Particle Production from Gravitational Inhomogeneities

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with Garani and Tesi

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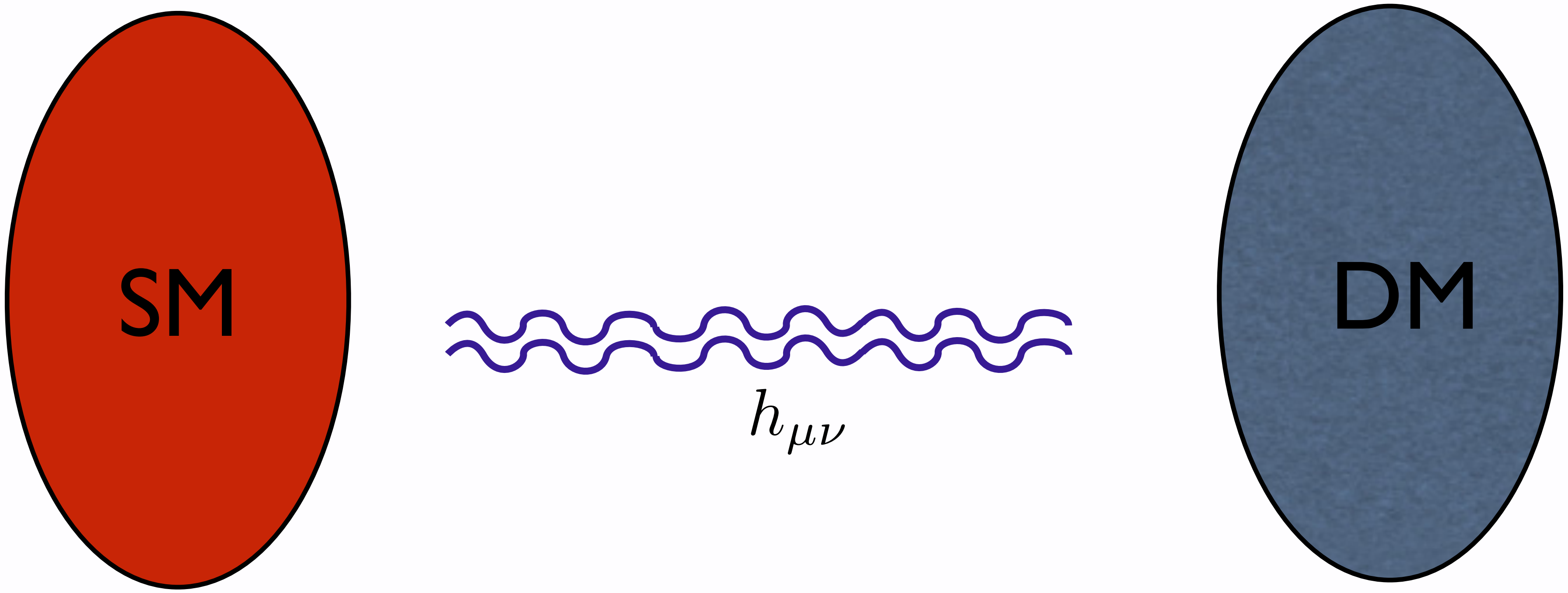
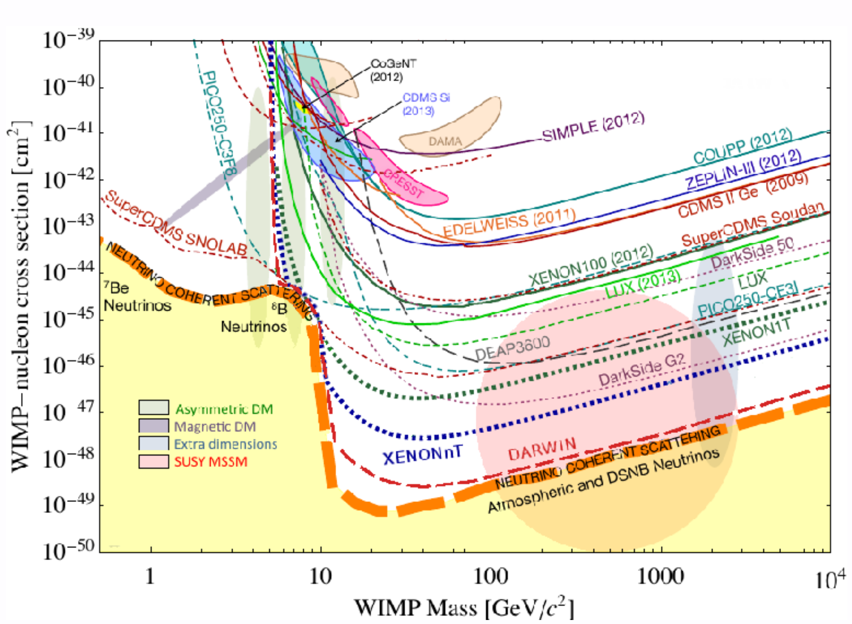
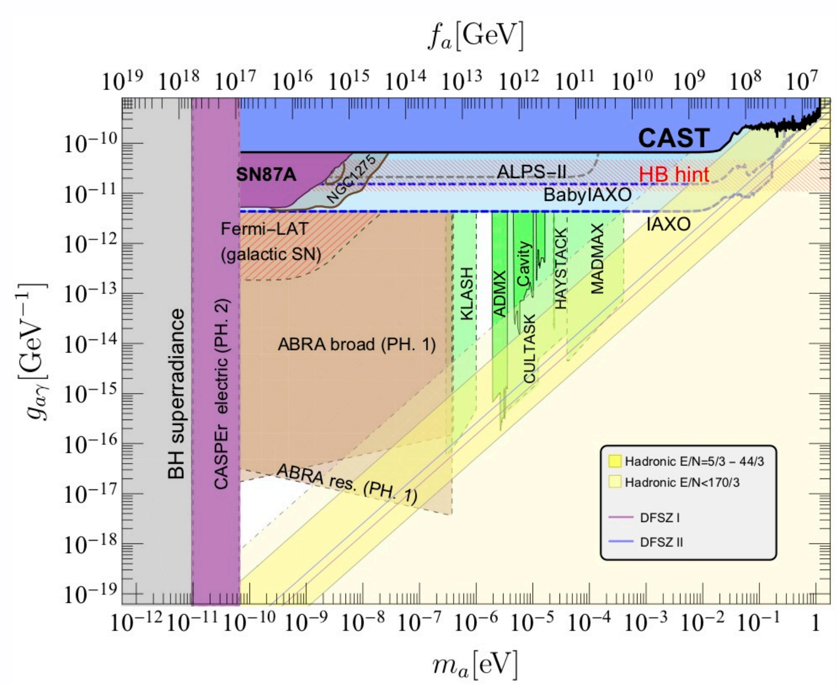
A fundamental question is how Dark Matter couples to the SM



It would be great if DM had other interactions with SM:

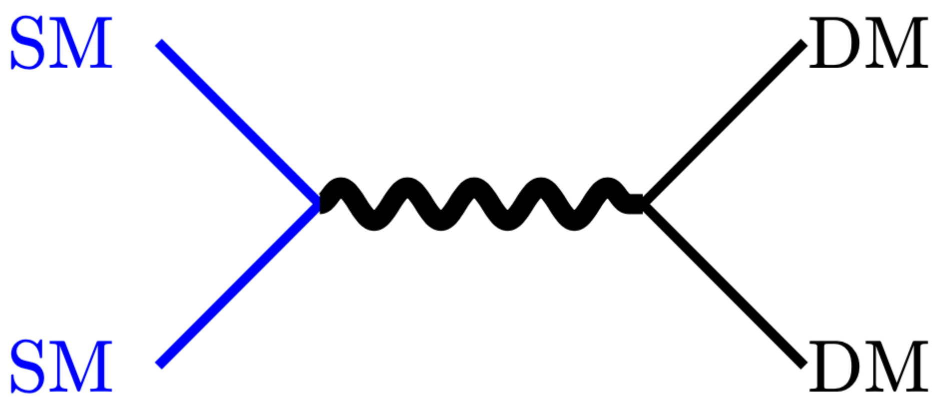




What if DM interacts only gravitationally?

