \\ \delta & M \end{pmatrix}$$

Non-invertible symmetries

QL, T. Yanagida :2505.05142

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- Winberg mass matrix

$$\bar{q}_L H d_R : \begin{pmatrix} 0 & \delta \\ \delta & M \end{pmatrix}$$

Ordinary symmetry $\begin{pmatrix} a+a' & a+b' \\ a'+b & b+b' \end{pmatrix}$

Accidental zero!

Non-invertible symmetries

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	$\mathbb{Z}_3$				$\widetilde{\mathbb{Z}}_3$		
Generator	$g^0\ ,g^1\ ,g^2$				$[g^0]\ ,[g^1]$		
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$\widetilde{\mathbb{Z}}_3$

$$[d_R] \quad \begin{pmatrix} [g^0] \\ [g^1] \end{pmatrix}$$

$$[\bar{q}_L] \quad ([g^0],[g^1]) \quad \begin{pmatrix} 0 & \checkmark \\ \checkmark & \checkmark \end{pmatrix}$$

$$H = [g^1]$$

Non-invertible symmetry for three-zero texture of quark mass matrix

QL, T. Yanagida :2505.05142

- We consider the non-invertible symmetry $\widetilde{\mathbb{Z}}_5 \times \widetilde{\mathbb{Z}}_3 \times \widetilde{\mathbb{Z}}_3$

$$\bar{u}_L H_1 u_R \qquad\qquad \bar{u}_L H_2 u_R$$

$$[\bar{q}_L] : (010, 111, 210) \begin{pmatrix} \checkmark & 0 & 0 \\ 0 & \checkmark & \checkmark \\ \checkmark & 0 & \checkmark \end{pmatrix} \quad [u_R] \begin{pmatrix} 101 \\ 000 \\ 201 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[H_1] = 111 \qquad\qquad [H_2] = 000$$

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$$\bar{d}_L H_1^\dagger d_R \qquad\qquad \bar{d}_L H_2^\dagger d_R$$

$$[\bar{q}_L] : (010, 111, 210) \quad \begin{pmatrix} 0 & \checkmark & 0 \\ \checkmark & 0 & \checkmark \\ 0 & \checkmark & \checkmark \end{pmatrix} \quad [d_R] \begin{pmatrix} 001 \\ 111 \\ 201 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & \star & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[H_1] = 111 \qquad\qquad [H_2] = 000$$

- Loop corrections are also manageable!

Non-invertible symmetry for three-zero texture of quark mass matrix

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- Discussion:
  1) Quantum behavior of non-invertible symmetry ? 2503.19964 2508.14970
  2) Origin of this symmetry? Higher dimensional compactification vs defect condensation?
  3) Possible prediction by extending to electroweak sector? 2508.12287
  4) mass hierarchy?
- We can use the three-zero texture of the quark matrix to solve strong CP problem, non-invertible symmetry can naturally give rise to this three-zero texture in 4D spacetime.

Thanks for your attention!