Pion production in neutrino collisions

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Autumn, 2025

Why do we care?

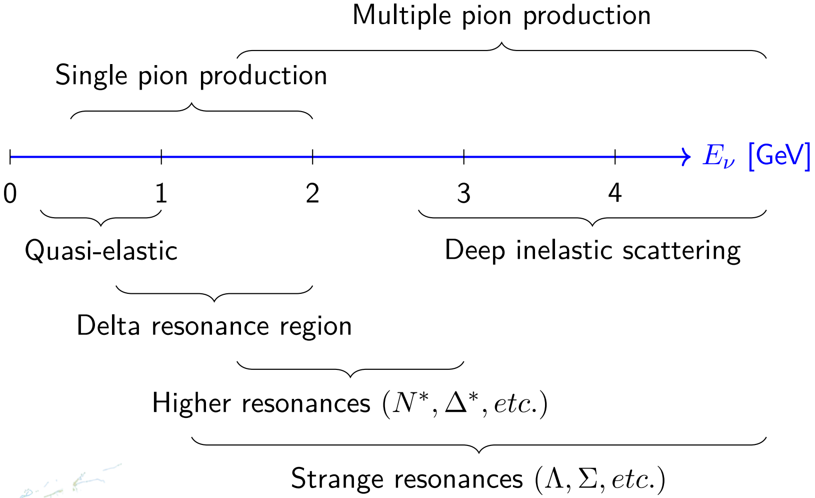
About neutrino collisions and the pions they can produce

- δ_{CP} and neutrino mass hierarchy
- 1σ uncertainties in cross sections from current neutrino event generators, partially due to bad π^0 handling. [1]
- Single pion production calculations involving Delta resonance are vitally important in the step from T2K to H2K. [2]

[1] arXiv:1905.13101 [hep-ph]

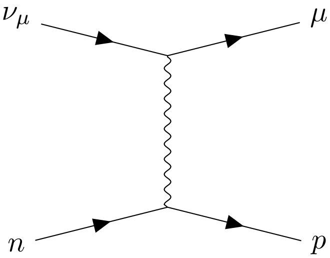
[2] arXiv:2303.03222 [hep-ex]

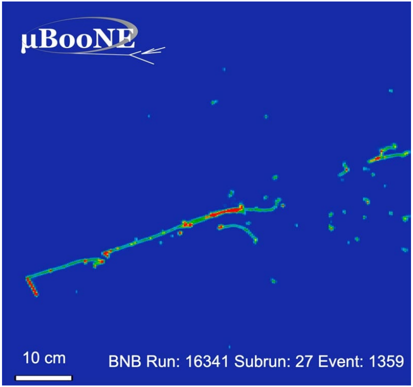
Outline


Multiple pion production
Single pion production
E_ν [GeV]
0
1
2
3
4
Quasi-elastic
Deep inelastic scattering
Delta resonance region
Higher resonances $(N^*, \Delta^*, etc.)$
Strange resonances $(\Lambda, \Sigma, etc.)$

Quasi-elastic scattering

$0.2\,\textbf{GeV} \lesssim E_\nu \lesssim 1\,\textbf{GeV}$





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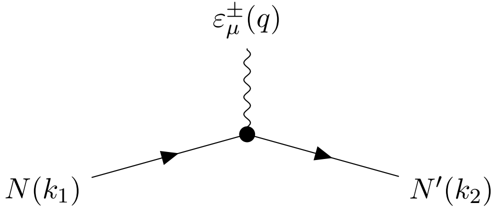

- Vertex needs structure that looks like Γ^μ, to contract with Lorentz indices of vector boson.