- Gravitational freeze-in:

[Garny-Sandora-Sloth '15
MR, Tesi, Tillim '20]



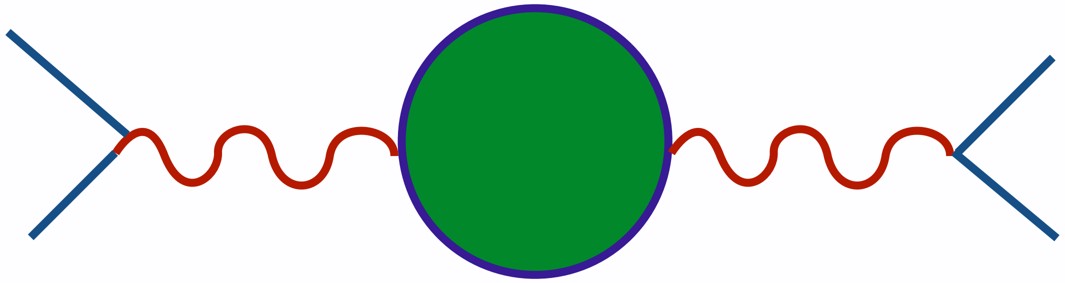
$$\mathcal{A} = \frac{1}{M_p^2 s}\left(T^{\rm SM}_{\mu\nu} T^{\rm DM}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}T^{\rm SM}T^{\rm DM}\right)$$

$$\frac{dY_D}{dT} = \frac{\langle\sigma v\rangle s(T)}{HT}(Y_D^2 - Y_{\rm eq}^2)$$

$$Y_D(0) = \int_0^{T_R}\frac{dT}{T}\,\frac{\langle\sigma v\rangle s}{H}Y_{\rm eq}^2$$

$$\langle\sigma v\rangle = 4\langle\sigma_0 v\rangle + 45\langle\sigma_{1/2} v\rangle + 12\langle\sigma_1 v\rangle$$

X-sec can be obtained through the optical theorem:



$$\sigma_{tot}(s) = \frac{\mathrm{Im}[\mathcal{A}(s)]_{\mathrm{forward}}}{\sqrt{s(s-4M^2)}}$$

$$\mathcal{A} \sim T_{SM} \frac{1}{p^2} \langle T(p) T(-p) \rangle \frac{1}{p^2} T_{SM}$$

In relativistic regime:

$$\langle T(p) T(-p) \rangle \sim c\, p^4 \log(-p^2)$$

$$\rho_D \approx 5 \cdot 10^{-4}\, c_D \left(\frac{T_R}{M_p}\right)^3 T^4$$

Free theory:

$$\frac{\Omega h^2}{0.12} = \frac{Y_D M}{0.4\,\mathrm{eV}} \approx \frac{c_D M}{2 \cdot 10^6\,\mathrm{GeV}} \left(\frac{T_R}{10^{15}\,\mathrm{GeV}}\right)^3$$

- Gravitational particle production:

[Schroedinger '39,
Ford '87,
Kolb, Riotto, Giudice '90s
Kolb, Long '24]

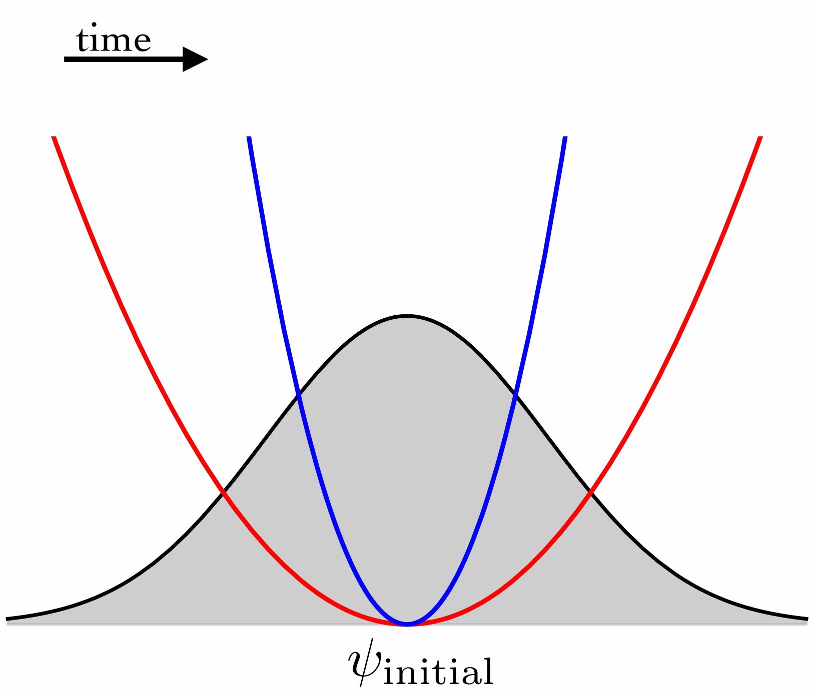
We live in an expanding universe:

$$ds^2 = a(\tau)^2(d\tau^2 - d\vec{x}^2)$$

In a time dependent background particles are produced due to the non-adiabatic evolution of the vacuum.

$$v''_{\vec{k}}(\tau) + \omega^2_{\vec{k}}(\tau) v_{\vec{k}}(\tau) = 0$$

$$\omega^2_{\vec{k}}(\tau) = |\vec{k}|^2 + M^2 a^2(\tau) - \frac{a''(\tau)}{a(\tau)}(1 - 6\xi)$$

