Probing CP violation for electroweak baryogenesis in 2HDM

素粒子原子核研究所
Institute of Particle and Nuclear Studies

Yushi Mura (KEK)

KEK

JHEP 07 (2025) 236, arXiv: 2504.07705 [hep-ph]
Collaborators: Masashi Aiko, Motoi Endo, Shinya Kanemura

JHEP 10 (2024) 041, arXiv: 2408.06863 [hep-ph]
Collaborators: Shinya Kanemura

The 5th AEI International Workshop, Durham, IPPP
2025/10/01

Introduction

- **SM was established by the Higgs discovery in 2012.** ATLAS, CMS (2012)

- **Some remaining problems**
  e.g., baryon asymmetry of the Universe (BAU) $\eta_B^{obs} = \frac{n_B - n_{\bar{B}}}{s} \simeq 8 \times 10^{-11}$
  PDG (2022)

- **Sakharov three conditions for baryon asymmetry** Sakharov (1967)

  ① Baryon # violation
  ② C and CP violation
  ③ Departure from thermal equilibrium

- **A promising scenario for baryogenesis: Electroweak baryogenesis**

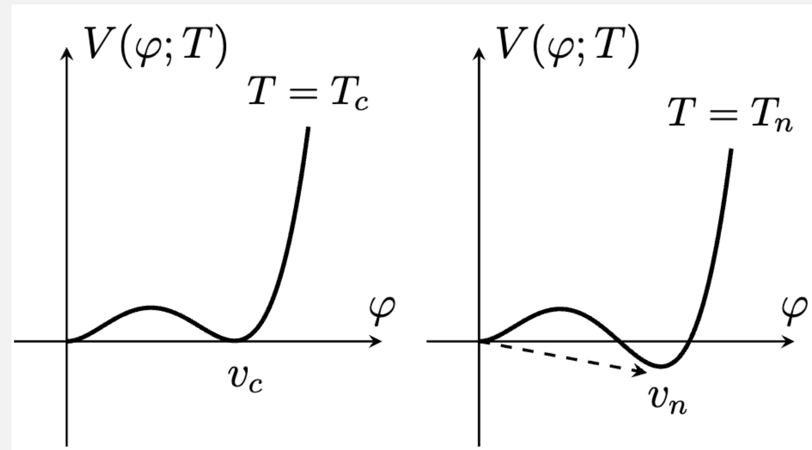
  Kuzmin, Rubakov and Shaposhnikov (1985)

  ① Sphaleron process
  ② EW interaction with CP phase
  ③ EW first order phase transition

Electroweak baryogenesis

- **Expanding bubble walls are created at first order PT.** Coleman, Callan and Coleman (1977) and many works

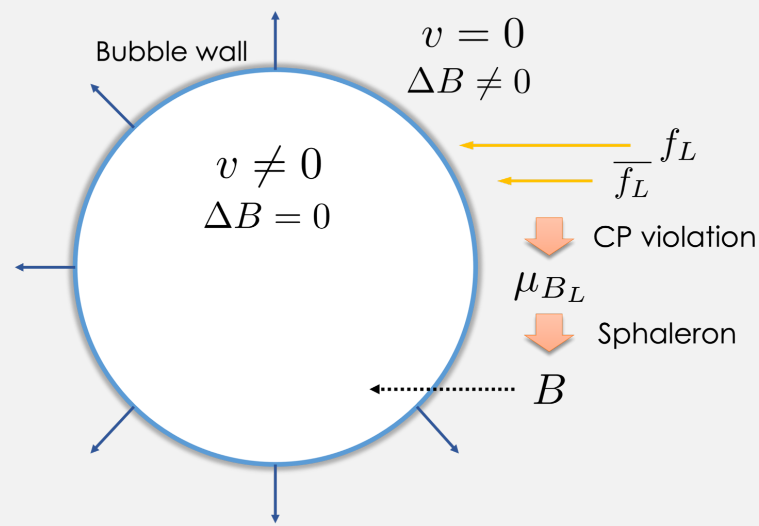
  First order PT is realized
  by tunneling process (vacuum decay).



- **Non-equilibrium sphaleron process around the bubble wall**

  Outside : baryon number violated
  Inside : baryon number conserved

  $\Gamma_{\mathrm{sph}}^{\mathrm{brk}} \propto e^{-v/T}$ Moore (1999)



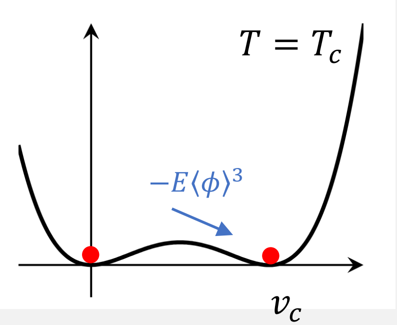
- **Generated baryon number is conserved inside the bubble.**

  $\Gamma_{\mathrm{sph}}^{\mathrm{brk}}(T_n) < H(T_n) \Longrightarrow v_n/T_n \gtrsim 1$ →Strongly first order PT

Is EWBG possible in the SM ?

- **Difficulties in the Standard Model**

  - SM cannot realize strongly first order EWPT.



$$V(\langle\phi\rangle, T) = \frac{1}{2} m_T^2 \langle\phi\rangle^2 - ET\langle\phi\rangle^3 + \frac{\lambda}{4}\langle\phi\rangle^4 + \cdots \qquad E \simeq \frac{1}{4\pi v^3}(2m_W^3 + m_Z^3)$$

$$\frac{v_n}{T_n} \sim \frac{v_c}{T_c} \simeq \frac{2E}{\lambda} = \frac{4Ev^2}{m_h^2} \gtrsim 1 \quad \text{->} \quad m_h \lesssim \sqrt{4Ev^2} \simeq 48 \text{ GeV}$$

Shaposhnikov (1987)
Dine et al. (1992), (1992)

Crossover like for $m_h \gtrsim 60$ GeV (lattice results)

Kajantie et al. (1996);
D'Onofrio and Rummukainen (2016);

  - SM Jarlskog invariant (from CKM matrix)

$$J_{CP} = \mathrm{Im}[V_{ud}V_{us}^* V_{cs} V_{cs}^*] = 3.12\times10^{-5}$$ PDG (2024) ➡ $$\eta_B < O(10^{-26})$$

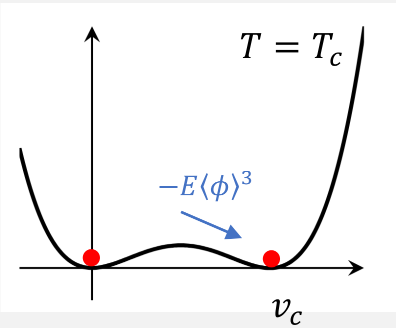
Gavela et al. (1994);
Huet and Sather (1995);

CPV and first order EWPT in 2HDM

- **EWPT can be changed by new physics effects.**

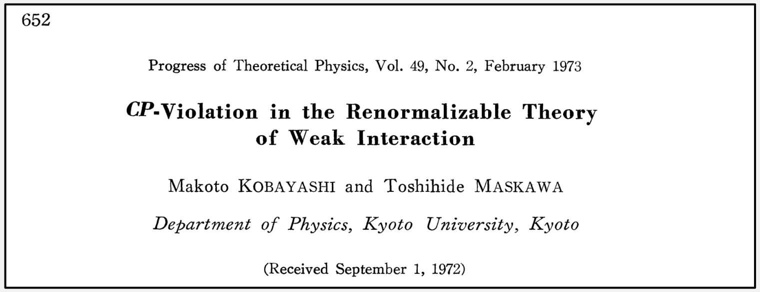
e.g.) Two Higgs doublet model (2HDM)

$\mathcal{L} \sim -\lambda \phi_{\mathrm{SM}}^{\dagger} \phi_{\mathrm{SM}} \phi^{\dagger} \phi$ -> $E \simeq \dfrac{1}{4\pi v^3}\left(2m_W^3 + m_Z^3 + (\lambda v^2)^{3/2}\right)$



Funakubo et al. (1994); Davies et al. (1994); Cline and Lemieux (1997) and more

- **New CP violation in new physics**


652

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

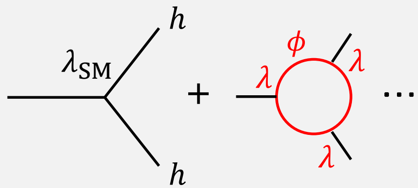
e.g.) CPV in 2HDM (SU(2) doublets: Φ_1 and Φ_2)

- Relative phase b/w Φ_1 and Φ_2
- New interaction b/w Φ_2 and SM fermions

Constraints and testability

- **New physics effects making EWPT first order**

e.g.) Triple Higgs coupling

$\lambda_{hhh} \simeq$ 

O(10)% deviation for first order EWPT

Kanemura, Okada and Senaha (2005), and more

	LHC	HL-LHC	ILC(1TeV)	FCC-ee/eh/hh
$\dfrac{\Delta\lambda_{hhh}}{\lambda_{hhh}}$	~600%	~50%	~10%	~5%

CMS (2022), ATLAS (2023), de Blas *et al.* (2020)

- **CP violating observables as a probe of EWBG**

  - Electric dipole moments

    Relation b/w EWBG and EDM

    Aiko, Endo, Kanemura and YM, JHEP 07 (2025) 236

EDMs	Current bounds	Expected limits
Electron	4.1×10^{-30} e cm JILA (2023)	$O(10^{-33})$ e cm Vutha, et al. (2018)
Neutron	1.8×10^{-26} e cm Abel, et al. (2020)	$O(10^{-28})$ e cm nEDM (2019)
Proton	2.1×10^{-25} e cm Sahoo (2017)	$O(10^{-29})$ e cm Alarcon, et al. (2022)

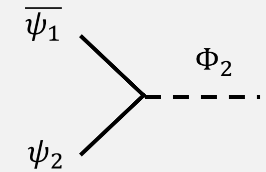
  - Direct CP violation in $H^\pm$ decays via loop-induced $H^\pm W^\mp Z$ vertex

    Probe of CPV in the Higgs sector Kanemura and YM, JHEP 10 (2024) 041

EDM predicted in 2HDM

- **Two regimes to avoid FCNC**

Discrete Z2 symmetry: $\Phi_2 \to -\Phi_2$



FCNC Yukawa

	Softly broken Z2 2HDM	General 2HDM
FCNC Yukawa	Forbidden	Needs to be small (e.g. MFV)
# of new CP phase	1	Many (more than 10)
eEDM to explain BAU	$O(10^{-28})\ e$ cm	Depends on Yukawa structure

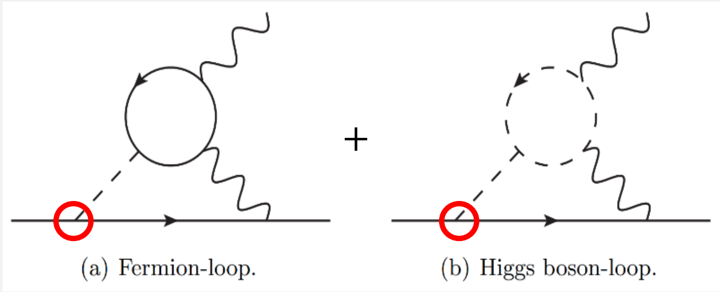
Fromme et al. (2006); Dorsch et al. (2017); Basler et al. (2021), and more

Fuyuto, Hou, and Senaha (2019); Enomoto, Kanemura and YM, JHEP 01 (2022) 104, Enomoto, Kanemura and YM, JHEP 09 (2022) 121, and more

- **What is the leading contributions for EDM ?**

Fuyuto, Hou and Senaha (2019); Kanemura Kubota and Yagyu (2020); and more

$d_e \simeq$



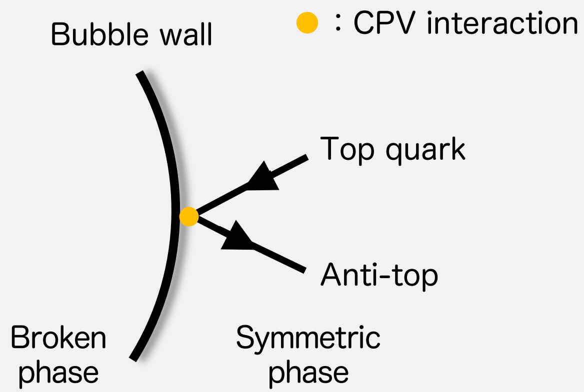
: couplings b/w electron and additional Higgs

- However, this CPV coupling is not important for BAU (Why? -> see next)

Top transport scenario

- **CPV force acting on top and anti-top ∝ y_t**

Cline, Joyce, and Kainulainen (2000); Fromme and Huber (2007); and more



Local mass and phase

$$m_t(r)e^{i\theta_t(r)}$$

Different dispersion relations for WKB quasi-(anti-)particle

$$\omega \sim \sqrt{p^2 + m_t^2} \pm \theta_t'(r)$$

Local asymmetry

$$n_L - n_{\bar{L}} \neq 0$$
$$n_R - n_{\bar{R}} \neq 0$$

Baryogenesis via sphaleron

$$\frac{\partial n_B}{\partial t} \sim \Gamma_{\text{sph}} \frac{\mu_{B_L}}{T} + (\text{washout})$$

- **Which EDMs constrain this scenario in g2HDM, and how?**

General two Higgs doublet model

- **Most general potential**

Higgs basis $\mathbf{\Phi}_1 = \begin{pmatrix} G^{\pm} \\ \frac{1}{\sqrt{2}}(v + h_1 + iG^0) \end{pmatrix}$ $\mathbf{\Phi}_2 = \begin{pmatrix} H^{\pm} \\ \frac{1}{\sqrt{2}}(h_2 + ih_3) \end{pmatrix}$

Davidson and Haber (2005)

$$V = -\mu_1^2 \mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1 + M^2 \mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2 - \left(\mu_3^2 \mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 + \mathrm{h.c.}\right)$$
$$+\frac{1}{2}\lambda_1\left(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right)^2 + \frac{1}{2}\lambda_2\left(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right)^2 + \lambda_3\left(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1\right)\left(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right) + \lambda_4\left(\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2\right)\left(\mathbf{\Phi}_2^\dagger \mathbf{\Phi}_1\right)$$
$$+\left\{\left(\frac{1}{2}\lambda_5 \mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 + \lambda_6 \mathbf{\Phi}_1^\dagger \mathbf{\Phi}_1 + \lambda_7 \mathbf{\Phi}_2^\dagger \mathbf{\Phi}_2\right)\mathbf{\Phi}_1^\dagger \mathbf{\Phi}_2 + \mathrm{h.c.}\right\} \qquad (\mu_3^2, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C})$$

