Probe Neutrino & Dark Matter with Cosmic Gravitational Focusing

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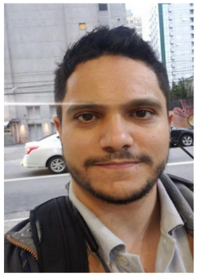

Pedro Pasquini

Liang Tan

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

SFG & Liang Tan, **PRD 111 (2025) 8, 083539** [arXiv:2409.11115]

SFG & Liang Tan [arXiv:2509.21213]

上海交通大學

SHANGHAI JIAO TONG UNIVERSITY

5th AEI @ Durham

October 1, 2025

李政道研究所

Tsung-Dao Lee Institute

Outline

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1) Dark Matter vs Neutrino

2) Cosmic Gravitational Focusing

3) 3rd Cosmological Way of Measuring ν Mass

4) Sensitive Probe of Light Dark Matter

5) Summary

Dark Matter vs Neutrino

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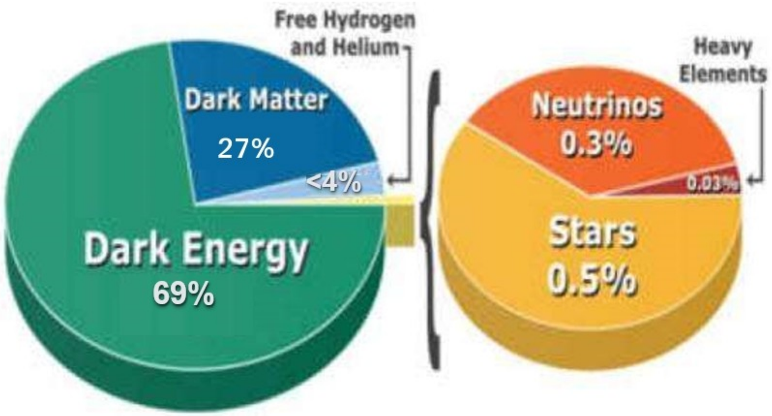

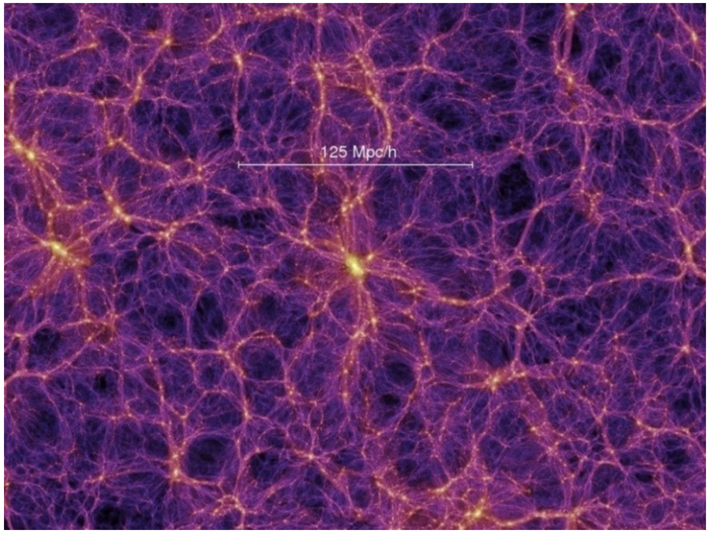

Matter Worlds in Universe

Dark Matter

- Largest fraction

Neutrino

- Largest number

- Important neutral components
- New Physics beyond SM
- Fundamentally unresolved issues

Minimal Neutrinos

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Georg G. Raffelt

Stars as Laboratories for Fundamental Physics

The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles

In the standard model, neutrinos have been assigned the most minimal properties compatible with experimental data: zero mass, zero charge, zero dipole moments, zero decay rate, zero almost everything.

Importance of Neutrino Masses

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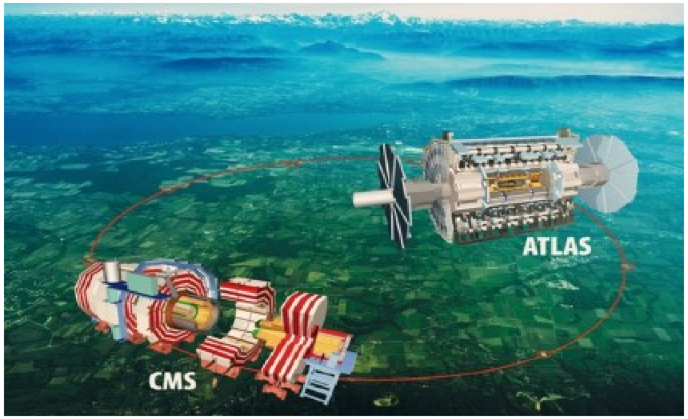
- **Higgs boson** ⇒ electroweak symmetry breaking & mass. ~ O(100)GeV
- **Chiral symmetry breaking** ⇒ majority of mass.
- The world seems not affected much by the tiny neutrino mass?
  - Neutrino mass ⇒ Mixing
  - 3 Neutrino ⇒ possible **CP violation**
  - CP violation ⇒ **Leptogenesis**
  - ⇒ **Matter-Antimatter Asymmetry**
  - There is something left in the Universe.
  - **EW Baryogenesis** is not enough.

Daya Bay @ **March 8, 2012**



LHC @ **July 4, 2012**

CMB for ν Mass Measurement

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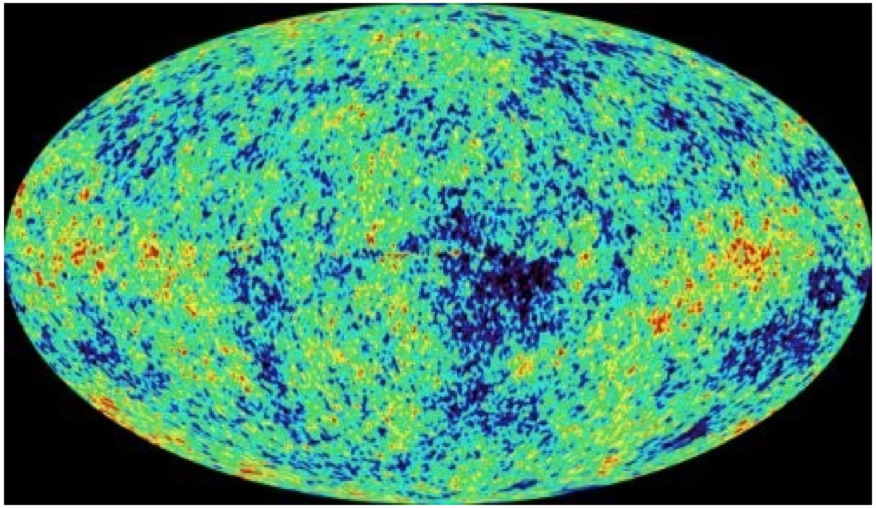
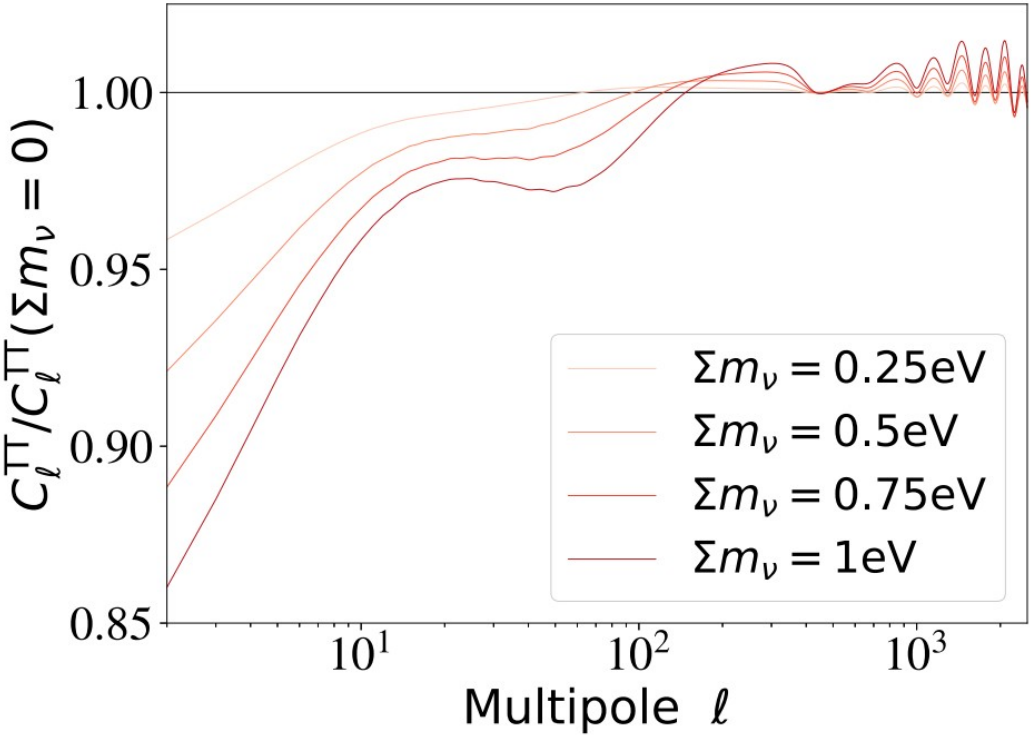
Massive neutrino decreasing CMB lensing power spectrum





$$C_\ell^{\kappa\kappa} \propto (\Omega_m h^2)^2 A_s \left(1 - 0.02 \frac{f_\nu}{4 \times 10^{-3}}\right) \qquad f_\nu = \Omega_\nu / \Omega_m$$

LSS for ν Mass Measurement

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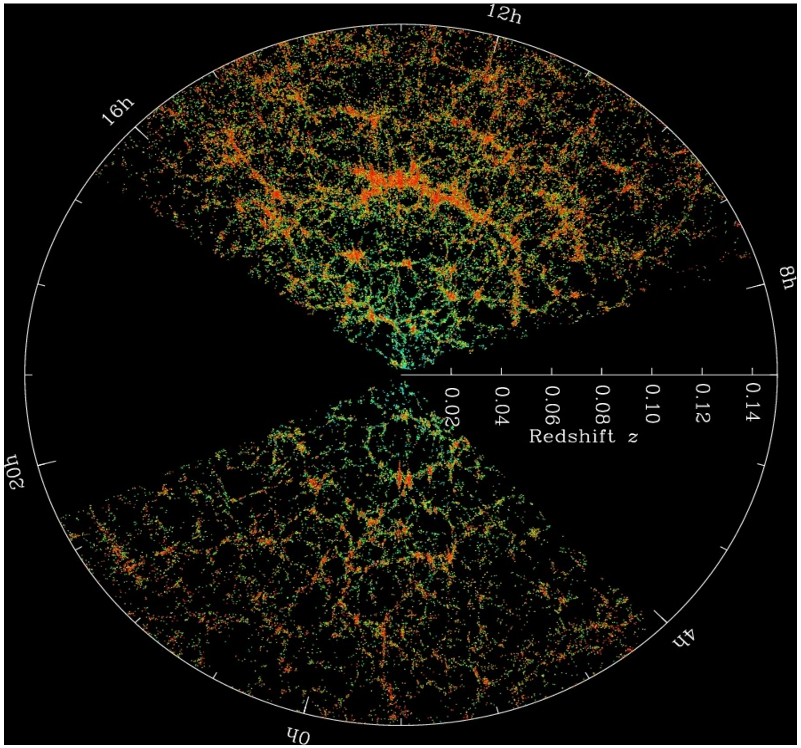
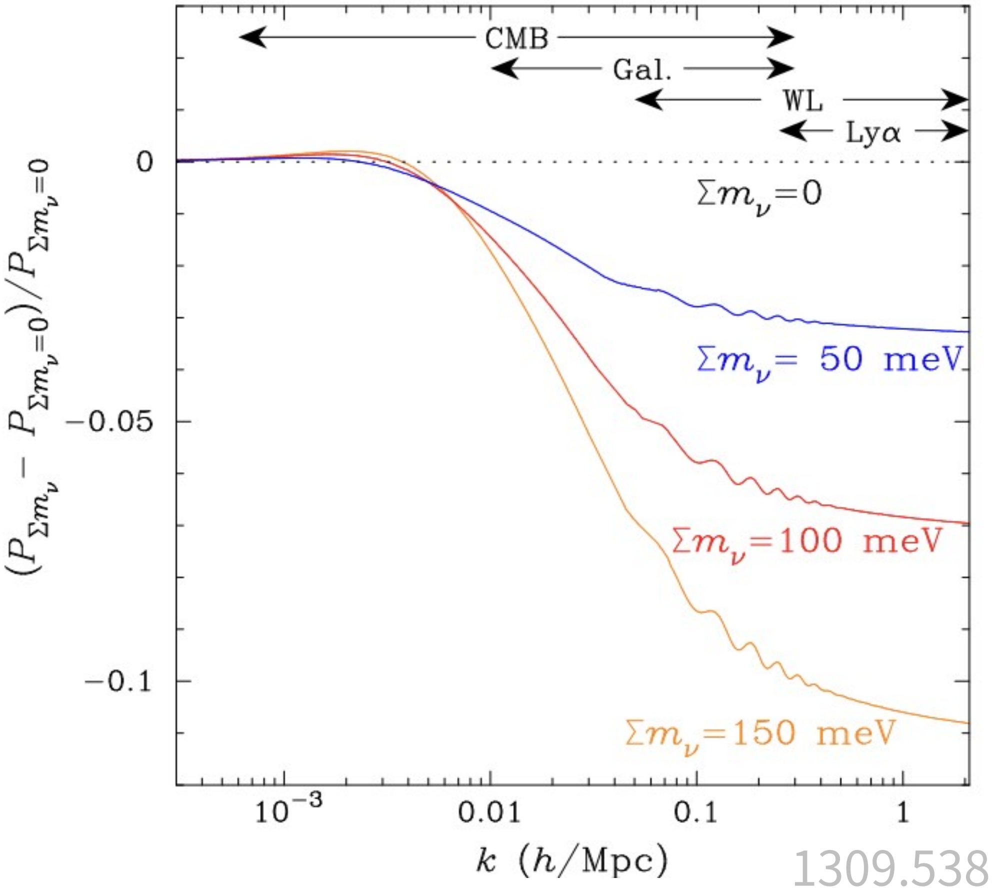
Suppress matter power spectrum





