STFC HEP Summer School Lecture Notes

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Contents

Preface

These lecture notes were prepared for the STFC HEP Summer School in September 2025, aimed at PhD students in experimental particle physics. They combine historical background, theoretical foundations, and experimental context into a coherent brief introduction to the Standard Model.

Students are encouraged to consult standard references for further details, including Peskin and Schroeder [1], Halzen and Martin [2], Thomson [3], Griffiths [4], Weinberg [5], Quigg [6], Langacker [7], and Aitchison & Hey [8]. Review articles such as the PDG Review [9], Altarelli [10], and Djouadi [11] are also recommended for complementary perspectives.

Chapter 1

Historical and Experimental Introduction

1.1 Origins of Particle Physics

The exploration of matter at smaller and smaller scales has shaped the field of high-energy physics. Rutherford's gold-foil experiment (1911) demonstrated that atoms consist of a dense, positively charged nucleus surrounded by electrons. This marked the birth of nuclear physics. The electron itself had been discovered earlier by J. J. Thomson (1897). The neutron, identified by Chadwick in 1932, explained nuclear stability and isotopes.

The positron was predicted by Dirac's relativistic wave equation and observed by Anderson in 1932, the first example of antimatter. Yukawa (1935) proposed the pion as the mediator of the strong nuclear force. The pion was discovered in 1947 in cosmic rays, confirming Yukawa's idea. The muon, discovered in 1937, was initially mistaken for the pion and famously led to the quip, "Who ordered that?"[1]

1.2 Neutrinos and Weak Interactions

Pauli proposed the neutrino in 1930 to preserve energy and momentum conservation in β-decay. Reines and Cowan detected the electron antineutrino in 1956, confirming its existence. The weak interaction showed peculiar features: it violated parity, as established by the Wu experiment in 1957, where the β-decay of ^{60}Co nuclei showed an asymmetry under mirror reflection. This discovery overturned the long-held assumption that nature is symmetric under P.

Fermi had already proposed a four-fermion point interaction to describe weak decays (1934). While successful at low energies, the theory is non-renormalisable: its coupling constant G_F has mass dimension -2, implying unitarity violation at energies $\gtrsim 300$ GeV. This motivated the search for a more fundamental description.

1.3 Quantum Electrodynamics

Quantum Electrodynamics (QED) became the paradigm of a successful relativistic quantum field theory. After the Lamb shift (1947) and anomalous magnetic moment measurements, renormalisation theory was developed (Tomonaga, Schwinger, Feynman, Dyson). The QED Lagrangian

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(iD - m)\psi, \tag{1.1}$$

[1] I.I. Rabi, upon learning of the muon's existence.

with $D_\mu = \partial_\mu + ieA_\mu$, describes photons and charged fermions. Its predictions, such as the electron $g-2$, agree with experiment at the level of 10^{-12}.

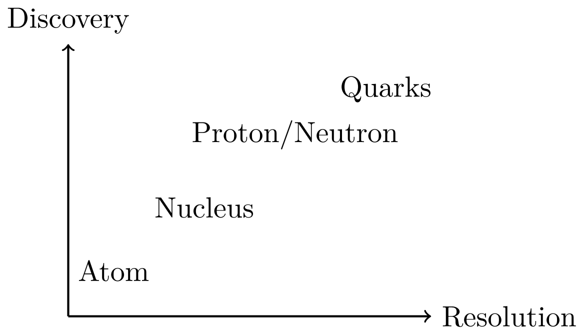

Figure 1.1: The "onion" view of matter: probing higher energies reveals deeper structure.

1.4 The Quark Model and QCD

The 1950s and 60s saw the "hadron zoo". Gell-Mann and Ne'eman proposed the Eightfold Way (1961), organising hadrons into $SU(3)$ multiplets. This led to the quark model (Gell-Mann, Zweig, 1964), where hadrons are bound states of u, d, s quarks. The discovery of the J/ψ meson in 1974 confirmed the charm quark; the bottom and top quarks followed in 1977 and 1995.

Deep inelastic scattering at SLAC (1968) revealed point-like constituents (partons) inside the proton, identified with quarks and gluons. Quantum Chromodynamics (QCD), based on $SU(3)_c$, emerged as the gauge theory of the strong interaction. Its key features are:

- Asymptotic freedom: the coupling $\alpha_s(Q^2)$ decreases at high Q^2.
- Confinement: quarks and gluons are never observed free, but bound in hadrons.

1.5 Electroweak Unification

The $V-A$ structure of the weak interaction was established in the late 1950s. Glashow (1961) introduced $SU(2)_L \times U(1)_Y$ as the gauge group of electroweak interactions. Weinberg and Salam (1967) proposed spontaneous symmetry breaking via the Higgs mechanism, predicting massive $W^\pm$ and Z bosons and a massless photon.

Neutral currents were discovered at CERN (Gargamelle, 1973), a decisive confirmation. The W and Z bosons were discovered by the UA1 and UA2 collaborations at CERN (1983)[2], cementing the electroweak theory.

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$
$$\downarrow$$
$$SU(2)_L \times U(1)_Y$$
$$\downarrow$$
$$U(1)_{\text{EM}}$$

Figure 1.2: Gauge symmetry breaking from the SM group down to electromagnetism.

[2] C. Rubbia and S. van der Meer were awarded the Nobel Prize in 1984.

1.6 Modern Colliders and Discoveries

Key milestones in accelerator-based physics:

- **SLAC (1968)**: deep inelastic scattering, discovery of quarks.
- **SPS (CERN)**: discovery of W and Z (1983).
- **Tevatron (Fermilab)**: discovery of the top quark (1995).
- **LEP (CERN)**: precision electroweak measurements, confirming 3 light neutrinos.
- **LHC (CERN)**: discovery of the Higgs boson (2012), precision Higgs measurements.

1.7 Experimental Methods

In natural units ($\hbar = c = 1$), all quantities are expressed in powers of energy. Cross sections are measured in barns ($1 \text{ b} = 10^{-28} \text{ m}^2$). The expected number of events is

$$N = \sigma \mathcal{L}_{\text{int}}, \tag{1.2}$$

where σ is the cross section and $\mathcal{L}_{\text{int}}$ is the integrated luminosity.

Detectors such as ATLAS and CMS consist of:

- **Tracking detectors** to reconstruct charged particle trajectories.
- **Electromagnetic and hadronic calorimeters** to measure energies.
- **Muon systems** to identify muons.
- **Trigger and data acquisition systems** to select interesting events.

