Do Neutrinos Dream in 5D? Towards a Comprehensive Extra-Dimensional Neutrino Phenomenology

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What does a particle in extra dimension mean for us?

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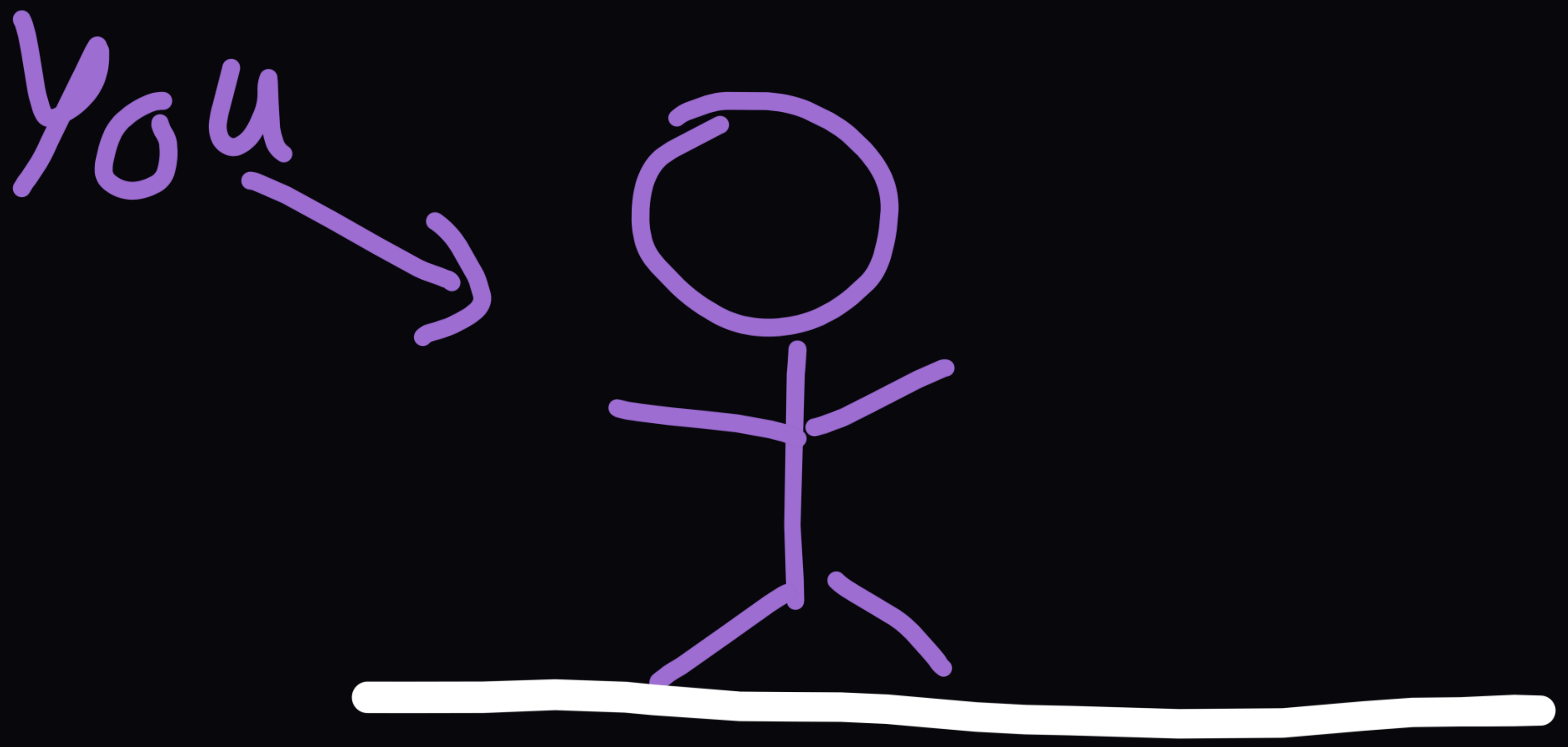