1309.5383

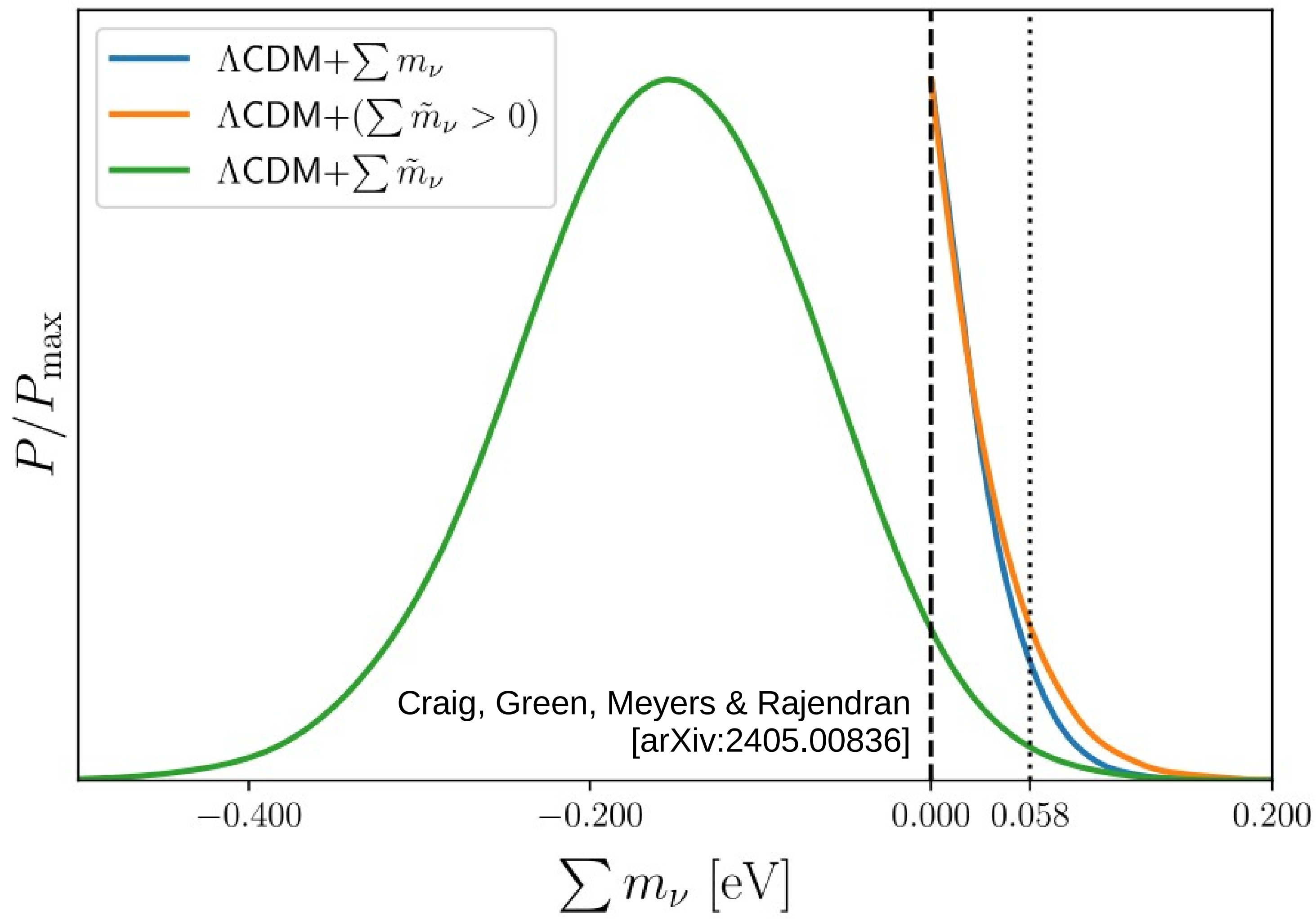
$$P^{(\sum m_\nu)}(k \gg k_{\rm fs}, z) \approx \left(1 - 2f_\nu - \frac{6}{5} f_\nu \log\frac{1+z_\nu}{1+z}\right) P^{(\sum m_\nu=0)}(k \gg k_{\rm fs}, z)$$

Cosmological Systematics

DESI [arXiv:2503.14744]

Model/Dataset	Ω_m	H_0 [km s^{-1} Mpc^{-1}]	$\sum m_\nu$ [eV]	N_{eff}	w or w_0	w_a
ΛCDM+$\sum m_\nu$						
DESI BAO+CMB (Baseline)	0.3009 ± 0.0037	68.36 ± 0.29	< 0.0642	—	—	—
DESI BAO+CMB (L-H)	0.2995 ± 0.0037	68.48 ± 0.30	< 0.0774	—	—	—
DESI BAO+CMB (plik)	0.2998 ± 0.0038	68.56 ± 0.31	< 0.0691	—	—	—
DESI BAO+CMB+Pantheon+	0.3021 ± 0.0036	68.27 ± 0.29	< 0.0704	—	—	—
DESI BAO+CMB+Union3	0.3020 ± 0.0037	68.28 ± 0.29	< 0.0674	—	—	—
DESI BAO+CMB+DESY5	0.3036 ± 0.0037	68.16 ± 0.29	< 0.0744	—	—	—
ΛCDM+N_{eff}						
DESI BAO+CMB	0.3004 ± 0.0042	69.2 ± 1.0	—	3.23 ± 0.18	—	—
ΛCDM+$\sum m_\nu$+N_{eff}						
DESI BAO+CMB	0.2996 ± 0.0042	69.00 ± 0.97	< 0.0741	3.16 ± 0.17	—	—
wCDM+$\sum m_\nu$						
DESI BAO+CMB	0.2943 ± 0.0073	69.28 ± 0.92	< 0.0851	—	-1.039 ± 0.037	—
DESI BAO+CMB+Pantheon+	0.3045 ± 0.0051	67.94 ± 0.58	< 0.0653	—	-0.985 ± 0.023	—
DESI BAO+CMB+Union3	0.3047 ± 0.0059	67.93 ± 0.69	< 0.0649	—	-0.985 ± 0.028	—
DESI BAO+CMB+DESY5	0.3094 ± 0.0049	67.34 ± 0.53	< 0.0586	—	-0.961 ± 0.021	—
wCDM+N_{eff}						
DESI BAO+CMB	0.2932 ± 0.0075	69.8 ± 1.1	—	3.13 ± 0.19	-1.047 ± 0.040	—
DESI BAO+CMB+Pantheon+	0.3039 ± 0.0052	68.7 ± 1.0	—	3.22 ± 0.19	-0.987 ± 0.024	—
DESI BAO+CMB+Union3	0.3039 ± 0.0059	68.8 ± 1.0	—	3.23 ± 0.19	-0.986 ± 0.029	—
DESI BAO+CMB+DESY5	0.3087 ± 0.0050	68.35 ± 0.98	—	3.27 ± 0.18	-0.960 ± 0.022	—
w_0w_aCDM+$\sum m_\nu$						
DESI BAO+CMB	0.353 ± 0.022	$63.7^{+1.7}_{-2.2}$	< 0.163	—	$-0.42^{+0.24}_{-0.21}$	-1.75 ± 0.63
DESI BAO+CMB+Pantheon+	0.3109 ± 0.0057	67.54 ± 0.59	< 0.117	—	-0.845 ± 0.055	$-0.57^{+0.23}_{-0.19}$
DESI BAO+CMB+Union3	0.3269 ± 0.0088	65.96 ± 0.84	< 0.139	—	-0.674 ± 0.090	$-1.06^{+0.34}_{-0.28}$
DESI BAO+CMB+DESY5	0.3188 ± 0.0058	66.75 ± 0.56	< 0.129	—	-0.758 ± 0.058	$-0.82^{+0.26}_{-0.21}$

Results from DESI BAO et al



Any independent measurement of neutrino masses?

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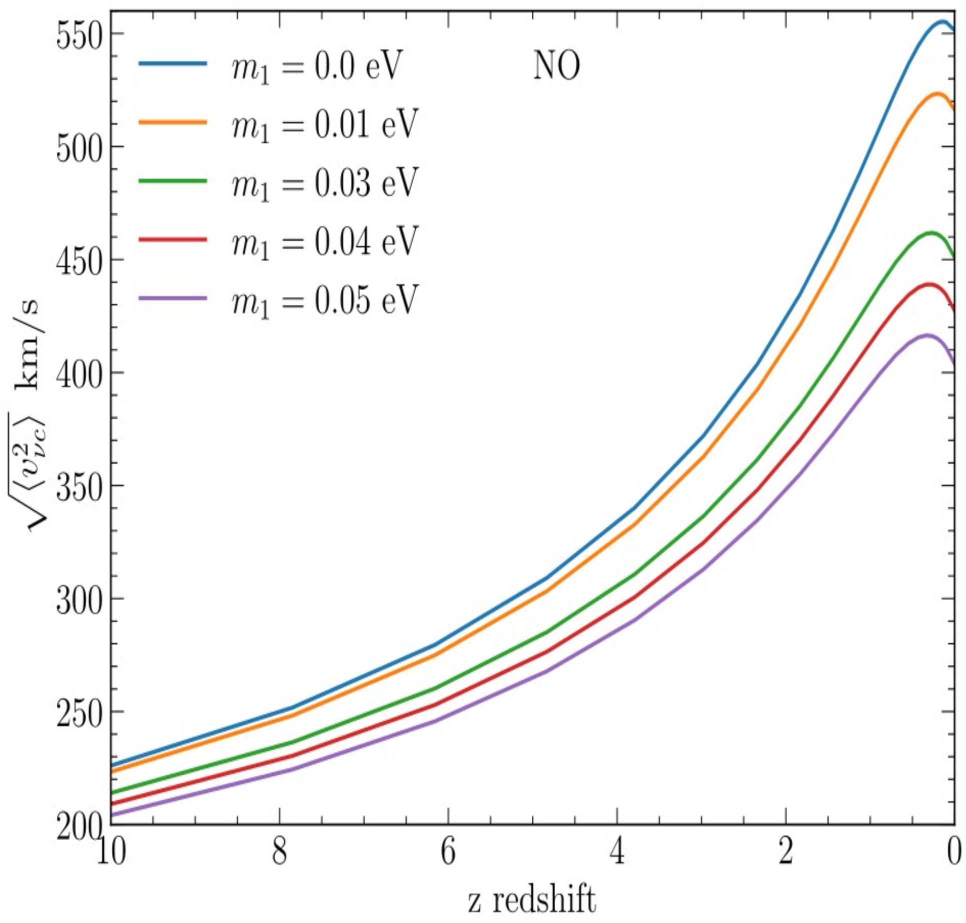
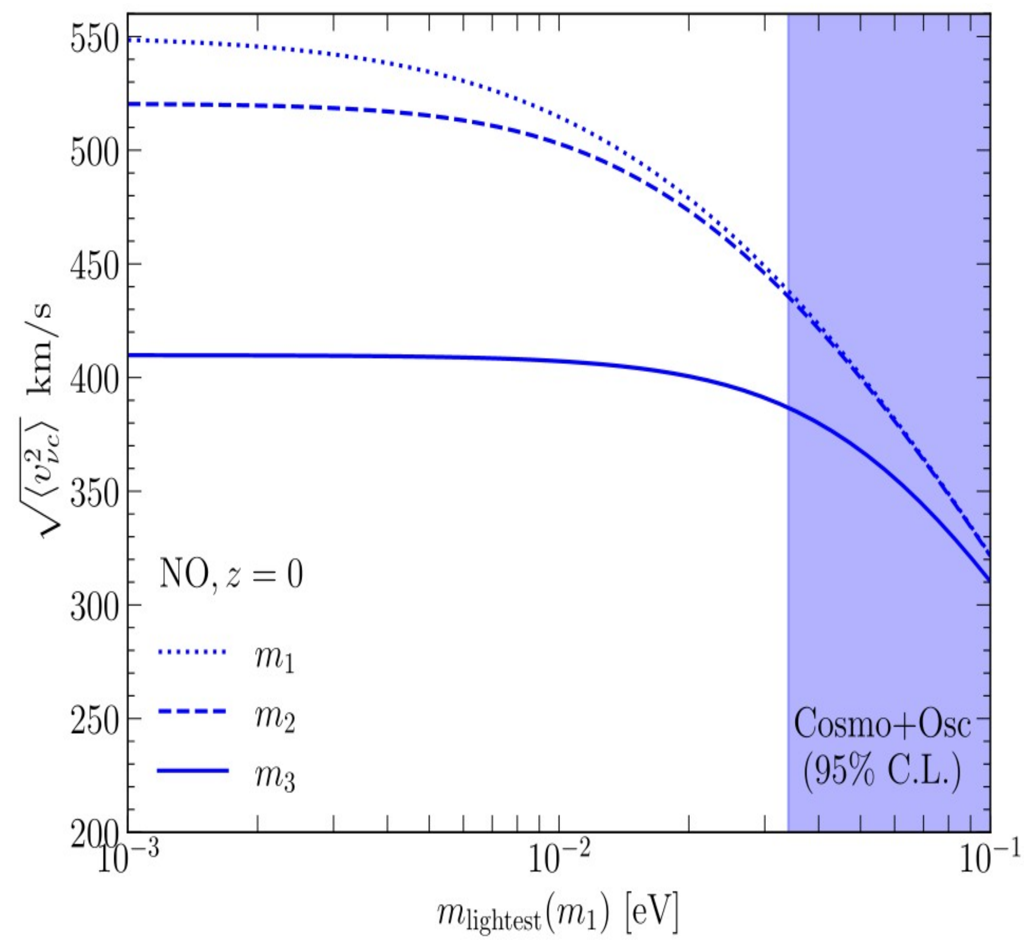
Cosmic ν Fluid (CνF) vs DM Halo





$$v_{\nu c}^{\rm bg} \approx \langle \boldsymbol{v}_{\nu c}^2(R) \rangle = \int \frac{d|\boldsymbol{k}|}{|\boldsymbol{k}|} \Delta_\zeta^2(\boldsymbol{k}) \left| \widetilde{W}(|\boldsymbol{k}|R) \right|^2 \frac{|T_{\theta_\nu} - T_{\theta_c}|^2}{|\boldsymbol{k}|^2}$$