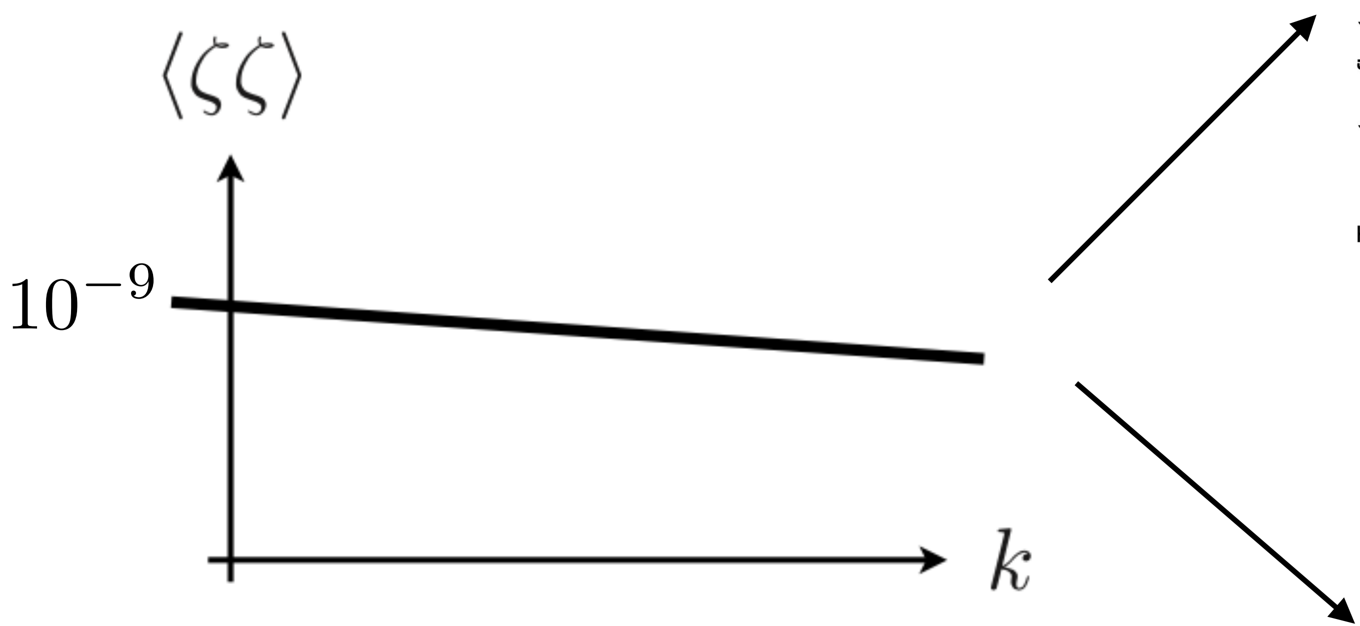


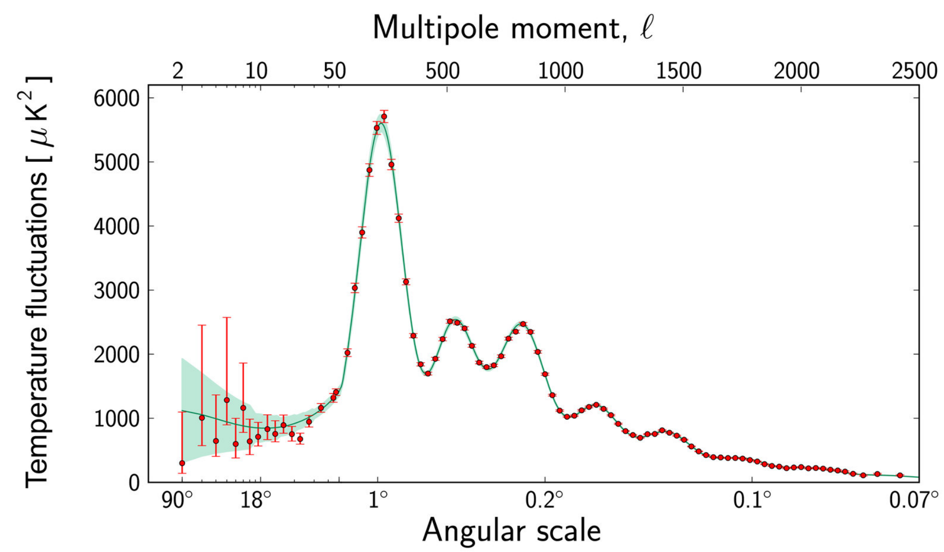
As for the time dependent harmonic oscillator the initial vacuum is interpreted as an excited state at late times.

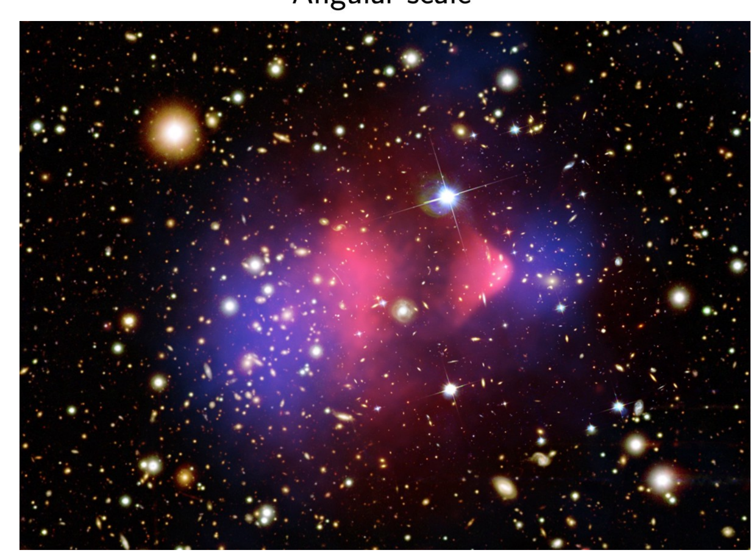
A phase of quasi de-Sitter — inflation — appears necessary to explain the initial conditions of our universe,

$$a(\tau) \approx -\frac{1}{H_I \tau}$$

Production of the inflaton generates the seeds of perturbations that eventually give rise to all that we see:







Could DM be produced by the expansion of the universe?

Cosmological particle production is associated to the breaking of Weyl invariance ($g_{\mu\nu}(x) \to \Omega(x)^2 g_{\mu\nu}(x)$):

- Kinetic term:

$$L = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

During inflation each mode is produced with an amplitude $H_I/(2\pi)$ that is constant till horizon re-entry. This gives a large abundance:

$$\frac{1}{s}\frac{d\rho}{d\log k} \approx \frac{H_I^2}{(2\pi)^2}\frac{\sqrt{M}}{M_{Pl}^{3/2}} \begin{cases} \dfrac{k_*}{k} & k \gg k_* \\ 1 & k \ll k_* \end{cases}$$

$$k_* = a_{\rm eq}\sqrt{M H_{\rm eq}} \sim 10^{-10}\sqrt{\frac{M}{10^{-5}\,{\rm eV}}}\,{\rm km}^{-1}$$

$$\Omega_{\rm DM} \sim \sqrt{\frac{M}{10^{-5}{\rm eV}}}\left(\frac{H_I}{10^{14} GeV}\right)^2$$

Inflationary production generates non-adiabatic perturbations in DM that are strongly constrained.

