Dilaton forbidden dark matter from the lattice

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Outline

1. Introduction
2. Dilaton EFT
3. Lattice Data
4. Dark Matter
5. Summary and Outlook

The Space of Nonabelian Gauge Theories

Consider $\mathrm{SU}(N_c)$ gauge theories with N_f fermions:

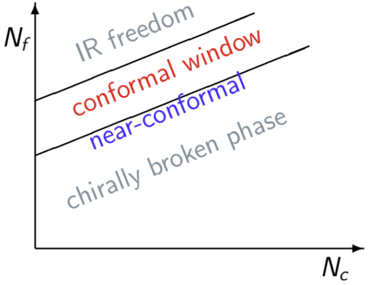

Figure: Gauge theory phase diagram PoS Lattice **2018**, 006 (2019).

- $N_f > \frac{11}{2} N_c$: Not asymptotically free.
- $\frac{11}{2} N_c > N_f > N_{fc}$: Asymptotically free, but approaches conformality in IR.
- $N_{fc} > N_f$: Confinement. Low energy states are colorless composites.
- Can generalize to other gauge groups and scalar matter.

Near–Conformal Gauge Theories

- Near–conformal gauge theories *confine*.
- But only just. The field content is chosen to ensure that they lie just beneath the boundary of the conformal window.
- There is also evidence for a light scalar composite forming in these gauge theories, unlike in QCD.

Evidence for a Light Scalar I

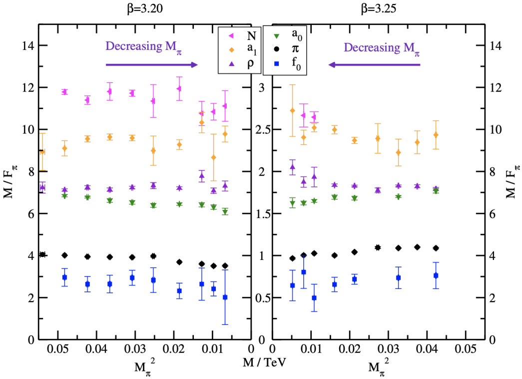

Figure: Lattice data for the masses of composites $\mathrm{SU}(3)$ gauge theories with $N_f = 2$ fermions in 2–index symmetric rep. From the LatHC collaboration: PoS LATTICE2015 (2016) 219.

Evidence for a Light Scalar II

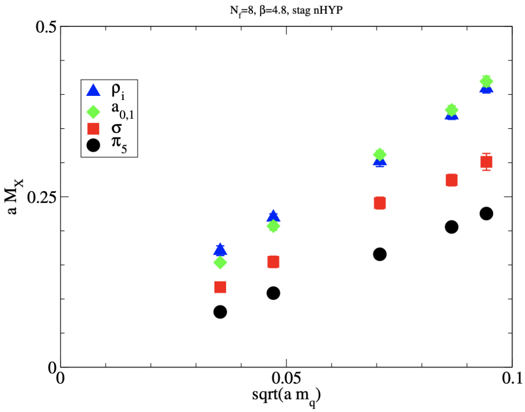

Figure: Lattice data for the masses of composites in the $\mathrm{SU}(3)$ gauge theory with $N_f = 8$ fundamental fermions from the LSD collaboration: PRD **110** (2024) 5, 054501

Dilaton EFT

Reviewed in Universe 9 (2023) 1, 10 with T. Appelquist, M. Piai and PRD **94** (2016) 5, 054502 by M. Golterman, Y. Shamir

Field Content

1. $N_f^2 - 1$ NGB fields π^a
   $\Sigma = \exp\{2i\pi^a T^a / F_\pi\}$
   $\langle \Sigma \rangle = \mathbb{1}$
2. Dilaton field χ
   $\langle \chi \rangle = F_d$

Symmetries

Chiral Symmetry

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$$
$$\Sigma \to L \Sigma R^\dagger$$

Scale Invariance

Scale $\times$ Poincaré $\to$ Poincaré

$$\chi(x) \to e^\lambda \chi(e^\lambda x)$$

Dilaton EFT

Leading order Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + \frac{m B_\pi f_\pi^2}{2}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] - V_\Delta(\chi)\,. \quad (1)$$

- pNGB terms are similar to those in chiral Lagrangian.
- Dependence on dilaton field χ is determined by scale invariance.
- See dilaton EFT of Golterman & Shamir: PRD **94** (2016) 5, 054502.