- **Most general Yukawa sector**

$$\mathcal{L}_Y = -\sum_{k=1,2}\left(\overline{Q_L} Y_{k,u}^\dagger \widetilde{\mathbf{\Phi}}_k u_R + \overline{Q_L} Y_{k,d} \mathbf{\Phi}_k d_R + \overline{L_L} Y_{k,l} \mathbf{\Phi}_k e_R + \mathrm{h.c.}\right)$$

$Y_{1,u} = \mathrm{diag}(y_u, y_c, y_t)$ $\quad Y_{1,d} = \mathrm{diag}(y_d, y_s, y_b)$ $\quad Y_{1,l} = \mathrm{diag}(y_e, y_\mu, y_\tau)$

- **$\boldsymbol{Y_2}$ is general complex matrix**

e.g.) Up type

$$Y_{2,u} = \begin{pmatrix} \rho_{uu} & \rho_{cu} & \rho_{tu} \\ \rho_{uc} & \rho_{cc} & \rho_{tc} \\ \rho_{ut} & \rho_{ct} & \rho_{tt} \end{pmatrix}$$

General two Higgs doublet model

- **Stationary conditions and mass spectra**

$$\frac{\partial V}{\partial h_i} = 0 \;\Leftrightarrow\; \mu_1^2 = \frac{1}{2}\lambda_1 v^2,\; \mu_3^2 = \frac{1}{2}\lambda_6 v^2$$

$$\frac{\partial^2 V}{\partial h_i \partial h_j} = \mathcal{M}_{ij}^n = \begin{pmatrix} \lambda_1 v^2 & \mathrm{Re}[\lambda_6]v^2 & -\mathrm{Im}[\lambda_6]v^2 \\ \mathrm{Re}[\lambda_6]v^2 & M^2 + \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)v^2 & -\frac{1}{2}\mathrm{Im}[\lambda_5] \\ -\mathrm{Im}[\lambda_6]v^2 & -\frac{1}{2}\mathrm{Im}[\lambda_5] & M^2 + \frac{1}{2}(\lambda_3+\lambda_4-\lambda_5)v^2 \end{pmatrix}$$

$$m_{H^\pm}^2 = M^2 + \frac{1}{2}\lambda_3 v^2$$

- **Mass eigenstate for neutral scalar bosons**

125 GeV Higgs

Orthogonal matrix R $\quad \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad R^T \mathcal{M}^n R = \mathrm{diag}(m_{H_1}, m_{H_2}, m_{H_3})$

- **Rephasing invariants in the model** $\quad \boldsymbol{\Phi}_2 \to e^{i\theta}\boldsymbol{\Phi}_2$

Potential: $\mathrm{Im}[\lambda_5^* \lambda_6^2]$, $\mathrm{Im}[\lambda_5^* \lambda_7^2]$, $\mathrm{Im}[\lambda_6^* \lambda_7]$

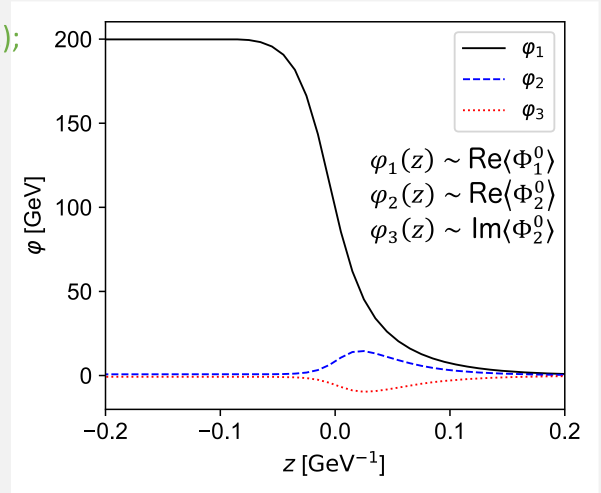
Yukawa: $\mathrm{Im}[\lambda_5 \rho_{tt}^2]$, $\mathrm{Im}[\lambda_6 \rho_{tt}]$, $\mathrm{Im}[\lambda_7 \rho_{tt}]$ (and other ρ_{ij} related invariants)

Essential CP violation

- **Discovered 125GeV Higgs is SM like.** ATLAS, Nature (2022); CMS, Nature (2022);

e.g.) H_1ZZ coupling $\kappa_Z \simeq 1$ ➡ $|\lambda_6| \ll 1$

➡ $\mathrm{Im}[\lambda_5^*\lambda_7^2]$, $\mathrm{Im}[\lambda_5\rho_{tt}^2]$, $\mathrm{Im}[\lambda_7\rho_{tt}]$



- **CPV force as a function of VEVs $\boldsymbol{\varphi}$**

$$S_{\mathrm{CPV}} \propto \frac{y_t|\rho_{tt}|}{2}\{(\varphi_1\varphi_2' - \varphi_2\varphi_1')\sin(\arg[\rho_{tt}]) + (\varphi_3\varphi_1' - \varphi_1\varphi_3')\cos(\arg[\rho_{tt}])\}$$

- If $\lambda_6 \simeq \lambda_7 \simeq 0$, tree level potential approximately has $\mathbb{Z}_2$ symmetry ($\boldsymbol{\Phi_2} \to -\boldsymbol{\Phi_2}$).
  ⇨ VEV of $\boldsymbol{\Phi_2}$ is suppressed, $\varphi_2, \varphi_3 \ll 1$.

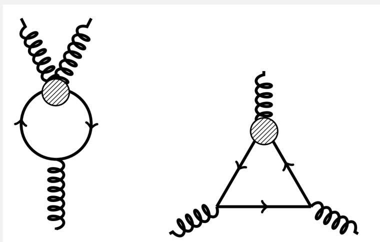
- For sufficient BAU, $\mathrm{Im}[\lambda_7\rho_{tt}]$ is necessary.

- **Minimal setup** $\rho_{ij} = 0$ (except for ρ_{tt}) and $\lambda_4 = \lambda_5 = \lambda_6 = 0$
  ($\lambda_4 = \lambda_5$ is for T parameter)

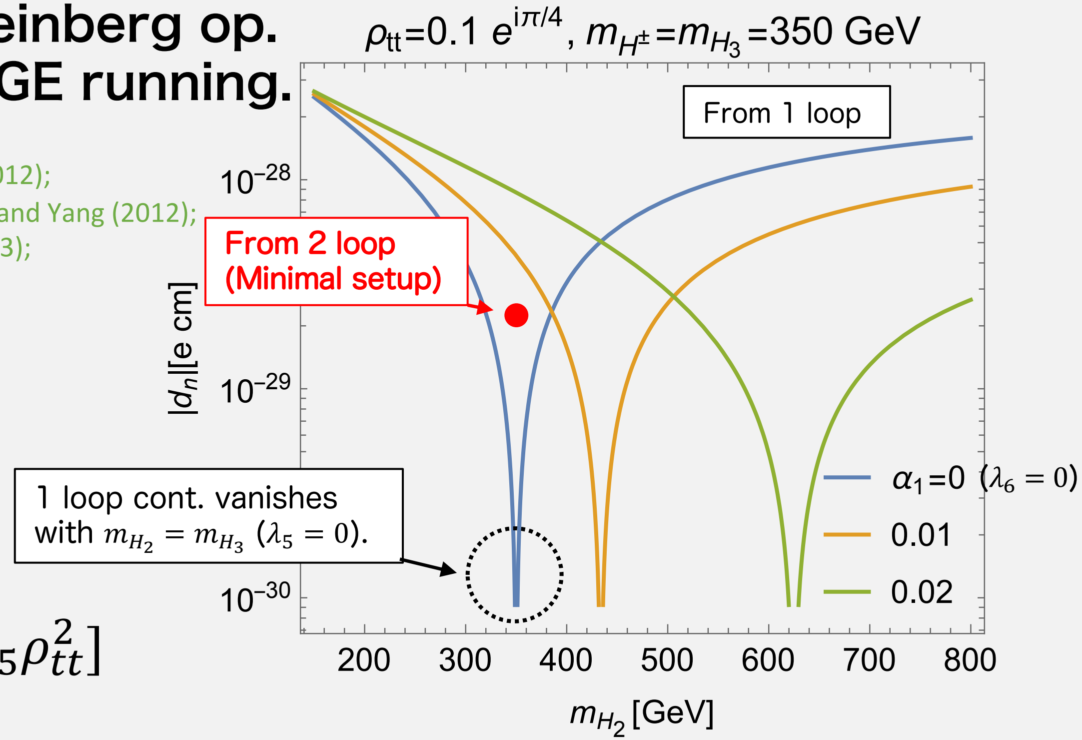
➡ $m_{H_2} = m_{H_3} = m_{H^\pm} \equiv m_\Phi$ One available CP phase: $\arg[\lambda_7\rho_{tt}]$

EDMs in the minimal setup

- **Top chromo EDM induces Weinberg op. and light fermion EDMs by RGE running.**

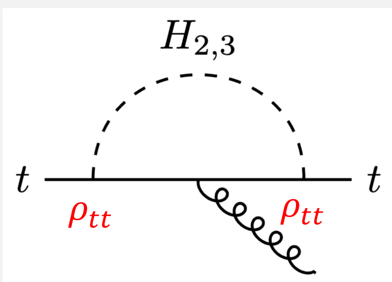


Kamenik et al. (2012);
Hisano, Tsumura and Yang (2012);
Kaneta et al. (2023);
and more works



- **At 1 loop level**

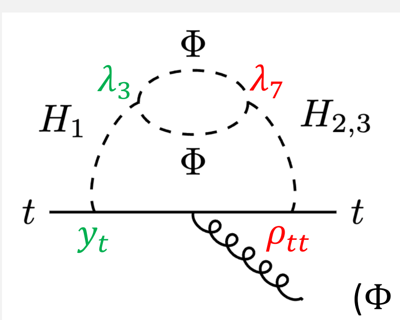
e.g.)



$\propto \mathrm{Im}[\lambda_5 \rho_{tt}^2]$

- **With the minimal setup, 2 loop diagrams are leading.**

e.g.)



$\propto \mathrm{Im}[\lambda_7 \rho_{tt}]$

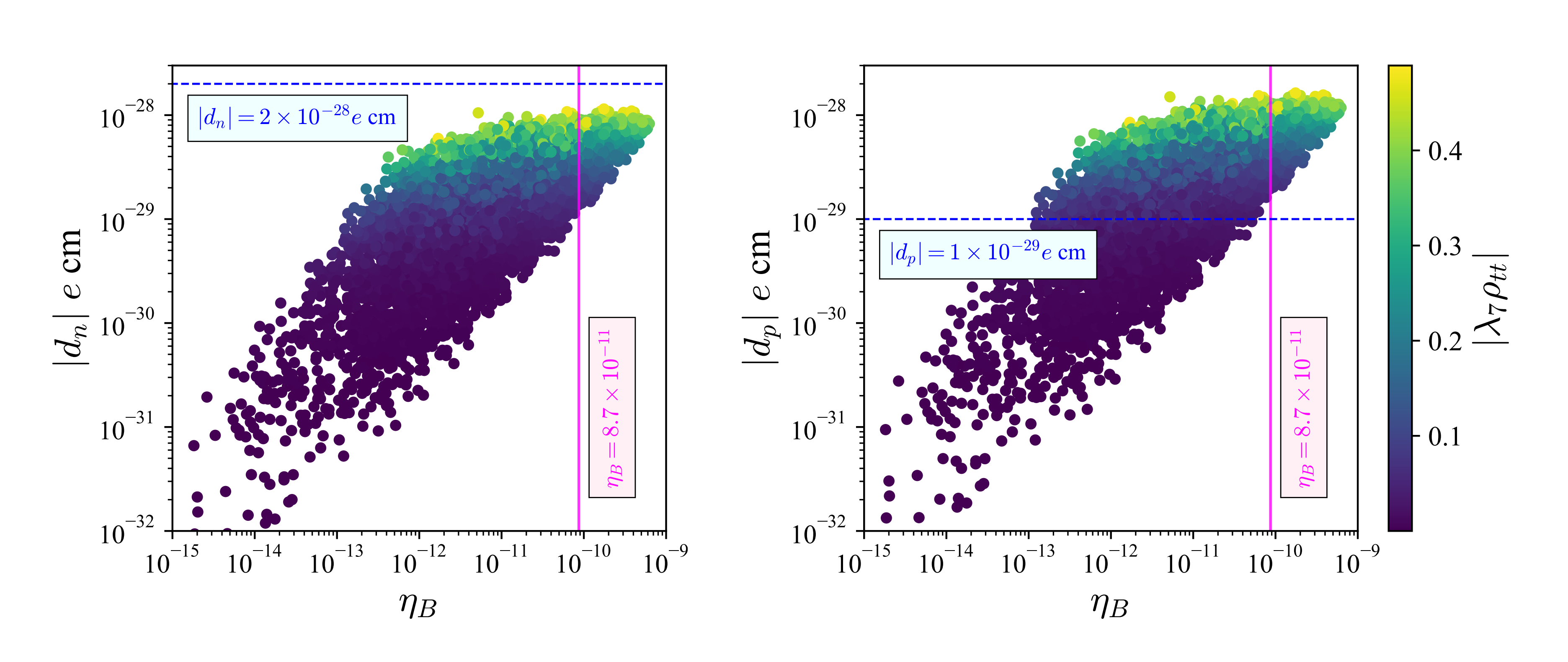
$(\Phi = H_2, H_3, H^\pm)$

At the red point,
$\lambda_7 = e^{i\pi/4}, -\mu_2^2 = 30^2\ \mathrm{GeV}^2$ are taken.