Cosmic ν Fluid (CνF) vs DM Halo

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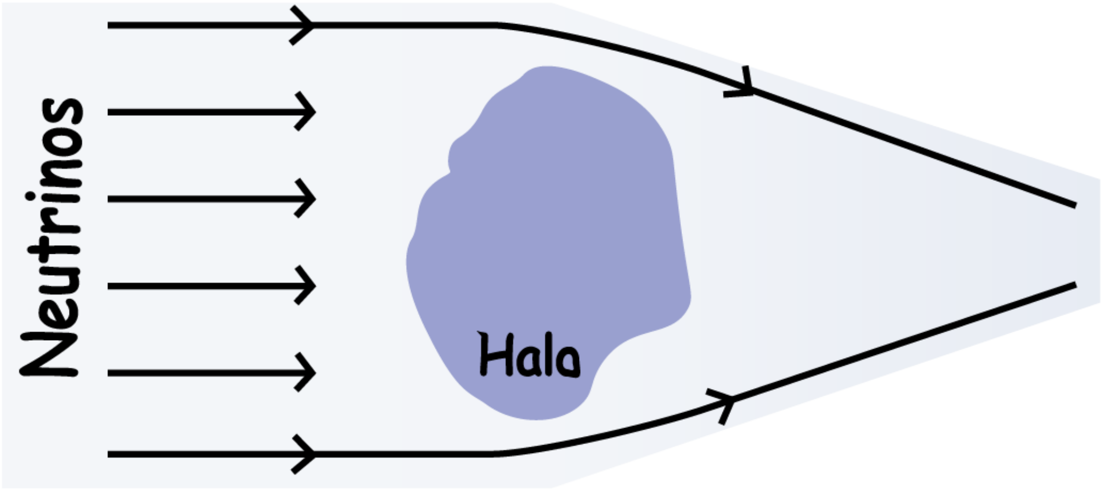
Gravitational attraction between DM halo & CνF



Cosmic ν Fluid **DM Halo**

Gravitational Deflection

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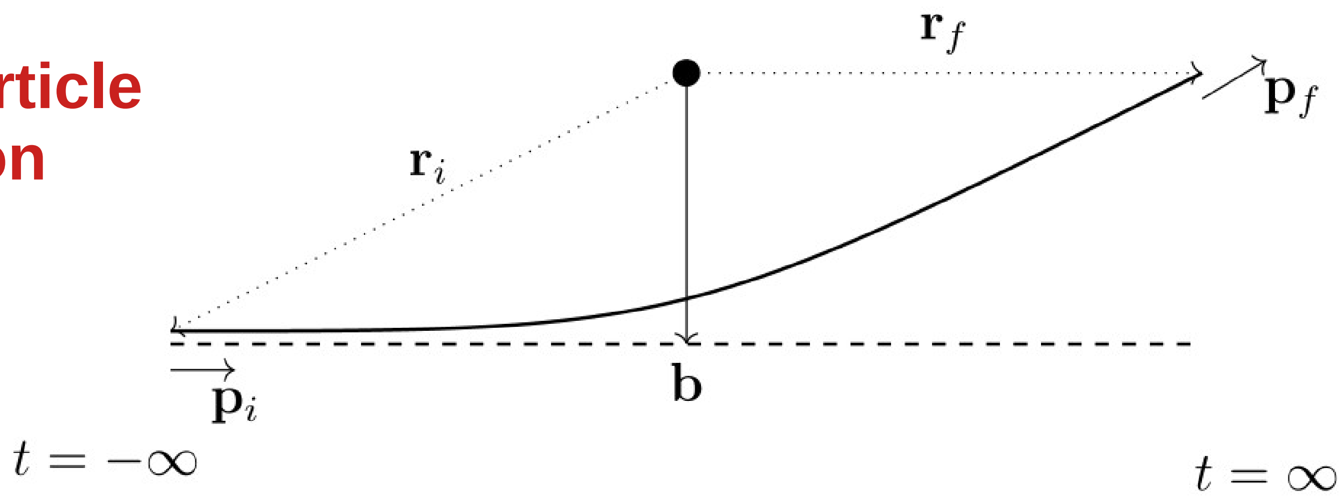
Single-particle description



$$ds^2 \equiv -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\Delta\phi = 2|\boldsymbol{b}| \int_{r_{\min}}^{\infty} \frac{dr}{r^2} \left[1 + \frac{2b_{90}}{r} - \frac{|\boldsymbol{b}|^2}{r^2} + \frac{2GM|\boldsymbol{b}|^2}{r^3}\right]^{-1/2}$$

$$b_{90} \equiv GM m_\nu^2 / |\boldsymbol{p}_i|^2 \qquad r_{\min}^3 + 2b_{90} r_{\min}^2 - |\boldsymbol{b}|^2 r_{\min} + 2GM|\boldsymbol{b}|^2 = 0$$

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Dynamical Friction

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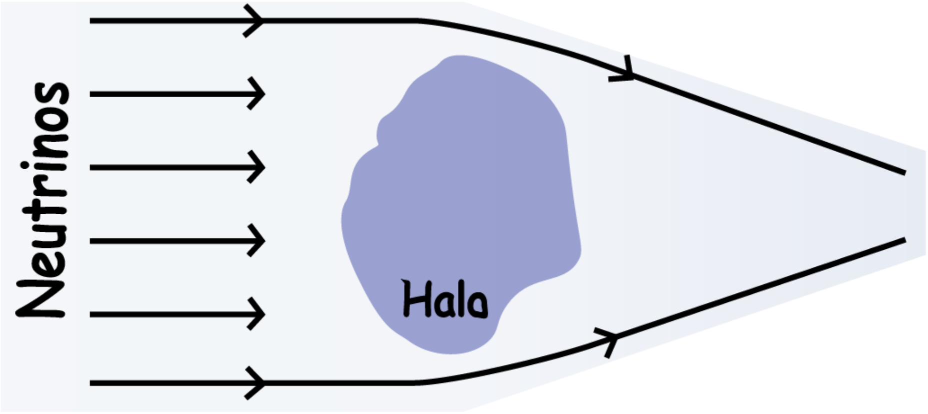
Single-particle description



$$\Delta\phi \approx \pi + 2\frac{GM}{|\boldsymbol{b}|}\left(\frac{m_\nu^2}{|\boldsymbol{p}_i|^2} + 2\right)$$

$$\Delta\boldsymbol{p}^{\|} \equiv \boldsymbol{p}_f^{\|} - \boldsymbol{p}_i = (-\cos\Delta\phi - 1)\,\boldsymbol{p}_i \approx -\frac{2G^2M^2}{|\boldsymbol{b}|^2}\left(\frac{m_\nu^2}{|\boldsymbol{p}_i|^2} + 2\right)^2 \boldsymbol{p}_i$$

vs Stone in Water Flow

Gravitational Focusing

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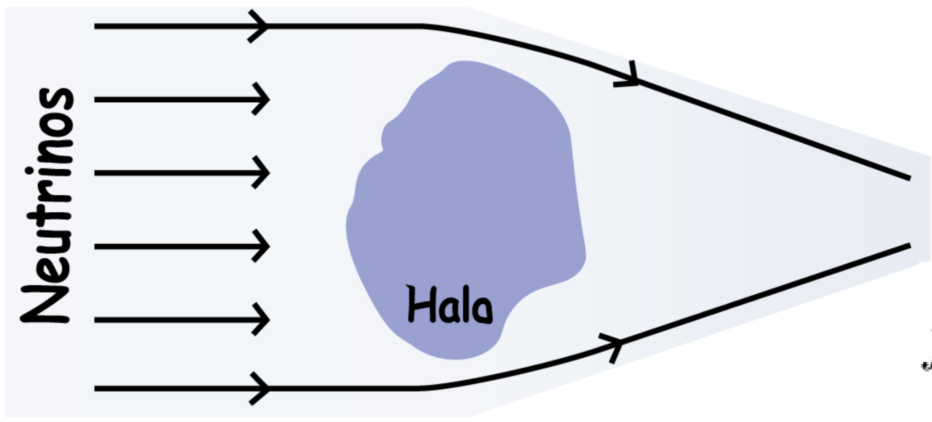

Boltzmann description

$$f_\nu(\boldsymbol{x},\boldsymbol{p}) \equiv \overline{f}_\nu(\boldsymbol{p}) + \delta f_\nu(\boldsymbol{x},\boldsymbol{p})$$

$$f_\nu(\boldsymbol{p}_i,\boldsymbol{v}) \approx \frac{2}{e^{|\boldsymbol{p}_i - E_{\boldsymbol{p}_i}\boldsymbol{v}_{\nu c}|/T}+1}$$

$$\left\{\partial_t + \frac{\boldsymbol{p}\cdot\nabla_{\boldsymbol{x}}}{aE_{\boldsymbol{p}}} - \left[(H+\dot{\Phi})\boldsymbol{p} + \frac{E_{\boldsymbol{p}}}{a}\nabla_{\boldsymbol{x}}\Psi - \frac{|\boldsymbol{p}|^2\nabla_{\boldsymbol{x}}\Phi - \boldsymbol{p}(\boldsymbol{p}\cdot\nabla_{\boldsymbol{x}}\Phi)}{aE_{\boldsymbol{p}}}\right]\cdot\nabla_{\boldsymbol{p}}\right\} f_\nu(\boldsymbol{x},\boldsymbol{p}) = 0$$

$$\delta\widetilde{f}_\nu(\boldsymbol{k},\boldsymbol{p}) = \widetilde{\Psi}(\boldsymbol{k})\left(\frac{m_\nu^2+2\boldsymbol{p}^2}{\boldsymbol{p}\cdot\boldsymbol{k}}\boldsymbol{k} - \boldsymbol{p}\right)\cdot\nabla_{\boldsymbol{p}}\overline{f}_\nu(\boldsymbol{p})$$

$$\mathbf{Im}[\delta\widetilde{\rho}_\nu] = -\frac{(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})\widetilde{\Psi}}{4\pi}\int d|\boldsymbol{p}'|\left(\mathbf{m}_\nu^4 + 3m_\nu^2|\boldsymbol{p}'|^2 + 2|\boldsymbol{p}'|^4\right)\frac{d\overline{f}_\nu}{d|\boldsymbol{p}'|}\Theta(|\boldsymbol{p}'| - E_{\boldsymbol{p}'}|\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}}|)$$

Density Dipole

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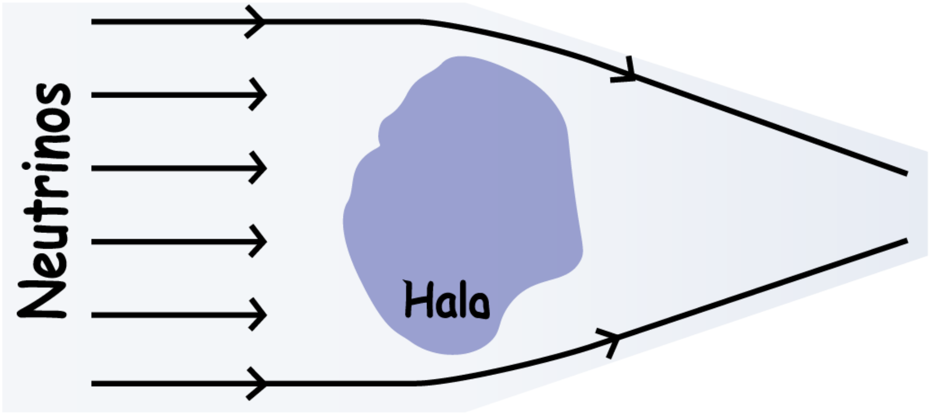

Density enhancement @ downwind!

$$\delta\rho_\nu(-\mathbf{x}) = -\delta\rho_\nu(\mathbf{x})$$

$$\widetilde{\delta}_m \equiv \widetilde{\delta}_{m0}(1 + i\widetilde{\phi})$$

$$[\widetilde{A}(\mathbf{k})]^* = \int d\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} A(\mathbf{x}) = \int d\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} A(-\mathbf{x})$$

$$= -\int d\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} A(\mathbf{x}) = -\widetilde{A}(\mathbf{k})$$

Density Dipole ⟺ **Imaginary in k-space**

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4) Sensitive Probe of Light Dark Matter