1.8 Outlook

The Standard Model is a triumph of 20th century physics, confirmed in experiments up to the TeV scale. Yet it leaves open questions: the origin of neutrino masses, the nature of dark matter, the matter–antimatter asymmetry, and the stability of the electroweak scale. These puzzles motivate searches for physics beyond the Standard Model at future colliders and other experiments.

Chapter 2

Symmetries and Group Theory Refresher

2.1 Lorentz and Poincaré Algebra

The Lorentz group $SO(1,3)$ preserves the Minkowski metric $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$. An infinitesimal Lorentz transformation is

$$x'^{\mu} = x^{\mu} + \omega^{\mu}{}_{\nu}x^{\nu}, \qquad \omega_{\mu\nu} = -\omega_{\nu\mu}. \tag{2.1}$$

The generators are defined by $M^{\mu\nu} = i(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu})$. Acting on fields, they satisfy

$$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho} + \eta^{\mu\sigma}M^{\nu\rho}). \tag{2.2}$$

Rotations are generated by $J_i = \frac{1}{2}\epsilon_{ijk}M^{jk}$ and boosts by $K_i = M^{0i}$. They obey

$$[J_i, J_j] = i\epsilon_{ijk}J_k, \tag{2.3}$$
$$[J_i, K_j] = i\epsilon_{ijk}K_k, \tag{2.4}$$
$$[K_i, K_j] = -i\epsilon_{ijk}J_k. \tag{2.5}$$

This shows the algebra is $su(2) \oplus su(2)$. Representations are labeled by (j_L, j_R).

Adding translations $P^{\mu} = i\partial^{\mu}$ yields the Poincaré algebra:

$$[M^{\mu\nu}, P^{\rho}] = i(\eta^{\nu\rho}P^{\mu} - \eta^{\mu\rho}P^{\nu}), \qquad [P^{\mu}, P^{\nu}] = 0. \tag{2.6}$$

Casimirs: P^2 (mass) and $W^{\mu}W_{\mu}$ with Pauli–Lubanski vector $W^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}M_{\nu\rho}P_{\sigma}$.

2.2 Representations

The (j_L, j_R) classification yields:

- Scalar: $(0,0)$.
- Weyl spinor: $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$.
- Vector: $(\frac{1}{2},\frac{1}{2})$.
- Dirac spinor: $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$.

A massive representation is labeled by (m, s). Massless reps are labeled by helicity.

2.3 Lie Groups and Algebras

A Lie group element can be written as $U = \exp(i\theta^a T^a)$ with generators T^a obeying

$$[T^a, T^b] = if^{abc}T^c. \tag{2.7}$$

Generators are normalised as $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$.

2.3.1 $SU(2)$:

The Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.8}$$

Define $t^a = \sigma^a/2$. Compute

$$[t^1, t^2] = \tfrac{1}{4}[\sigma^1, \sigma^2] = \tfrac{1}{4}(2i\sigma^3) = it^3, \tag{2.9}$$

and cyclic permutations. Thus $f^{abc} = \epsilon^{abc}$.

Tensor product decomposition: $2 \otimes 2 = 3 \oplus 1$. Explicitly,

$$|1,1\rangle = |\uparrow\uparrow\rangle, \tag{2.10}$$
$$|1,0\rangle = \tfrac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \tag{2.11}$$
$$|1,-1\rangle = |\downarrow\downarrow\rangle, \tag{2.12}$$
$$|0,0\rangle = \tfrac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle). \tag{2.13}$$

2.3.2 $SU(3)$:

$SU(3)$ has 8 generators $t^a = \lambda^a/2$. For example,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.14}$$

Explicitly,

$$[t^1, t^2] = it^3, \qquad [t^4, t^5] = i\left(\tfrac{1}{2}t^3 + \tfrac{\sqrt{3}}{2}t^8\right). \tag{2.15}$$

The adjoint representation is 8-dimensional with matrices $(T^a_{\mathrm{adj}})_{bc} = -if^{abc}$.

2.4 The Gauge Principle

Start with global $U(1)$: $\psi \to e^{i\alpha}\psi$, invariant under constant α. For local $\alpha(x)$,

$$\delta\mathcal{L} = \bar{\psi}i\partial\psi \to \bar{\psi}i\partial\psi + \bar{\psi}\gamma^\mu(\partial_\mu\alpha)\psi, \tag{2.16}$$

breaking invariance. Introduce A_μ transforming as $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$, and define $D_\mu = \partial_\mu + ieA_\mu$. Then

$$F_{\mu\nu} = \frac{i}{e}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.17}$$

For $SU(N)$,

$$D_\mu = \partial_\mu + igA^a_\mu t^a, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu. \tag{2.18}$$

Gauge fields transform as

$$A_\mu \to UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.19}$$

2.5 Standard Model Gauge Structure

The SM gauge group is

$$SU(3)_c \times SU(2)_L \times U(1)_Y. \tag{2.20}$$

Representations:

$$Q_L = (u_L, d_L)^T \sim (3, 2, +1/6), \tag{2.21}$$
$$u_R \sim (3, 1, +2/3), \quad d_R \sim (3, 1, -1/3), \tag{2.22}$$
$$L_L = (\nu_L, e_L)^T \sim (1, 2, -1/2), \quad e_R \sim (1, 1, -1). \tag{2.23}$$

Electric charge:

$$Q = T^3 + \tfrac{1}{2}Y. \tag{2.24}$$

Check: u_L: $T^3 = +1/2$, $Y = +1/6$, so $Q = +2/3$. d_L: $T^3 = -1/2$, $Y = +1/6$, so $Q = -1/3$.

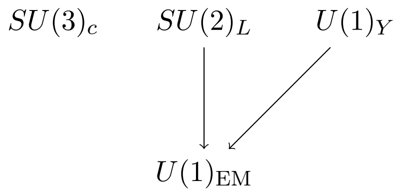

Figure 2.1: The Standard Model gauge group and its breaking to electromagnetism.

2.6 Anomaly Cancellation

Gauge anomalies spoil consistency. For one generation, compute $[SU(2)_L]^2 U(1)_Y$ anomaly:

$$\sum_{\text{doublets}} Y = 3\left(\tfrac{1}{6}\right) + \left(-\tfrac{1}{2}\right) = 0. \tag{2.25}$$

Similarly, $[SU(3)_c]^2 U(1)_Y$: contribution from Q_L, u_R, d_R cancels. For $[U(1)_Y]^3$, sum over all fermions in a generation:

$$3\Big[2\left(\tfrac{1}{6}\right)^3 + \left(\tfrac{2}{3}\right)^3 + \left(-\tfrac{1}{3}\right)^3\Big] + \Big[2\left(-\tfrac{1}{2}\right)^3 + (-1)^3\Big] = 0. \tag{2.26}$$

Thus gauge anomalies cancel, a highly nontrivial consistency condition.