Quasi-elastic scattering

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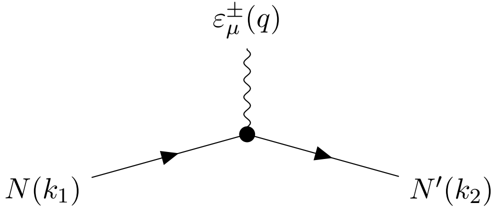

- Vertex needs structure that looks like Γ^μ, to contract with Lorentz indices of vector boson.
- For the W-boson with point-particle nucleons:

$$\Gamma^{\mu}_{NNW} = g_{NNW}\,\gamma^{\mu}\left(1-\gamma^{5}\right)$$

Quasi-elastic scattering

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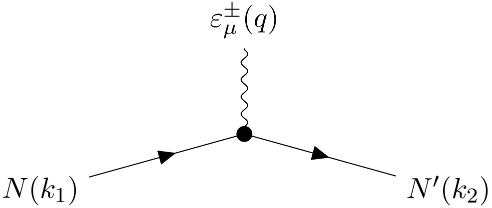

- Vertex needs structure that looks like Γ^μ, to contract with Lorentz indices of vector boson.
- For the W-boson with ~~point-particle~~ nucleons:

$$\Gamma^{\mu}_{NNW} = g_{NNW}(q^2)\,\gamma^\mu\left(1-\gamma^5\right)$$

Form factors

General structure

$$V^\mu(Q^2) = F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m} + F_S(Q^2)\frac{q^\mu}{2m}$$

$$A^\mu(Q^2) = F_A(Q^2)\gamma^\mu\gamma^5 + F_T(Q^2)\frac{i\sigma^{\mu\nu}q_\nu\gamma^5}{2m} + F_P(Q^2)\frac{q^\mu\gamma^5}{2m}$$

- Symmeteries: Time, $\mathcal{G}$-parity, CVC, and PCAC

Scherer, S. (2003). Introduction to Chiral Perturbation Theory

C.H. Llewellyn-Smith, (1972). Neutrino reactions at accelerator energies

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- Symmeteries: Time, $\mathcal{G}$-parity, CVC, and PCAC
- Nucleon form factors can be determined from electron scattering.

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C.H. Llewellyn-Smith, (1972). Neutrino reactions at accelerator energies

Form factors

$0.2\,\textbf{GeV} \lesssim E_\nu \lesssim 1\,\textbf{GeV}$

$$
\begin{aligned}
\langle N_i(k_2)|\, J^{\mu}_{N,W^\pm}(0)\, |N_j(k_1)\rangle \\
&= \bar{u}_i(k_2)\, \Gamma^{\mu}_{NNW}(Q^2)\tau^{\pm}_{ij}\, u_j(k_1) \\
&= \bar{u}_i(k_2)\, g_{NNW}\Bigg(\bigg(F_A(Q^2)\gamma^\mu + F_P(Q^2)\frac{q^\mu}{2m_N}\bigg)\gamma^5 \\
&\qquad + F_1^W(Q^2)\gamma^\mu + F_2^W(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\Bigg)\tau^{\pm}_{ij}\, u_j(k_1)
\end{aligned}
$$

$$
g_{NNW} = -\frac{g}{\sqrt{2}}(V_{ud})
$$

arXiv:2110.15319 [hep-ph]

Form factors

0.2 **GeV** $\lesssim E_\nu \lesssim 1$ **GeV**

$$
\begin{aligned}
\langle N_i(k_2)|\, J^\mu_{N,W^\pm}(0)\, |N_j(k_1)\rangle & \\
&= \bar{u}_i(k_2)\, \Gamma^\mu_{NNW}(Q^2) \tau^\pm_{ij}\, u_j(k_1) \\
&= \bar{u}_i(k_2)\, g_{NNW} \Bigg(\Big(F_A(Q^2)\gamma^\mu + F_P(Q^2) \frac{q^\mu}{2m_N} \Big) \gamma^5 \\
&\qquad + F_1^W(Q^2)\gamma^\mu + F_2^W(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m} \Bigg) \tau^\pm_{ij}\, u_j(k_1)
\end{aligned}
$$

$$
g_{NNW} = -\frac{g}{\sqrt{2}} (V_{ud})
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Form factors