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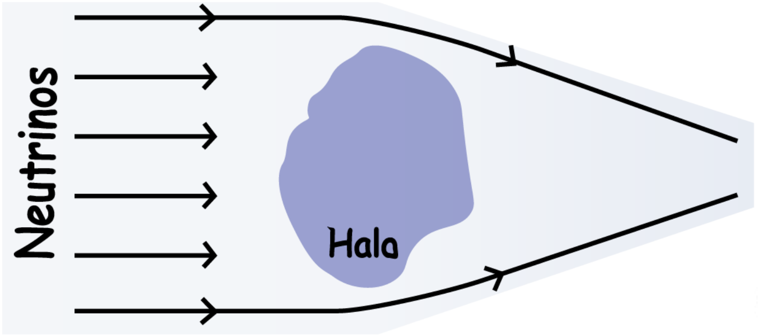
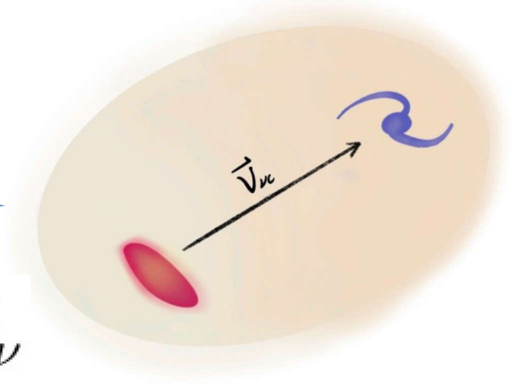
Galaxy Correlation & Bias



$$\delta_m = \delta_{m0} + \sum_{i=1}^{3} \frac{\delta \rho_{\nu_i}}{\rho_m}$$

$$\delta_{g\alpha} = b_c^\alpha F_c \delta_c + b_\nu^\alpha F_\nu \delta_\nu$$



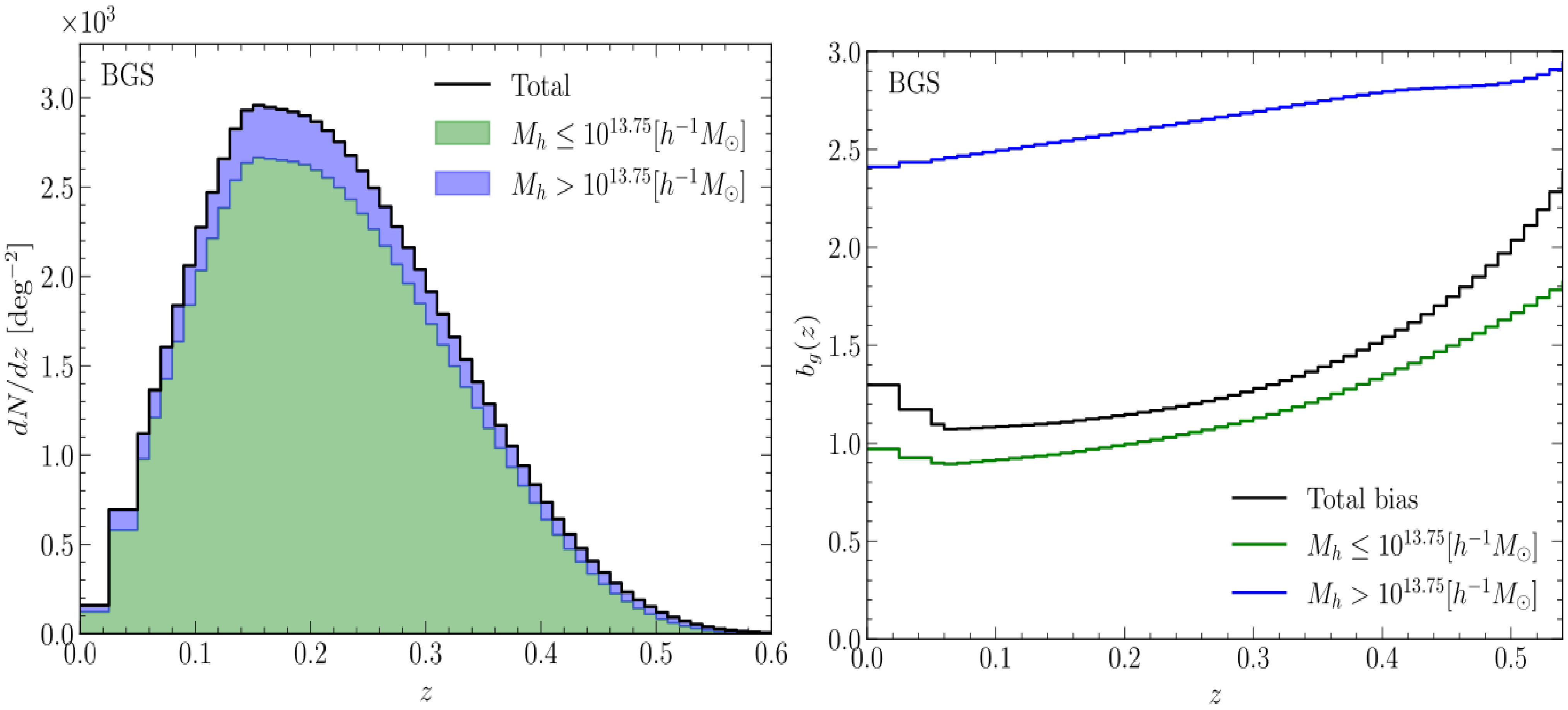
Density dipole can affect the galaxy formation!

$$\delta_{g\alpha,\mathrm{RSD}}(\boldsymbol{x}) \equiv \delta_{g\alpha}(\boldsymbol{x}) - \frac{\partial}{\partial x}\left(\frac{\boldsymbol{u}_m \cdot \hat{\boldsymbol{x}}}{aH}\right)$$

$$\mathrm{Im}[\widetilde{\delta}_{g\alpha,\mathrm{RSD}} \widetilde{\delta}^*_{g\beta,\mathrm{RSD}}] = -i\Delta b \left[\mu_{\boldsymbol{k}}^2 \frac{\dot{\widetilde{\phi}}}{H} + (f\mu_{\boldsymbol{k}}^2 + 1)\widetilde{\phi}\right] \widetilde{\delta}_{m0}^2$$

Galaxies with different bias $\Delta b \equiv b_c^\alpha - b_c^\beta$

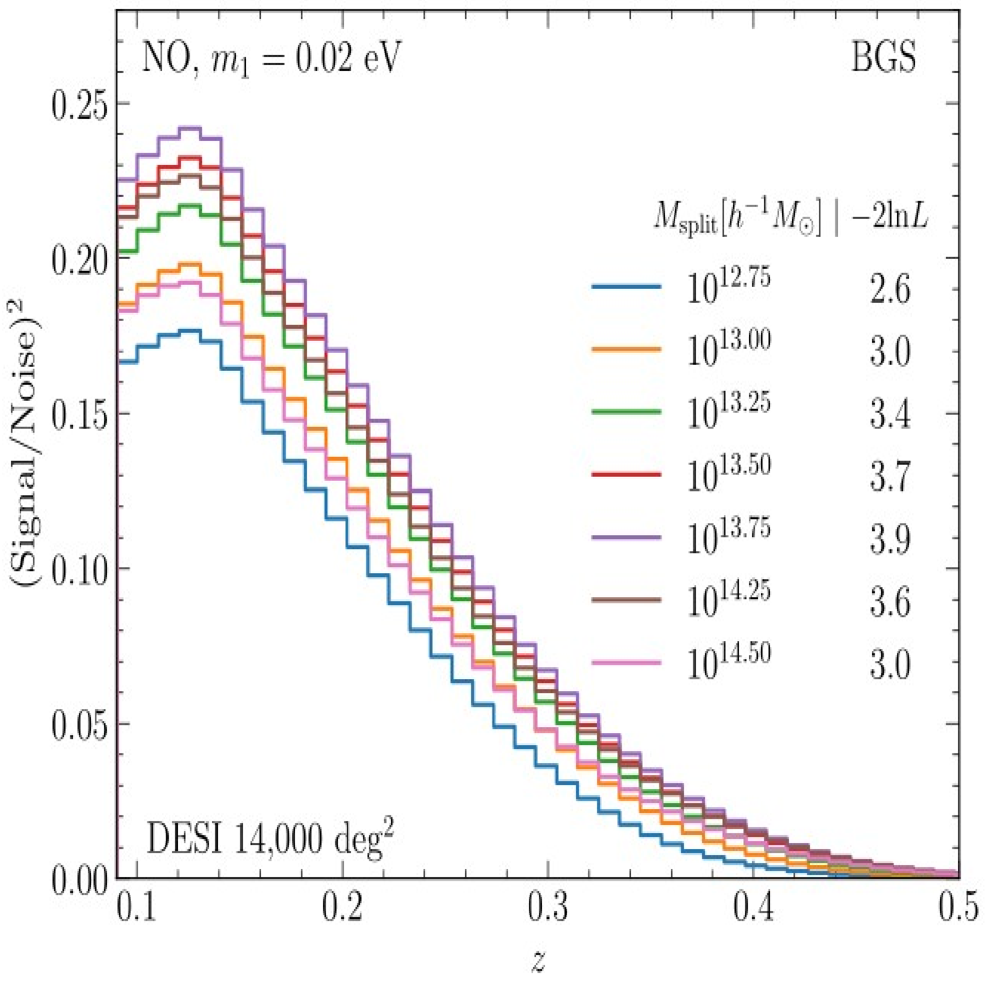
Halo Mass Function & HOD for BGS

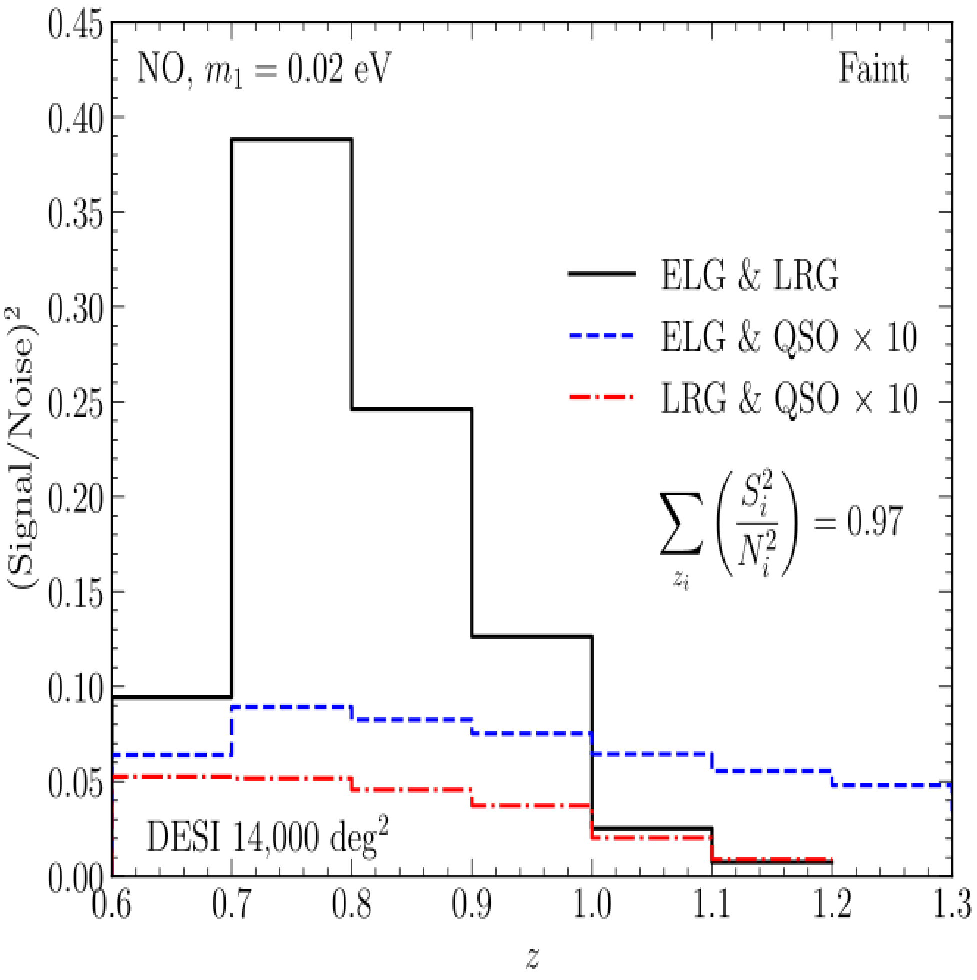


$$n_g(z) \equiv \int d\ln M_h \frac{dn(z)}{d\ln M_h} \langle N(M_h) \rangle \qquad b_g(z) \equiv \frac{1}{n_g} \int d\ln M_h \frac{dn(z)}{d\ln M_h} \langle N(M_h) \rangle b_h(M_h, z)$$

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

Signal-Noise-Ratio @ DESI





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Scale & Mass Distributions

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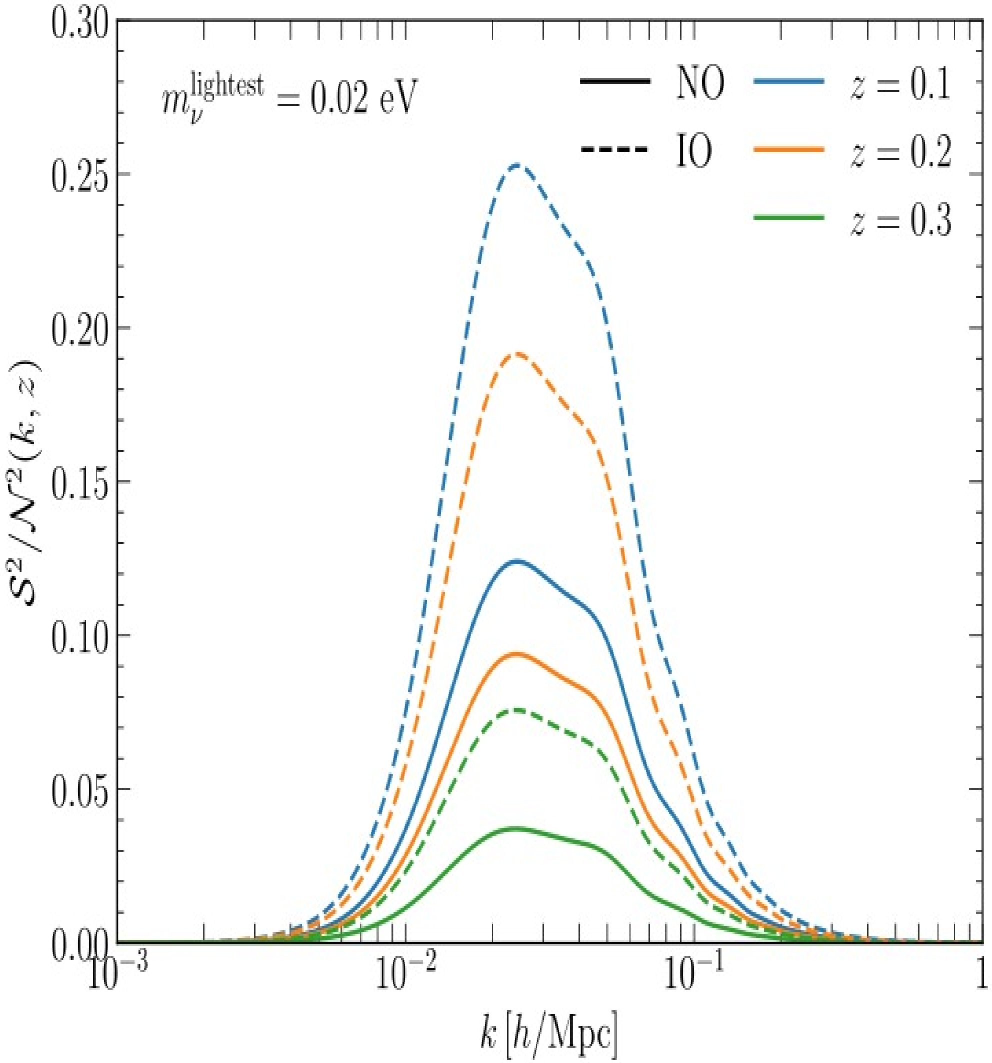