2.7 Discrete Symmetries

Parity acts as $(t, \vec{x}) \to (t, -\vec{x})$, implemented by $\psi(t, \vec{x}) \to \gamma^0 \psi(t, -\vec{x})$. Charge conjugation: $\psi \to C\bar{\psi}^T$, with $C = i\gamma^2\gamma^0$. Time reversal is antiunitary: $\psi(t, \vec{x}) \to i\gamma^1\gamma^3\psi(-t, \vec{x})$.

Weak interactions are $V - A$, coupling only to $\psi_L = \frac{1}{2}(1 - \gamma^5)\psi$. Hence they maximally violate P and C, as shown by Wu (1957)[1]. CP is almost conserved but violated in kaon decays (1964)[2]. The CPT theorem ensures local Lorentz-invariant QFT is invariant under CPT.

[1] Wu's experiment with ^{60}Co decay demonstrated parity violation in weak interactions.
[2] Christenson, Cronin, Fitch, and Turlay observed CP violation in K^0 decays.

2.8 Global Symmetries and Baryon/Lepton Number

The SM Lagrangian conserves B and L. At the classical level,

$$J_B^\mu = \frac{1}{3}\sum_q \bar{q}\gamma^\mu q, \quad J_L^\mu = \sum_\ell \bar{\ell}\gamma^\mu \ell, \tag{2.27}$$

are conserved. Quantum anomalies imply

$$\partial_\mu J_{B+L}^\mu \propto F\tilde{F}, \tag{2.28}$$

violating $B + L$ through non-perturbative sphalerons, but preserving $B - L$.

2.9 Summary

We derived the Lorentz and Poincaré algebras, representations of $SU(2)$ and $SU(3)$, the gauge principle, and anomaly cancellation. We showed how the SM representations ensure consistency, and reviewed discrete and global symmetries with historical footnotes.

Chapter 3

QED, QCD, and Yang–Mills Theories

3.1 Introduction

Quantum Electrodynamics (QED) is the simplest gauge theory, describing electrons and photons. Its success motivated the generalisation to non-Abelian Yang–Mills theories, culminating in Quantum Chromodynamics (QCD). This chapter develops QED in detail, generalises to Yang–Mills, and then focuses on QCD, including one-loop computations, renormalisation, and experimental tests.

3.2 Quantum Electrodynamics

3.2.1 Global to Local Symmetry

Start from the free Dirac Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - m)\psi. \tag{3.1}$$

It is invariant under global $U(1)$: $\psi \to e^{i\alpha}\psi$. Localising $\alpha = \alpha(x)$ spoils invariance:

$$\delta\mathcal{L}_0 = \bar{\psi}\gamma^\mu(\partial_\mu\alpha)\psi. \tag{3.2}$$

Introduce A_μ with transformation $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$ and covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu. \tag{3.3}$$

Then

$$\mathcal{L}_{\text{QED}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(iD\!\!\!\!/ - m)\psi. \tag{3.4}$$

3.2.2 Noether Current

For global $U(1)$, $\psi \to e^{i\alpha}\psi$, the variation $\delta\psi = i\alpha\psi$ gives

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta\psi = \bar{\psi}\gamma^\mu\psi, \tag{3.5}$$

with $\partial_\mu j^\mu = 0$. This is the electric current.

3.2.3 Photon Mass Term

A naive mass term $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$ transforms non-invariantly, showing gauge invariance forbids a photon mass. Indeed, experiments constrain $m_\gamma < 10^{-18}$ eV.

3.2.4 Gauge Fixing and Propagator

Add gauge fixing term

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2. \tag{3.6}$$

Photon propagator:

$$D_{\mu\nu}(p) = \frac{-i}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right). \tag{3.7}$$

In Feynman gauge $\xi = 1$: $-ig_{\mu\nu}/p^2$.

3.2.5 Ward Identity

Gauge invariance implies $q_\mu \mathcal{M}^\mu = 0$, ensuring charge conservation and cancellation of unphysical polarisations.

3.2.6 Worked Example: $e^+e^- \to \mu^+\mu^-$

Amplitude:

$$\mathcal{M} = \bar{v}(p_2)(-ie\gamma^\mu)u(p_1)\frac{-ig_{\mu\nu}}{q^2}\bar{u}(k_1)(-ie\gamma^\nu)v(k_2). \tag{3.8}$$

Spin-averaged squared matrix element:

$$\overline{|\mathcal{M}|^2} = \frac{2e^4}{q^4}\left[(p_1 \cdot k_1)(p_2 \cdot k_2) + (p_1 \cdot k_2)(p_2 \cdot k_1)\right]. \tag{3.9}$$

Cross section in CM frame:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta), \qquad \sigma = \frac{4\pi\alpha^2}{3s}. \tag{3.10}$$

3.2.7 One-Loop Corrections in QED

Electron self-energy diagram contributes

$$\Sigma(p) = -ie^2 \int \frac{d^dk}{(2\pi)^d}\gamma^\mu \frac{p - k + m}{(p-k)^2 - m^2}\gamma_\mu \frac{1}{k^2}, \tag{3.11}$$

evaluated with dimensional regularisation $d = 4 - \epsilon$. Divergences appear as $1/\epsilon$ poles. Renormalisation absorbs divergences into redefined parameters.

Vertex correction gives anomalous magnetic moment $a_e = (g-2)/2 = \alpha/(2\pi)$[1].

3.3 Yang–Mills Theory

3.3.1 General Non-Abelian Construction

Local $SU(N)$: $\psi \to U(x)\psi$, $U(x) = e^{i\alpha^a(x)t^a}$. Define

$$D_\mu = \partial_\mu + igA_\mu^a t^a. \tag{3.12}$$

Field strength from commutator:

$$[D_\mu, D_\nu] = igF_{\mu\nu}, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc}A_\mu^b A_\nu^c. \tag{3.13}$$

Lagrangian:

$$\mathcal{L}_{\mathrm{YM}} = -\tfrac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}(iD - m)\psi. \tag{3.14}$$

[1]First computed by Schwinger (1948).

3.3.2 Self-Interactions

Expanding F^2 yields 3- and 4-gluon vertices with strength fixed by g.