$0.2\,\textbf{GeV} \lesssim E_\nu \lesssim 1\,\textbf{GeV}$

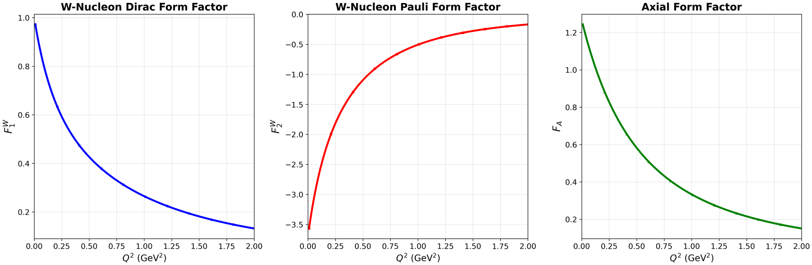
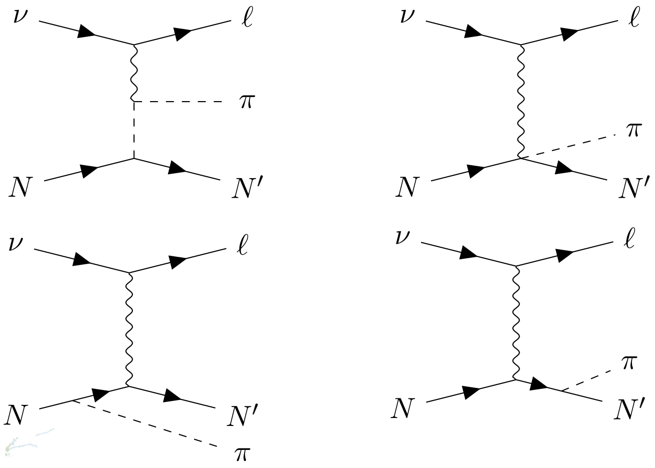


Figure: W-boson nucleon form factors

Pion production

$0.5\,\textbf{GeV} \lesssim E_\nu \lesssim 2\,\textbf{GeV}$



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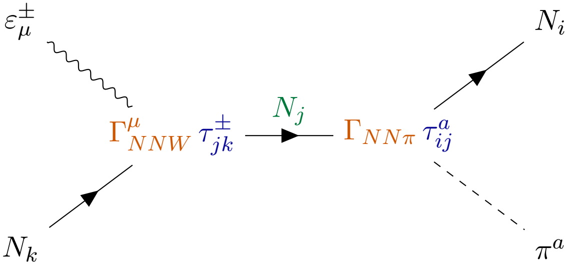

$$\langle N_i, \pi^a |\, J^\mu_{s-channel}\, | N_k \rangle = \bar{u}_i\, \Gamma_{NN\pi}\, \tau^a_{ij}\, D(p_\pi + p_{N_i})\, \Gamma^\mu_{NNW}\, \tau^\pm_{jk}\, u_k$$

Pion production

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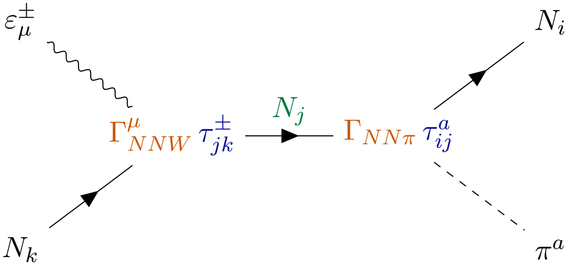

$$\langle N_i, \pi^a |\, J^\mu_{s-channel}\, | N_k \rangle = \bar{u}_i\, \Gamma_{NN\pi}\, \tau^a_{ij}\, D(p_\pi + p_{N_i})\, \underbrace{\Gamma^\mu_{NNW}}_{g_{NNW}\left(F_1^W(Q^2)\gamma^\mu + F_2^W(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m} + F_A(Q^2)\gamma^\mu\gamma^5\right)}\, \tau^\pm_{jk}\, u_k$$

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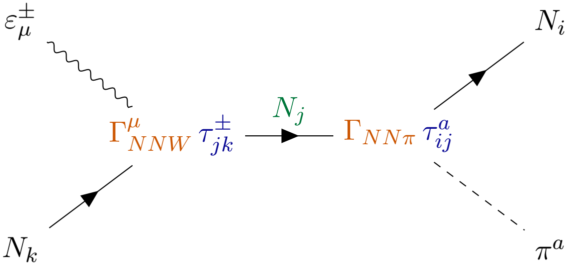