Correlation b/w EDMs and BAU

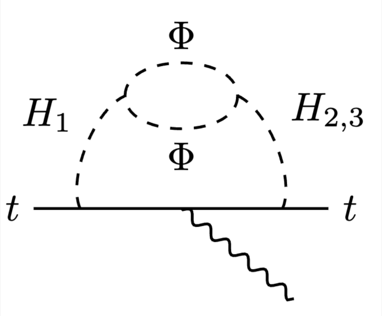
- **Scanning parameter space** $m_\Phi = [200,500]\ \mathrm{GeV}, \mu_2^2 = \left[-m_\Phi^2, 0\right], |\rho_{tt}| = [0,0.5]$
  $|\lambda_7| = [0,1], \lambda_2 = [0,1], \arg[\lambda_7 \rho_{tt}] = -\pi/2\,, v_w = \left[0.1, 1/\sqrt{3}\right]$

- **Neutron and proton EDMs**

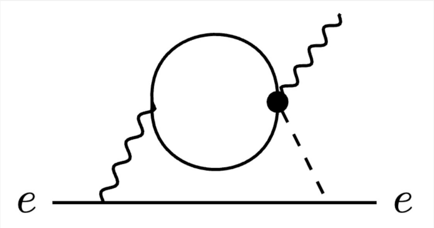


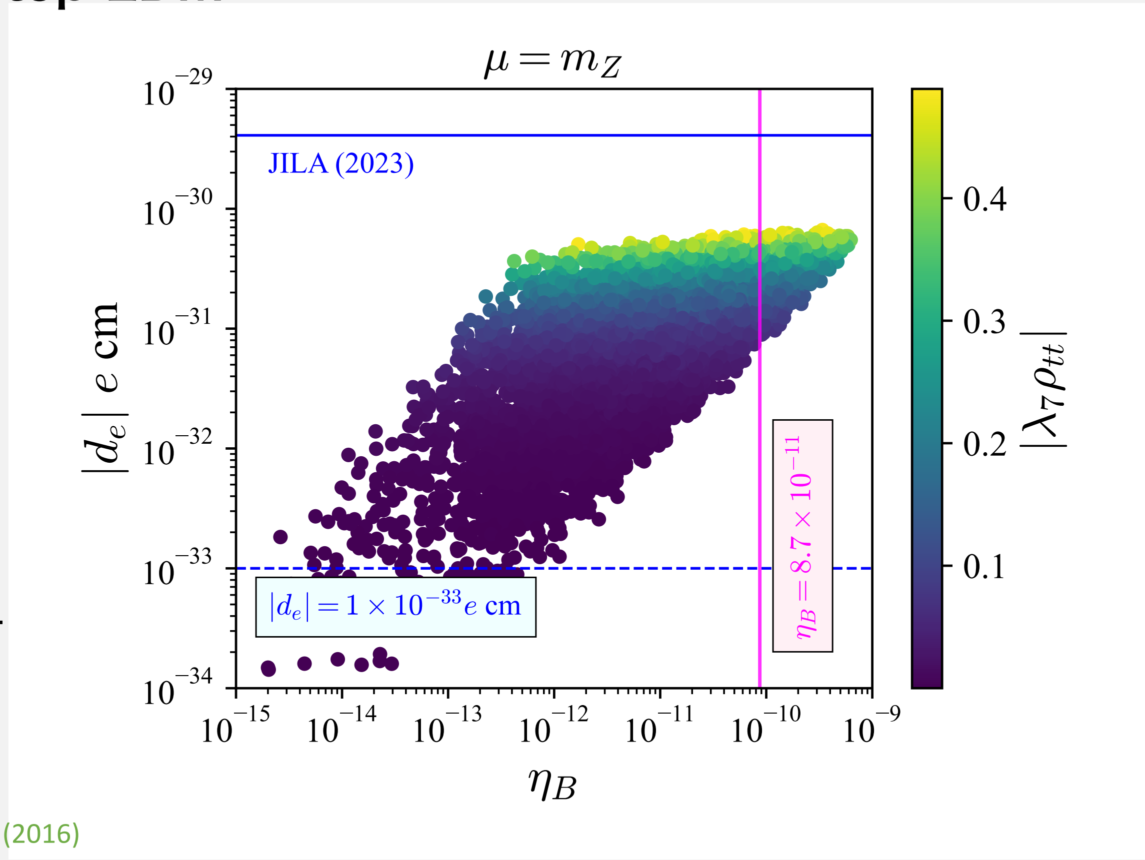
Correlation b/w EDMs and BAU

- **Electron EDM induced by top EDM**
  - Dipole operators for top are induced below $\Lambda \simeq m_\Phi$.



  - Matching to d_e at $\mu \sim m_Z$, where top, Higgs, W and Z are decoupled.

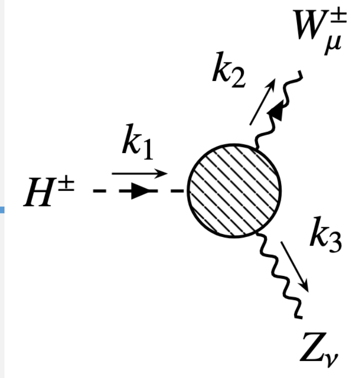




Cirigliano et al. (2016)
Fuyuto and Ramsey-Musolf (2017)

- Small effects by other couplings (e.g. ρ_{ee}) can change EDMs, but not η_B very much.

$H^{\pm}W^{\mp}Z$ vertex



- **Custodial symmetry in the SM** $\mathbb{M}_{\rm SM} = (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{-} & \phi^{0} \end{pmatrix}$

$$\mathcal{L} = \mathrm{Tr}\left[(D_{\mu}\mathbb{M}_{\rm SM})^{\dagger}(D^{\mu}\mathbb{M}_{\rm SM})\right] - V(\mathrm{Tr}[\mathbb{M}_{\rm SM}^{\dagger}\mathbb{M}_{\rm SM}])$$

$\mathbb{M}_{\rm SM} \to L\mathbb{M}_{\rm SM}R^{\dagger}$ $\quad L \in \mathrm{SU}(2)_L, R \in \mathrm{SU}(2)_R$ $\quad$ L=R symmetry -> Custodial symmetry

Sikivie et al. (1980)

  - ρ parameter (relation b/w W and Z bosons mass) $\rho = m_W^2/m_Z^2 c_W^2 = 1$ (Tree level)

- **An important vertex : $H^{\pm}W^{\mp}Z$ vertex** Grifols and Mendez (1980)

  - WZ system (two bosons system) ~ $\mathbf{3} \times \mathbf{3} = \mathbf{1}_S + \mathbf{3}_A + \mathbf{5}_S$

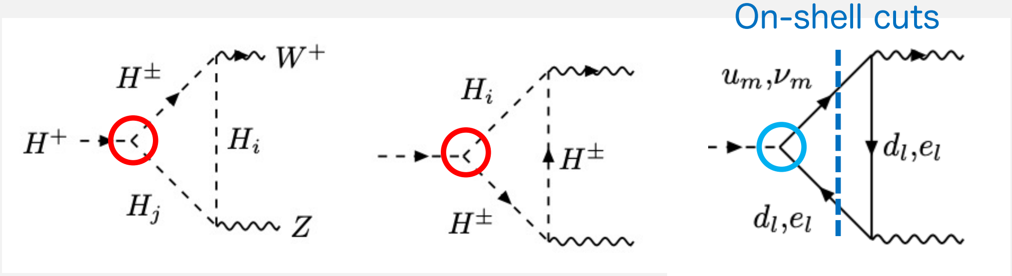
$\rho = 1$ models	Scalar reps. (L,R)	Quintuplet $H_5^{\pm}$	Triplet $H_3^{\pm}$	$H^{\pm}W^{\mp}Z$ vertex
Georgi-Machacek model	(3,3)	✓	✓	$H_5^{\pm}W^{\mp}Z$ (tree-level)
2HDM	(2,2)*2	×	✓	$H_3^{\pm}W^{\mp}Z$ (loop-induced)

- **Relation to CP violation (2HDM)** Pomarol and Vega (1994)

  - To have CP phase in the potential, the custodial symmetry must be broken.

Direct CPV in $H^{\pm}$ decays

- **Interference of scalar and fermion contributions (most general 2HDM)**



Kanemura and YM, JHEP 10 (2024) 041

$$\Delta \equiv \Gamma(H^{+} \to W^{+}Z) - \Gamma(H^{-} \to W^{-}Z)$$

$$\Delta \simeq \rho_{tt}^{R} Z_7^{I} \left(m_{H^{\pm}}^2 - m_{H_2}^2\right) \times f_3 \, \mathrm{Im}[f_1^*] + \rho_{tt}^{I} Z_7^{R} \left(m_{H^{\pm}}^2 - m_{H_3}^2\right) \times f_2 \, \mathrm{Im}[f_1]$$

f_1: loop function in fermion loop f_2: loop function in scalar loop

- **Can we test direct CPV in future colliders?**

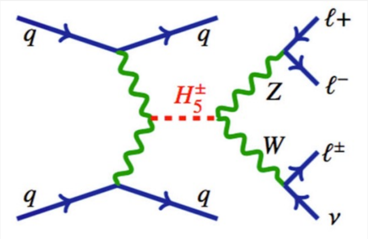
Humphrey, Kanemura and YM, work in progress

Current bound

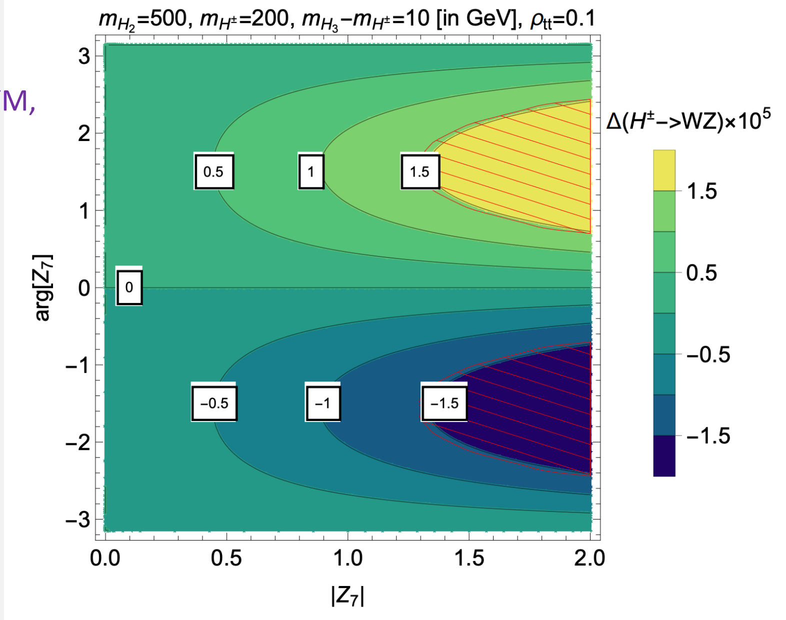
ATLAS, Eur. Phys. J. C (2023) 83:633;
CMS, Eur. Phys. J. C 81 (2021) 8, 723

$\sigma_{WZjj} \lesssim 80$ fb at $m_{H^{\pm}} = 300$ GeV

Tree level HWZ: GM model



Georgi and Machacek (1985);
Chanowitz and Golden (1985);



Summary

- **New physics is necessary for EWBG**
  - New source of CPV can be introduced in 2HDM.

- **Minimal setup for EWBG in general 2HDM**
  - EDMs and BAU correlated by $\mathrm{Im}[\lambda_7 \rho_{tt}]$
  - It is viable under current bounds but would be tested in the future.

- **$\boldsymbol{H^\pm \to W^\pm Z}$ decay in general 2HDM**
  - We calculated $H^\pm \to W^\pm Z$ with the most general setup in 2HDM.
  - We find the decay asymmetry is sensitive to the CP phases.

Back up

WKB method

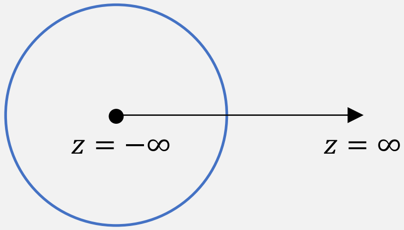
Cline, Joyce, and Kainulainen (2000);
Fromme and Huber (2007); and more

- **Transport equation for chemical potential**

$$C_1\,\mu''(z) + C_2\,\mu'(z) + C_3\,\mu(z) = S_{\mathrm{CPV}}$$

where, $\mu(z) = \mu_\psi - \mu_{\bar{\psi}}$

Bubble



- **By solving Dirac eq. for ψ with WKB approximation, we have**

$$S_{\mathrm{CPV}} = C_4\left(m_\psi^2 \theta_\psi'\right)' + C_5\, m_\psi^2 \theta_\psi' \left(m_\psi^2\right)'.$$

C_i are functions of z, T, m, and v_w.

- **Final BAU:** $$\eta_B \simeq (\mathrm{const.})\, \Gamma_{\mathrm{sph}}^{\mathrm{sym}} \int_0^\infty dz\ \mu_{B_L} e^{-(\mathrm{const.})\, \Gamma_{\mathrm{sph}}^{\mathrm{sym}}\, z}$$

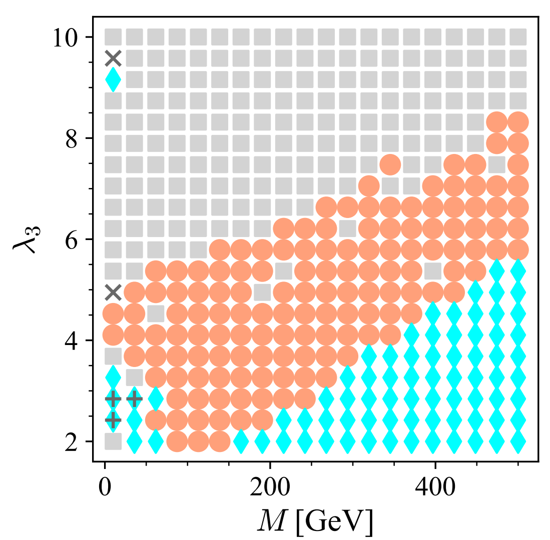
washout

- **Comments on VEV-Insertion-Approximation (VIA) method** Riotto (1996) (1998);
  - BAU evaluated by VIA method tends to be larger than that by WKB method.
  - Main difference: CPV source is derived by SK formalism. Cline and Laurent (2021); Basler (2023); and more
  - Long-term used CPV source (considered as LO) vanishes by correct resummation.