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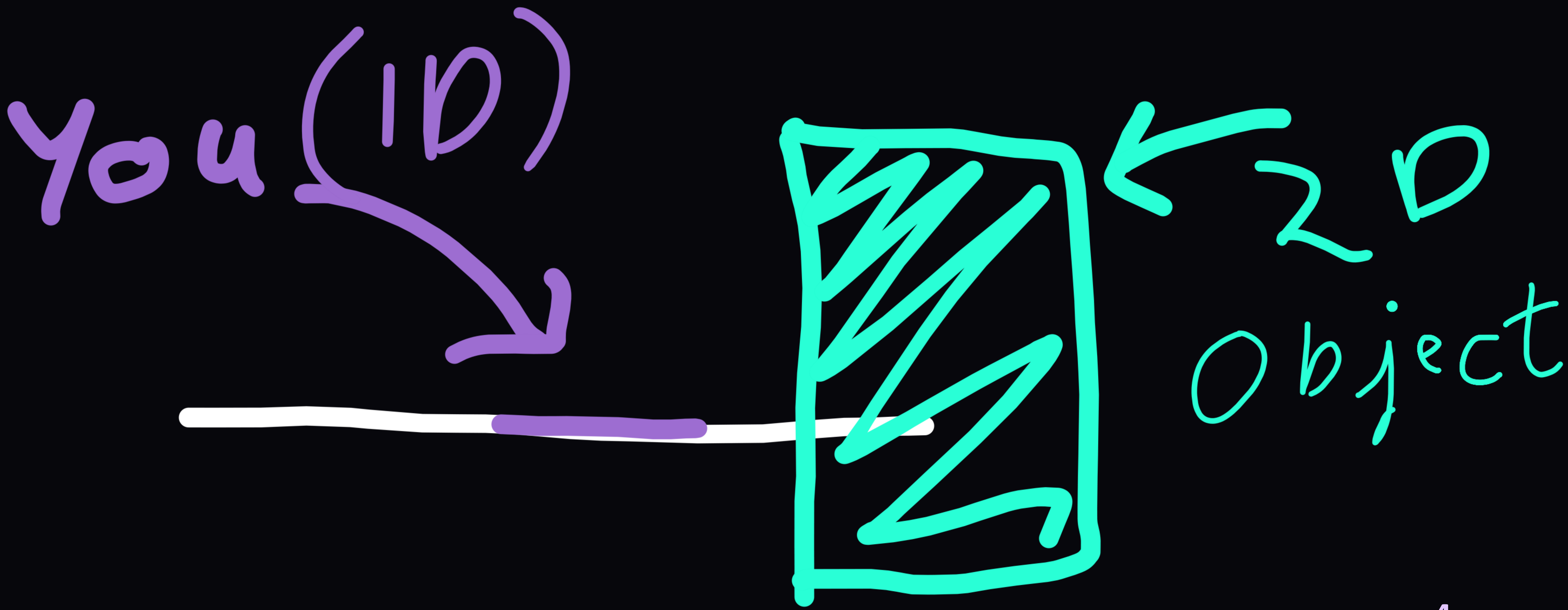