- Mass term:

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi$$

Particles are produced during reheating or radiation when the mass becomes relevant leading to smaller abundance,

$$\Omega_{\rm DM} \sim \left(\frac{M}{10^9\,{\rm GeV}}\right)^{5/2}$$

Massless fermions and in general conformally coupled theories are not produced in a homogeneous FLRW background

$$ds^2 = a(\tau)^2(d\tau^2 - d\vec{x}^2) \longrightarrow \Box\chi = 0$$

$$ds^2 = a^2(\tau)[\eta_{\mu\nu} + h_{\mu\nu}(\tau, \vec{x})]dx^\mu dx^\nu$$

[Maleknejad, Kopp '24
Garani, MR, Tesi '24, '25]

With inhomogeneities the background is not Weyl flat and particle production takes place even if Weyl invariance is not broken by mass or kinetic term. Take conformal scalar:

$$\Box\varphi = \frac{1}{2}h_{\mu\nu}\frac{\delta T^{\mu\nu}}{\delta\varphi} \equiv \partial^\mu(h_{\mu\nu}\partial^\nu\varphi) + \frac{\varphi}{12}(2\partial^\mu\partial^\nu + \eta^{\mu\nu}\Box)h_{\mu\nu}$$

The abundance can be computed from the Bogoliubov transformation induced by the background. To first order:

$$\delta\varphi(\vec{k},\tau) = \frac{1}{2}\int d\tau' G_R(k,\tau-\tau')\int\frac{d^3q}{(2\pi)^3}h_{\mu\nu}(q,\tau')\frac{\delta T^{\mu\nu}}{\delta\varphi}(\vec{\omega},\tau')\Big|_{\varphi=\varphi_0}$$

$$G_R(k,\tau) = -\theta(\tau)\frac{e^{ik\tau} - e^{-ik\tau}}{2ik} \qquad \vec{\omega} = \vec{k} - \vec{q}$$

Therefore:

$$\bar{a}_{\vec{k}} = a_{\vec{k}} + \sum_\omega \beta^*_{\vec{k}\vec{\omega}}a^+_{-\vec{\omega}} \longrightarrow N_{\vec{k}} = \sum_{\vec{\omega}}|\beta_{\vec{k}\vec{\omega}}|^2$$

spin-0
$$\beta^{0*}_{\vec{k}\vec{\omega}} = \frac{h_{\mu\nu}(\tilde{q})}{2\sqrt{k\omega}}\left[\tilde{k}^{\mu}\tilde{\omega}^{\nu} + \frac{1}{12}(2\tilde{q}^{\mu}\tilde{q}^{\nu} + \tilde{q}^{2}\eta_{\mu\nu})\right]$$

spin-1/2
$$\beta^{-*}_{\vec{k}\vec{\omega}} = h_{\mu\nu}(\tilde{q})\ \xi^{-*}_{\vec{k}}\frac{\sigma^{\alpha}\tilde{k}_{\alpha}}{2k}\left[\frac{1}{16}\bar{\sigma}^{\mu}(\sigma^{\rho}\bar{\sigma}^{\sigma} - \sigma^{\sigma}\bar{\sigma}^{\rho})(\tilde{q}_{\rho}\eta^{\nu}_{\sigma} - \tilde{q}_{\sigma}\eta^{\nu}_{\rho}) + \frac{1}{2}\bar{\sigma}^{\nu}\tilde{\omega}^{\mu}\right]\xi^{+}_{-\vec{\omega}}$$

spin-1
$$\beta^{\pm*}_{\vec{k}\vec{\omega}} = \frac{h_{\mu\nu}(\tilde{q})}{2\sqrt{k\omega}}\,\varepsilon^{\rho*}_{\vec{k},\pm} Z^{\mu\nu}_{\sigma\rho}\varepsilon^{\sigma*}_{-\vec{\omega},\pm}$$

Scalar and tensor perturbations can be parametrized as

$$ds^2 = a^2(\tau)[1 + 2\Phi(\tau,\vec{x})]d\tau^2 - a^2(\tau)[\delta_{ij}(1 - 2\Psi(\tau,\vec{x})) - h_{ij}(\tau,\vec{x})]dx^i dx^j$$

We decompose:

$$h_{ij}(\tau,\vec{x}) = \sum_{\pm}\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} h^{\pm}(\tau,\vec{q})\epsilon^{\pm}_{ij}(\vec{q}) + h.c.$$

$$\Theta(\tau,\vec{x}) \equiv \Phi + \Psi\,, \qquad \Sigma(\tau,\vec{x}) \equiv \Phi - \Psi$$

Due to Weyl invariance Σ does not contribute

$$\beta^{\lambda}_{\vec{k}\vec{\omega}} = \frac{\Theta(k+\omega,\vec{q})}{2\sqrt{k\omega}}(B_J)_{\vec{k}\vec{\omega}} + \frac{h^{+}(k+\omega,\vec{q})}{2\sqrt{k\omega}}(C_J^{+})_{\vec{k}\vec{\omega}} + \frac{h^{-}(k+\omega,\vec{q})}{2\sqrt{k\omega}}(C_J^{-})_{\vec{k}\vec{\omega}}$$

$J=0$		
B_0	C_0^+	C_0^-
$\frac{3(k-\omega)^2-q^2}{6}$	$\frac{4k^2q^2-\left(k^2+q^2-\omega^2\right)^2}{4\sqrt{2}q^2}$	$\frac{4k^2q^2-\left(k^2+q^2-\omega^2\right)^2}{4\sqrt{2}q^2}$
$J=1/2$		
$B_{1/2}$	$C_{1/2}^+$	$C_{1/2}^-$
$\frac{1}{2}(\omega-k)\sqrt{q^2-(k-\omega)^2}$	$\frac{((k+\omega)^2-q^2)(k-q-\omega)\sqrt{q^2-(k-\omega)^2}}{4\sqrt{2}q^2}$	$\frac{((k+\omega)^2-q^2)(k+q-\omega)\sqrt{q^2-(k-\omega)^2}}{4\sqrt{2}q^2}$
$J=1$		
B_1	C_1^+	C_1^-
$\frac{1}{2}(q^2-(k-\omega)^2)$	$\frac{(k+q-\omega)^2((k+\omega)^2-q^2)}{4\sqrt{2}q^2}$	$\frac{(-k+q+\omega)^2((k+\omega)^2-q^2)}{4\sqrt{2}q^2}$

With these formulas we can compute fully differential quantities from scalar and tensor backgrounds.

An alternative approach is to follow Schwinger.

One can compute the vacuum persistence as the overlap of in and out vacuum

$$\langle \text{out} | \text{in} \rangle = e^{i\Gamma_{1\text{PI}}} \longrightarrow |\langle \text{out} | \text{in} \rangle|^2 = e^{-2\text{Im}\Gamma_{1\text{PI}}}$$

The number of decays is:

$$N \approx 2\text{Im}[\Gamma_{1\text{PI}}]$$

Contrary to Schwinger pair production it is very easy to compute $\Gamma_{1\text{PI}}$

$$S_{\text{int}} = \frac{1}{2}\int d^4x h_{\mu\nu}(x) T^{\mu\nu}(x)$$

To second order in the perturbations:

$$\Gamma_{1PI} = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4} h^{\mu\nu}(-q)\langle T_{\mu\nu}(q) T_{\rho\sigma}(-q)\rangle h^{\rho\sigma}(q)$$

For conformally coupled particles <TT> is completely fixed!

$$\langle T_{\mu\nu}(q) T_{\rho\sigma}(q')\rangle = (2\pi)^4 \delta^4(q+q') \frac{c_J}{7680\pi^2} \Pi_{\mu\nu\rho\sigma}(q) \log(-q^2)$$

$$\Pi_{\mu\nu\rho\sigma}(q) \equiv (2\pi_{\mu\nu}\pi_{\rho\sigma} - 3\pi_{\mu\rho}\pi_{\nu\sigma} - 3\pi_{\mu\sigma}\pi_{\nu\rho}) \; , \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu} q^2 - q_\mu q_\nu$$

Now $\log x = \log|x| + i\pi\theta(-x)$ so that,

$$N_{\text{particles}} = 4\text{Im}\Gamma_{1PI} = \frac{c_J}{15360\pi} \int \frac{d^4q}{(2\pi)^4} \theta(q^2) h^{\mu\nu}(q) \Pi_{\mu\nu\rho\sigma}(q) h^{\rho\sigma}(-q)$$

There is one last surprise... the theory is Weyl invariant so the answer must contain the Weyl tensor!

$$N_{\text{particles}} = \frac{c_J}{2560\pi} \int \frac{d^4q}{(2\pi)^4} \theta(q^2) W^{\mu\nu\rho\sigma}(q) W_{\mu\nu\rho\sigma}(-q)$$

Bogoliubov vs. Schwinger

Formulas look very different...

$$\text{Bogo} \sim \int d^3k \int d^3q \, |h(q\,, k+\omega)|^2 K(k\,, q\,, \cos\theta)$$

$$\text{Schwinger} \sim \int dq_0 \int d^3q |h(q\,, q_0)|^2 \tilde{K}(q, q_0)$$

This suggests the change of variables:

$$k + \omega = q_0 \quad \longrightarrow \quad q_0 = k + \sqrt{k^2 + q^2 - 2qk\cos\theta}$$

With this change of variable the angular integral can be done and one can check that the result agrees with Schwinger.