3.3.3 Ghosts and Faddeev–Popov

Gauge fixing introduces Faddeev–Popov determinant. Introducing ghost fields c^a yields ghost Lagrangian

$$\mathcal{L}_{\text{ghost}} = \bar{c}^a(-\partial^\mu D_\mu^{ab})c^b. \tag{3.15}$$

Ghosts cancel unphysical gluon degrees of freedom.

3.4 Quantum Chromodynamics

3.4.1 Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f. \tag{3.16}$$

Quarks in triplet rep, gluons in adjoint.

3.4.2 Colour Factors

Generators satisfy $\text{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$. Define

$$t^a t^a = C_F I, \quad f^{acd}f^{bcd} = C_A\delta^{ab}, \quad \text{Tr}(t^a t^b) = T_F\delta^{ab}. \tag{3.17}$$

For $SU(N)$: $C_F = \frac{N^2-1}{2N}, C_A = N, T_F = \frac{1}{2}$. For QCD ($N = 3$): $C_F = 4/3, C_A = 3$.

3.4.3 One-Loop β-Function

Contributions from gluon loop, ghost loop, quark loop yield

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right). \tag{3.18}$$

For $SU(3)$:

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(11 - \tfrac{2}{3}n_f\right). \tag{3.19}$$

Thus

$$\alpha_s(Q^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\left(Q^2/\Lambda^2_{\text{QCD}}\right)}. \tag{3.20}$$

This is asymptotic freedom[2].

3.4.4 Confinement

At low energies, α_s grows, leading to confinement. Wilson loop:

$$W(C) = \langle 0|\text{Tr}\,P\exp\left(ig\oint_C A_\mu dx^\mu\right)|0\rangle, \tag{3.21}$$

with area law $W(C) \sim e^{-\sigma RT}$ giving $V(R) = \sigma R$.

[2] Discovered by Gross, Wilczek, Politzer in 1973.

3.4.5 Deep Inelastic Scattering

Hadronic tensor:

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \frac{\hat{p}^\mu \hat{p}^\nu}{p \cdot q} F_2(x, Q^2), \tag{3.22}$$

with Bjorken $x = Q^2/(2p \cdot q)$. Callan–Gross relation:

$$F_2(x) = 2xF_1(x). \tag{3.23}$$

Scaling violations arise via DGLAP equations.

3.4.6 Jets and Evidence

At PETRA (1979), 3-jet events revealed gluon emission[3]. Running α_s measured at many scales confirms QCD.

3.5 Anomalies

Axial anomaly in QED:

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}. \tag{3.24}$$

In non-Abelian theories, gauge anomalies must cancel; in the SM they do via interplay of quarks and leptons.

3.6 Summary

We built QED from symmetry, computed scattering and loop corrections, developed Yang–Mills theory, derived gluon self-interactions and ghosts, and studied QCD in detail: colour factors, β-function, confinement, DIS, jets. These theories form the backbone of the Standard Model.

[3] First direct evidence of gluons.

Chapter 4

Electroweak Theory

4.1 Motivation and History

The weak interaction was originally described by Fermi's four-fermion theory of β-decay (1934). Although successful at low energies, it is non-renormalisable: the amplitude for processes like $\nu e \to \nu e$ grows with energy, violating unitarity around $E \sim 300$ GeV. This hinted at the need for a more fundamental, renormalisable description. The discovery of parity violation in weak interactions (Wu experiment, 1957[1]) and the universality of weak interactions motivated a gauge theory.

The electroweak theory, developed by Glashow (1961), Salam, and Weinberg (1967), unifies weak and electromagnetic interactions into an $SU(2)_L \times U(1)_Y$ gauge theory. It predicted neutral currents (observed in Gargamelle, 1973[2]) and massive $W^\pm$ and Z bosons (discovered by UA1/UA2, 1983[3]). t'Hooft and Veltman proved its renormalisability (1971[4]), establishing the Standard Model as a consistent quantum field theory.

4.2 Gauge Structure

The gauge group is

$$SU(2)_L \times U(1)_Y. \tag{4.1}$$

Left-handed fermions form doublets, right-handed fermions are singlets:

$$L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (2, -1), \qquad e_R \sim (1, -2), \tag{4.2}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2, +\tfrac{1}{3}), \qquad u_R \sim (1, +\tfrac{4}{3}),\ d_R \sim (1, -\tfrac{2}{3}). \tag{4.3}$$

Here (n, Y) denotes an $SU(2)$ representation and hypercharge Y.

The electric charge is given by the Gell-Mann–Nishijima relation:

$$Q = T^3 + \tfrac{1}{2}Y, \tag{4.4}$$

where T^3 is the third $SU(2)$ generator.

[1] C.S. Wu observed that the β-decay of ^{60}Co violates mirror symmetry, establishing maximal parity violation.
[2] The Gargamelle bubble chamber at CERN discovered neutral current interactions in ν scattering.
[3] The W and Z bosons were discovered in proton–antiproton collisions at the CERN SPS.
[4] t'Hooft and Veltman demonstrated that spontaneously broken gauge theories are renormalisable.

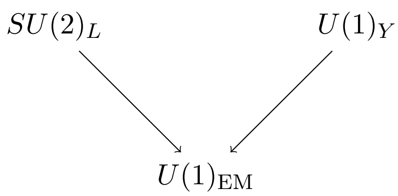

Figure 4.1: Electroweak gauge group breaking to electromagnetism.

4.3 Spontaneous Symmetry Breaking

Introduce a complex scalar Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{4.5}$$

with $\mu^2 < 0$. The vacuum expectation value (vev) is

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{-\mu^2/\lambda}. \tag{4.6}$$

This breaks $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$. Three Goldstone bosons are eaten to give mass to $W^\pm$ and Z.

4.4 Mass Eigenstates and Mixing

Gauge bosons before symmetry breaking: W^a_μ $(a = 1, 2, 3)$ from $SU(2)_L$, and B_μ from $U(1)_Y$. Charged bosons:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu), \quad m_W = \tfrac{1}{2} g v. \tag{4.7}$$

Neutral bosons mix:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}, \tag{4.8}$$

with weak mixing angle $\tan\theta_W = g'/g$. Mass:

$$m_Z = \tfrac{1}{2} v \sqrt{g^2 + g'^2}, \quad m_\gamma = 0. \tag{4.9}$$

4.4.1 Couplings

Couplings satisfy

$$e = g \sin\theta_W = g' \cos\theta_W. \tag{4.10}$$

The Z coupling to a fermion f is

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W} \bar{f} \gamma^\mu \left[(T^3 - Q \sin^2\theta_W) P_L - Q \sin^2\theta_W P_R \right] f \, Z_\mu. \tag{4.11}$$

4.5 Charged and Neutral Currents

4.5.1 Charged Currents

From $W^\pm$ exchange:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{u}_L \gamma^\mu d_L W^+_\mu + \bar{\nu}_L \gamma^\mu e_L W^+_\mu) + \text{h.c.} \tag{4.12}$$

This reproduces Fermi theory at low energies with $G_F/\sqrt{2} = g^2/(8 m_W^2)$.