doi/10.1103/PhysRevD.105.094022

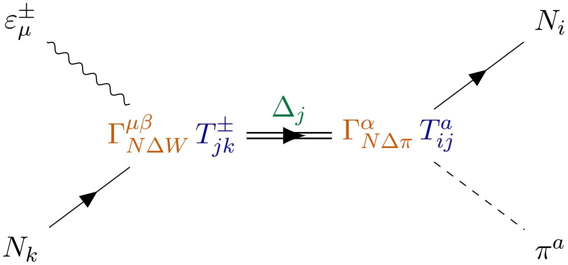
$$g_{NN\pi}\left(G_A(p_\pi^2)\gamma^\mu + G_P(p_\pi^2)\frac{p_\pi^\mu}{2m_N}\right)(p_\pi)_\mu\gamma^5$$

$$\langle N_i, \pi^a|\, J^\mu_{s-channel}\, |N_k\rangle = \bar{u}_i\, \overbrace{\Gamma_{NN\pi}}\, \tau^a_{ij}\, D(p_\pi + p_{N_i})\, \underbrace{\Gamma^\mu_{NNW}}\, \tau^\pm_{jk}\, u_k$$

$$g_{NNW}\Big(F_1^W(Q^2)\gamma^\mu + F_2^W(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m} + F_A(Q^2)\gamma^\mu\gamma^5\Big)$$

Delta resonances

$0.5\,\textbf{GeV} \lesssim E_\nu \lesssim 2\,\textbf{GeV}$



$$\langle N_i, \pi^a |\, J^\mu_{s-channel}\, | N_k \rangle = \bar{u}_i\, \Gamma^\alpha_{N\Delta\pi}\, T^a_{ij}\, S_{\alpha\beta}(p_\pi + p_{N_i})\, \Gamma^{\mu\beta}_{N\Delta W}\, T^\pm_{jk}\, u_k$$

Delta resonances

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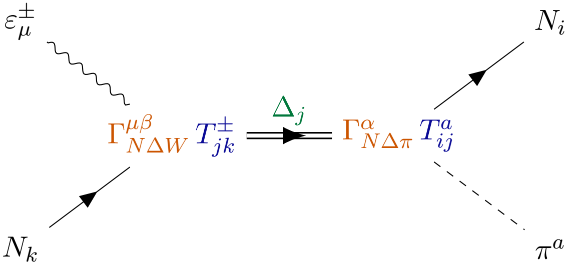

$$\langle N_i, \pi^a |\, J^{\mu}_{s-channel}\, | N_k \rangle = \bar{u}_i\, \Gamma^{\alpha}_{N\Delta\pi}\, T^{a}_{ij}\, \underbrace{S_{\alpha\beta}(p_\pi + p_{N_i})}_{\frac{-(\not{p}+M_\Delta)}{p^2 - M_\Delta^2 + i M_\Delta \Gamma_\Delta}\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p_\alpha p_\beta}{M_\Delta^2} + \frac{1}{3M_\Delta}(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha)\right]}\, \Gamma^{\mu\beta}_{N\Delta W}\, T^{\pm}_{jk}\, u_k$$

arXiv:hep-ph/0701149

Delta resonances

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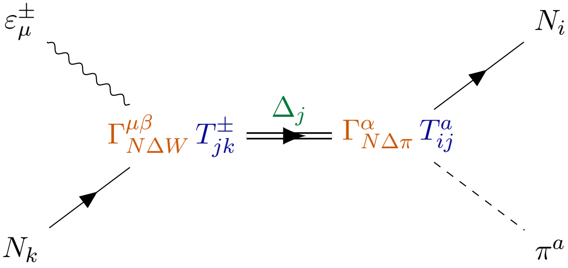