Kainulainen (2021); Postma et al. (2022);

Electroweak phase transition

- Several fates of the vacuum (We used `CosmoTransitions`) Wainwright, Comput. Phys. Commun. 183 (2011)



(a) Pink filled circle: The phase transition from phase A to phase B is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *below* than 1 GeV.

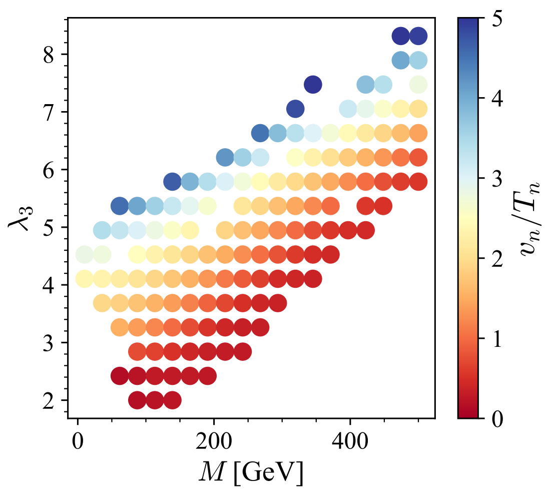
(b) Gray $+$: The phase transition $A \to B$ is first order, B is the vacuum at $T = 0$, B is the electroweak vacuum, and the VEV in A is *larger* than 1 GeV.

(c) Gray $\times$: The phase transition $A \to B$ is first order, B is the vacuum at $T = 0$, and B is *not* the electroweak vacuum.

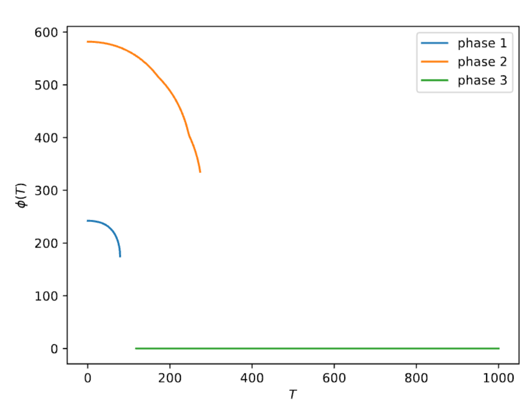
(d) Black dot: The phase transition $A \to B$ is first order, and B is *not* the vacuum at $T = 0$.

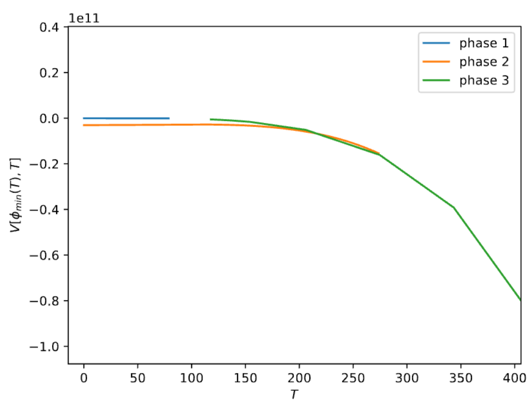
(e) Blue diamond: The phase transition $A \to B$ is second order.

(f) Gray box: No phase transition is returned, including the case of $\Gamma/H^4 < 1$.



- Example of unrealistic phase transition



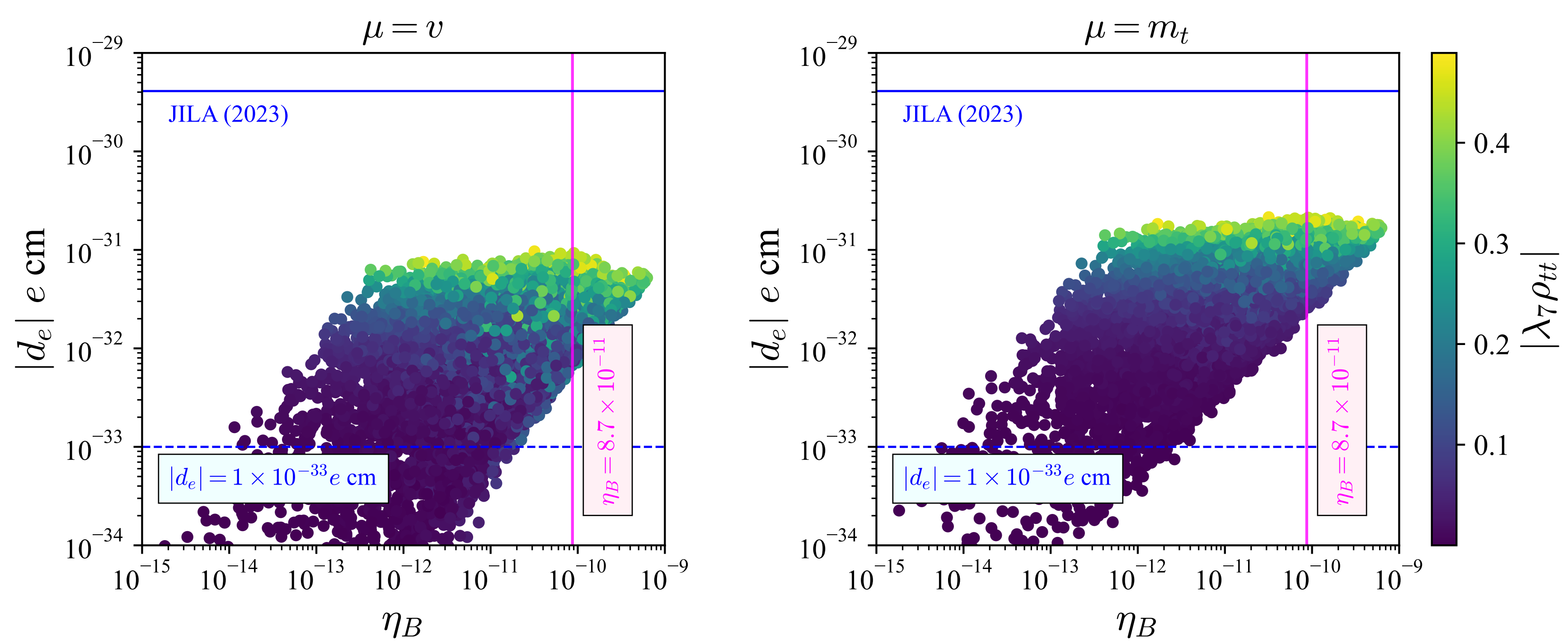


Electron EDM

- Effective Lagrangian $$\mathcal{L}_{\rm eff} = -\frac{1}{\Lambda^2}\left(\frac{g'}{\sqrt{2}}C_{tB}\overline{Q_L}\sigma^{\mu\nu}t_R\tilde{\Phi}_1 B_{\mu\nu} + \frac{g}{\sqrt{2}}C_{tW}\overline{Q_L}\sigma^{\mu\nu}t_R\tau^a\tilde{\Phi}_1 W^a_{\mu\nu} + \text{h.c.}\right),$$
- Electron EDM induced by top EDM
- Matching at Λ $$d_t^{B_\mu} = \frac{g_1 v}{\Lambda^2}\text{Im}[C_{tB}],\quad d_t^{W^3_\mu} = \frac{g_2 v}{\Lambda^2}\text{Im}[C_{tW}].$$

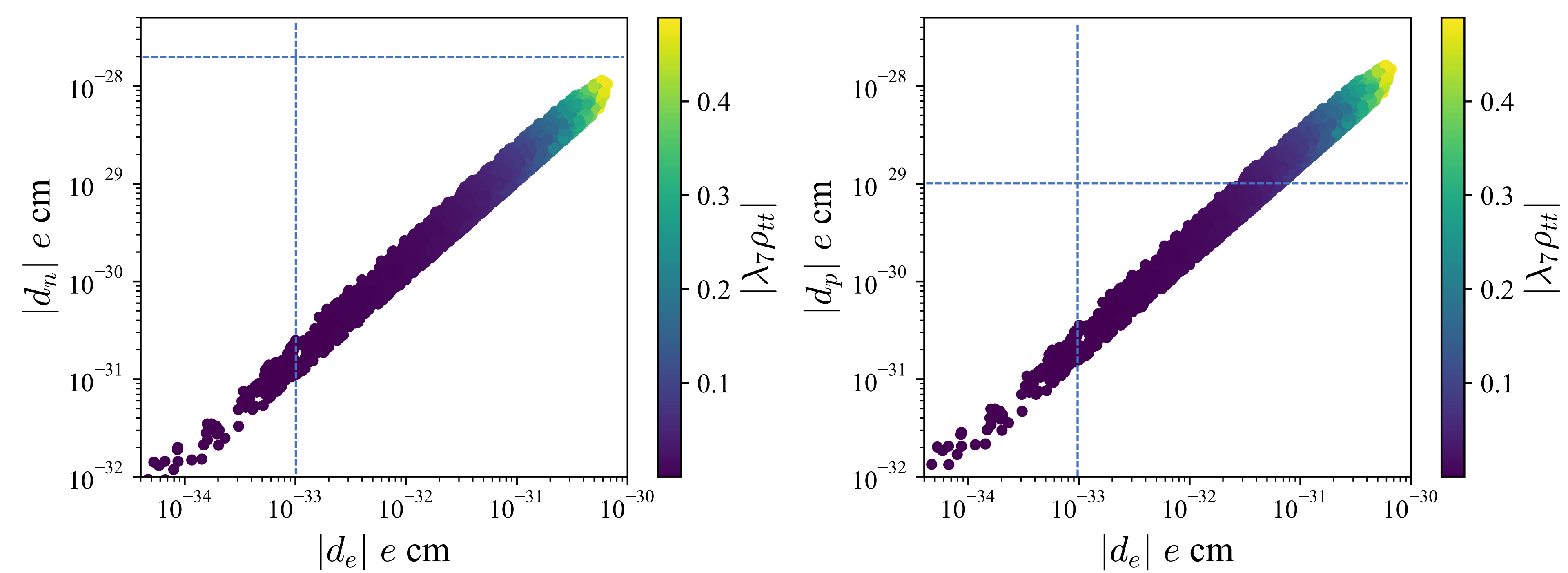
$$d_e = -\frac{e}{2v}\left(\frac{v}{\Lambda}\right)^2\left(\log\frac{\Lambda}{\mu}\right)^2\left[(A_e - D_e)\text{Im}[C_{tB}] + (B_e - E_e)\text{Im}[C_{tW}]\right],$$

Fuyuto and Ramsey-Musolf (2017)



Correlation among EDMs

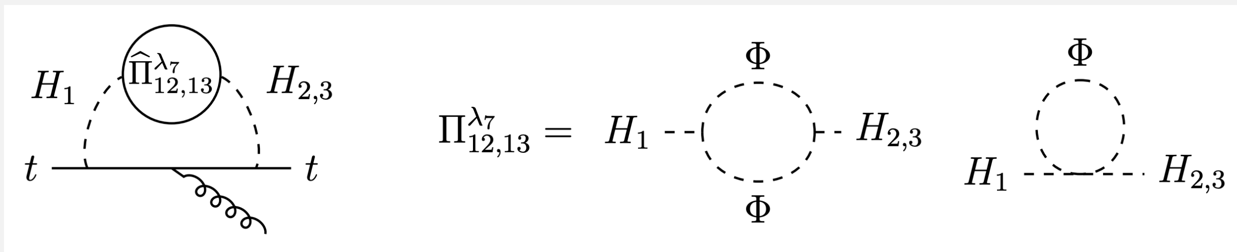
- **Strong correlation among EDMs (dashed: future prospect bounds)**



- **All of CPV quantities are correlated by** $\mathrm{Im}[\lambda_7 \rho_{tt}]$**.**
  ⇨Characteristic prediction of our scenario

Renormalization

- **UV divergence in mixing-self-energy diagrams**



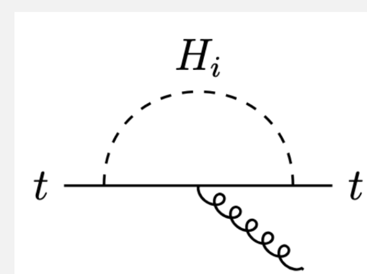
- **Effective potential renormalization**

$$\left.\frac{\partial V}{\partial \varphi_i}\right|_{\varphi=v_{\mathrm{EW}}} = 0, \quad \left.\frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j}\right|_{\varphi=v_{\mathrm{EW}}} = \mathcal{M}_{ij} \quad \Rightarrow \quad \begin{array}{l} \Gamma_i^{(1)}(p^2=0) + \delta\Gamma_i^{(1)} = 0 \\ \Gamma_{ij}^{(2)}(p^2=0) + \delta\Gamma_{ij}^{(2)} = 0 \end{array}$$

- **Other renormalization schemes?** e.g.) $\overline{\mathrm{MS}}$ scheme $d_t^{\overline{\mathrm{MS}},(2)} = d_t^{\mathrm{EP},(2)} + \Delta d_t^{(2)}$ (?)
  - Scheme conversion $\lambda_6^{\overline{\mathrm{MS}},(1)} = \lambda_6^{\mathrm{EP},(1)} + \Delta\lambda_6^{(1)}$

    ⇨ $\lambda_6^{\mathrm{EP},(1)} = 0$ does not mean $\lambda_6^{\overline{\mathrm{MS}},(1)} = 0$.
  - Finally, $d_t^{\overline{\mathrm{MS}},(2)} = d_t^{\mathrm{EP},(2)} + \Delta d_t^{(2)} \underline{- \Delta d_t^{(2)}} + O(\hbar^3)$

    from one-loop diagram

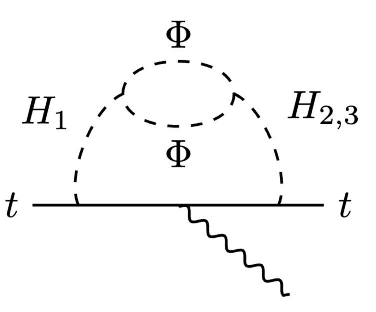


Scheme dependence

- Renormalization scheme for effective potential (EP scheme)

$$\left.\frac{\partial V^{\rm eff}_{T=0}}{\partial \varphi_i}\right|_{\varphi=\varphi_{vac}} = 0, \quad (i=1,2,3),$$
$$\left.\frac{\partial^2 V^{\rm eff}_{T=0}}{\partial \varphi_i \partial \varphi_j}\right|_{\varphi=\varphi_{vac}} = \mathcal{M}^2_{ij}, \quad (i,j=1,2,3),$$

make

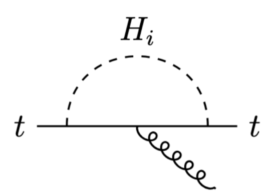


UV finite.