4.5.2 Neutral Currents

Neutral currents arise from Z exchange:

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} \sum_f \bar{f}\gamma^\mu (g_V^f - g_A^f \gamma^5) f \, Z_\mu, \tag{4.13}$$

where $g_V^f = T^3 - 2Q\sin^2\theta_W$, $g_A^f = T^3$.

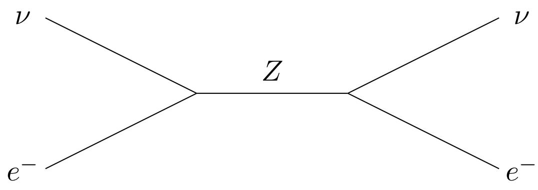

Figure 4.2: Neutrino–electron scattering via Z exchange.

4.6 Experimental Tests

4.6.1 Neutrino Scattering

Neutral currents observed in $\nu N \to \nu X$ confirmed electroweak unification. Cross section ratios tested g_V^f, g_A^f.

4.6.2 LEP Precision Measurements

At LEP, Z production in e^+e^- annihilation allowed measurement of its width. The invisible width corresponds to 3 light neutrino species:

$$\Gamma(Z \to \nu\bar{\nu}) = \frac{G_F m_Z^3}{12\pi\sqrt{2}}. \tag{4.14}$$

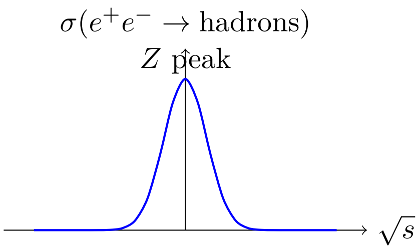

Figure 4.3: Resonant Z production at LEP.

4.6.3 Discovery of W and Z

UA1/UA2 discovered W, Z in 1983 in $p\bar{p}$ collisions, with masses consistent with electroweak predictions.

4.7 Renormalisability

t'Hooft and Veltman proved that spontaneously broken gauge theories are renormalisable. This ensures predictive power at arbitrarily high energies, unlike Fermi theory.

4.8 Summary

We derived the electroweak theory from $SU(2)_L \times U(1)_Y$, showed how the Higgs mechanism gives mass to W and Z while keeping the photon massless, derived charged and neutral currents, and discussed experimental confirmations: neutral currents, LEP precision tests, and discovery of W, Z. The theory is renormalisable and a central pillar of the Standard Model.

Chapter 5

The Higgs Mechanism

5.1 Motivation

Gauge invariance forbids explicit mass terms for gauge bosons and fermions. Yet the $W^\pm$ and Z are massive, and fermions have masses. A mechanism is required to generate masses without spoiling renormalisability.

The Higgs mechanism achieves this through spontaneous symmetry breaking (SSB) of the electroweak gauge group. The idea originates in condensed matter physics (superconductivity, Anderson 1963[1]). Higgs (1964) proposed a relativistic version, introducing a scalar field whose vacuum expectation value (vev) breaks symmetry.

5.2 Spontaneous Symmetry Breaking in Scalar Theory

Consider a complex scalar ϕ with Lagrangian

$$\mathcal{L} = |\partial_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4, \qquad \lambda > 0. \tag{5.1}$$

If $\mu^2 > 0$, vacuum at $\phi = 0$. If $\mu^2 < 0$, the potential has a circle of minima:

$$V(\phi) = \lambda \left(|\phi|^2 - \frac{v^2}{2} \right)^2, \quad v = \sqrt{-\mu^2/\lambda}. \tag{5.2}$$

Choose $\phi = \frac{1}{\sqrt{2}}(v + h(x))$ in unitary gauge. The spectrum: one massive scalar h (Higgs), no massless Goldstones.

5.3 The Higgs Mechanism in $U(1)$

Abelian Higgs model: complex scalar ϕ charged under $U(1)$:

$$\mathcal{L} = |D_\mu \phi|^2 - \mu^2 |\phi|^2 - \lambda |\phi|^4 - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad D_\mu = \partial_\mu + igA_\mu. \tag{5.3}$$

Expand about vev: $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$. Then

$$|D_\mu \phi|^2 = \tfrac{1}{2}(\partial_\mu h)^2 + \tfrac{1}{2} g^2 v^2 A_\mu A^\mu + \cdots. \tag{5.4}$$

Thus $m_A = gv$, the gauge boson acquires mass, while one scalar h remains.

[1] Anderson showed that gauge fields can acquire mass in a superconductor via spontaneous symmetry breaking.

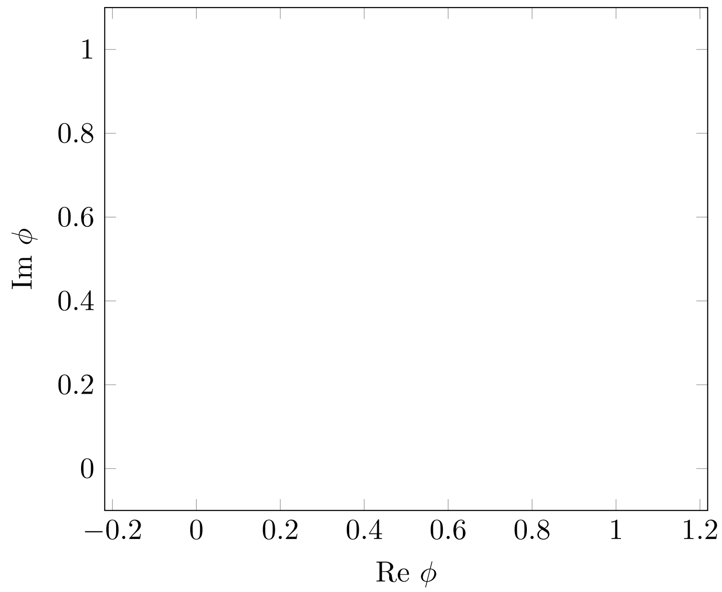

Figure 5.1: Mexican hat potential with degenerate vacua.