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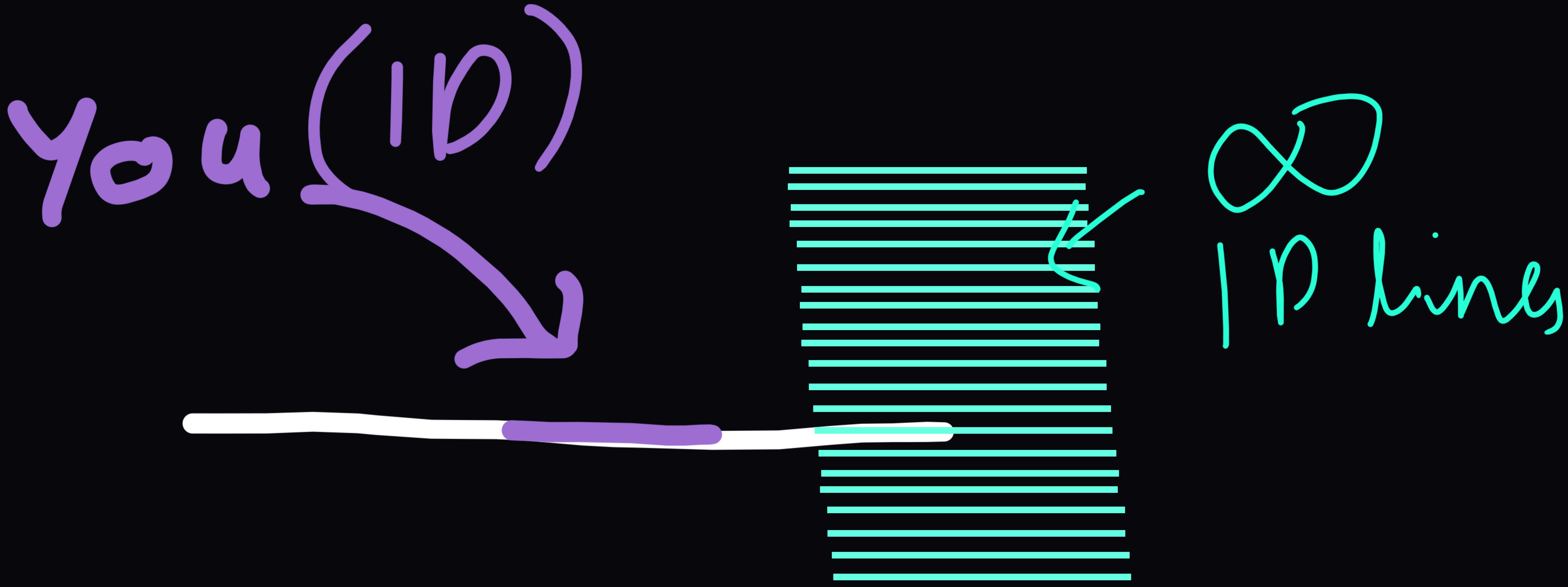

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Ingredients for particle in extra dimension

$$S^{\Psi}_{\text{free}} = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G} \left[i\overline{\Psi}\Gamma^A \nabla_A \Psi - \text{sgn}(y) M_D \overline{\Psi}\Psi - \frac{M_J}{2}\overline{\Psi}\Psi^c \right]$$

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Brane

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Ingredients for particle in extra dimension

Compactify on circle of radius R

$$S^{\Psi}_{\text{free}} = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G}\left[i\overline{\Psi}\Gamma^A\nabla_A\Psi - \text{sgn}(y)M_D\overline{\Psi}\Psi - \frac{M_J}{2}\overline{\Psi}\Psi^c\right]$$

Brane

Bulk

Ingredients for particle in extra dimension

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Brane

Bulk

5D Kinetic Term

$$\Gamma^{\mu} = \gamma^{\mu}\,, \qquad \Gamma^5 = i\gamma^5$$

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Ingredients for particle in extra dimension

Compactify on circle of radius R

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Brane

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5D Kinetic Term

Dirac Mass Term

Majorana Mass Term

$$\Gamma^\mu = \gamma^\mu \,, \qquad \Gamma^5 = i\gamma^5$$

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Ingredients for particle in extra dimension

Solve for EOMs

$$\Psi(x,y) = \begin{pmatrix} \psi_L(x,y) \\ \psi_R(x,y) \end{pmatrix} ,$$

$$S^{\Psi}_{\text{free}} = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G} \left[i\Psi\Gamma^A \nabla_A \Psi - y M_D \Psi\Psi - \frac{M_J}{2} \Psi\Psi^c \right] , \qquad (1)$$

For convenience, $M_D = M_J = 0$

$$i\sigma^\mu \partial_\mu \Psi_R + \partial_5 \Psi_L = 0$$
$$i\bar{\sigma}^\mu \partial_\mu \Psi_R - \partial_5 \Psi_R = 0$$