- Scheme difference (MS bar and EP scheme)

$$\tilde{d}_t^{(2),\overline{\rm MS}} = \tilde{d}_t^{(2)} + \Delta \tilde{d}_t^{(2),\overline{\rm MS}},$$
$$\Delta \tilde{d}_t^{(2),\overline{\rm MS}} = \frac{{\rm Im}[\lambda_7 \rho_{tt}]}{\sqrt{2}} \frac{3\lambda_3 v}{(16\pi^2)^2} \frac{2m_t^2}{m_\Phi^2 - m_{H_1}^2} \Big(C_{11}[\Phi,t,t] - C_{11}[H_1,t,t] \Big) \log \left.\frac{\mu^2}{m_\Phi^2}\right|_{\overline{\rm MS}},$$

- Scheme conversion and one-loop EDM



$$\lambda_6^{\overline{\rm MS}} = \lambda_6 - \frac{3}{16\pi^2}\lambda_3\lambda_7 \log\frac{\mu^2}{m_\Phi^2} + \ldots \Big|_{\rm EP}$$

causes

$$\tilde{d}_t^{(1),\overline{\rm MS}} = \frac{{\rm Im}[\lambda_6 \rho_{tt}]}{\sqrt{2}} \frac{v}{16\pi^2} \frac{2m_t^2}{m_\Phi^2 - m_{H_1}^2} \Big(C_{11}[\Phi,t,t] - C_{11}[H_1,t,t] \Big)\Big|_{\overline{\rm MS}}$$

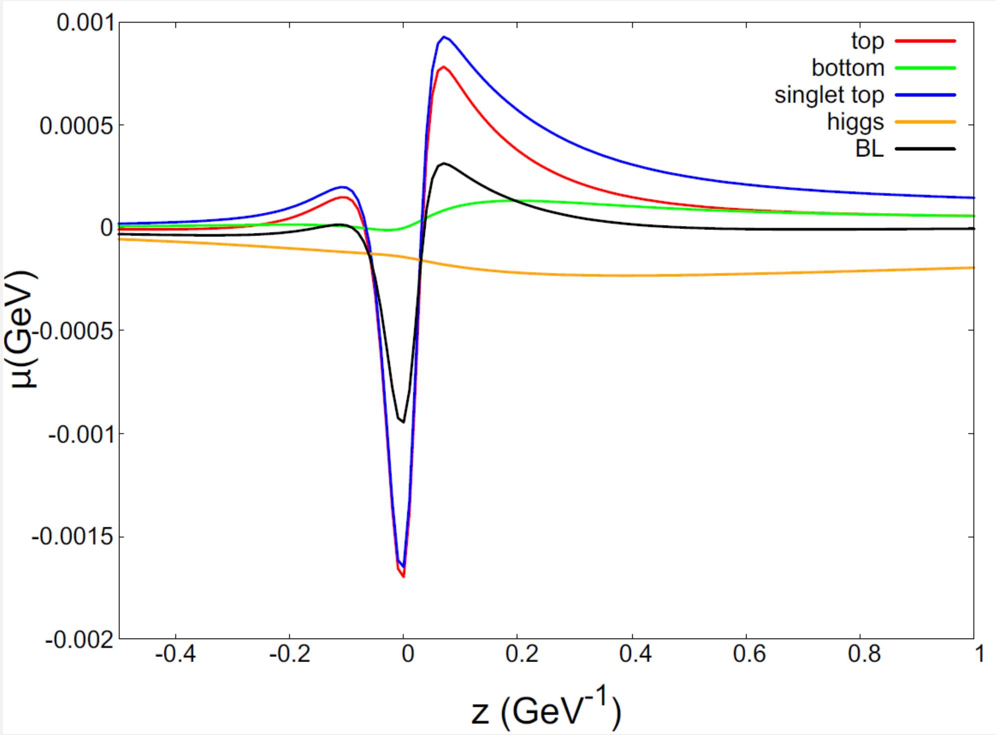
- Consequently, we have $\tilde{d}_t^{(1),\overline{\rm MS}} + \tilde{d}_t^{(2),\overline{\rm MS}} = \tilde{d}_t^{(2)} + \mathcal{O}(\hbar^3)\Big|_{\rm EP}$ and there are no scale dependence.

Estimation of baryon asymmetry

- Boltzmann equation with perturbations from thermal equilibrium

$$(\partial_t + \boldsymbol{v}_g \cdot \partial_{\boldsymbol{x}} + \boldsymbol{F} \cdot \partial_{\boldsymbol{p}}) f_i = C[f_i, f_j, \ldots] \qquad f_i = \frac{1}{e^{\beta[\gamma_w(E_i + v_w p_z) - \mu_i]} \pm 1} + \delta f_i$$

- Chemical potentials (particle – anti-particle)



- μ_{B_L} affects rate of $\Delta B \neq 0$ process

Cline, Joyce and Kainulainen, JHEP 07 (2000)

In plasma flame

$$\frac{\partial n_B}{\partial t} = \frac{3}{2}\Gamma_{\rm sph}\left(\frac{3\mu_{B_L}}{T} - \frac{A}{T^3}n_B\right)$$

Integrated in wall flame

$$\eta_B \equiv \frac{n_B - n_{\overline{B}}}{s} = \frac{405\Gamma_{\rm sph}}{4\pi^2 v_w g_* T}\int_0^\infty dz\ \mu_{B_L} e^{-({\rm const.})\Gamma_{\rm sph} z}$$

CP violating bubble

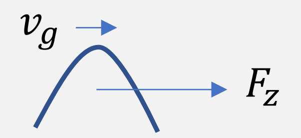
- "Semi classical force approach" (WKB method)

Cline, Joyce and Kainulainen, JHEP 07 (2000);
Cline and Kainulainen Phys. Rev. D 101 (2020)

$$\left(i\partial_\mu\gamma^\mu - m(z)P_L - m^*(z)P_R\right)\psi = 0$$

$$v_g = \frac{p_z}{E} \pm (\partial_z\theta \text{ corrections})$$

$$F_z = -\frac{\partial_z|m^2|}{2E} \pm (\partial_z\theta \text{ corrections})$$

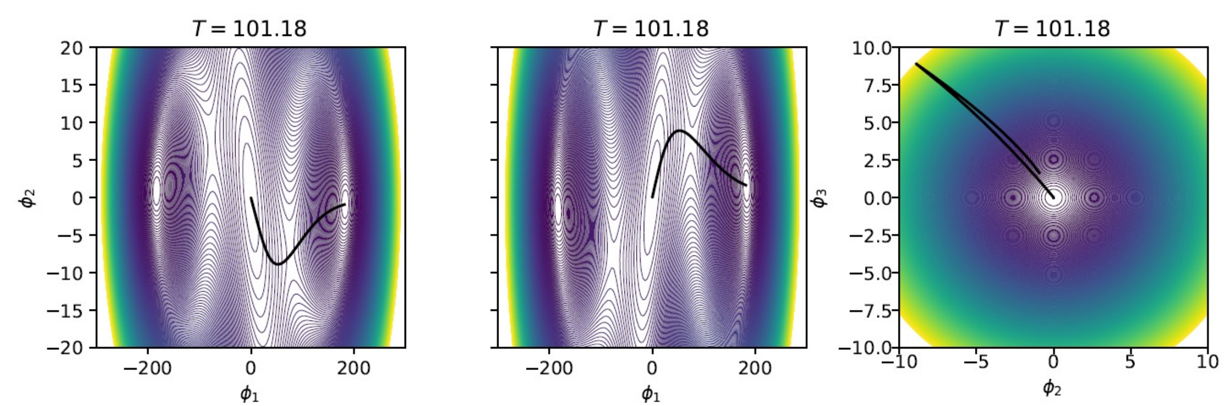


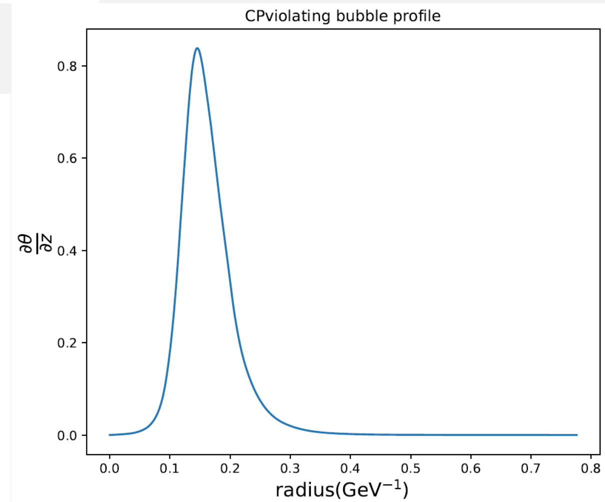
WKB wave packet

$$m_t^2 = \frac{1}{2}\Big(y_t^2\varphi_1^2 + |\rho_{tt}|^2(\varphi_2^2+\varphi_3^2) + 2y_t|\rho_{tt}|\varphi_1\big(\varphi_2\cos\theta_{tt} + \varphi_3\sin\theta_{tt}\big)\Big),$$

$$\theta_t' = \frac{1}{2m_t^2}\left\{y_t|\rho_{tt}|\Big((\varphi_3\varphi_1' - \varphi_1\varphi_3')\cos\theta_{tt} + (\varphi_1\varphi_2' - \varphi_2\varphi_1')\sin\theta_{tt}\Big) + |\rho_{tt}|^2(\varphi_3\varphi_2' - \varphi_2\varphi_3')\right\}$$
$$+ \frac{1}{\varphi_1^2+\varphi_2^2+\varphi_3^2}(\varphi_3\varphi_2' - \varphi_2\varphi_3'), \qquad (3.10)$$

- Order parameters $\varphi_1, \varphi_2, \varphi_3$ (Black lines: path of PT)



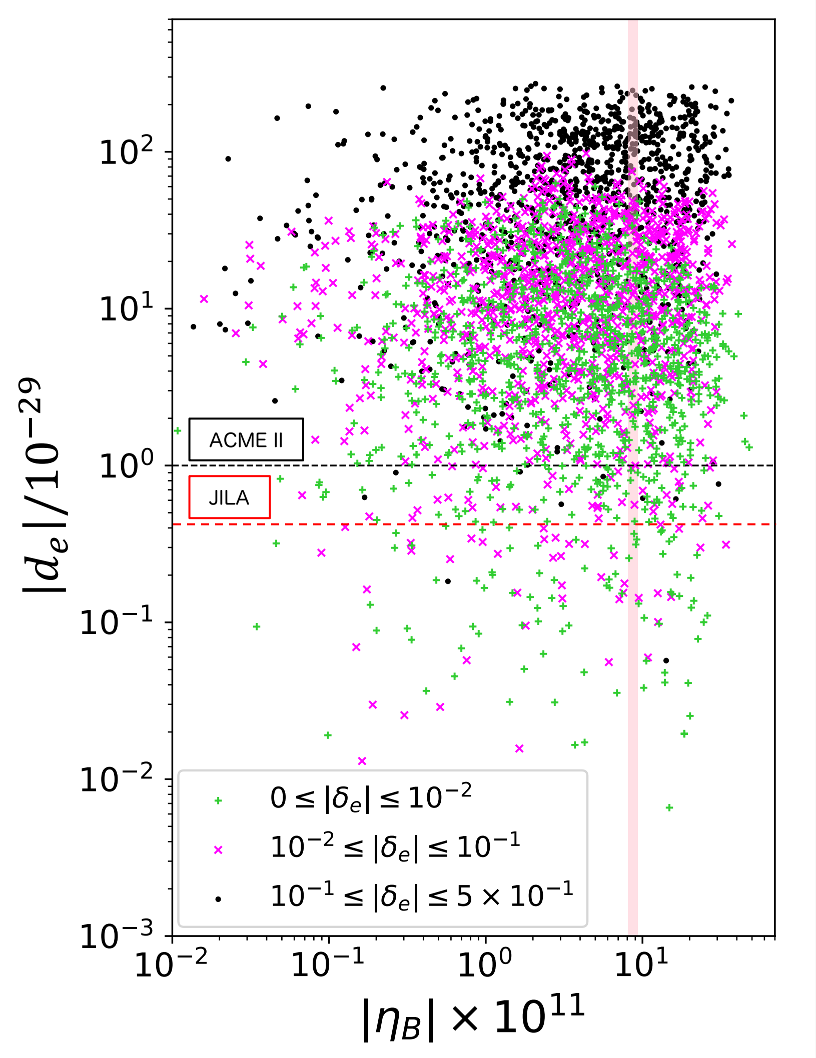


- Large φ_2, φ_3 during PT are needed to enhance BAU.

With Yukawa alignment

Enomoto, Kanemura and YM, JHEP 09 (2022) 121

$$\lambda_2 = 0.1,\ m_\Phi = 350\ \mathrm{GeV},\ M = 30\ \mathrm{GeV},\ v_w = 0.1,$$
$$\theta_u = \theta_d = [0, 2\pi),\ |\zeta_d| = |\zeta_e| = [0, 10],\ |\lambda_7| = [0.5, 1.0],\ \theta_7 = [0, 2\pi).$$



These points are allowed from various constraints.