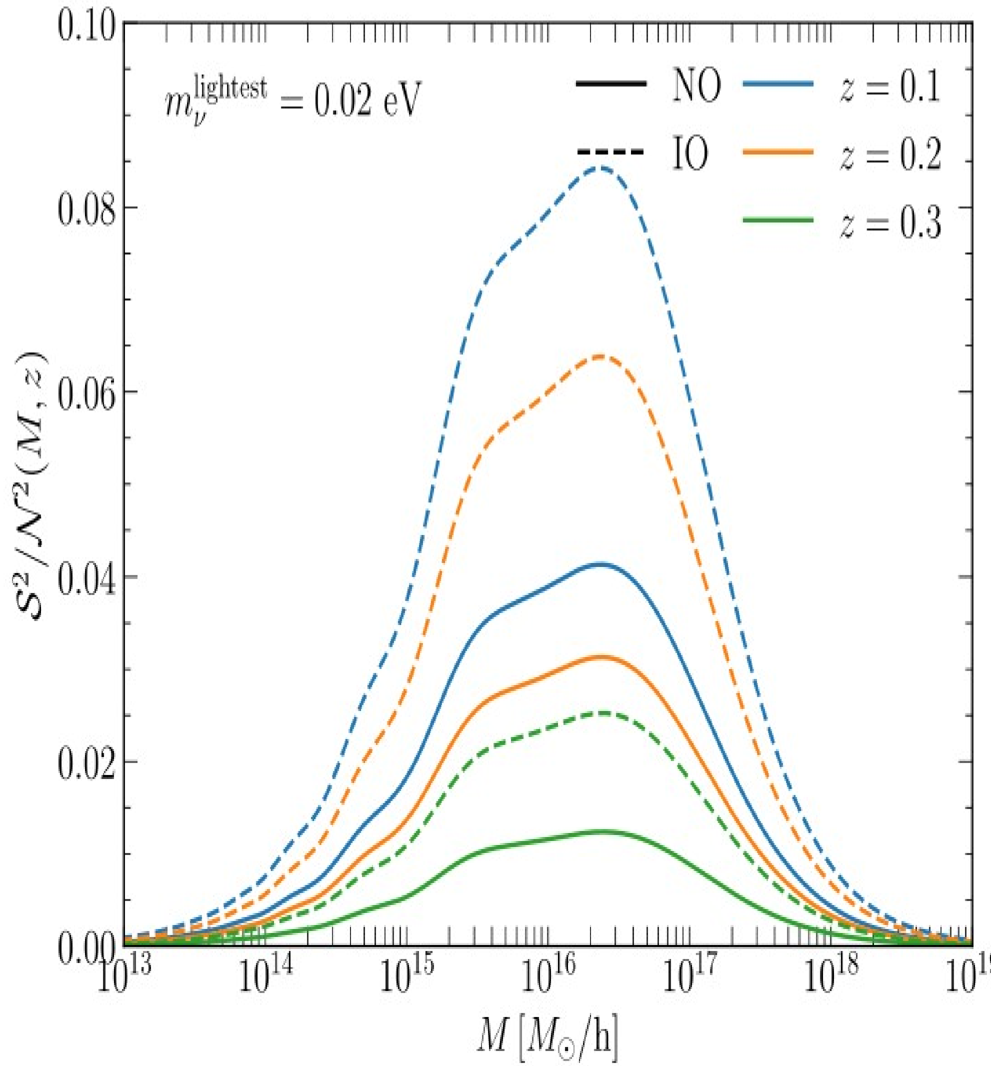

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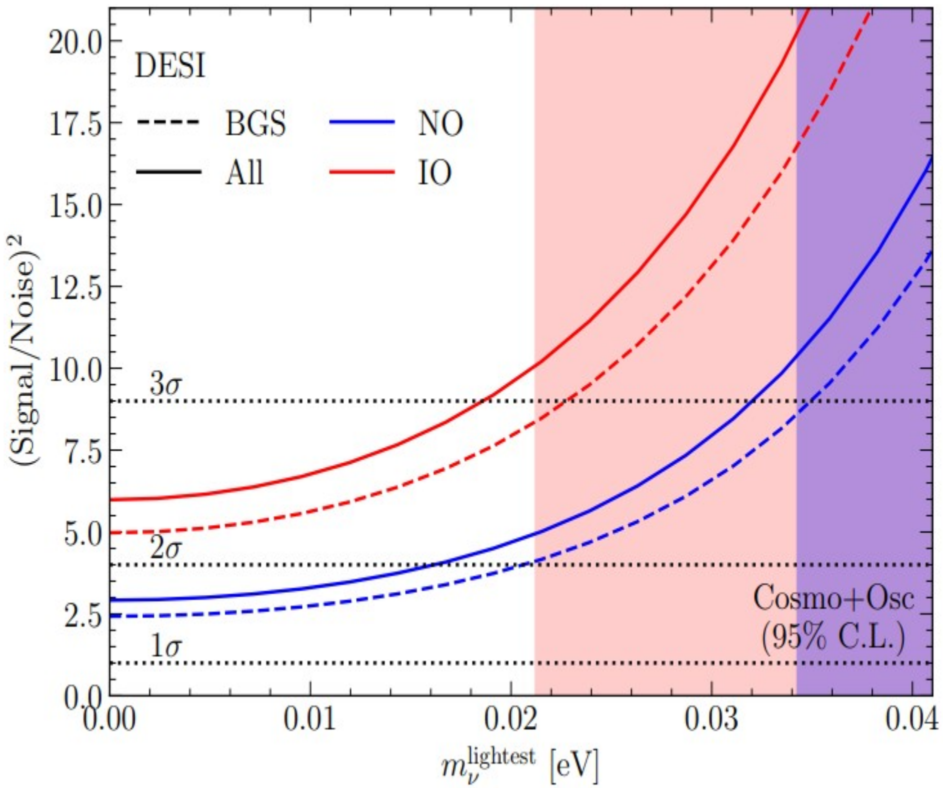
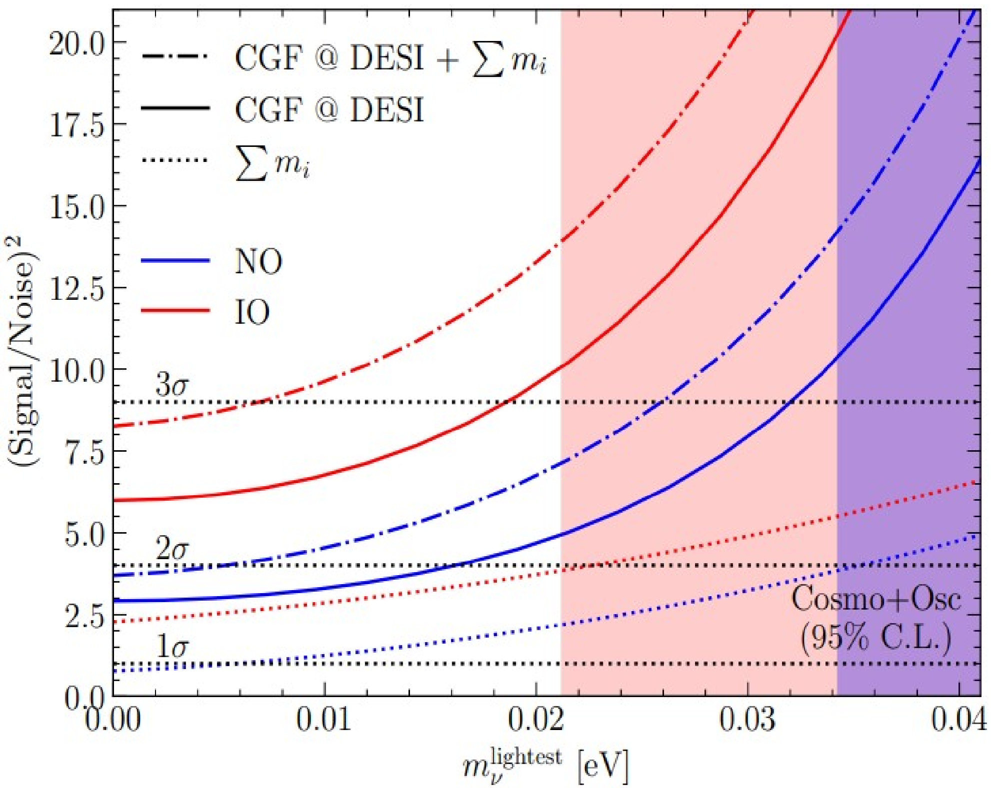
Sensitivities on ν Masses

$$\mathrm{Im}\left[\tilde{\delta}_{g\alpha}\left(\tilde{\delta}_{g\beta}\right)^*\right] \propto \left(\mathbf{v}_{\nu c}\cdot\hat{\mathbf{k}}\right)\left(f_0 m_\nu^4 + f_1 m_\nu^2 T^2 + f_2 T^4\right)$$





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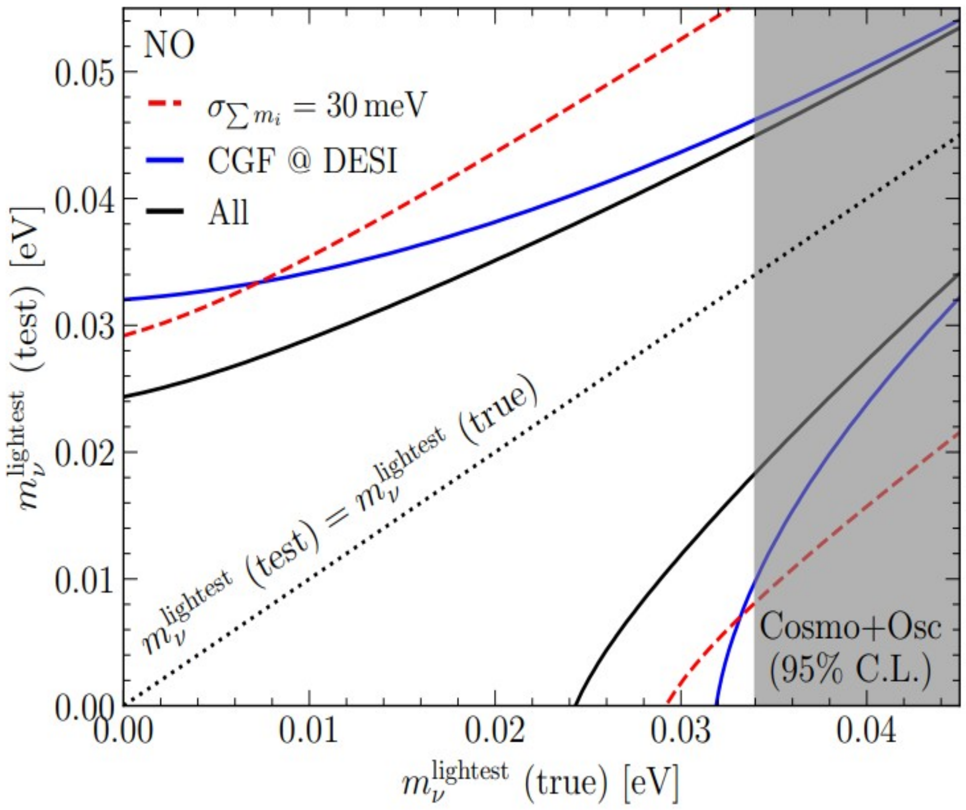