This allows to compute only inclusive quantities.

In cosmology we are often interested in stochastic backgrounds.

scalar perturbations:

$$ds^2 = a^2 d\tau^2[1 + 2\Psi(\tau, \vec{x})] - a^2[1 - 2\Psi(\tau, \vec{x})]d\vec{x}^2 \,.$$

$$\langle \Psi_{\vec{q}}(\tau)\Psi^*_{\vec{q}'}(\tau')\rangle = (2\pi)^3\delta^3(\vec{q} - \vec{q'})\frac{2\pi^2}{q^3}\Delta_\Psi(q, \tau, \tau')$$

Taking the average of previous formulas,

$$\frac{d(na^3)}{d\log k} = \frac{k^3}{4\pi^2}\int d(\log q)\int d(\cos\theta)\int d\tau \int d\tau' \times$$
$$\times\, e^{-i(k+\omega)(\tau-\tau')}\Delta(q, \tau, \tau')K^s_J[k\,, q, \cos\theta]$$

$$K_0 = \frac{(q^2 - 3(k-\omega)^2)^2}{36k\omega} \quad K_{1/2} = \frac{(k-\omega)^2(q^2 - (k-\omega)^2)}{2k\omega} \quad K_1 = \frac{(q^2 - (k-\omega)^2)^2}{2k\omega}$$

For small q formulas simplify:

$$\lim_{q\to 0}\int d\cos\theta K_J = \frac{1}{30}\,c_J\,\frac{q^4}{k^2}+\cdots \qquad\qquad c_J=\left(\frac{4}{3}\,,4\,,16\right)$$

The inclusive abundance is in fact exactly,

$$\frac{d(na^3)}{dq_0}=\frac{c_J}{240\pi^2}\int d(\log q)\,\theta(q_0^2-q^2)q^4\Delta_\Psi(q,q_0,-q_0)$$

tensor perturbations:

$$\frac{d(na^3)}{dq_0}=\frac{c_J}{640\pi^2}\int d(\log q)\,\theta(q_0^2-q^2)(q_0^2-q^2)^2\Delta_h(q,q_0,-q_0)$$

Abundance is more UV sensitive.

Stochastic Dark Matter

This mechanism allows to produce cosmologically dark sectors when the mass scale is negligible:

- massless fermions and gauge fields
- interacting CFTs

If these sectors are gapped by mass or confinement they will contribute to DM.

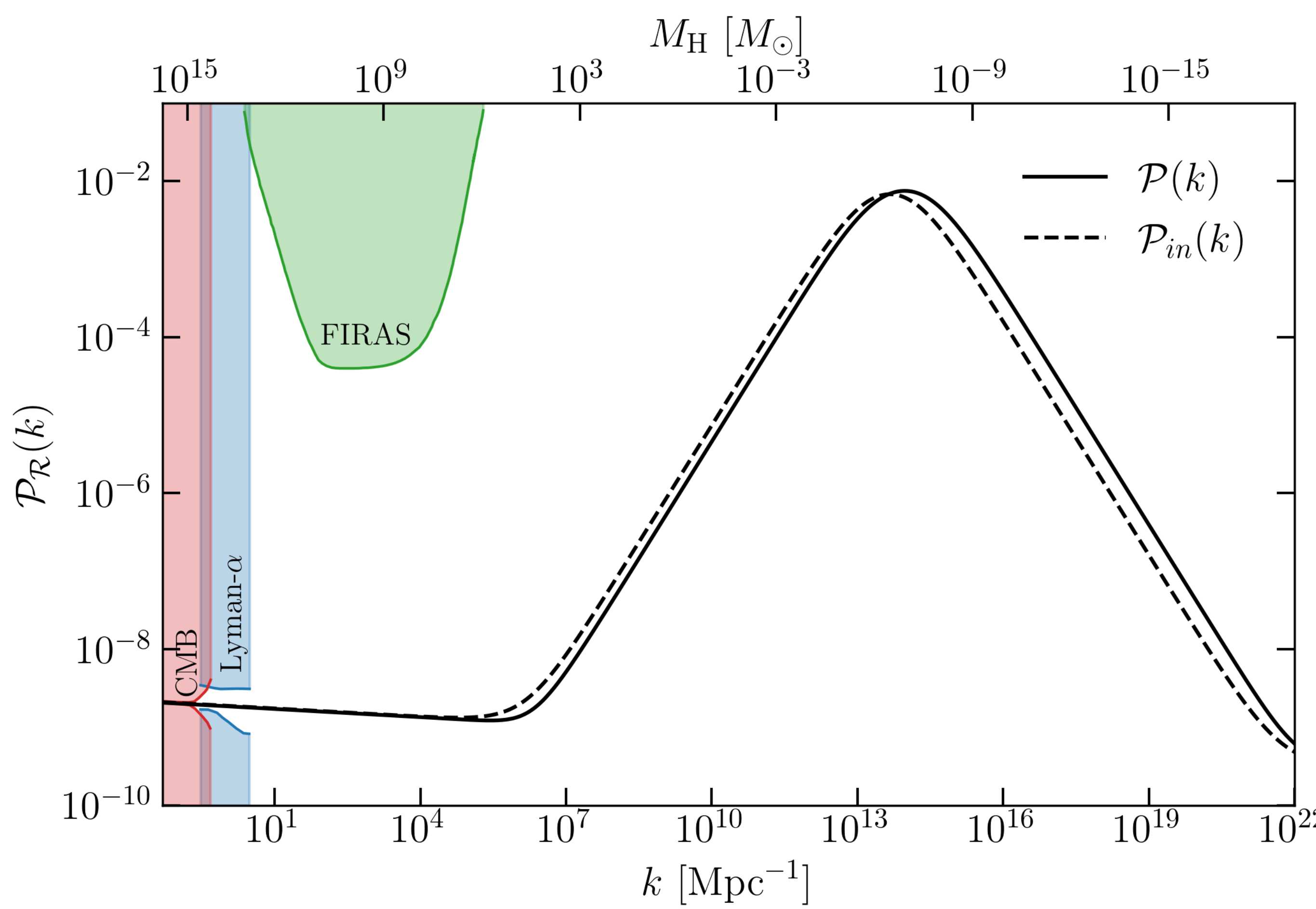
The background could be produced by inflation, first order phase transitions or any other violent event in the early universe.