Fermion loop contributions are proportional to $|\zeta_u||\zeta_e|\sin\delta_e$.
$(\delta_e \equiv \theta_u - \theta_e)$

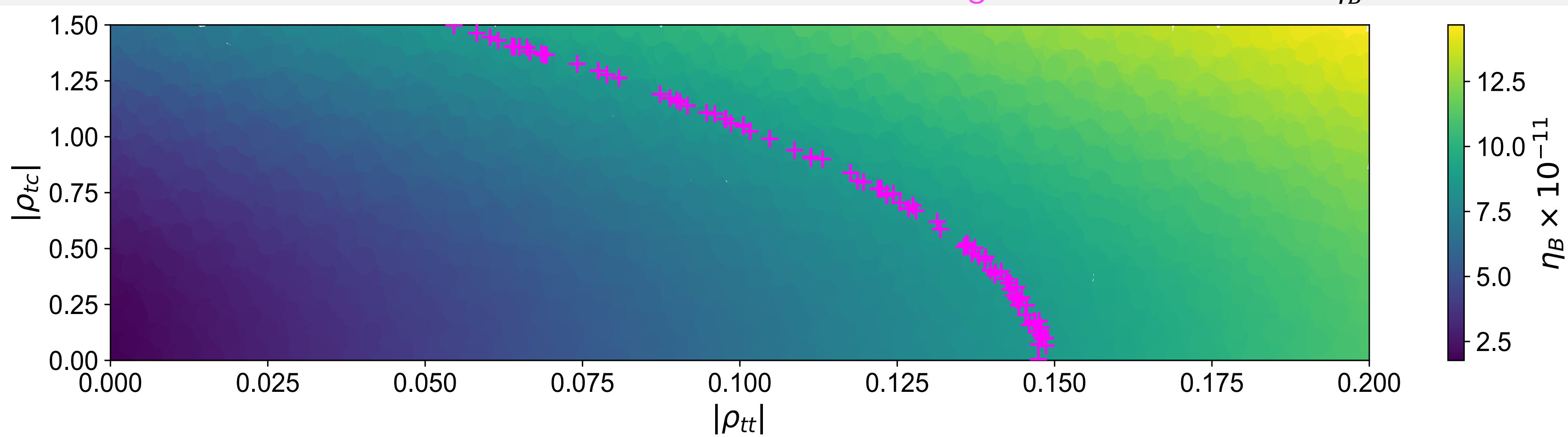
Many points are satisfied from eEDM data and they generate sufficient BAU.

Top-charm transport scenario

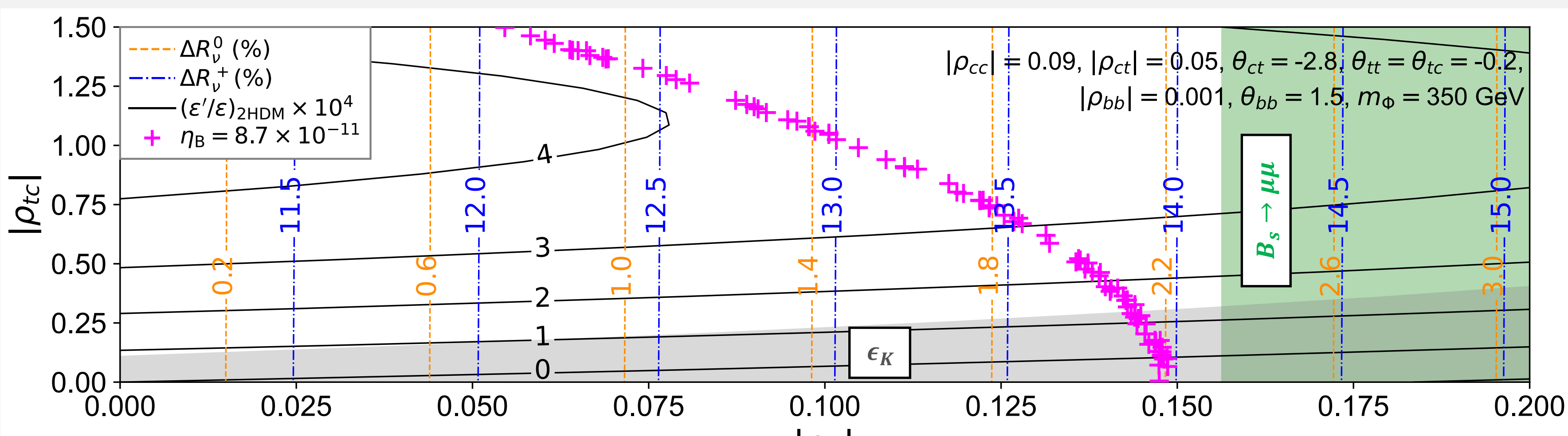
Kanemura and YM, JHEP 09 (2023) 153

magenta : observed BAU $\eta_B = 8.7\times10^{-11}$

Top-charm transport



Top transport



$H^{\pm}W^{\mp}Z$ vertex

- Charged Higgs $H^{\pm}$ are introduced in some classes of extended Higgs models.

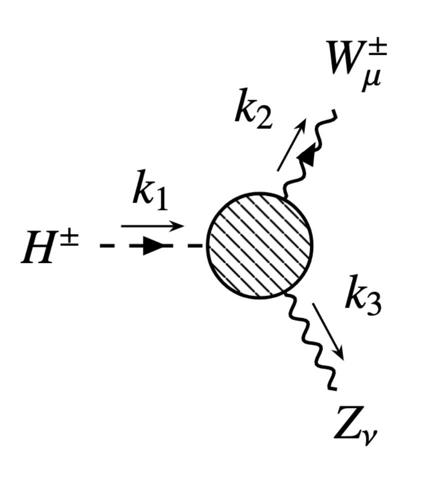
- An important vertex : $H^{\pm}W^{\mp}Z$ vertex
  - Consequence of custodial symmetry violation in 2HDM
  - At tree level

    Georgi-Machacek model, etc

    Georgi and Machacek (1985);
    Chanowitz and Golden (1985);
  - Beyond tree level

    Minimal Supersymmetric Standard Model
    CP conserving Two Higgs doublet model

    Mendez and Pomarol (1991); Kanemura (2000); and more



- Even at one loop order, effects from $H^{\pm}W^{\mp}Z$ vertex are enhanced.

What is new ?

Kanemura and YM, JHEP 10 (2024) 041

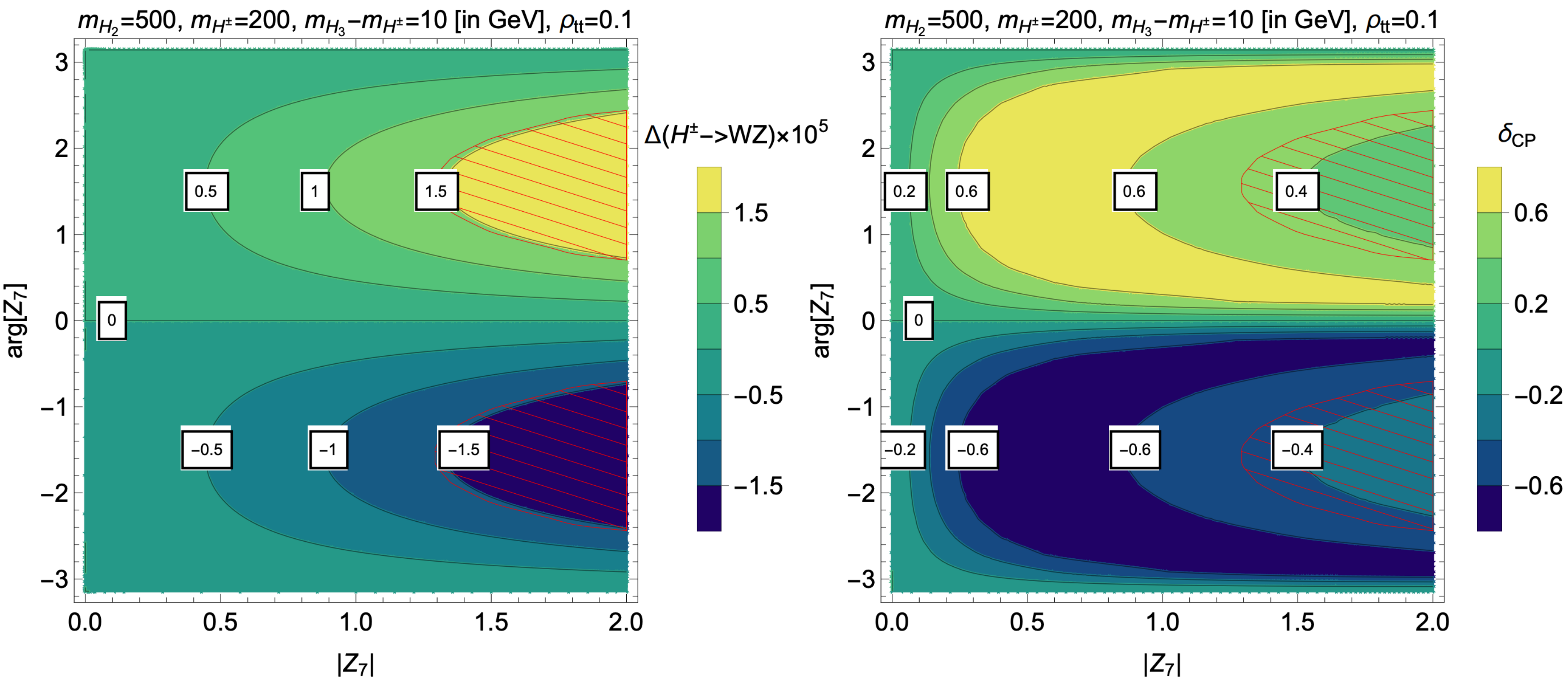
- Giving full formulae at one-loop level in the most general 2HDM

- As a probe of the CP violation in the general 2HDM

Testing CP violation

Kanemura and YM, JHEP 10 (2024) 041

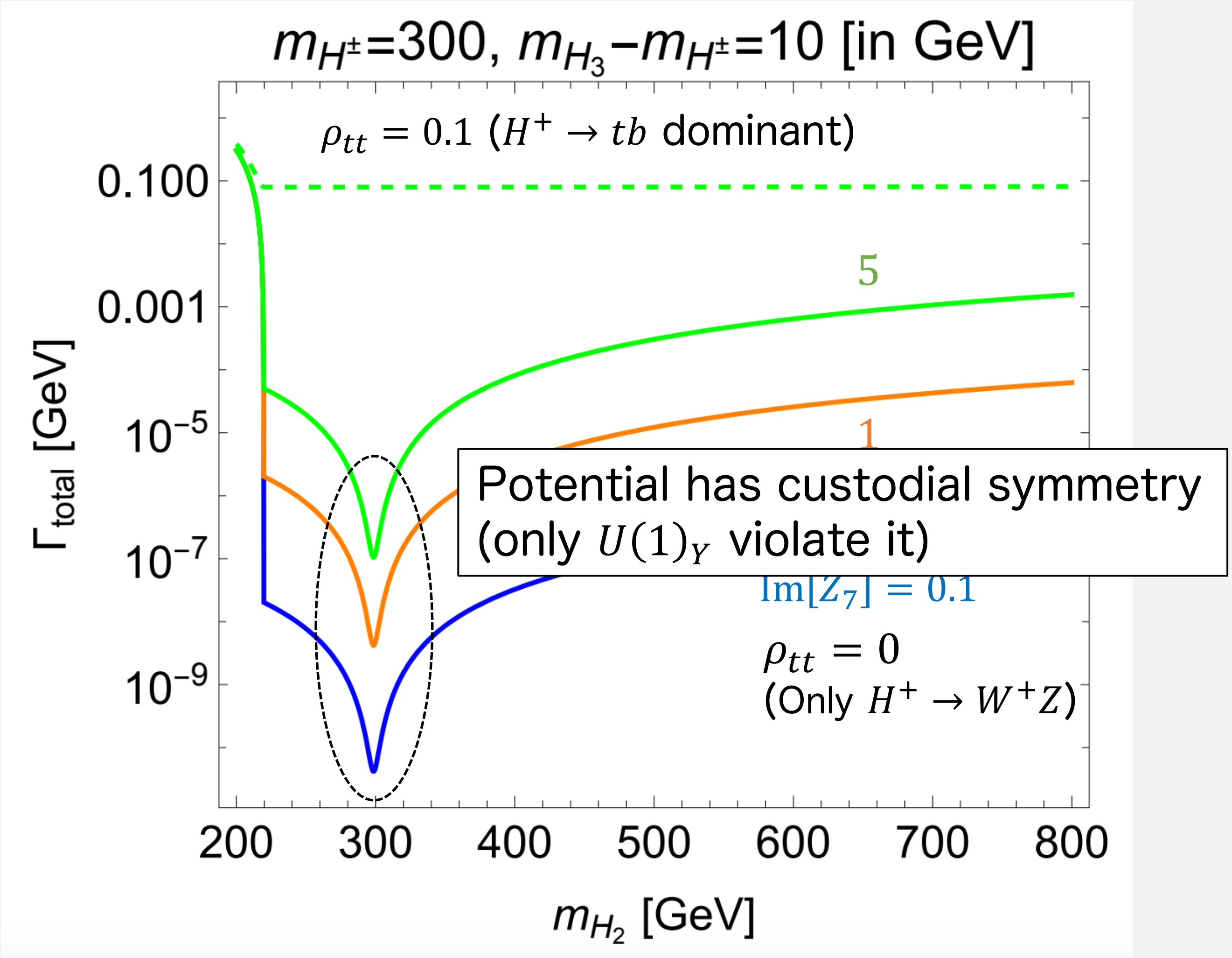
Left: $\Delta \equiv \Gamma(H^+ \to W^+Z) - \Gamma(H^- \to W^-Z)$ Right: $\delta_{CP} \equiv \dfrac{\Delta}{\Gamma(H^+ \to W^+Z) + \Gamma(H^- \to W^-Z)}$



- At $\delta_{CP} \simeq 0.6$, $\Gamma(H^- \to W^-Z) \simeq \Gamma(H^+ \to W^+Z)/4$ is shown by definition

Decay rate for $H^{\pm} \to W^{\pm}Z$

- Non-zero $\mathrm{Im}[Z_7]$ and $Z_4 + Z_5 \propto m_{H_2}^2 - m_{H^{\pm}}^2$ case