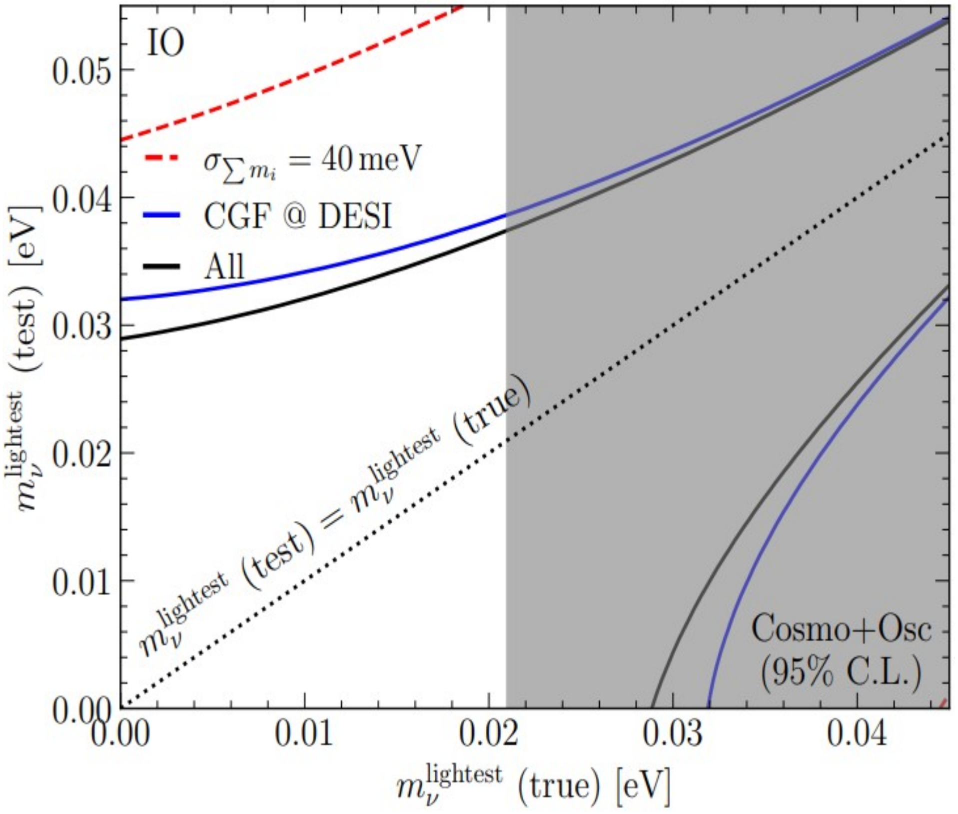

DESI, Euclid, Subaru PFS, **CSST**

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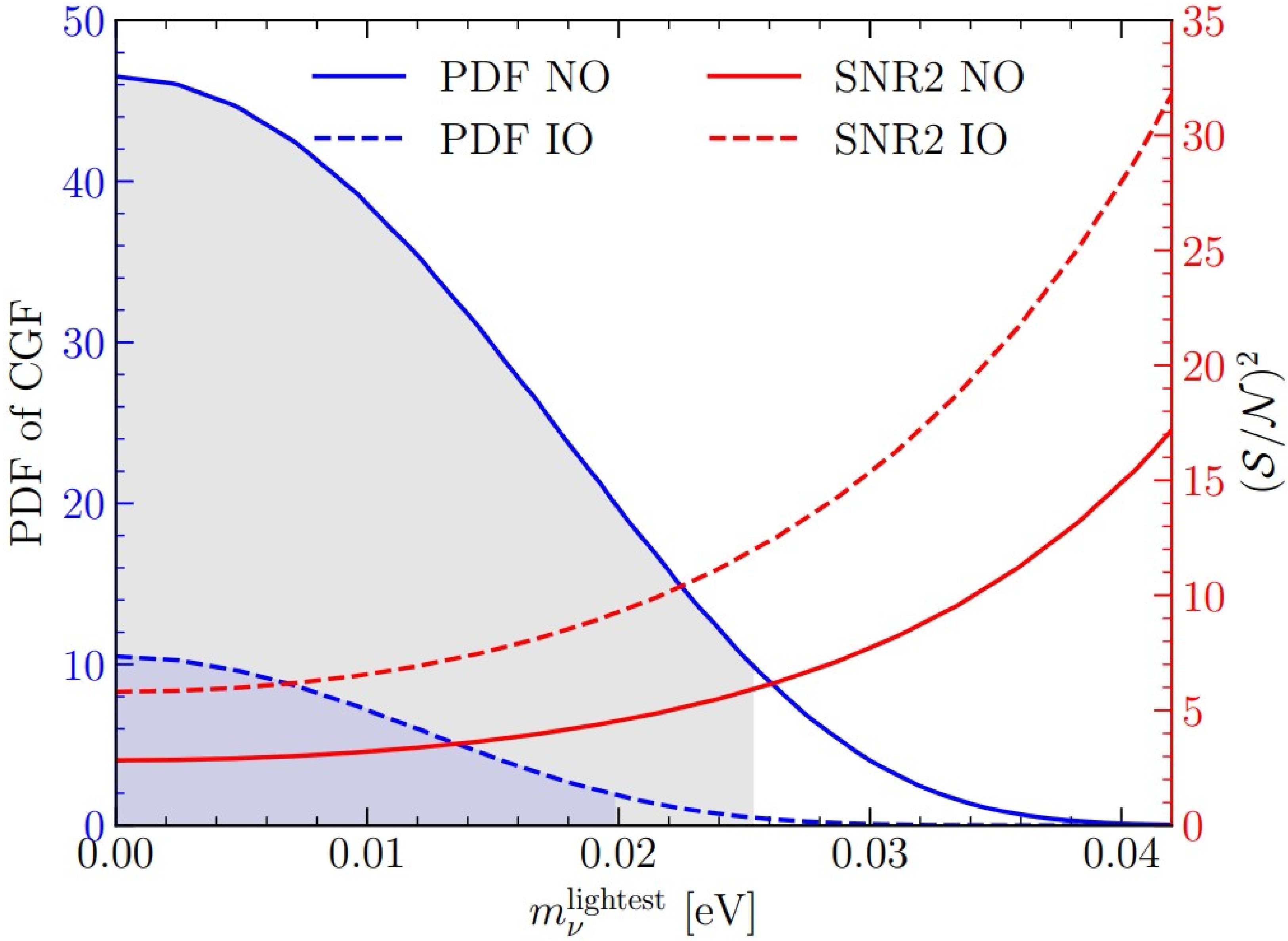
Neutrino Mass Orderings



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Sensitivities on ν Mass Ordering

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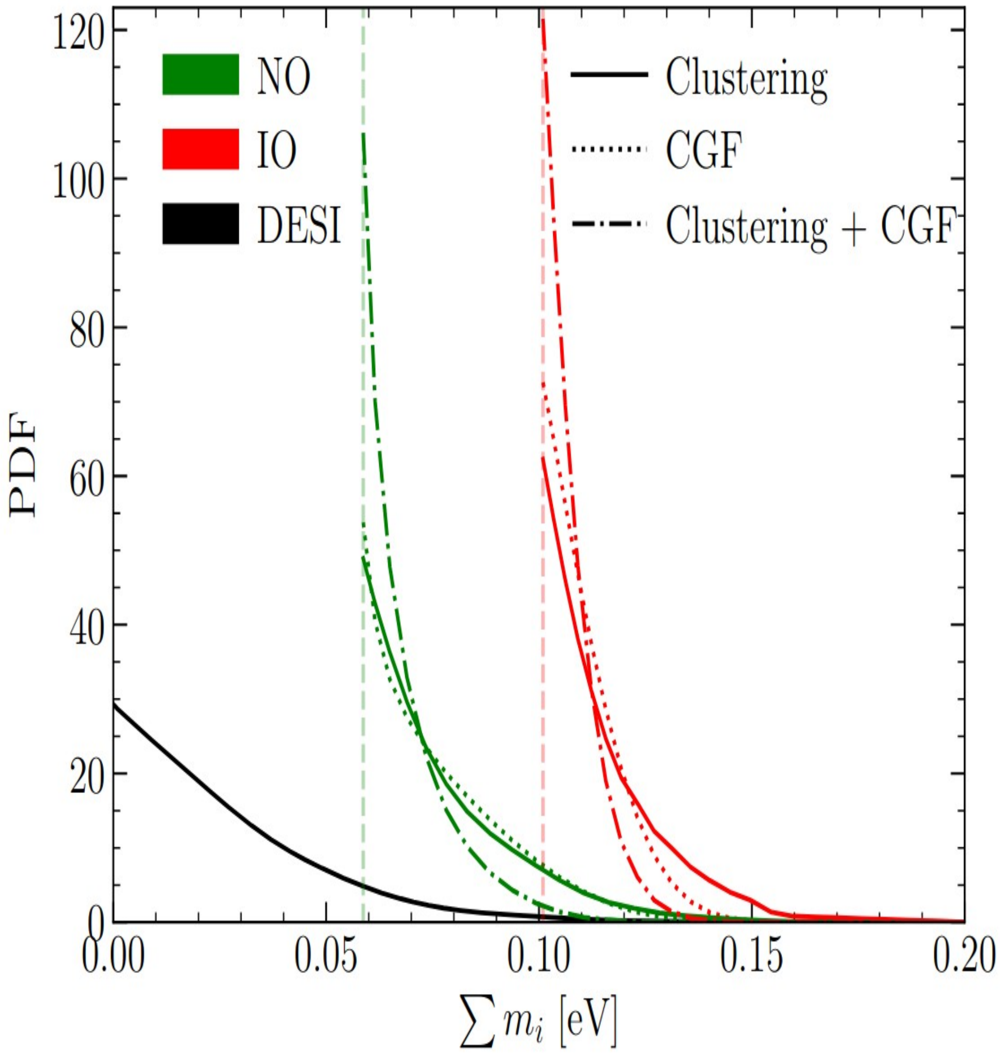

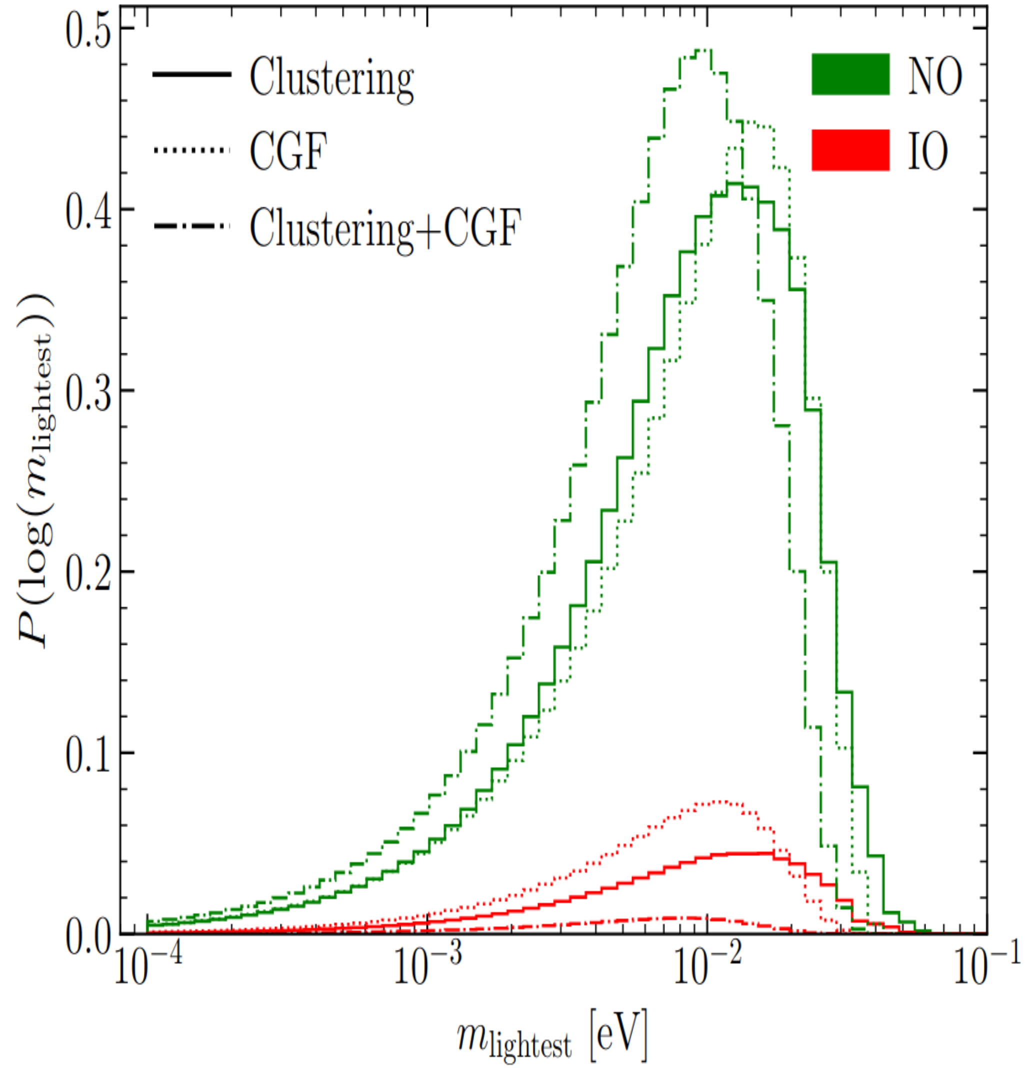

89.9% vs 10.1% → **98.2% vs 1.8%**

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SFG & Liang Tan [arXiv:2509.21213]

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Multi-Components for Dark Matter?

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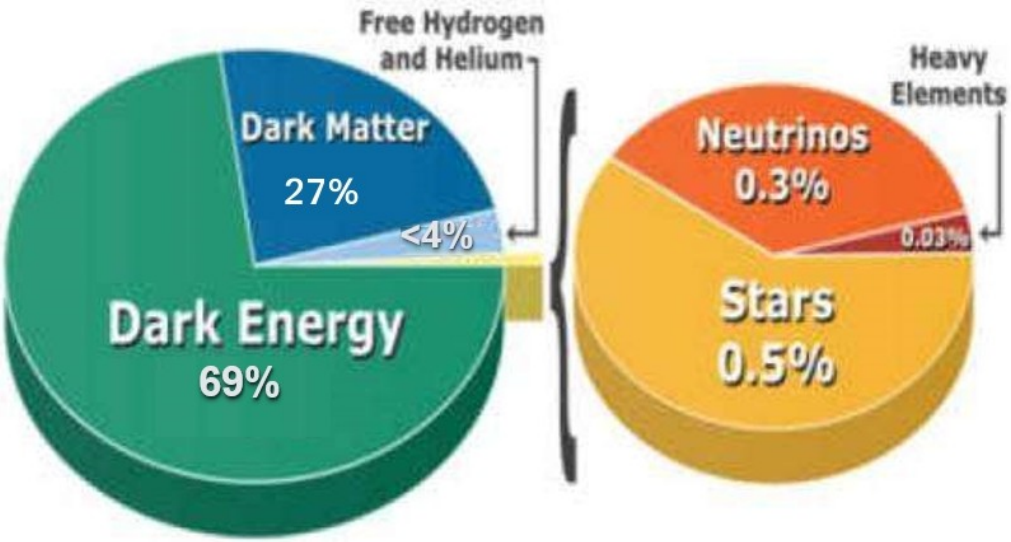


