Neutrino

Exercise 1

Consider a system composed of a left-handed and a right-handed neutrino:

$$\nu = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} .$$

Assume that the system has a Dirac mass term denoted by m_D, and that the right-handed component also has a Majorana mass term m_M. The Lagrangian of the mass term is given by:

$$\mathcal{L}_{\text{mass}} = -\overline{\nu_L} m_D \nu_R + \frac{1}{2} \overline{\nu_R} C^\dagger \nu_R \, m_M + \text{h.c.}$$

(a) Considering only two massive neutrino states, determine the mixing matrix as a function of m_M.

(b) Determine the oscillation probability of the system in the regime where $m_M \ll m_D$.

(c) Estimate the oscillation length for $m_M = 10^{-14}$ eV. You can consider $m_D = 0.1$ eV.

Exercise 2

Consider a scenario with four neutrinos that propagate through matter:

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} ,$$

(a) Describe the matter potential matrix.

(b) Consider a simplified two-neutrino scenario, where one of the neutrinos is sterile. Assume a mass-squared splitting between the active and sterile states of $\Delta m^2 = 1$ eV^2, and that the matter potential is given by the charged-current (CC) term. Also assume a mixing angle between the active and sterile states of $\theta = 10°$. Determine the energy and matter density at which the mixing angle in matter becomes maximal.