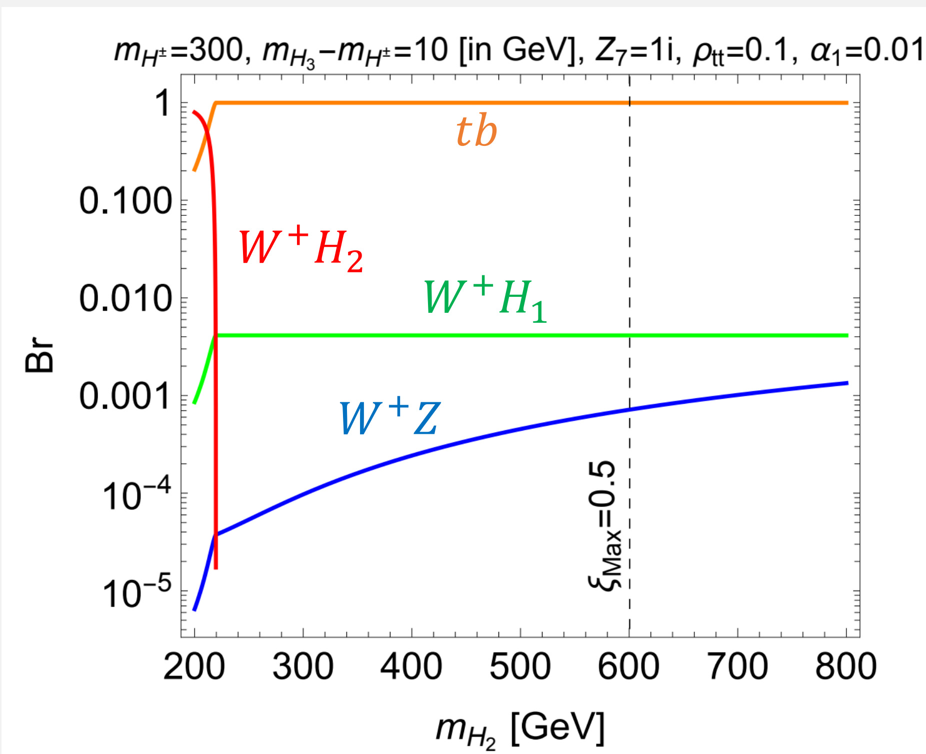


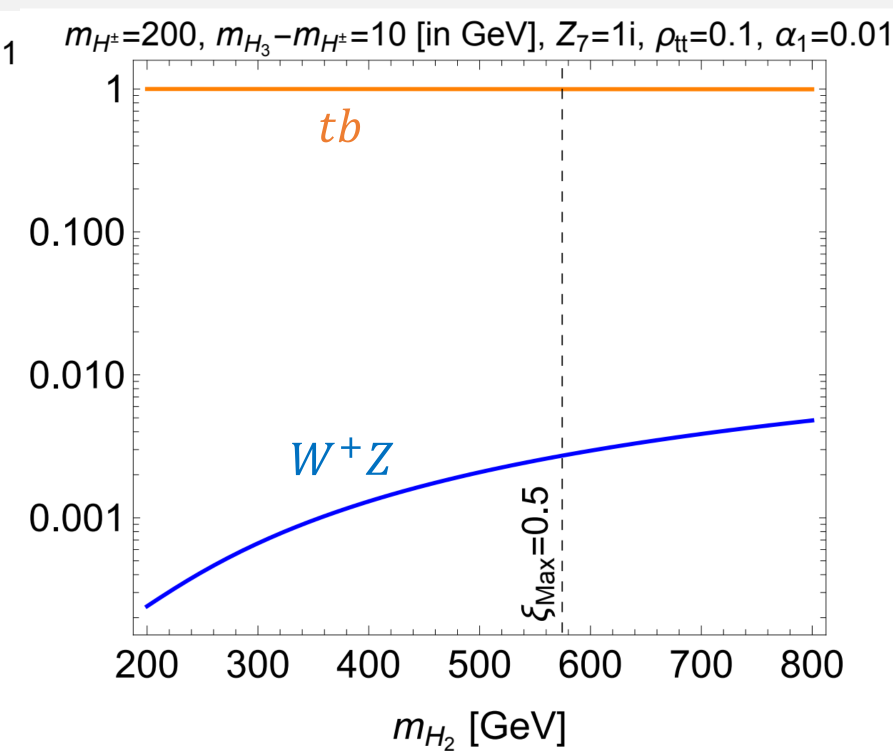
Branching ratio

cf.) Mixing angle

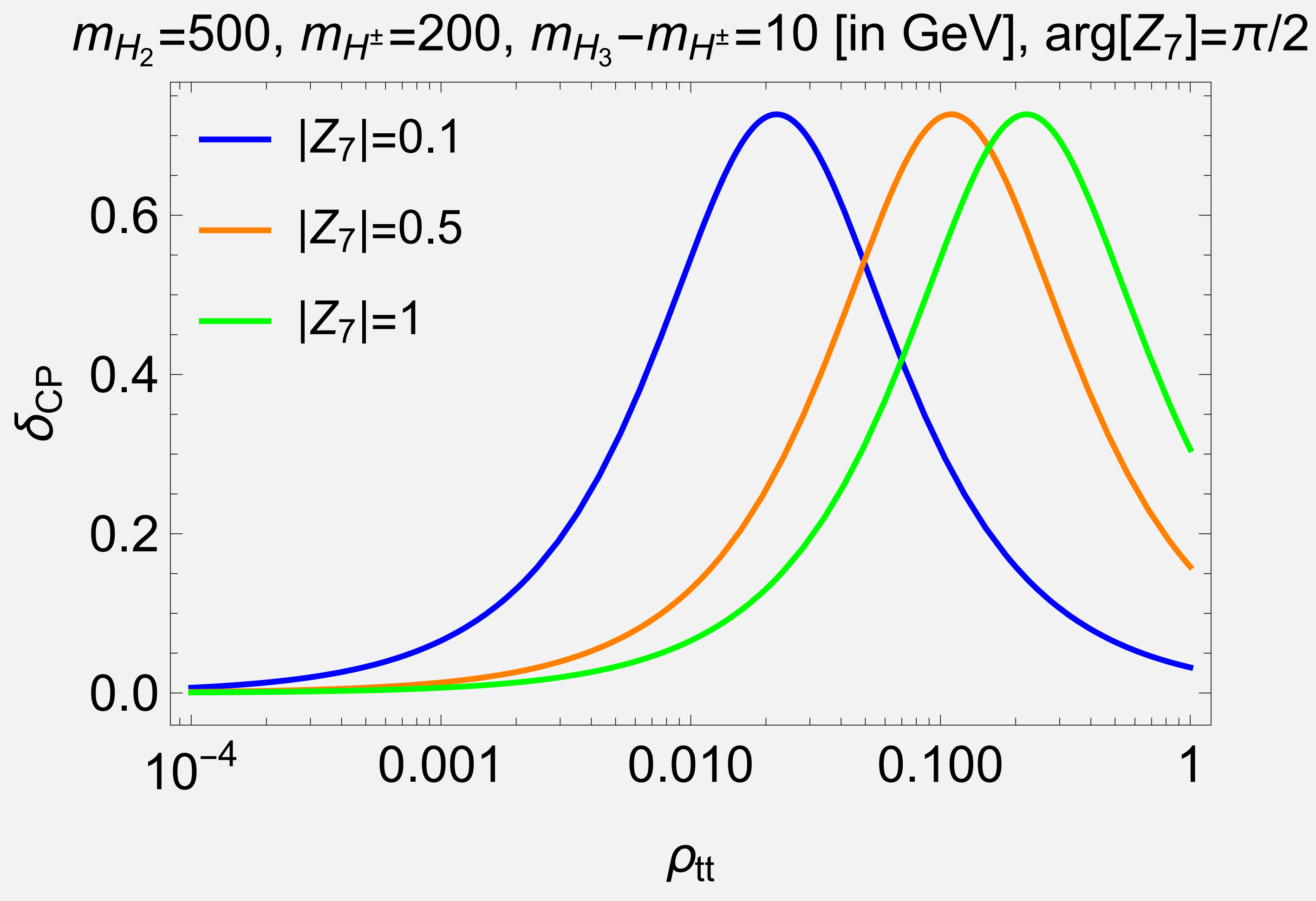
$$H_1 = h_1 \cos\alpha_1 - h_2 \sin\alpha_1$$
$$H_2 = h_1 \sin\alpha_1 + h_2 \cos\alpha_1$$

- **Branching ratio for $H^+ \to XY$ ($\rho_{ij} = 0$ except for ρ_{tt})**
  - Custodial symmetry violation $\propto m_{H_2} - m_{H^\pm}$
  - For $m_{H^\pm} < m_{H_2}$, main modes are $H^\pm \to tb, WZ, WH_1$
  - If $m_{H^\pm} < m_W + m_{H_1}$, $\mathrm{Br}(H^\pm \to W^\pm Z)$ can be efficiently large





δ_{CP} as a function of Z_7 and ρ_{tt}



$\overline{\text{MS}}$ beta function and threshold correction

- **Connecting $\overline{\text{MS}}$ parameters and observables** $\lambda^0 = \lambda^{\overline{\text{MS}}} - \delta\lambda^{\overline{\text{MS}}} = \lambda_{\text{OS}} - \delta\lambda_{\text{OS}}$

Matching condition $\lambda^{\overline{\text{MS}}}(\mu) = \lambda_{\text{OS}} - \delta\lambda_{\text{OS}} + \delta\lambda^{\overline{\text{MS}}} = \lambda_{\text{OS}} - \delta\lambda_{\text{OS}}\Big|_{\text{fin}} + \Delta\lambda$

Tree | ≧ 1 loop (log form) | ≧ 2 loop

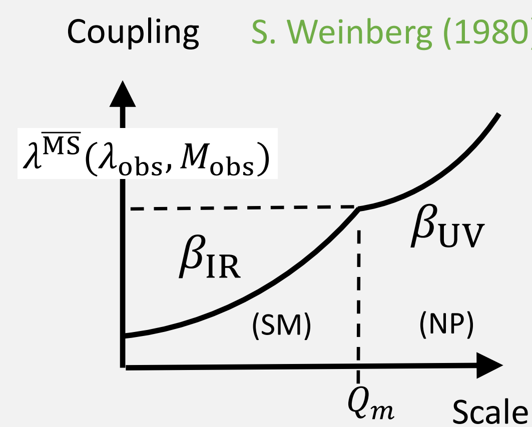
Using this relation, let $\lambda^{\overline{\text{MS}}}(\mu = Q_m)$ match observables (Usually, $Q_m \simeq \text{mass}$)

Ex) Tree level matching condition $\lambda^{\overline{\text{MS}}}(Q_m) = \lambda_{\text{OS}}$

$\beta(\lambda, Q) = \beta_{\text{IR}}(\lambda) + \theta_{\text{step}}(Q/Q_m)\,\beta_{\text{UV}}(\lambda)$

Q: Energy scale
Q_m: Matching scale

S. Weinberg (1980)

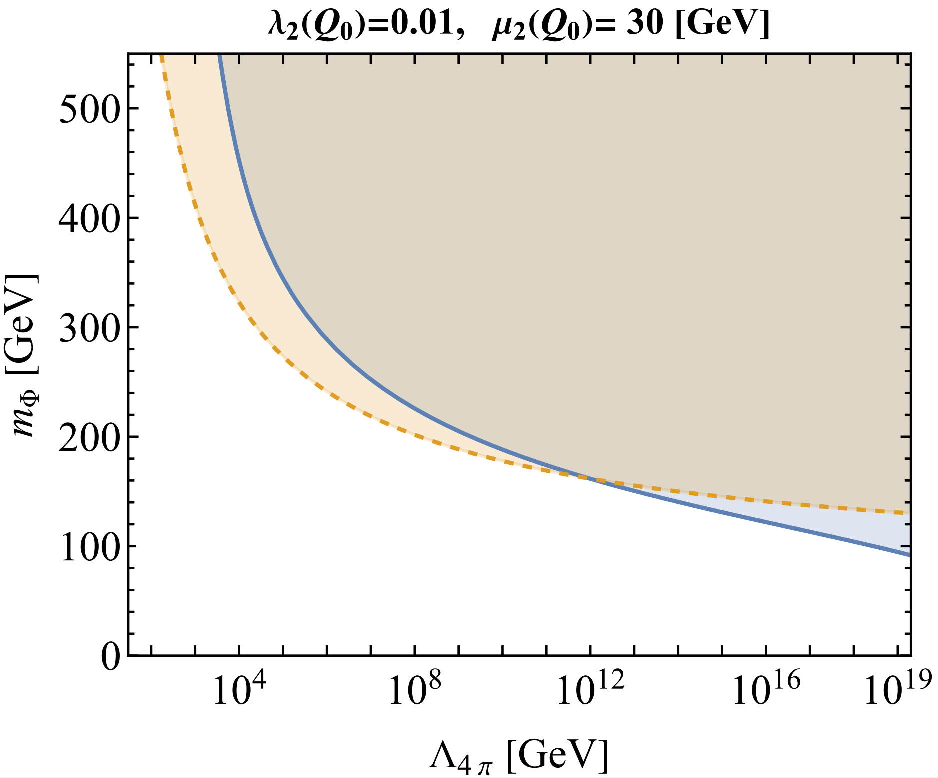


- **Less care has been taken in studies for EWBG**
  - It suffers higher order threshold corrections
  - In principle, we only know appropriate value of Q_m

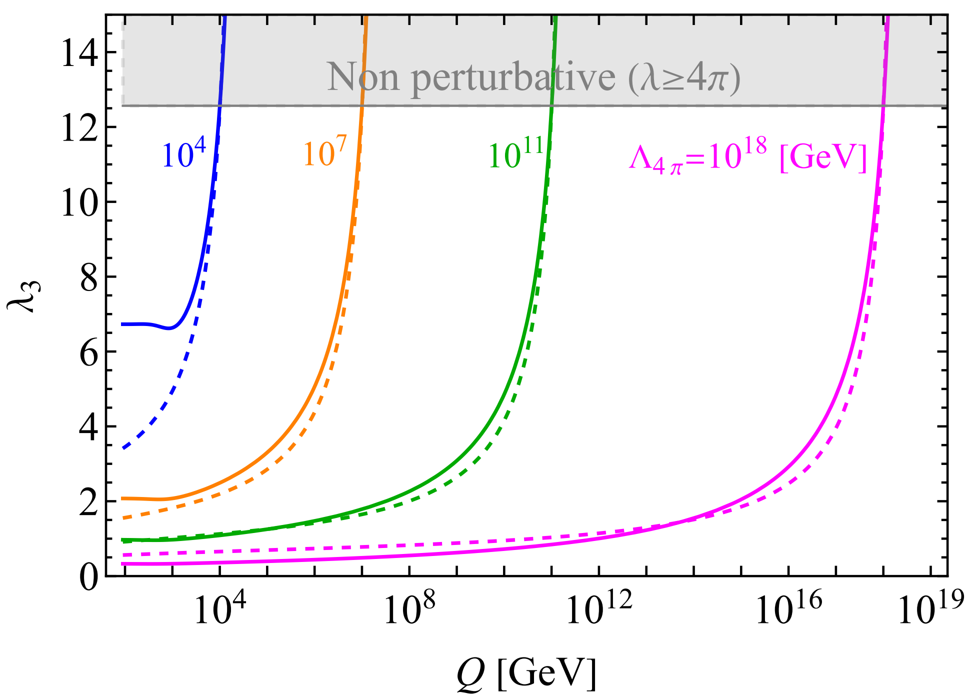
- **Physical scheme without threshold uncertainty will be introduced to evaluate perturbative region of EWBG models**