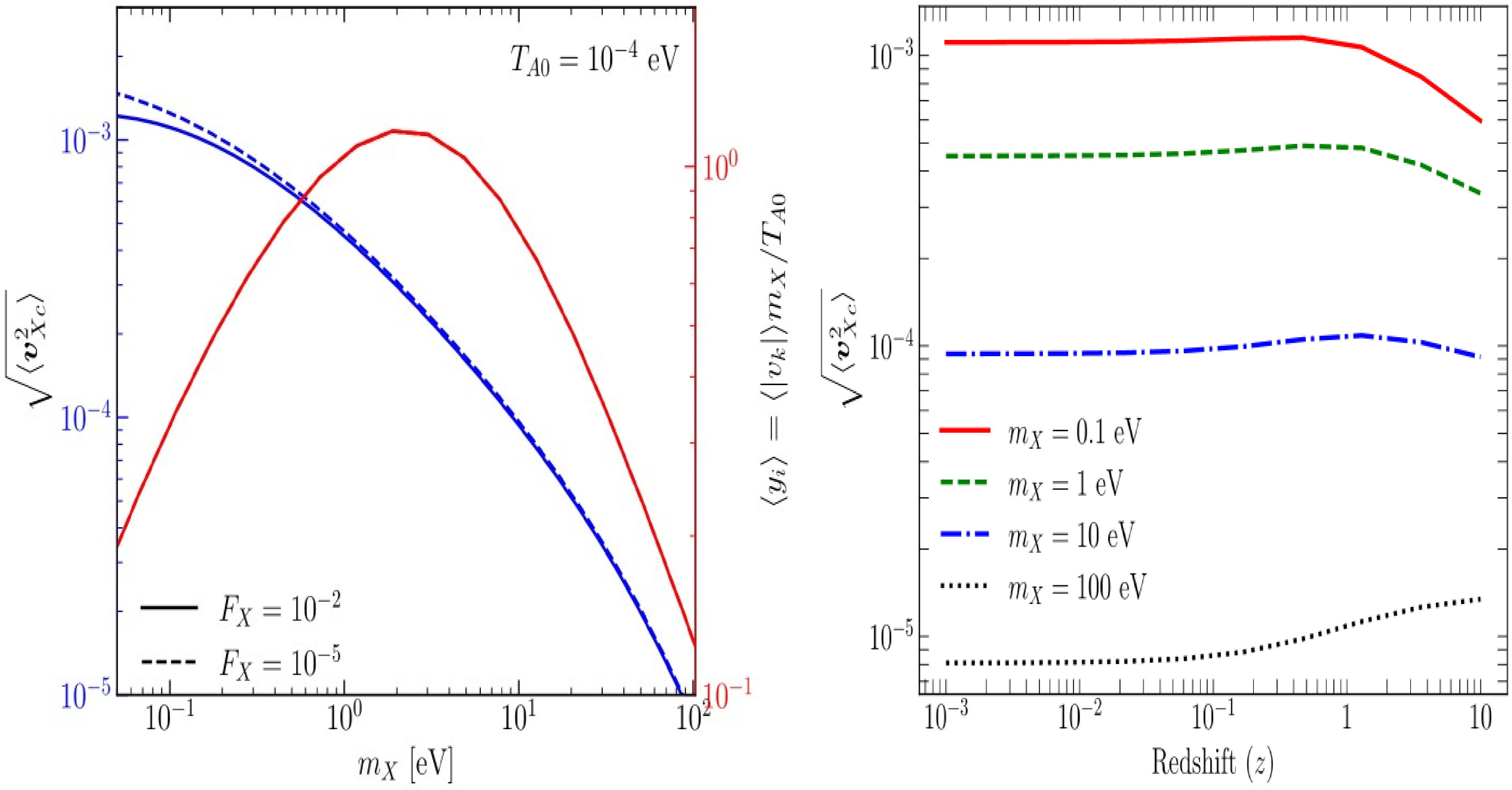
- Matter dominates the formation & evolution of our Universe.

- The <5% matter world is already complicated enough.

***Christoph Englert*: "Why should a hidden sector be trivial?"**

- What if there is a **major heavy DM** + a **minor light DM**?

Relative Velocity between Species



Relative velocity decreases with the light DM mass m_X!

Mass & Sensitivity Enhancement

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$$\tilde{\phi} = \frac{Ga^2}{|\boldsymbol{k}|^2} \sum_i (\boldsymbol{v}_{\nu_i \mathbf{c}} \cdot \hat{\boldsymbol{k}}) \left[\mathbf{m_i^4} f_0(y_i) + 3\mathbf{m_i^2} T_\nu^2 f_1(y_i) + 2T_\nu^4 f_2(y_i)\right]$$

$$\tilde{\phi}_X \equiv \frac{Ga^2}{|\boldsymbol{k}|^2} (\boldsymbol{v}_{\mathbf{Xc}} \cdot \hat{\boldsymbol{k}}) \left(\mathbf{m_X^4} f_0 + 3\mathbf{m_X^2} T_A^2 f_1 + 2T_A^4 f_2\right)$$

- **For neutrino ~ 0.1eV**

$$\Omega_\nu \approx 0.3\% \qquad \Omega_{\rm DM} \approx 27\% \qquad \frac{\Omega_\nu}{\Omega_{\rm DM}} \approx 10^{-2}$$

- **For light DM ~ 1eV**

$$\frac{\Omega_X}{\Omega_{\rm DM}} \sim \frac{m_\nu^4}{m_X^4} \frac{\Omega_\nu}{\Omega_{\rm DM}}$$

Significant enhancement!

Freeze-In DM Phase Space Distribution

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$$\tilde{\phi}_X \equiv \frac{Ga^2}{|\boldsymbol{k}|^2}(\boldsymbol{v}_{\mathbf{Xc}} \cdot \hat{\boldsymbol{k}}) \left(\mathbf{m_X^4} f_0 + 3\mathbf{m_X^2} T_A^2 f_1 + 2T_A^4 f_2 \right)$$

$$f_n(y_i) \equiv g_X \int_{y_i}^{\infty} dy y^{2n} df_X(y)/dy \qquad y \equiv \frac{|\mathbf{p}|}{T_A}$$

Freeze-in Phase Space Distribution

$$f_X(\boldsymbol{p}) \approx C_X \frac{e^{-|\boldsymbol{p}|/T_A(a)}}{\sqrt{|\boldsymbol{p}|/T_A(a)}} \quad \Rightarrow \quad f_0 = -g_X C_X \frac{e^{-y_i}}{2\sqrt{y_i}}$$

For $m_X \gg T_{A0}$, the 1st term dominates

$$\Rightarrow \quad \tilde{\phi}_X \approx -\frac{4\pi^{3/2}}{3} \frac{Ga^2}{|\boldsymbol{k}|^2}(\boldsymbol{v}_{\mathbf{Xc}} \cdot \hat{\boldsymbol{k}}) \rho_{X0} \left(\frac{\mathbf{m_X}}{T_{A0}} \right)^{\mathbf{3}} \frac{e^{-y_i}}{\sqrt{y_i}}$$

Mass power reduces when replacing C_X by ρ_{X0}

Free-Streaming Limit

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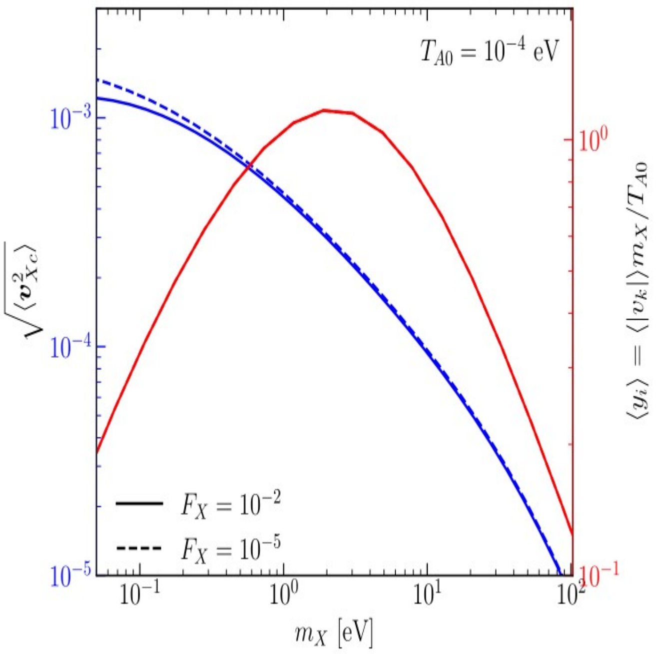
$$\tilde{\phi}_X \approx -\frac{4\pi^{3/2}}{3}\frac{Ga^2}{|\boldsymbol{k}|^2}(\boldsymbol{v_{Xc}} \cdot \hat{\boldsymbol{k}})\rho_{X0}\left(\frac{\mathbf{m_X}}{T_{A0}}\right)^{\mathbf{3}}\frac{e^{-y_i}}{\sqrt{y_i}}$$

$$y_i = m_X \frac{|v_{Xc} \cdot \hat{k}|}{T_A}$$

$$\propto |\boldsymbol{v_{Xc}}|^{\frac{1}{2}} \mathbf{m_X}^{\frac{5}{2}}$$



$$|\boldsymbol{v_{Xc}}| \propto \frac{1}{\mathbf{m_X}}$$

$$\tilde{\phi}_X \propto \mathbf{m_X^2}$$

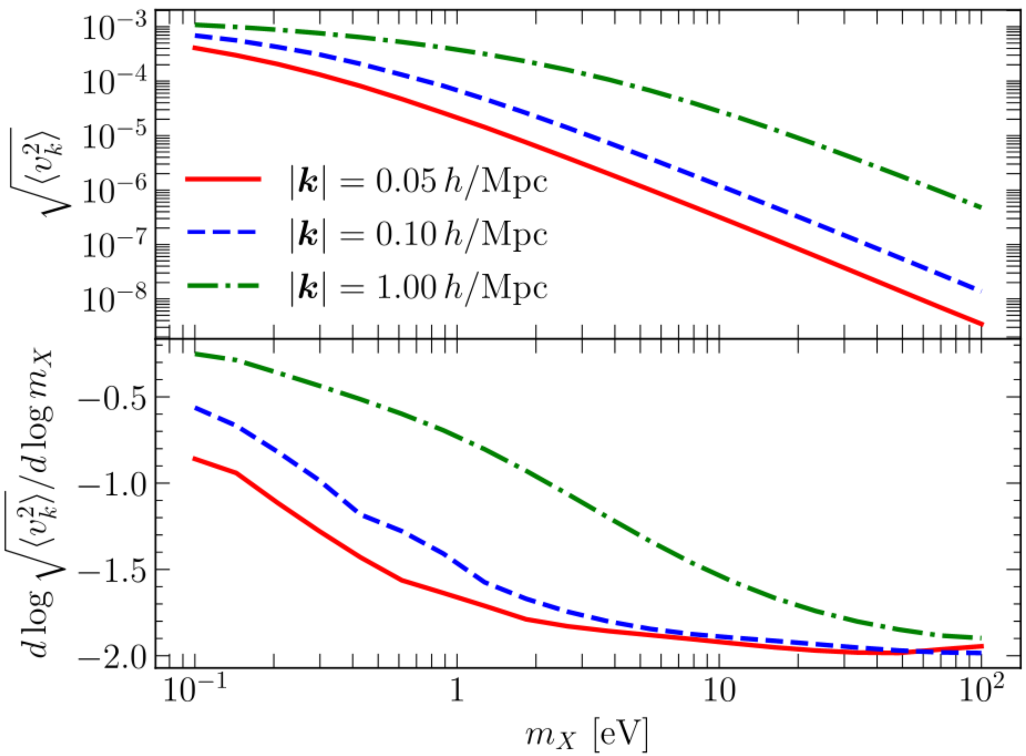
$$\frac{\Omega_X}{\Omega_{\rm DM}} \sim \frac{m_\nu^2}{m_X^2}\frac{\Omega_\nu}{\Omega_{\rm DM}}$$

Clustering Limit

$$k_{\rm fs}^{-1} \equiv 2\pi\sqrt{\frac{2}{3}}\frac{\langle|p_X|\rangle}{m_X H_0} \approx 0.384\frac{10\,{\rm eV}}{m_X}\,{\rm Mpc}/h$$

For light DM m_X ~ O(10eV), free-streaming scale << 1Mpc

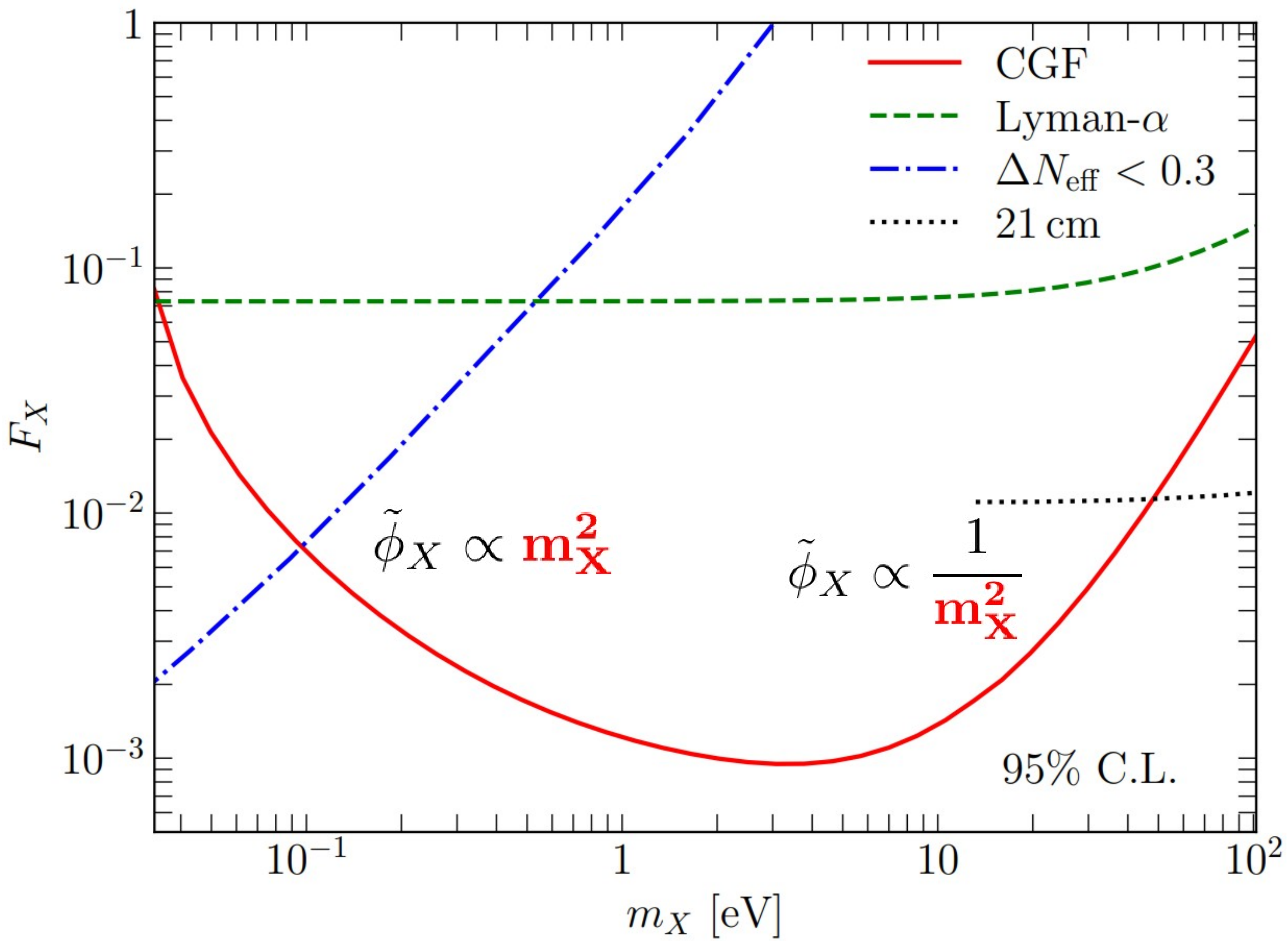
$$\tilde{\phi}_X = -4\pi G\rho_{X0}(\boldsymbol{v}_{\mathbf{Xc}}\cdot\hat{\boldsymbol{k}})\int_{s_i}^{s} ds' a^2(s')(s-s')^2$$



