

STFC HEP summer school - Standard Model

Problem Set (Problems Only)

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Problem 1. Gauge Invariance Check Show explicitly that the kinetic term for a left-handed quark doublet,

$$\mathcal{L} = i \bar{Q}_L \gamma^\mu D_\mu Q_L, \quad D_\mu = \partial_\mu + ig W_\mu^a \frac{\tau^a}{2} + ig' Y B_\mu, \quad (1)$$

is invariant under $SU(2)_L \times U(1)_Y$ gauge transformations. Write the transformation of Q_L , W_μ^a , B_μ , and $D_\mu Q_L$.

Problem 2. $\tau^- \rightarrow \pi^- \nu_\tau$ Diagram and Amplitude Draw the tree-level Feynman diagram for $\tau^- \rightarrow \pi^- \nu_\tau$ and derive the amplitude in terms of G_F , V_{ud} and f_π .

Hint: Use the matrix element $\langle 0 | \bar{d} \gamma^\mu \gamma^5 u | \pi^-(p) \rangle = i f_\pi p^\mu$.

Problem 3. CKM Counting and CP Show that for n generations the CKM matrix contains $\frac{n(n-1)}{2}$ mixing angles and $\frac{(n-1)(n-2)}{2}$ physical phases. Explain why CP violation in the SM requires $n \geq 3$.

Problem 4. QCD Field Strength from the Commutator Starting from $D_\mu = \partial_\mu + ig_s G_\mu^a T^a$, derive $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c$. Illustrate graphically the non-Abelian three-gluon vertex that arises.

Problem 5. Fermion Masses from Yukawa Couplings Explain why direct mass terms like $\bar{e}_L e_R$ are forbidden by $SU(2)_L \times U(1)_Y$ gauge invariance. Show how the Yukawa interaction $-y_e \bar{L}_L \Phi e_R + \text{h.c.}$ generates m_e after EWSB.

Problem 6. Anomaly Cancellation: $[SU(2)_L^2 U(1)_Y]$ For one SM generation, compute the $[SU(2)_L]^2 U(1)_Y$ anomaly coefficient using left-handed fields only. Show that the quark and lepton contributions cancel.

Problem 7. Higgs Potential and Self-Coupling Starting from $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$ with $\mu^2 < 0$, expand about the vacuum and show $m_h^2 = 2\lambda v^2$. Sketch a “Mexican hat” potential.

Problem 8. Running Couplings and (Near) Unification Qualitatively discuss the one-loop running of the SM gauge couplings g_s , g , g' , and sketch $\alpha_i^{-1}(\mu)$ versus $\log \mu$ showing near unification.