- Inflationary production:

[Garani, MR, Tesi '24]

Inflation produces curvature perturbations that depend on the evolution of the inflaton. This is described by the power spectrum

$$\Delta_\zeta(k) \sim \frac{1}{2\epsilon M_{\rm pl}^2}\left(\frac{H_I}{2\pi}\right)^2$$



At large scales the perturbations are small but they could be large at smaller scales.

Metric perturbations are induced by curvature perturbations,

$$\Delta_\Psi(q,\tau,\tau') = T(q,\tau)\,T(q,\tau')\Delta_\zeta(q)$$

$$T''(q\,,\tau) + 3\frac{a'}{a}(1+u_s^2)T'(q,\tau) + u_s^2 q^2 T(q\,,\tau) = 0$$

The power spectrum that controls particle production is,

$$\Delta(\vec{q},q_0) = \Delta_\zeta(\vec{q}) \times |\mathcal{I}(q\,,q_0)|^2\,, \quad \text{with} \quad \mathcal{I}(q\,,q_0) \equiv \int_{-\infty}^{\infty} d\tau\, e^{-iq_0\tau}\,T(q,\tau)$$

Performing the q_0 integral,

$$na^3 = c_J \frac{A^\zeta}{4\pi^2}\int d(\log q) q^3 \Delta_\zeta(q) \qquad A^\zeta \equiv \frac{1}{240}\int_1^\infty dx |\mathcal{I}(1,x)|^2 \approx 0.008$$

A^ζ is only mildly dependent on cosmology. This gives,

$$n_{\rm DM} \approx 10^{-4}\, c_J \times \Delta_\zeta(q_{\rm peak}) q_{\rm peak}^3$$

Abundance is dominated by modes that exit horizon towards the end of inflation

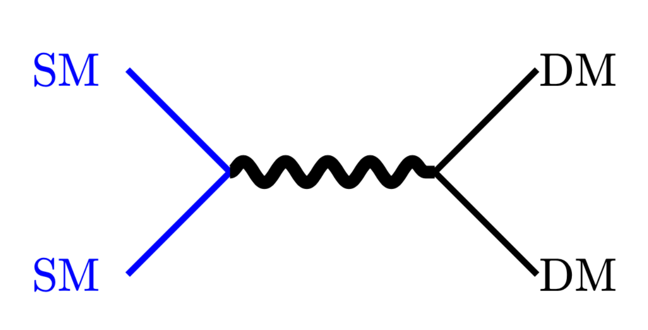
$$\Omega_{\rm DM} \approx 10^{-4} c_J \frac{M q_{\rm peak}^3}{3 M_{\rm Pl}^2 H_0^2} \Delta_\zeta(q_{\rm peak}) \approx c_J \frac{\Delta_\zeta(q_{\rm max})}{10^{-2}} \frac{M}{10^7\,{\rm GeV}}$$

$$q_{\rm max} \sim 10^{-7}\,{\rm eV}$$

Other contributions can easily be smaller,

- Gravitational freeze-in:

[Garny-Sandora-Sloth '15
MR-Tesi-Tillim '20]

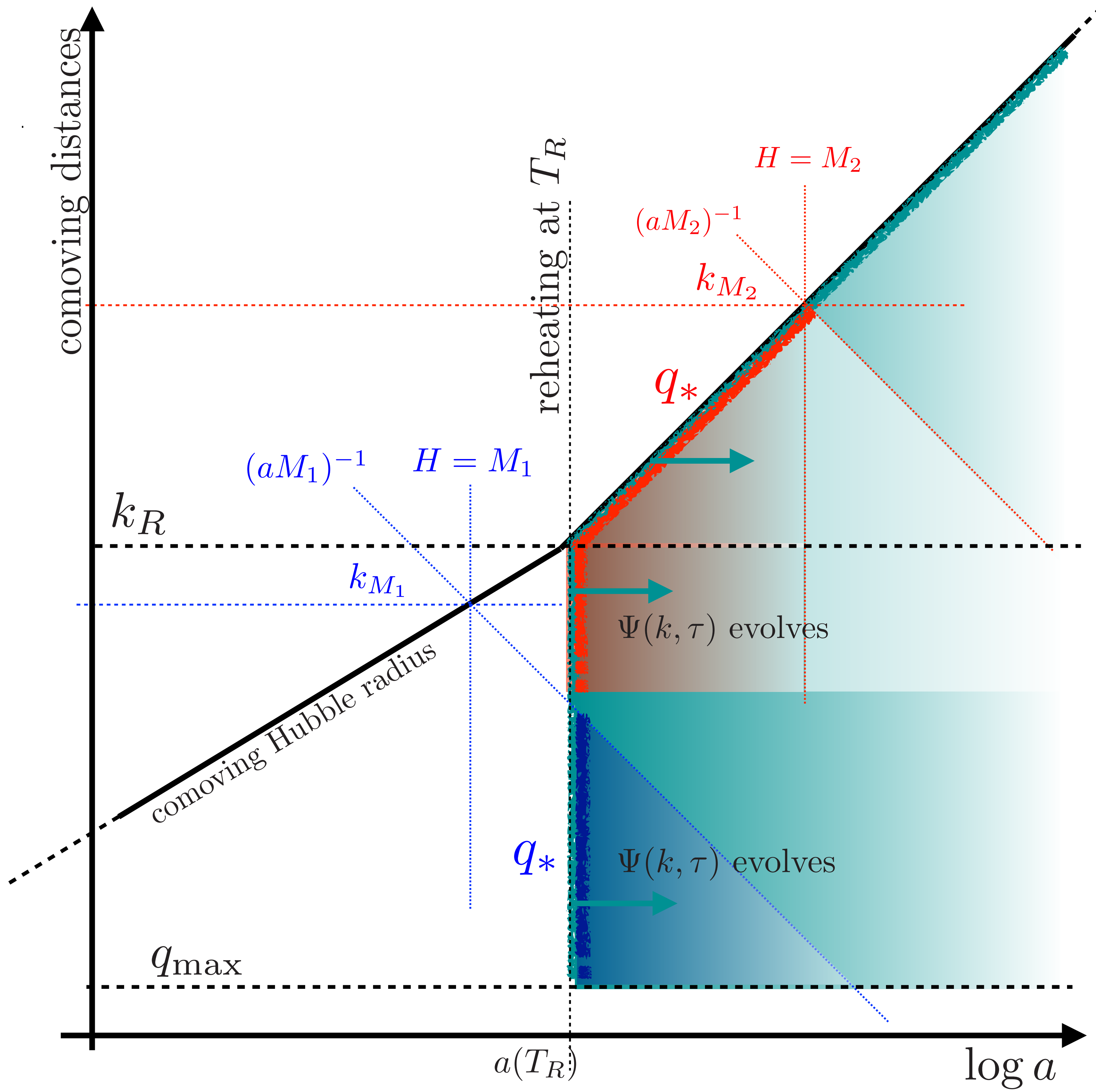


$$\Omega_{\rm DM}|_{\rm GFI} \approx 10^{-5} \frac{M k_R^3}{3 M_{\rm Pl}^2 H_0^2} \times c_J$$

- Time dependent background:

[Redi-Tesi '22]

$$\Omega_{\rm DM}|_{\rm GPP} \approx 10^{-2} \frac{M\, k_M^3}{3 M_{\rm Pl}^2 H_0^2} \times \eta_{\rm eff} \lesssim \left(\frac{M}{10^9\,{\rm GeV}}\right)^{5/2}$$



Modes re-entering the horizon during reheating are not diluted!