$$|\boldsymbol{v}_{\mathbf{Xc}}| \propto \frac{1}{\mathbf{m_X^2}}$$

$$\tilde{\phi}_X \propto \frac{1}{\mathbf{m_X^2}}$$

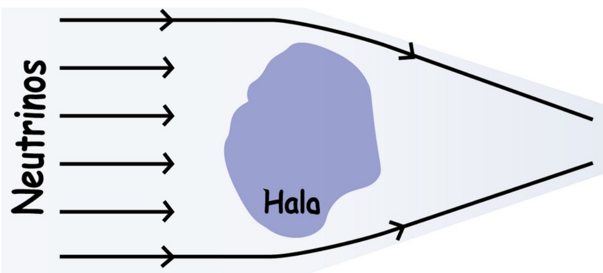
Projected Sensitivity @ DESI

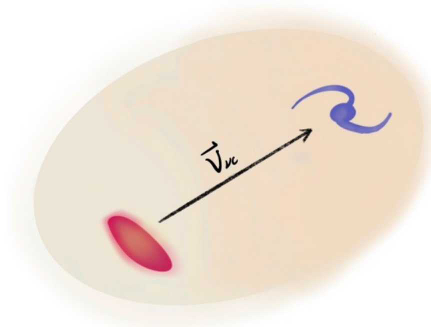


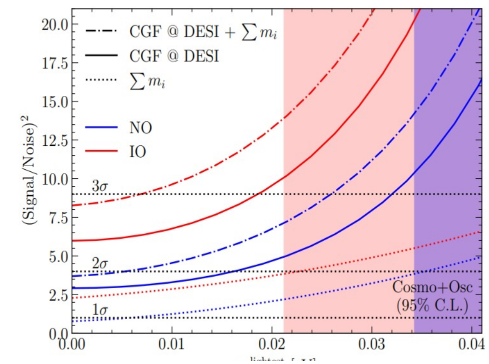
Summary

1) Neutrino & DM have many similarities

2) 3rd cosmological way of measuring ν masses







$$\tilde{\phi} = \frac{Ga^2}{|\boldsymbol{k}|^2} \sum_i (\boldsymbol{v}_{\nu_i \mathbf{c}} \cdot \hat{\boldsymbol{k}}) \left[\mathbf{m_i^4} f_0(y_i) + 3\mathbf{m_i^2} T_\nu^2 f_1(y_i) + 2T_\nu^4 f_2(y_i) \right]$$

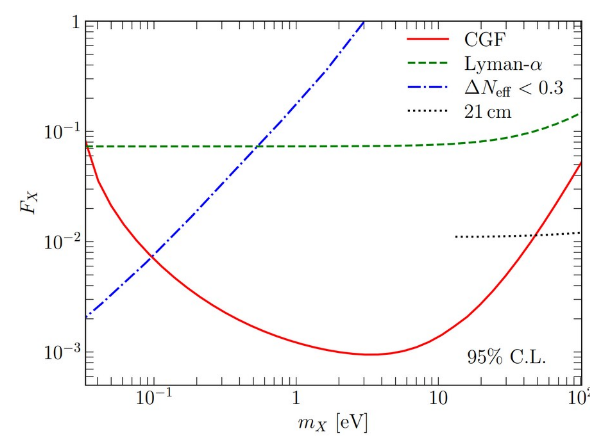
Dipole vs Imaginary **Fourth power dependence!**

3) Light DM fluid vs Major CDM halo

$$\tilde{\phi}_X \propto \mathbf{m_X^2}$$

$$\tilde{\phi}_X \propto \frac{1}{\mathbf{m_X^2}}$$

Free-streaming Limit **Clustering Limit**



Thank You

Imaginary Density & Average

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$$\mathrm{Im}[\delta\rho_\nu] = \frac{\widetilde{\Psi}}{4\pi}\sum_i (\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})\left[m_i^4\left(1-\frac{m_i|\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}}|}{2T_\nu}\right)+\pi^2 m_i^2 T_\nu^2+\frac{14}{15}\pi^4 T_\nu^4\right]$$

$$\langle(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})^2\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left|\frac{T_{\theta_{\nu_i c}}(\boldsymbol{k}',z)}{|\boldsymbol{k}'|}\right|^2,$$

$$\langle(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})[\partial_z(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})]\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left[\frac{T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\frac{\partial_z T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\right],$$

$$\langle[\partial_z(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})]^2\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left[\frac{\partial_z T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\right]^2,$$

Decoherence & Non-Linear

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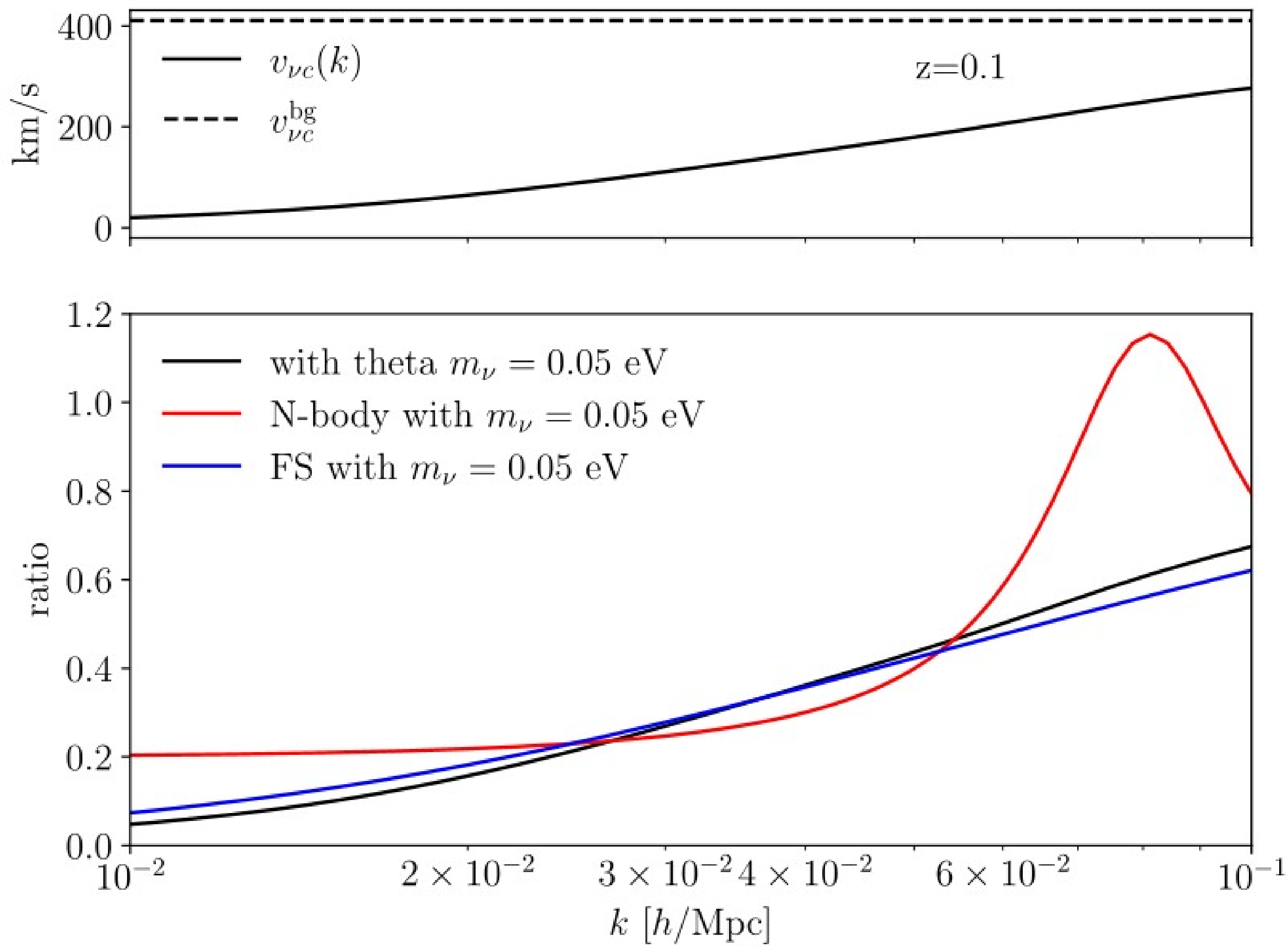
$$\langle(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})^2\rangle = \frac{1}{3}\int \frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2 \Delta_\zeta^2(\boldsymbol{k}')\left|\frac{T_{\theta_{\nu_i c}}(\boldsymbol{k}',z)}{|\boldsymbol{k}'|}\right|^2$$



Interpolation between Limits

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Free-streaming Limits

$$\tilde{\phi}_X \equiv \frac{Ga^2}{|\boldsymbol{k}|^2}(v_{\mathbf{Xc}} \cdot \hat{\boldsymbol{k}})\left(\mathbf{m_X^4} f_0 + 3\mathbf{m_X^2} T_A^2 f_1 + 2T_A^4 f_2\right)$$

Clustering Limits

$$\tilde{\phi}_X = -4\pi G \rho_{X0}(v_{\mathbf{Xc}} \cdot \hat{\boldsymbol{k}}) \int_{s_i}^{s} ds' a^2(s')(s-s')^2$$

$$\tilde{\phi}_X \equiv -4\pi G F_X \rho_{\mathrm{DM0}}(\boldsymbol{v}_{Xc} \cdot \boldsymbol{k}) g(|\boldsymbol{k}|)$$

$$g(|\boldsymbol{k}|) \equiv A \frac{k_{\mathrm{fs}}^3}{(|\boldsymbol{k}| + k_{\mathrm{fs}})^3} + (B-A)\frac{k_{\mathrm{fs}}^4}{(|\boldsymbol{k}|^2 + k_{\mathrm{fs}})^4} \Rightarrow \begin{cases} A\, k_{\mathrm{fs}}^3/|\boldsymbol{k}|^3 \\ B \end{cases}$$

$$A \equiv \frac{\sqrt{\pi}}{3}\frac{a^2}{k_{\mathrm{fs}}^3}\left(\frac{m_X}{T_{A0}}\right)^3 \frac{e^{-y_i}}{\sqrt{y_i}} \qquad B \equiv \int_{s_i}^{s} ds' a^2(s')(s-s')^2$$

《 Physics Letters A 》

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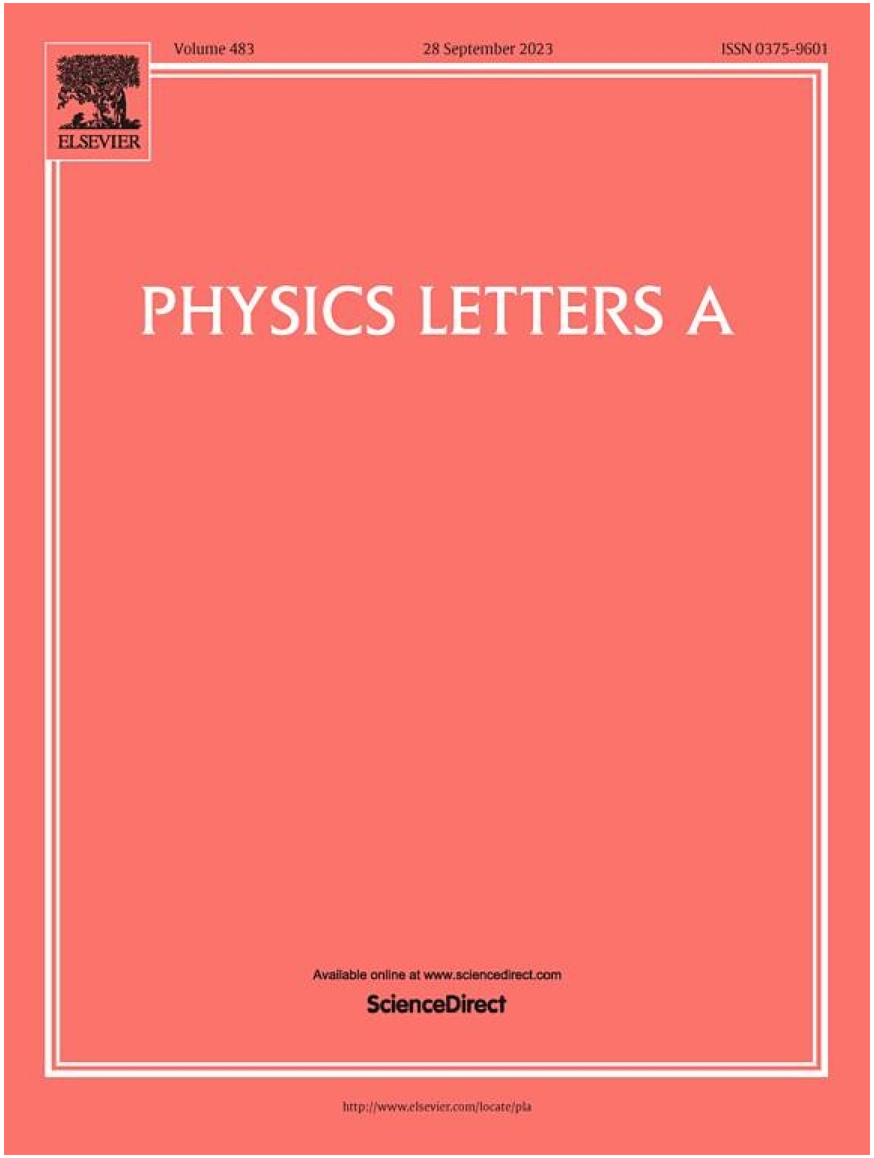

Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.