Dilaton Potential

$$V_\Delta = \frac{m_d^2 \chi^4}{4(4-\Delta) f_d^2} \left[1 - \frac{4}{\Delta} \left(\frac{f_d}{\chi} \right)^{4-\Delta} \right] . \tag{2}$$

- Potential contains a scale invariant term ($\sim \chi^4$) and a deformation ($\sim \chi^\Delta$), which explicitly violates scale invariance. We treat Δ as a floating parameter that can take a range of values.

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- This potential has a minimum at $\chi = f_d$, and a weak curvature $m_d^2 \ll (4\pi f_d)^2$.
- Potentials of this form are discussed in e.g: Rattazzi & Zaffaroni JHEP **0104**, 021 (2001), GGS PRL.**100** 111802, (2008), CCT PRD.**100** 095007 (2019).

Fit to Lattice Data

for SU(3) gauge theory with $N_f = 8$ Dirac fermions

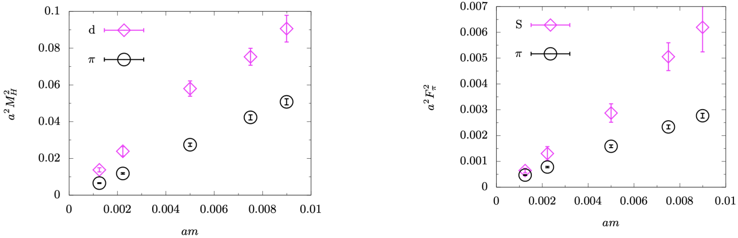
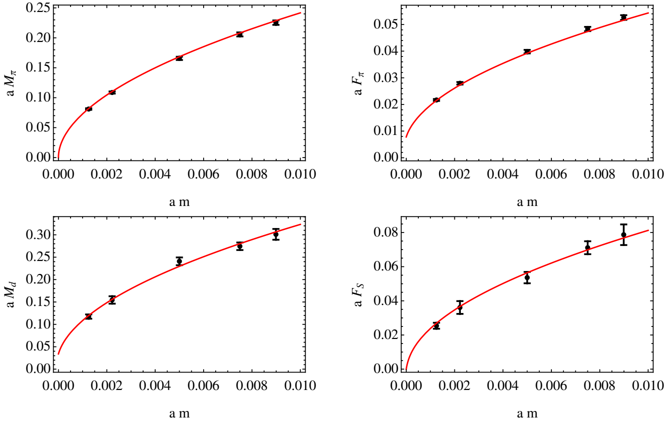


Figure: Lattice data for M_π^2, M_d^2, F_π^2 and F_S^2 from LSD Collab: PRD **110** (2024) 5, 054501. The lattice spacing is denoted by a.

We also include data for the π–π scattering length in the I=2, $\ell = 0$ channel from LSD Collab: PRD **105** (2022) 3, 034505

Result Of Global Fit to dEFT

Presented in LSD Collab: PRD **108** (2023) 9, 9



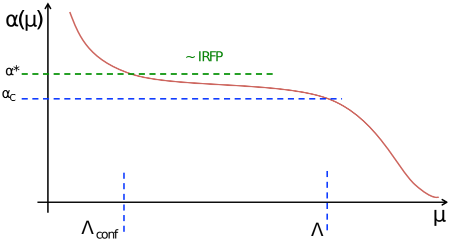
Result Of Global Fit to dEFT

Presented in LSD Collab: PRD **108** (2023) 9, 9

Parameter	Value and Uncertainty
y	2.091(32)
aB_π	2.45(13)
Δ	3.06(41)
$a^2 f_\pi^2$	$6.1(3.2) \times 10^{-5}$
f_π^2/f_d^2	0.1023(35)
m_d^2/f_d^2	1.94(65)
χ^2/dof	21.3/19=1.12

Table: Central values of fit parameters obtained in a six parameter global fit to LSD data for $M^2_{\pi,d}$, $F^2_{\pi,S}$ and scattering length.

Interpretation of Δ



1. Strongly coupled over large interval of scales $\Longrightarrow$ possibility of large anomalous dimensions. Note our lattice fits showed $y \approx 2$.