Gravity waves from inflation are unavoidably small leading to negligible particle production.

$$\Omega_{\rm DM} \sim c_J \left(\frac{H_I}{M_{\rm Pl}}\right)^2 \frac{M}{10^6\,{\rm GeV}}$$

First order phase transitions also produce large background inhomogeneities especially with supercooling,

$$h_{\mu\nu} \sim \left(\frac{H}{\beta}\right)^2 \frac{\delta\rho}{\rho_{\rm tot}}$$

Both scalar perturbations and gravity waves are produced that can source the production of spectator fields. The latter might be enhanced for rapid transitions.

If the source is classical a much larger effect can be produced similar to axion-photon conversion.

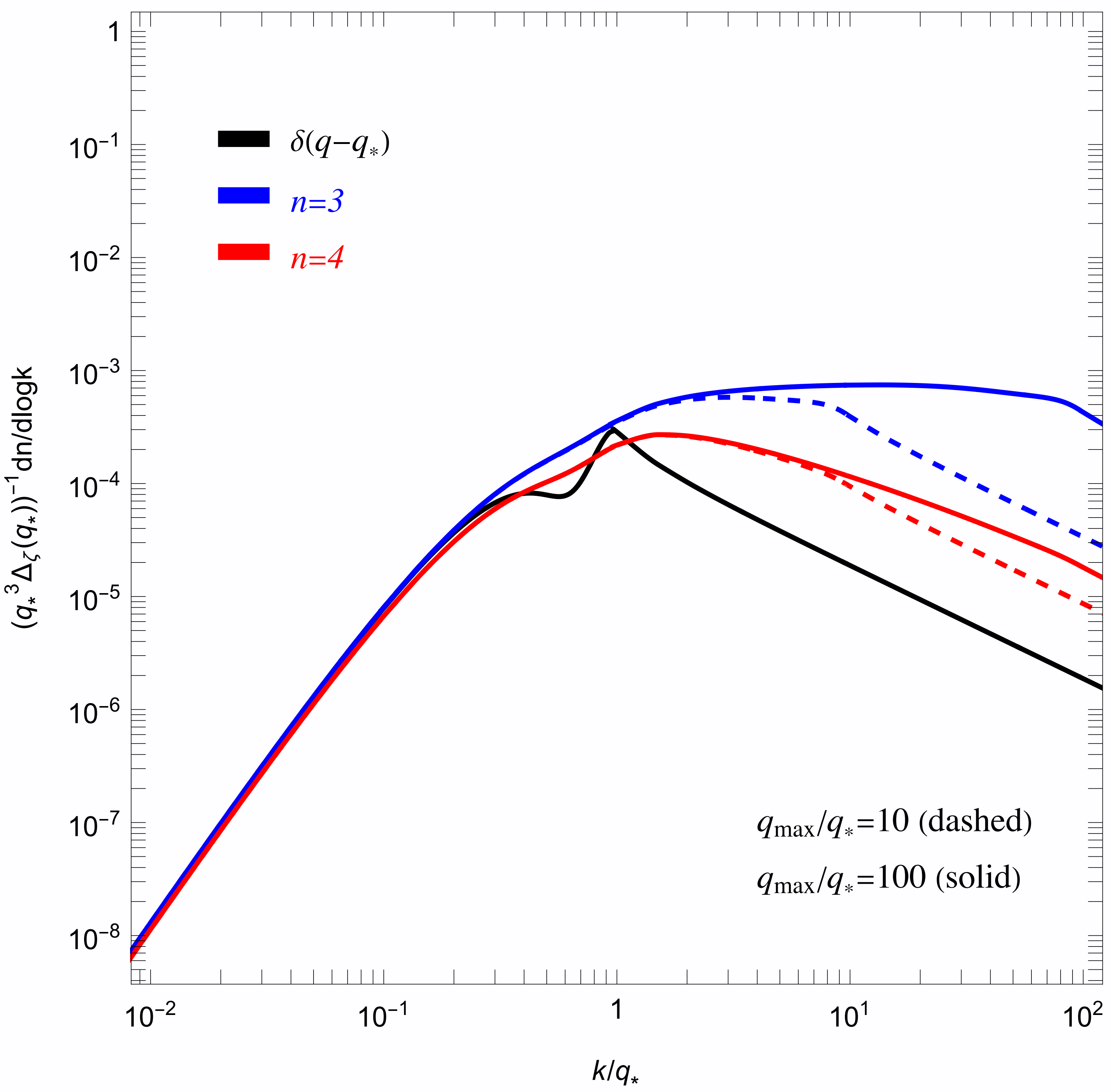
[Eroncel et al. '25]

SUMMARY

- Inhomogeneities allow for a new mechanism of particle production that works even for conformally coupled sectors such as fermions and gauge fields.

- Inclusive quantities are simply determined by the central charge.

- This mechanism can be applied to the production of dark sectors that can host (heavy) dark matter. The formalism can be also applied to production from astrophysical systems or other defects and more generally to different perturbations.



$\delta(q-q_*)$
n=3
n=4
$q_{\rm max}/q_*$=10 (dashed)
$q_{\rm max}/q_*$=100 (solid)
$(q_*^3\Delta_\zeta(q_*))^{-1}$dn/dlogk
k/q_*

tensor perturbations:

$$\Delta L = \frac{1}{2} h_{ij} T^{ij} \qquad\qquad \partial^i h_{ij} = h_i^i = 0$$

Kernels:

$$K_0 = \frac{\left(k^4 - 2k^2\left(q^2+\omega^2\right) + \left(q^2-\omega^2\right)^2\right)^2}{64kq^4\omega}$$

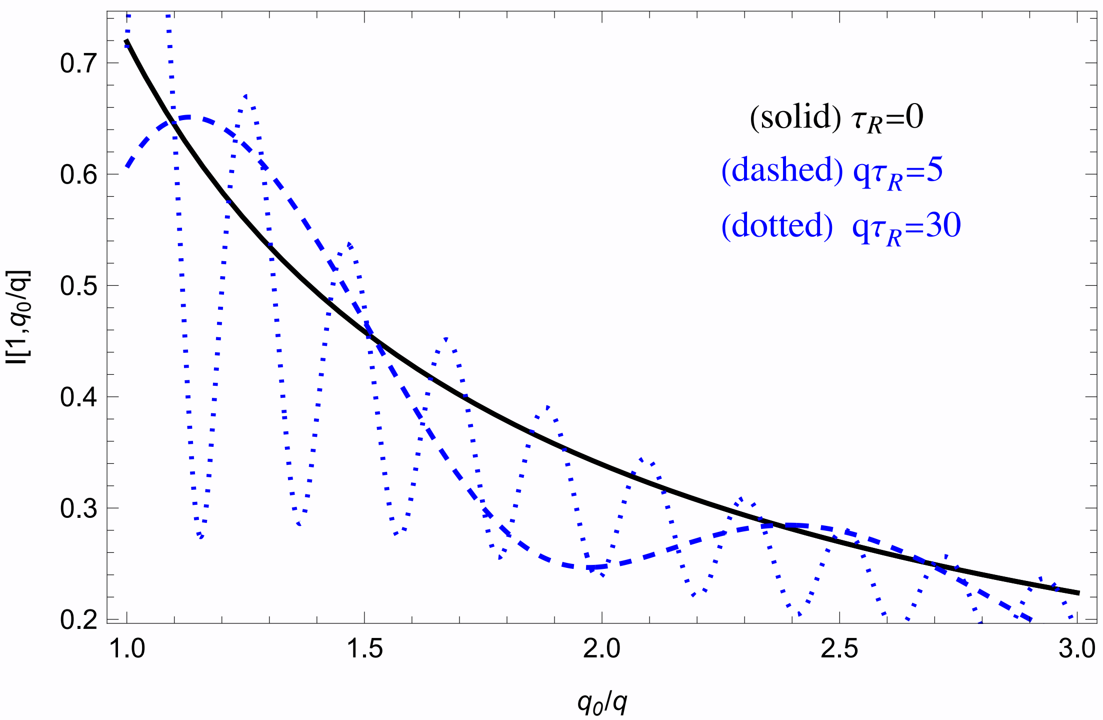
$$K_{1/2} = \frac{\left((k-\omega)^2+q^2\right)(k-q-\omega)(k+q-\omega)(k-q+\omega)^2(k+q+\omega)^2}{64kq^4\omega}$$