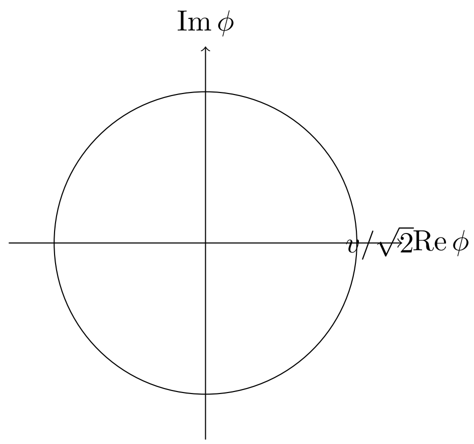

Figure 5.2: Degenerate vacuum manifold $|\phi| = v/\sqrt{2}$ in the complex ϕ plane (circle of minima).

5.4 Electroweak Higgs Mechanism

Higgs doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2. \tag{5.5}$$

With $\langle \Phi \rangle = (0, v/\sqrt{2})^T$, symmetry breaking gives

$$m_W = \tfrac{1}{2} g v, \tag{5.6}$$

$$m_Z = \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v, \tag{5.7}$$

$$m_\gamma = 0. \tag{5.8}$$

One physical scalar remains: the Higgs boson h with

$$m_h^2 = 2\lambda v^2. \tag{5.9}$$

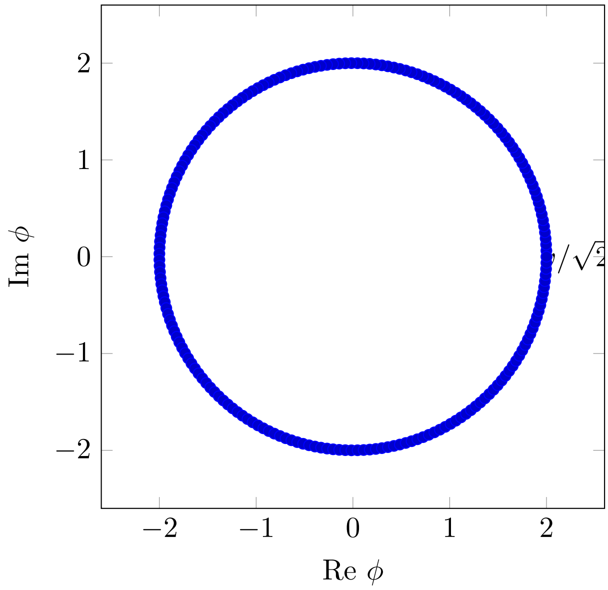

Figure 5.3: Degenerate vacuum manifold $|\phi| = v/\sqrt{2}$ in the complex ϕ plane (circle of minima).

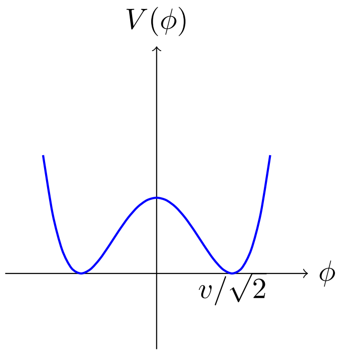

Figure 5.4: Higgs potential: symmetry breaking gives a nonzero vacuum expectation value.

5.5 Fermion Masses via Yukawa Couplings

Direct fermion mass terms $\bar{\psi}_L \psi_R$ are not gauge invariant. Introduce Yukawa couplings:

$$\mathcal{L}_Y = -y_d \bar{Q}_L \Phi d_R - y_u \bar{Q}_L \tilde{\Phi} u_R - y_e \bar{L}_L \Phi e_R + \text{h.c.}, \tag{5.10}$$

where $\tilde{\Phi} = i\sigma^2 \Phi^*$. After SSB, $\Phi = (0, (v+h)/\sqrt{2})^T$, yielding

$$m_f = \frac{y_f v}{\sqrt{2}}, \qquad \mathcal{L} \supset -\frac{m_f}{v} h \bar{f} f. \tag{5.11}$$

Thus fermion masses are proportional to their Yukawa couplings.

5.6 Renormalisability and Unitarity

Higgs exchange cancels bad high-energy behaviour in WW scattering. Without Higgs, $\sigma(W_L W_L \to W_L W_L)$ grows with s, violating unitarity at ~ 1 TeV. Including Higgs restores unitarity.

t'Hooft and Veltman showed the full electroweak theory with Higgs is renormalisable. This makes it predictive at all energies.

5.7 Experimental Evidence

5.7.1 Indirect Evidence

Before discovery, electroweak precision data (LEP, SLC) constrained Higgs mass via loop effects.

5.7.2 Discovery of Higgs Boson

ATLAS and CMS observed a new boson in 2012 at $m_h \approx 125$ GeV via $h \to \gamma\gamma$, $h \to ZZ^* \to 4\ell$, $h \to WW^* \to 2\ell 2\nu$[2].

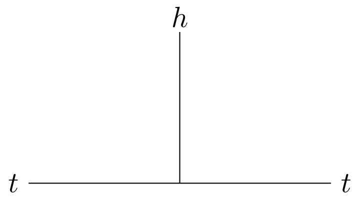

Figure 5.5: Yukawa coupling of Higgs to top quark.

5.8 Theoretical Implications

The Higgs potential raises questions:

- Hierarchy problem: quantum corrections to m_h are quadratically divergent.
- Stability of the vacuum: running λ may become negative at high scales.
- Naturalness: why is $v \ll M_{\text{Planck}}$?

These motivate physics beyond the Standard Model (supersymmetry, compositeness, etc.).

5.9 Summary

The Higgs mechanism explains how gauge bosons and fermions acquire mass while preserving gauge invariance and renormalisability. The discovery of the Higgs at 125 GeV confirmed this mechanism experimentally, though theoretical puzzles remain.

[2] The 2013 Nobel Prize was awarded to Higgs and Englert.

Chapter 6

Flavour Physics

6.1 Generations of Fermions

The Standard Model contains three generations of quarks and leptons:

$$\text{Quarks:} \quad \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \tag{6.1}$$

$$\text{Leptons:} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}. \tag{6.2}$$

Each generation has identical gauge quantum numbers, differing only by mass. The replication of generations is unexplained by the Standard Model, but is essential for CP violation and anomaly cancellation.