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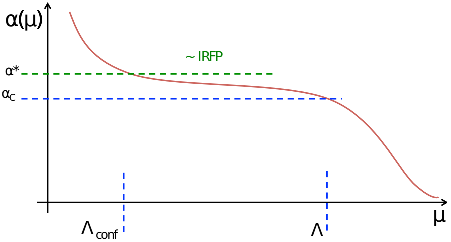

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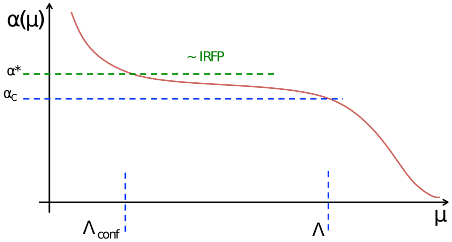


1. Strongly coupled over large interval of scales $\implies$ possibility of large anomalous dimensions. Note our lattice fits showed $y \approx 2$.
2. Allows for new relevant interactions besides (near marginal) gauge interaction.
3. Δ should be identified with the engineering plus anomalous dimension of this new relevant operator.

Composite Dark Matter

PRD **110** (2024) 3, 035013 with T. Appelquist and M. Piai.

I want to talk about a description of DM, in which the DM is a composite particle that forms in a new dark sector gauge theory.

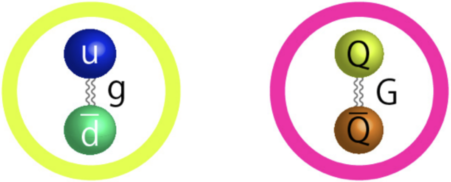
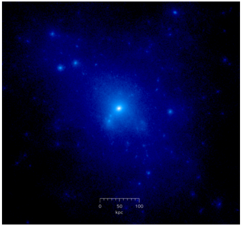


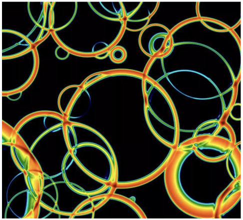
Figure: Dark pion (image: Kavli IPMU).

The dark sector gauge theory interacts feebly with the standard model. Dark matter is a composite state, analogous to the pion of QCD.

Why Compositeness?

The standard model has three gauge interactions. A fourth may be out there as a hidden sector.





Features

- Sizable self interactions can affect small scale structure anomalies.
- Confining phase transition may generate observable grav waves.

Our dark matter will be a thermal relic.

Our Composite Dark Matter Framework

- Suppose the dark sector is a near–conformal gauge theory, and dark matter is the pNGB.
- The low energy spectrum of these gauge theories have a light scalar. Unlike the pNGB, the light scalar carries no conserved charges and so can decay (slowly) to standard model.
- Nevertheless, the pNGBs can annihilate readily into the scalars, so the freezeout of this process can set the relic density of pNGBs.
- We describe these low energy states using dilaton EFT.

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- We describe these low energy states using dilaton EFT.
- In the following, specialise to $\mathrm{SU}(3)$ gauge theory with $N_f = 8$ fermions for concreteness.

Dilaton Self-Interactions

The dilaton field χ experiences a net potential of

$$W(\chi) \equiv V(\chi) - \frac{M_\pi^2 F_\pi^2 N_f}{2} \left(\frac{\chi}{F_d}\right)^y . \tag{3}$$

Expand potential around its minimum $\chi = F_d + \bar{\chi}$:

$$W(\bar{\chi}) = \text{constant} + \frac{M_d^2}{2}\bar{\chi}^2 + \frac{\gamma}{3!}\frac{M_d^2}{F_d}\bar{\chi}^3 + \ldots , \tag{4}$$

where $\gamma \geq 2$ (from unitarity bound) GGS PRL.**100** 111802, (2008) and γ cannot be too large for EFT to remain weakly coupled.

The functional form of the dilaton potential will not matter in the following. Only M_d, F_d and γ impact on our study.

Freezeout

The relic density is set by $\pi\pi \to \chi\chi$ annihilations freezing out.