Triviality bound

- **Triviality bounds : Upper bound of the mass respect to $\Lambda_{4\pi}$**
- **Analysis in Inert doublet model**



Kanemura and YM, PRD 110 (2024) 7, 075016

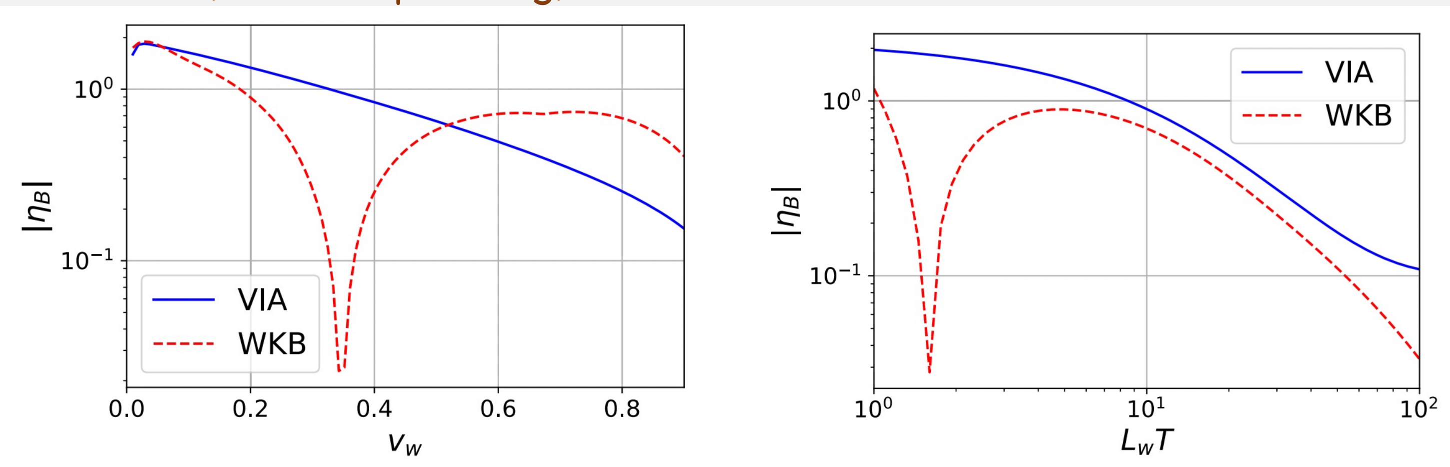


Discrepancy between WKB and VIA

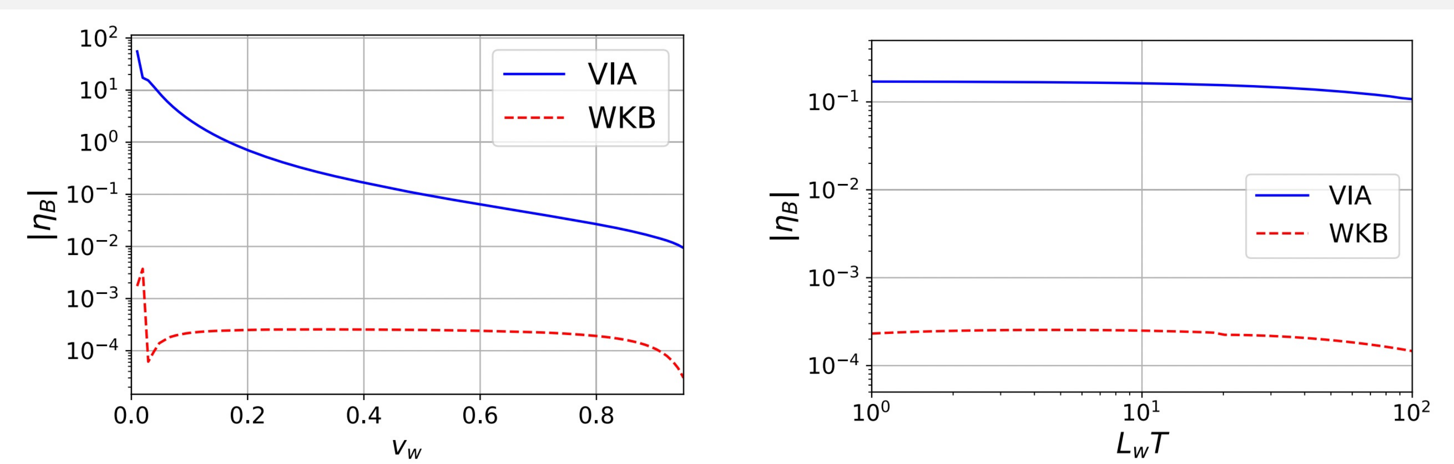
- Each method provides different results. Cline and Laurent (2021)

v_w : wall velocity
L_w : wall width

Quark (charm-top mixing) case



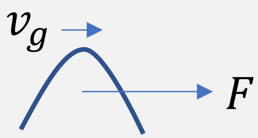
Lepton (tau) case



Discrepancy between WKB and VIA

- **WKB method**

Cline, Joyce and Kainulainen (2000)



WKB wave packet

Boltzmann eq.

$$(\partial_t + \boldsymbol{v}_g \cdot \partial_{\boldsymbol{x}} + \boldsymbol{F} \cdot \partial_{\boldsymbol{p}}) f_i = C[f_i, f_j, \ldots]$$

$$v_g = \frac{p}{E} \pm (\text{from wall})$$
$$F = (\text{from wall}) \pm (\text{correction})$$

Group velocity v_g and force F are derived from Dirac or Klein-Gordon equation.

- **VIA method** A. Riotto (1995), (1997), (1998)

Schwinger-Dyson eq. $$-\left(\partial_u^2 + m^2\right) i\Delta(u,v) = i\delta^4(u-v) + \int d^4w \, \Pi(u,w) i\Delta(w,v)$$

Self energy contains CPV with VEV insertion approx.

Quantum diffusion eq. $$\partial_\mu j^\mu = \partial_t n + \nabla \cdot \boldsymbol{j} = (Source)$$

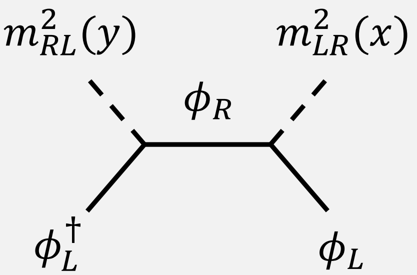
$$\boldsymbol{j} \simeq -D\nabla n \qquad \text{(Fick's low)}$$

Usual VEV insertion approximation

- Perturbation about space dependent off-diagonal mass

Self energy at 2nd order

$$\Pi^{(2)}_{LL} \supset$$



$$= \; -m^2_{LR}\, G_{RR}\, m^2_{RL}$$

- CP violating source term in usual VIA

$$g(x,y) = m^2_{LR}(x) m^2_{RL}(y)$$

$$S^{(2)}_{LL}(x) = \int d^4y \Big(\mathrm{Re}[g(x,y)+g(y,x)]\mathrm{Re}[G^{<}_{RR}(x,y)G^{>}_{LL}(y,x) - G^{>}_{RR}(x,y)G^{<}_{LL}(y,x)]$$
$$-\mathrm{Im}[g(x,y)-g(y,x)]\mathrm{Im}[G^{<}_{RR}(x,y)G^{>}_{LL}(y,x) - G^{>}_{RR}(x,y)G^{<}_{LL}(y,x)]\Big)$$

CP conserving part ⇨ relaxation term

CP violating part ⇨ source term

$$S^{(2)}_{LL}(x) \supset S^{(2)}_{\mathrm{CPV}} = 2\mathrm{Im}[m^2_{LR}\partial_\mu m^2_{RL}]$$
$$\times \int d^4y\, (y-x)^\mu \mathrm{Im}[G^{<}_{RR}(x,y)G^{>}_{LL}(y,x) - G^{>}_{RR}(x,y)G^{<}_{LL}(y,x)]$$

with further approximation: $m^2_{IJ}(y) = m^2_{IJ}(x) + (x-y)^\mu \partial_\mu m^2_{IJ}(x) + O(\partial^2)$

KMS relation

$\boldsymbol{M^2}$ ⇨ mass matrix including background field interaction
$\boldsymbol{\Pi}$ ⇨ thermal self energy
$\boldsymbol{G}$ ⇨ Green functions obtained from constraint eq.

- In thermal equilibrium and single flavor system, Wightman functions and self energies satisfy

$$(1+n)G^{<} = nG^{>}, \ \ (1+n)\Pi^{<} = n\Pi^{>}, \qquad \text{where,} \quad n = \frac{1}{e^{(E-\mu)/T}-1} \ .$$

(bosonic case of Kubo-Martin-Schwinger relation)

- We assume self energies satisfy this relation and are diagonal even in two flavor system.

- At the 0th order in VIA (free for the background field), Green functions can be obtained from constraint eq. such as

$$G^{>}_{(0),IJ} = (n_I+1)\rho_{(0),I}\delta_{IJ}, \qquad (I, J \text{ is flavor indices})$$

$$G^{<}_{(0),IJ} = n_I\rho_{(0),I}\delta_{IJ}, \qquad \text{where,} \quad \rho_{(0),I} = \frac{\gamma_I}{(k^2-m_I^2-\Pi^h)^2-\gamma_I^2/4}, \quad \gamma_I = -4ik_0\Gamma_I$$

spectral function ... thermal width

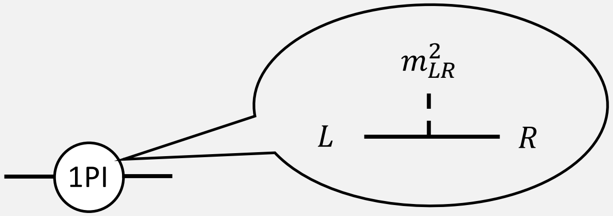
Of course, we can find these 0th order Wightman functions satisfy KMS relation.

2^{nd} order in VIA

- Green function at 1^{st} and 2^{nd} order in VIA

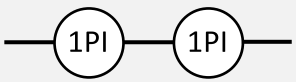
$$G^{ab}_{(1),IJ} = c\sum_{c} G^{ac}_{(0),II}\, m^2_{IJ}\, G^{cb}_{(0),JJ},$$

(off diag.)



$$G^{ab}_{(2),IJ} = cd\sum_{cd} G^{ac}_{(0),II}\, m^2_{IJ}\, G^{cd}_{(0),JJ}\, m^2_{JI}\, G^{db}_{(0),II}.$$

(diag.)



$(a, b, c, d = \pm)$

- Source term, for example diagonal component, is

$$M^2 = M_d^2 + \delta M^2$$

(diag.) + (off diag.)

$$\begin{aligned}\overline{S}^{(2)}_{LL} =& \underline{[\delta M^2, G^>_{(1)} + G^<_{(1)}]_{LL}} + \underline{[M_d^2, G^>_{(2)} + G^<_{(2)}]_{LL}} \\ & \underline{+[\Pi^> + \Pi^<, G^h_{(2)}]_{LL}} + \underline{\left(\{\Pi^>, G^<_{(2)}\} - \{\Pi^<, G^>_{(2)}\}\right)_{LL}}\end{aligned}$$

$= 0$ ($\because$ both are diagonal)

$$= \underline{2|m_{LR}|^4 \rho_{(0),L}\rho_{(0),R}(n_L - n_R)} - \underline{2|m_{LR}|^4 \rho_{(0),L}\rho_{(0),R}(n_L - n_R)} = 0.$$

- The source term of right and off diagonal components also vanish.

Full order in VIA

- We can solve constraint equations exactly at leading order in derivative expansion.

$$k^2 G^\lambda = \frac{1}{2}\Big(\{M^2, G^\lambda\} + \{\Pi^\lambda, G^h\} + \frac{1}{2}\big([\Pi^>, G^<] - [\Pi^<, G^>]\big)\Big),$$

$$k^2 G^t = 1 + \frac{1}{2}\Big(\{M^2 + \Pi^t - \Pi^h, G^t\} - \Pi^< G^> - G^< \Pi^>\Big).$$

Solution $$G^\lambda_{LL} = \frac{\gamma_R \gamma_L}{\mathcal{D}_+ \mathcal{D}_- \rho_{(0),R}}\Big(g^\lambda_L + g^\lambda_R \frac{\rho_R}{\gamma_L}|m_{LR}|^4\Big),$$

$$G^\lambda_{LR} = \frac{m^2_{LR}}{\mathcal{D}_+ \mathcal{D}_-}\Big(\gamma_R g^\lambda_R (k^2 - m^2_L) + \gamma_L g^\lambda_L (k^2 - m^2_R) + \frac{1}{2}\gamma_R \gamma_L (g^\lambda_R - g^\lambda_L)\Big).$$

- By substituting these into kinetic equation, we can find source term exactly vanishes: $\overline{\boldsymbol{S}} = \boldsymbol{0}$.

- In conclusion, VIA source does not appear <u>at the leading order in derivative expansion</u>.
($\boldsymbol{\partial_x} \ll \boldsymbol{k}$)

Possibility

- Thermal corrections possibly provide off diagonal element of self energy.
- We have used $\boldsymbol{e^{-i\diamond} \to 1}$, and next to leading order correction has not been calculated yet.

⇨ Being key to solve discrepancy between WKB and VIA ?