CGC

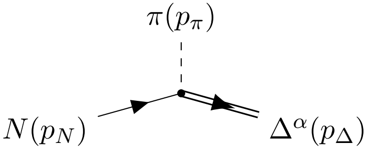
$$\langle N_i, \pi^a | \, J^\mu_{s-channel} \, | N_k \rangle = \bar{u}_i \, \Gamma^\alpha_{N\Delta\pi} T^a_{ij} \underbrace{S_{\alpha\beta}(p_\pi + p_{N_i})} \Gamma^{\mu\beta}_{N\Delta W} T^\pm_{jk} u_k$$

$$\frac{-(\not{p} + M_\Delta)}{p^2 - M_\Delta^2 + i M_\Delta \Gamma_\Delta} \left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p_\alpha p_\beta}{M_\Delta^2} + \frac{1}{3M_\Delta}\left(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha\right)\right]$$

arXiv:hep-ph/0701149

Delta resonances

Form factors and Lorentz structures for N, Δ, π vertex



$$\frac{i\kappa}{2m_\pi}(g^{\alpha\varrho} - z\gamma^\alpha\gamma^\varrho)(p_\pi)_\varrho \qquad\qquad \frac{h}{0.96 * m_\pi m_\Delta}\epsilon^\alpha{}_{\rho\sigma\delta}\left(p_\pi^\rho\gamma^\sigma p_\Delta^\delta\right)\gamma^5$$

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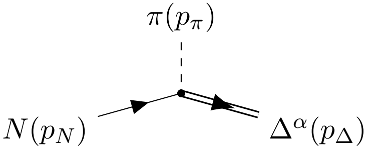


$$\frac{i\kappa}{2m_\pi}(g^{\alpha\varrho} - z\gamma^\alpha\gamma^\varrho)(p_\pi)_\varrho$$

- Heavy coupling dependence
- Not consistent with high energy particles

doi/10.1103/PhysRevC.60.042201,

arXiv:0804.2750[nucl-th], arXiv:1905.13101 [hep-ph]

$$\frac{h}{0.96 * m_\pi m_\Delta}\epsilon^\alpha{}_{\rho\sigma\delta}\left(p_\pi^\rho\gamma^\sigma p_\Delta^\delta\right)\gamma^5$$

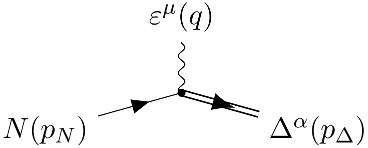
- Invariant under gauge-like transformation $\Delta^\alpha \rightarrow \Delta^\alpha + \partial^\alpha\epsilon$
- No unphysical spin-$1/2$ components

arXiv:nucl-th/0509020, arXiv:2106.09031[hep-ph],

doi/10.1103/PhysRevD.58.096002

Delta resonances

Form factors and Lorentz structures for N, Δ EW vertex



$$\Gamma^{\mu\alpha}_{N\to\Delta} = \mathcal{O}^{\mu\alpha} = \mathcal{O}^{\mu\alpha}_V + \mathcal{O}^{\mu\alpha}_A$$

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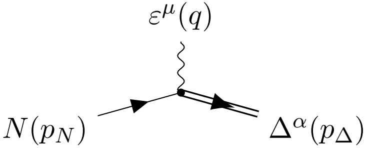

$$\mathcal{O}_V^{\alpha\mu} = \left(\frac{C_3^V(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^V(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + \frac{C_5^V(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_N)_\beta + \frac{C_6^V(Q^2)}{m_N^2} q^\mu q^\alpha \right)\gamma^5$$

$$\mathcal{O}_A^{\alpha\mu} = \left(\frac{C_3^A(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^A(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{m_N^2} q^\mu q^\alpha \right)1$$

$$t^{\mu\alpha\beta} = (g^{\mu\alpha}q^\beta - g^{\mu\beta}q^\alpha)$$

doi/10.1016/0370-1573(72)90010-5, arXiv:hep-ph/0701149, arXiv:hep-ph/0310363, arXiv:1807.08314 [hep-ph]

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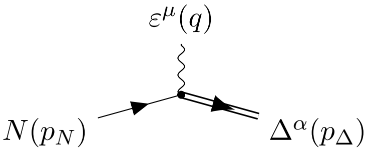
Form factors and Lorentz structures for N, Δ EW vertex



CVC

$$\mathcal{O}_V^{\alpha\mu} = \left(\frac{C_3^V(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^V(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + \frac{C_5^V(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_N)_\beta + \cancel{\frac{C_6^V(Q^2)}{m_N^2}} q^\mu q^\alpha \right)\gamma^5$$

$$\mathcal{O}_A^{\alpha\mu} = \left(\frac{C_3^A(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^A(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{m_N^2} q^\mu q^\alpha \right)1$$