Boltzmann Equation

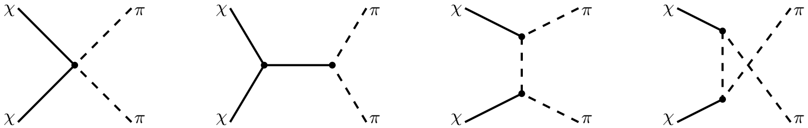
$$\frac{\partial n_\pi}{\partial t} + 3Hn_\pi = - \langle \sigma_{2\pi \to 2\chi} v \rangle n_\pi^2 + \langle \sigma_{2\chi \to 2\pi} v \rangle \left(n_\chi^{\text{eq}} \right)^2 . \tag{5}$$

We solve numerically to get the relic density of pNGBs today.

We have taken $\Gamma_{\chi \to \text{SM}}$ large enough to maintain dilatons in thermal equilibrium with SM so that $n_\chi = n_\chi^{\text{eq}}(T_{\text{SM}})$. More details on SM couplings to follow.

Thermally Averaged Cross Sections

The inverse annihilation process $\chi\chi \to \pi\pi$ can happen for zero kinetic energy in the initial state, because $\Delta > 0$. We compute its cross section using dilaton EFT:



For $T \ll M_\pi$, the thermal averaged x-section $\approx$ x-section at $\vec{p} = 0$:

$$\langle \sigma_{2\chi \to 2\pi} v \rangle = \frac{M_\pi^2 N_\pi}{36\pi F_d^4} \sqrt{\Delta(2+\Delta)}(1+\Delta)(5+\gamma)^2 \,, \tag{6}$$

where the mass splitting is $\Delta \equiv (M_d - M_\pi)/M_\pi$. We take $0 < \Delta < 1/2$, as seen in lattice data.

Forbidden Dark Matter

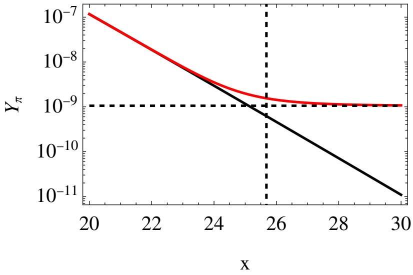
However, the calculation of the thermal average $\langle \sigma_{2\pi \to 2\chi} v \rangle$ is less straightforward, as this reaction is kinematically forbidden when pions have zero momentum (or at $T = 0$).

In this case, taking the thermal average leads to an exponential suppression of the cross section. For $x = M_\pi / T$, we have

$$\langle \sigma_{2\pi \to 2\chi} v \rangle = \frac{(1 + \Delta)^3}{N_\pi^2} e^{-2\Delta x} \langle \sigma_{2\chi \to 2\pi} v \rangle \,. \tag{7}$$

The dark matter relic abundance is set through annihilations to heavier states that are kinematically forbidden at $T = 0$. This framework is an example of forbidden DM Griest & Seckel: PRD **43**, 3191 (1991), D'Agnolo & Ruderman: PRL **115**, 061301 (2015)

Solving the Boltzmann Equation



We plot solution taking the scale as $M_\pi = 1$ GeV, with parameters $M_\pi/F_\pi = 4$, $F_\pi^2/F_d^2 = 0.1$, $\Delta = 0.3$, $\gamma = 3$ and $y = 2$.

Plot using convenient variables:

$$Y_\pi = n_\pi/s$$
$$x = M_\pi/T$$

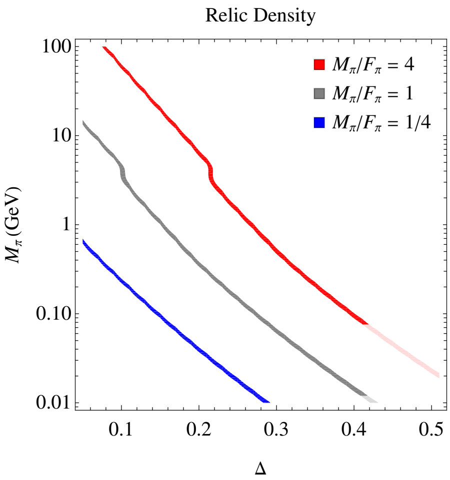
High temp boundary condition $Y_\pi(T_i) = n_\pi^{\text{eq}}(T_i)/s(T_i)$.

Before freezeout, $Y_\pi \approx n_\pi^{\text{eq}}(T)/s(T)$.