6.2 Quark Mixing and the CKM Matrix

In the weak basis, Yukawa couplings generate quark mass matrices:

$$\mathcal{L}_Y = -\bar{Q}_L Y_d \Phi d_R - \bar{Q}_L Y_u \tilde{\Phi} u_R + \text{h.c.} \tag{6.3}$$

After electroweak symmetry breaking, mass matrices $M_{u,d} = Y_{u,d} v/\sqrt{2}$ are diagonalised by unitary rotations:

$$u_L \to U_u u_L, \quad d_L \to U_d d_L, \quad V_{\text{CKM}} = U_u^\dagger U_d. \tag{6.4}$$

The charged current becomes

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu V_{\text{CKM}} d_L W_\mu^+ + \text{h.c.} \tag{6.5}$$

6.2.1 Parametrisation

For n generations, V has n^2 parameters. Removing unphysical phases, physical parameters are

$$\frac{n(n-1)}{2} \text{ angles}, \quad \frac{(n-1)(n-2)}{2} \text{ phases}. \tag{6.6}$$

For $n = 3$: 3 angles, 1 CP-violating phase.

Standard parametrisation:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{6.7}$$

6.3 CP Violation

CP violation requires a nonzero complex phase in the CKM matrix. For two generations, all phases can be rotated away; for three, one physical phase remains. This was confirmed by CP violation in kaons (1964[1]) and later in B mesons (BaBar, Belle).

6.3.1 Jarlskog Invariant

A basis-independent measure of CP violation is

$$J = \mathrm{Im}(V_{us}V_{cb}V_{ub}^{*}V_{cs}^{*}). \tag{6.8}$$

Nonzero J signals CP violation. For SM values, $J \sim 3 \times 10^{-5}$.

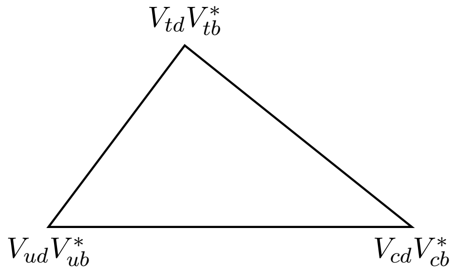

Figure 6.1: Unitarity triangle: graphical representation of CKM unitarity.

6.4 Flavour-Changing Neutral Currents (FCNCs)

In the SM, neutral currents are flavour diagonal due to GIM mechanism (Glashow, Iliopoulos, Maiani, 1970). Box and penguin diagrams generate FCNCs at loop level, suppressing processes like $K^0-\bar{K}^0$ mixing, $b \to s\gamma$.

6.5 Lepton Flavour and Neutrinos

Originally, neutrinos were massless in the SM. Observation of oscillations (Super-Kamiokande 1998[2]) implies nonzero masses and mixing. The leptonic mixing matrix (PMNS) is analogous to CKM:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\bar{\nu}_L U_{\mathrm{PMNS}}\gamma^{\mu}e_L W_{\mu}^{+} + \mathrm{h.c.} \tag{6.9}$$

U_{PMNS} contains large mixing angles, in contrast to CKM.

6.6 Experimental Status

- Kaon physics: CP violation, rare decays ($K_L \to \pi^0\nu\bar{\nu}$).
- B factories: CKM unitarity triangle tested, confirming SM picture.
- Neutrino oscillations: solar, atmospheric, reactor experiments measure mass-squared differences and angles.

[1] Discovered by Christenson, Cronin, Fitch, Turlay.

[2] Discovery of atmospheric neutrino oscillations.

6.7 Open Questions

- Why three generations?
- Origin of Yukawa couplings and mass hierarchies.
- Is CP violation sufficient for baryogenesis? (Probably not.)
- Dirac or Majorana neutrinos?

6.8 Summary

Flavour physics arises from the replication of fermion generations and the structure of Yukawa couplings. The CKM matrix governs quark mixing and CP violation; the PMNS matrix governs neutrino mixing. Experiments confirm the SM structure, but fundamental questions remain unanswered.

Chapter 7

Neutrino Physics

7.1 Introduction

Neutrinos are electrically neutral, weakly interacting fermions. In the original Standard Model, they were massless and only left-handed states appeared. However, the discovery of oscillations implies nonzero masses, requiring an extension of the SM. Neutrino physics thus provides direct evidence for physics beyond the Standard Model.

7.2 Dirac and Majorana Mass Terms

A Dirac mass term couples left- and right-handed components:

$$\mathcal{L}_D = -m_D \bar{\nu}_L \nu_R + \text{h.c.} \tag{7.1}$$

This requires the existence of right-handed neutrinos, singlets under SM gauge group.

A Majorana mass term couples a spinor to itself:

$$\mathcal{L}_M = -\frac{1}{2} m_M \bar{\nu}_L^c \nu_L + \text{h.c.}, \tag{7.2}$$

where $\nu^c = C\bar{\nu}^T$ is the charge-conjugated field. Majorana masses violate lepton number by two units.

7.2.1 See-Saw Mechanism

Introduce both Dirac and heavy Majorana mass terms:

$$\mathcal{L} = -m_D \bar{\nu}_L \nu_R - \tfrac{1}{2} M \bar{\nu}_R^c \nu_R + \text{h.c.} \tag{7.3}$$

Mass matrix in (ν_L, ν_R^c) basis:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}. \tag{7.4}$$

Diagonalisation yields eigenvalues $\sim m_D^2/M$ (light) and $\sim M$ (heavy). For $m_D \sim 100$ GeV, $M \sim 10^{14}$ GeV, we obtain $m_\nu \sim 0.1$ eV. This explains small neutrino masses naturally.

7.3 Neutrino Oscillations

If flavour eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ differ from mass eigenstates (ν_1, ν_2, ν_3), oscillations occur. Relation:

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \tag{7.5}$$

with $U = U_{\text{PMNS}}$ unitary.

7.3.1 Two-Flavour Oscillations

For two flavours (ν_e, ν_μ),

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{7.6}$$

Probability for $\nu_e \to \nu_\mu$ after distance L:

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \ \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \tag{7.7}$$

where $\Delta m^2 = m_2^2 - m_1^2$, E neutrino energy.

7.3.2 Three-Flavour Oscillations

The PMNS matrix parametrised as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \mathrm{diag}(1, e^{i\alpha_1/2}, e^{i\alpha_2/2}), \tag{7.8}$$

with mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, CP phase δ, Majorana phases $\alpha_{1,2}$.

7.4 Matter Effects

In matter, ν_e experiences forward scattering on electrons, giving effective potential $V = \sqrt{2}G_F n_e$. This modifies oscillations (MSW effect[1]), explaining solar neutrino deficit.