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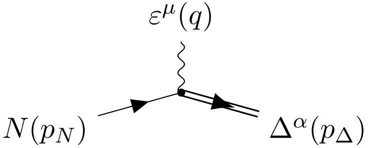

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PCAC

$$\mathcal{O}_A^{\alpha\mu} = \left(\frac{C_3^A(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^A(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{m_N^2} q^\mu q^\alpha \right)\mathbb{1}$$

$$t^{\mu\alpha\beta} = (g^{\mu\alpha}q^\beta - g^{\mu\beta}q^\alpha)$$

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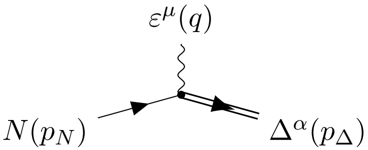

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PCAC

$$\mathcal{O}_A^{\alpha\mu} = \left(\frac{C_3^A(Q^2)}{m_N} t^{\mu\alpha\beta}\gamma_\beta + \frac{C_4^A(Q^2)}{m_N^2} t^{\mu\alpha\beta}(p_\Delta)_\beta + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{m_N^2} q^\mu q^\alpha \right)1$$

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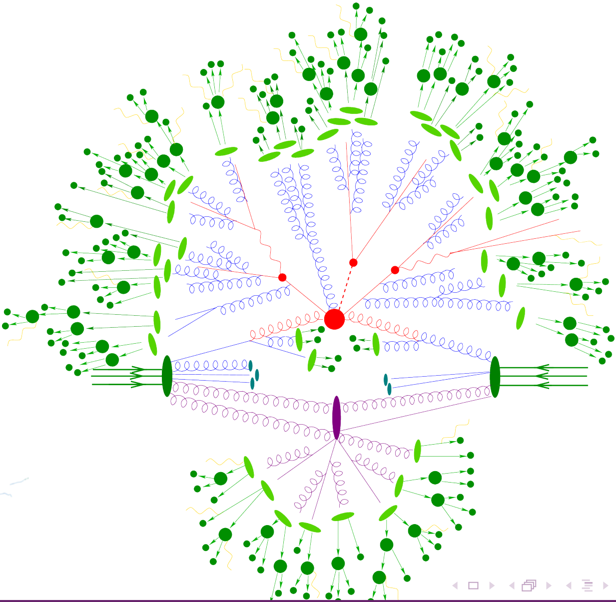
Higher energy effects

E_ν dependant

- Higher resonances such as N^*, Δ^*: $1.5\,\text{GeV} \lesssim E_\nu \lesssim 3\,\text{GeV}$
  - Multiple pions can be easily produced
- Strange mesons and resonances: $2\,\text{GeV} \lesssim E_\nu \lesssim 5\,\text{GeV}$
- DIS: $E_\nu \gtrsim 3\,\text{GeV}$

SHERPA

A Monte-Carlo event generator



MC technology

What Sherpa has:

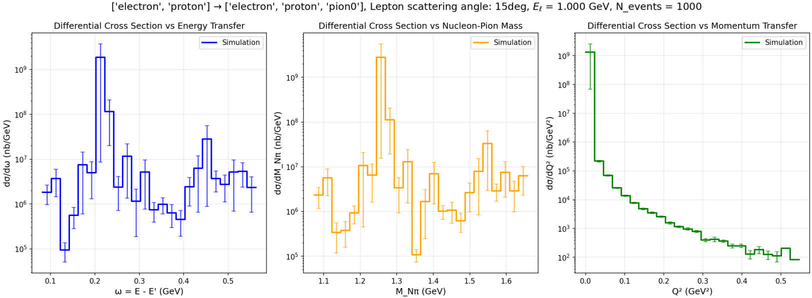
- Great phase space integration
- Framework for DIS neutrino scattering
- Hadronisation

What needs implementing:

- Ability to do incoming neutrino flux
- Form factors
- GI Delta couplings

Invariant mass plot

$ep \to ep\pi^0$



What's next?

- Implementing consistent theory into MC event generator
- Comparison against electron scattering data, and other neutrino event generators
- Including nuclear effects beyond Fermi gas model