After freezeout Y_π roughly constant.

$$\Omega_{\text{CDM}} h^2 = \frac{M_\pi s_0 Y_\pi(\infty)}{\rho_c/h^2},$$

Parameter Space



- Bands indicate parameter space for which $\Omega_{\text{CDM}} h^2$ is within 10% of its observed value.
- Range of DM masses allowed. Lighter than typical WIMPs, due to forbidden mechanism.
- Pale shaded regions excluded due to upper bounds on $\frac{\sigma}{M_\pi}(\pi\pi \to \pi\pi)$ e.g. from bullet cluster...

Coupling to Visible Sector

At the level of dilaton EFT, the necessary couplings take the form

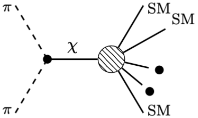
$$\mathcal{L}_{\text{int}} = \epsilon F_d^{4-d_{\text{SM}}} \left(\frac{\bar{\chi}}{F_d}\right) \mathcal{O}_{\text{SM}} \,, \tag{8}$$

where ϵ are weak, dimensionless constants, and $\mathcal{O}_{\text{SM}}$ are singlet, scalar operators involving light SM fields (e.g $\mathcal{O} = F_{\mu\nu}F^{\mu\nu}$).

1. The dilaton couplings are not constrained by the form of the SM energy momentum tensor (as dilaton is a composite of dark sector, and not SM dofs).
2. Bounds exist for specific subsets of these couplings from astrophysics, CMB, collider experiments. We leave this for future work.
3. We can however derive a more model independent constraint...

Consistency Condition

1. The inclusive decay rate $\Gamma_{\chi\to\mathrm{SM}}$ must be large enough to bring the dark sector and SM into thermal equilibrium long before freezeout.
2. The decay rate must also be small enough so that direct annihilations $\pi\pi\to$ SM do not overwhelm forbidden annihilations to dilatons.



Two-Sided Bound on the Inclusive Decay Rate

$$H_{T=M_\pi} \lesssim \Gamma_{\chi\to\mathrm{SM}} \lesssim H_{T=T_f}\frac{M_\pi N_\pi F_d^2}{n_\pi^{\mathrm{eq}}(T_f)}, \tag{9}$$

Summary and Outlook

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Summary and Outlook

- We have fitted dilaton EFT to lattice data for a particular near–conformal gauge theory, finding a good fit.
- We proposed a description of composite DM based on dilaton EFT.
- The DM is the pNGB of a nearly conformal gauge theory, and the dilaton plays the role of a mediator with the standard model.
- Our framework naturally implements the forbidden dark matter mechanism. The DM is a thermal relic with abundance set by forbidden $\pi\pi \to \chi\chi$ annihilations. The framework accommodates a wide range of DM masses: $M_\pi \sim 10$ MeV $-$ 100 GeV.

Thank you!

Lattice Action

- Our numerical calculations use improved nHYP smeared staggered fermions with smearing parameters $\alpha = (0.5, 0.5, 0.4)$. [LSD PRD 99(2019)014509]
- $\beta_A/\beta_F = -0.25$ where $\beta_F = 4.8$.
- After taste splitting, only $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ flavor symmetry preserved in massless theory (3 exact NGBs).
- Spectral study has revealed that the taste splitting of the 63-plet masses are on the order of 20–30%. [LSD PRD 99(2019)014509]

Summary of Improvements to Lattice Dataset

Presented in 2306.06095

Since the previous LSD study of the $N_f = 8$ theory PRD **99** (2019) 014509, we have made some changes.

1. We have data for a new observable: The scalar decay constant F_S.
2. We have extrapolated the quantities M_π, F_π, M_σ (and also F_S) to the infinite volume limit.
3. We have improved our estimates of systematic uncertainties using Bayesian Model Averaging Jay, Neil PRD **103** (2021) 114502

The $N_f = 8$ spectrum has also been calculated before in LatKMI PRD **96** (2017) 014508

Scalar Decay Constant

Measured by LatKMI in PRD 96 (2017) 014508

Define scalar decay constant using the matrix element

$$\langle 0| J_S(x) |\chi(p)\rangle \equiv F_S M_d^2 e^{-p\cdot x} \,, \tag{10}$$

where

$$J_S(x) \equiv m \sum_{i=1}^{N_f} \bar{\psi}_i \psi_i \,. \tag{11}$$

1. F_S can be extracted from lattice measurement of correlator $\langle J_S(x) J_S(0)\rangle$, which is used already to measure M_d.
2. It is a true decay constant: It would control the decay rate of the dilaton if there was a heavy scalar mediator coupled to $\bar{\psi}\psi$ along with light states. Analogous to f_π for the QCD pion decaying to leptons via $W^\pm$.