7.5 Experimental Evidence

- Solar neutrinos: Homestake experiment (1968), SNO (2001) confirmed flavour change.
- Atmospheric neutrinos: Super-Kamiokande (1998) discovered oscillations in $\nu_\mu \to \nu_\tau$.
- Reactor neutrinos: KamLAND, Daya Bay measured θ_{13}.
- Accelerator neutrinos: K2K, MINOS, T2K, NOvA confirmed oscillations and measure parameters.

Figure 7.1: Schematic of neutrino oscillations: flavour change due to mass mixing.

7.6 Current Status and Open Questions

Neutrino parameters (approximate 2024 values):

$$\Delta m_{21}^2 \approx 7.4 \times 10^{-5}\ \mathrm{eV}^2, \tag{7.9}$$

$$|\Delta m_{31}^2| \approx 2.5 \times 10^{-3}\ \mathrm{eV}^2, \tag{7.10}$$

$$\theta_{12} \approx 33^\circ,\ \theta_{23} \approx 45^\circ,\ \theta_{13} \approx 8^\circ. \tag{7.11}$$

Open questions:

[1] Proposed by Mikheyev, Smirnov, Wolfenstein.

- Normal or inverted mass hierarchy?
- Absolute neutrino mass scale (KATRIN, cosmology).
- Are neutrinos Dirac or Majorana? (Neutrinoless double beta decay experiments).
- Size of CP violation in lepton sector.

7.7 Summary

Neutrino oscillations demonstrate physics beyond the SM, requiring neutrino masses and mixing. The see-saw mechanism provides a natural explanation for small masses. Experiments confirm oscillations and measure mixing angles, but fundamental questions remain.

Chapter 8

Anomalies, Consistency, and Beyond the Standard Model

8.1 Introduction

The Standard Model (SM) is a quantum field theory built on gauge invariance. Gauge symmetry must be exact at the quantum level: any violation (anomaly) renders the theory inconsistent. Remarkably, the SM particle content ensures anomaly cancellation. This chapter develops the theory of anomalies, their cancellation in the SM, and open problems motivating physics beyond the SM.

8.2 Chiral Anomalies

A classical global symmetry may be broken by quantum effects. Consider the axial current $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$. Classically $\partial_\mu J_5^\mu = 0$ for $m = 0$. At one loop (triangle diagram),

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{8.1}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. This is the Adler–Bell–Jackiw anomaly (1969[1]).

8.3 Gauge Anomalies

Gauge anomalies are fatal: they spoil current conservation and gauge invariance. In a general gauge theory, triangle diagrams with three gauge currents may generate anomalies. For $U(1)$, the anomaly is proportional to $\sum_f Q_f^3$, sum over chiral fermions. For mixed anomalies, e.g. $[SU(2)_L]^2 U(1)_Y$, the coefficient involves $\sum Y_f$ over doublets.

8.4 Anomaly Cancellation in the Standard Model

SM fermions come in generations. Check anomalies for one generation:

- $[SU(3)_c]^2 U(1)_Y$: quarks contribute, cancel among Q_L, u_R, d_R.
- $[SU(2)_L]^2 U(1)_Y$: $3 \times Q_L$ and L_L contributions cancel.
- $[U(1)_Y]^3$: sum over all fermions vanishes.
- Gravitational anomaly with $U(1)_Y$: $\sum Y_f = 0$.

[1]Discovered independently by Adler and by Bell–Jackiw.

Explicitly, for $[SU(2)_L]^2U(1)_Y$:

$$3\left(\tfrac{1}{6}\right)+\left(-\tfrac{1}{2}\right)=0. \tag{8.2}$$

For $[U(1)_Y]^3$ anomaly:

$$3\Big[2\left(\tfrac{1}{6}\right)^3+\left(\tfrac{2}{3}\right)^3+\left(-\tfrac{1}{3}\right)^3\Big]+\Big[2\left(-\tfrac{1}{2}\right)^3+(-1)^3\Big]=0. \tag{8.3}$$

This cancellation is nontrivial and constrains the allowed fermion content.

8.5 Global Anomalies

In addition to perturbative anomalies, there are global anomalies (Witten anomaly). For $SU(2)$, consistency requires an even number of fermion doublets. In the SM each generation has 4 doublets (3 quark + 1 lepton), so the condition is satisfied.

8.6 Accidental Symmetries and Their Violation

At renormalisable level, the SM conserves baryon (B) and lepton (L) numbers. These are accidental global symmetries. However, nonperturbative effects (instantons, sphalerons) violate $B+L$ but conserve $B-L$. This has cosmological implications: electroweak baryogenesis relies on sphaleron processes.

8.7 Limits of the Standard Model

Despite its successes, the SM leaves open puzzles:

- **Neutrino masses:** absent in minimal SM, require new physics (see-saw, etc.).
- **Dark matter:** no SM particle is a viable candidate.
- **Baryon asymmetry:** SM CP violation too small to explain observed matter–antimatter asymmetry.
- **Hierarchy problem:** Higgs mass quadratically sensitive to high scales.
- **Unification:** SM couplings nearly but not exactly unify at high energy.

8.8 Theoretical Directions Beyond the SM

- **Supersymmetry (SUSY):** cancels quadratic divergences, provides dark matter candidate, improves unification.
- **Grand Unified Theories (GUTs):** embed SM group in larger simple group (SU(5), SO(10)), predict proton decay.
- **Extra dimensions:** Kaluza–Klein modes, possible solutions to hierarchy problem.
- **Composite Higgs and Technicolor:** Higgs as bound state, new strong dynamics.
- **Seesaw and Leptogenesis:** explain neutrino masses and baryon asymmetry.

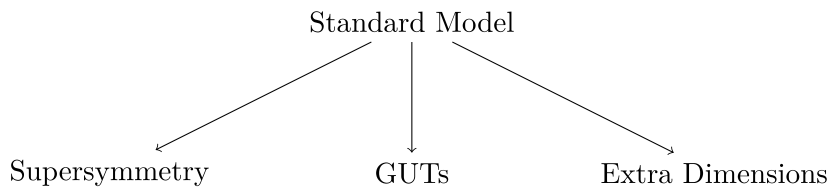

Figure 8.1: Possible extensions of the Standard Model.

8.9 Summary

Anomalies play a crucial role in consistency of gauge theories. The SM is anomaly free thanks to its specific fermion content. Yet it has theoretical and experimental shortcomings, motivating extensions such as SUSY, GUTs, and new mechanisms for neutrino masses, dark matter, and baryogenesis.

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