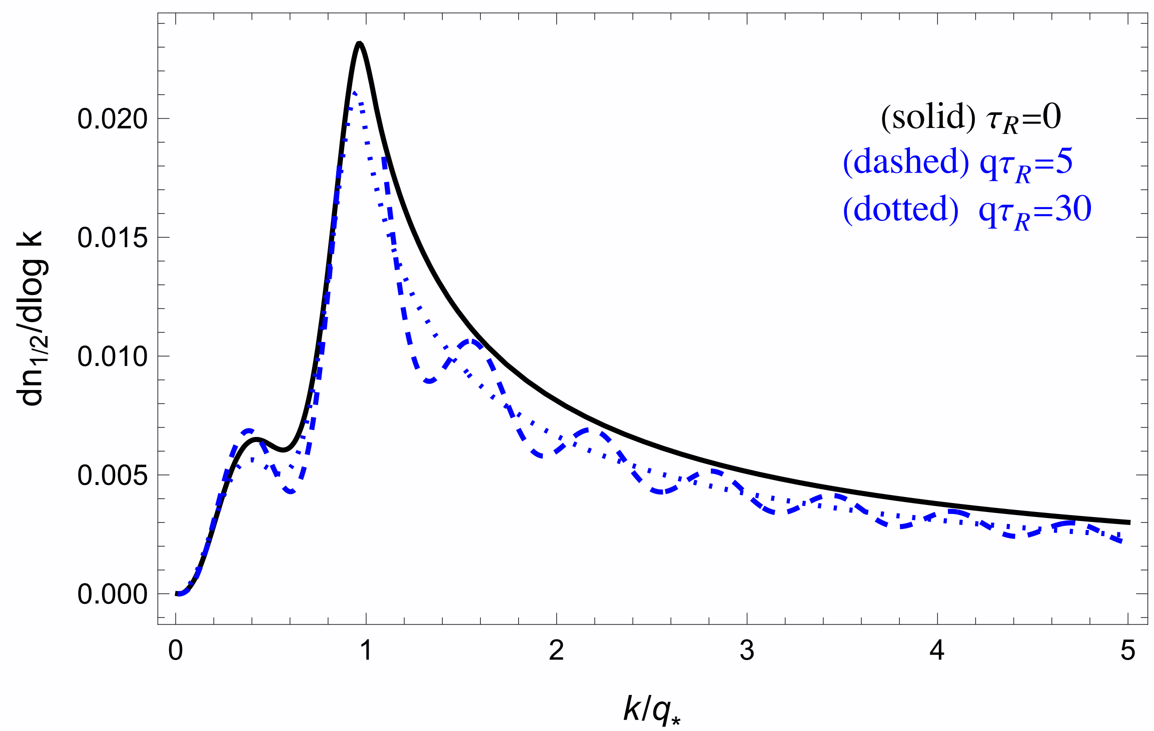
$$K_1 = \frac{(k-q+\omega)^2(k+q+\omega)^2\left(k^4-4k^3\omega+6k^2\left(q^2+\omega^2\right)-4k\left(3q^2\omega+\omega^3\right)+q^4+6q^2\omega^2+\omega^4\right)}{32kq^4\omega}$$

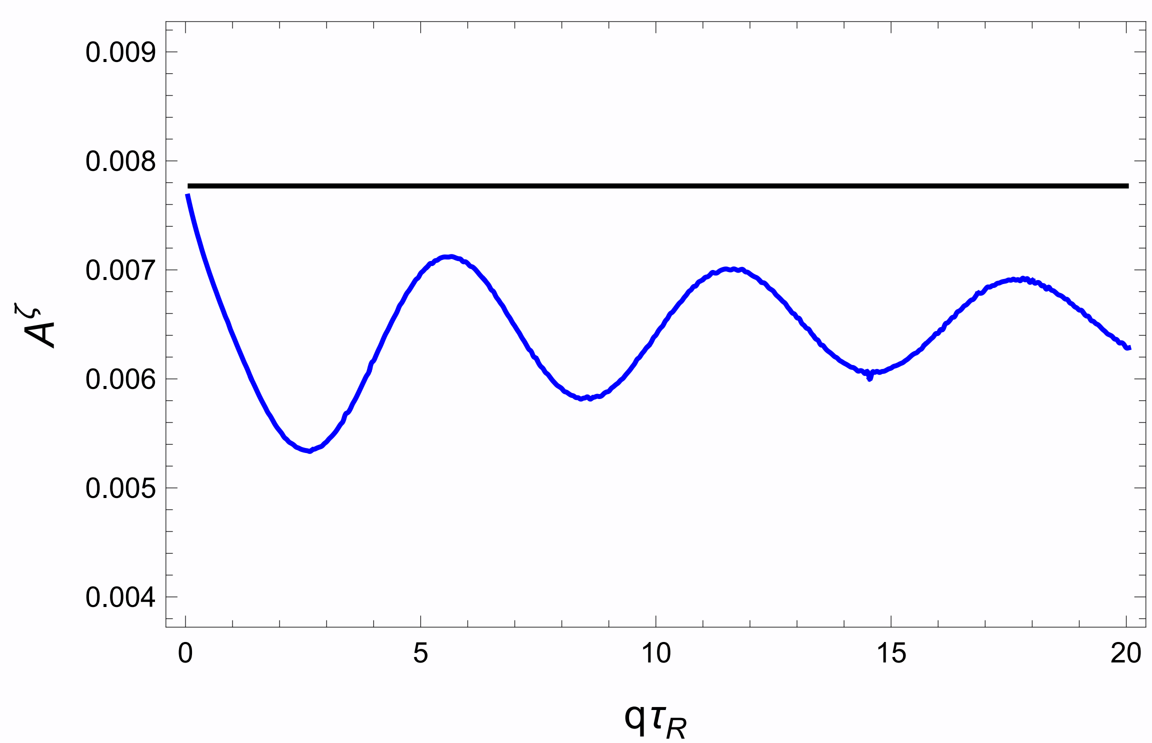
Small q:

$$\lim_{q\to 0}\int d\cos\theta K_J = \frac{1}{5}\, c_J\, k^2 + \cdots$$

slow reheating:







$$A^\zeta \equiv \frac{1}{240}\int_1^\infty dx |\mathcal{I}(1,x)|^2$$