Scalar Decay Constant

This quantity can also be calculated in dilaton EFT:

$$|F_S| = \frac{yN_f M_\pi^2 F_\pi}{2M_d^2} \frac{f_\pi}{f_d}. \tag{12}$$

- Incorporating Eq. (12) into our EFT fit provides a direct test of the coupling between the light scalar and the fermion mass, treated as an external source.

Lattice Calculation of Scattering Phase Shift

Phys.Rev.D **105** (2022) with LSD Collaboration

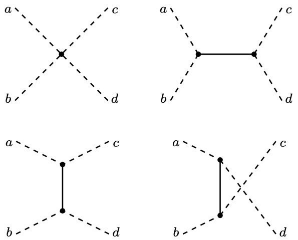
M. Lüscher NPB 354 (1991)

$$k^2 = \frac{1}{4}E_{\pi\pi}^2 - M_\pi^2 \tag{13}$$

$$k \cot \delta(k) = \frac{2\pi}{L}\pi^{-3/2} Z_{00}\left(1, \frac{k^2 L^2}{4\pi^2}\right) \tag{14}$$

- Restrict ourselves to $I = 2$ channel.
- $E_{\pi\pi}$ is the two–PNGB ground state energy.
- Measured at finite volume (L) on the lattice from a fit to a two point correlation function of two PNGB operators. Schematically: $C(t) \sim \langle \mathcal{O}^{I=2}(t)\mathcal{O}^{\dagger\, I=2}(0)\rangle$ where $\mathcal{O}^{I=2} \sim \pi\pi$.

$I = 2$ Scattering Length



- Scattering amplitude at threshold $= M_\pi a^{I=2}$
- First diagram, same as χPT. The others only arise for light scalar (dilaton).

$$M_\pi a^{I=2} = -\frac{M_\pi^2}{16\pi F_\pi^2}\left(1 - (y-2)^2 \frac{f_\pi^2}{f_d^2}\frac{M_\pi^2}{M_d^2}\right). \tag{15}$$

Simplifies to χPT result when $y \to 2$ or $f_\pi^2/f_d^2 \to 0$.

Scaling Relations at Leading Order

We also want to test the alternate possibility - that the $N_f = 8$ theory is *inside* the conformal window.

Assuming the gauge coupling g has reached its fixed point value $g^\star$, physical quantities may be fitted to scaling relations Zwicky, del Debbio PLB **700** (2011)

$$M_X = C_X m^{[1/(1+\gamma^\star)]}, \quad (16)$$

$$F_Y = C_Y m^{[1/(1+\gamma^\star)]}, \quad (17)$$

$$1/a_0^{(2)} = C_a m^{[1/(1+\gamma^\star)]}. \quad (18)$$

Result of Global Fit to Mass–Deformed CFT

Fitting to the same set of lattice data as in the dilaton case, we find:

Parameter	Value and Uncertainty
C_{M_π}	2.121(78)
C_{F_π}	0.522(19)
C_{M_d}	2.97(12)
C_{F_S}	0.706(33)
C_a	-5.88(22)
γ^*	1.073(28)
χ^2/dof	$48.1/19 = 2.53$

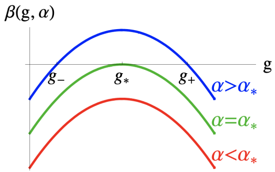
The χ^2/dof is larger than for the dEFT fit, while the number of fit parameters is the same. This indicates a lower quality fit.

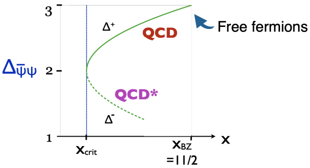
Marginality Crossing

Gies and Jaeckel: Eur.Phys.J.C46 (2006)

Kaplan, Lee, Son and Stephanov: Phys.Rev.D80 (2009)

Gukov: Nucl.Phys.B.919 (2017)





$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + \sum_i \bar{\psi}_i \not{D} \psi_i + \mathcal{L}_{4\ \mathrm{fermi}} \quad (19)$$

The conformal window is exited when a 4 fermi operator becomes relevant.

Freezeout Temperature