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Ingredients for particle in extra dimension

Solve for EOMs

$$\Psi(x,y) = \begin{pmatrix} \psi_L(x,y) \\ \psi_R(x,y) \end{pmatrix},$$

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For convenience, $M_D = M_J = 0$

Comparing to SM

$$i\sigma^\mu\partial_\mu\Psi_R + \partial_5\Psi_L = 0$$
$$i\bar{\sigma}^\mu\partial_\mu\Psi_R - \partial_5\Psi_R = 0$$

$$i\sigma^\mu\partial_\mu\Psi_R - m\Psi_L = 0$$
$$i\bar{\sigma}^\mu\partial_\mu\Psi_R - m\Psi_R = 0$$

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Ingredients for particle in extra dimension

Solve for EOMs

$$\boxed{\Psi(x,y) = \begin{pmatrix} \psi_L(x,y) \\ \psi_R(x,y) \end{pmatrix},}$$

$$S^{\Psi}_{\text{free}} = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{G} \left[i\Psi\Gamma^A \nabla_A \Psi - y M_D \Psi\Psi - \frac{M_J}{2}\Psi\Psi^c \right], \qquad (1)$$

For convenience, $M_D = M_J = 0$

Comparing to SM

$$i\sigma^\mu \partial_\mu \Psi_R + \partial_5 \Psi_L = 0$$
$$i\bar{\sigma}^\mu \partial_\mu \Psi_R - \partial_5 \Psi_R = 0$$

$$i\sigma^\mu \partial_\mu \Psi_R - m\Psi_L = 0$$
$$i\bar{\sigma}^\mu \partial_\mu \Psi_R - m\Psi_R = 0$$

5D Kinetic Term = 4D Mass Term

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Ingredients for particle projected from extra dimension

$$-\mathcal{L}_{\text{brane}} \supset \overline{L_L}\tilde{Y}_D \widetilde{H}\psi_R + \frac{1}{2}\overline{\psi^c_R}\tilde{B}\psi_R + \text{h.c.}$$

Bulk fermions interaction with active (brane) neutrinos via Higgs

Mixing between KK modes via Brane Majorana mass term

Ingredients for oscillation in presence of extra dimension

Oscillation Dictated by mass matrices

For Vanilla Case ($M_D = M_J = B = 0$):

$$\mathcal{L} \supset -\overline{\nu_L} \begin{pmatrix} m_D & \sqrt{2}m_D & \sqrt{2}m_D & \dots & \sqrt{2}m_D \\ 0 & \mu_1 & 0 & \dots & 0 \\ 0 & 0 & \mu_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \mu_N \end{pmatrix} \nu_R \equiv -\overline{\nu_L} \mathrm{M} \nu_R$$

KK mass = n/(Radius of extra dimension)

Ingredients for oscillation in presence of extra dimension

Oscillation Dictated by mass matrices

For Vanilla Case ($M_D = M_J = B = 0$):

$$\mathcal{L} \supset -\overline{\nu_L}\begin{pmatrix} m_D & \sqrt{2}m_D & \sqrt{2}m_D & \dots & \sqrt{2}m_D \\ 0 & \mu_1 & 0 & \dots & 0 \\ 0 & 0 & \mu_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \mu_N \end{pmatrix}\nu_R \equiv -\overline{\nu_L}\mathbf{M}\nu_R$$

$$|\nu_{\alpha,n}\rangle = \sum_i \sum_m U_{\alpha i} V^i_{nm} |\hat{\nu}_{i,m}\rangle$$

Flavour eigenstate

PMNS rotation: Diagonalises m_D

KK rotation: Diagonalises mass matrix

Mass eigenstate

$$\boxed{P_{\alpha\to\alpha}(t) \equiv |\langle \nu_{\alpha,0}(t)|\nu_{\alpha,0}(0)\rangle|^2}$$

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Collecting the ingredients

$\boxed{\text{Parameters: } m_D^{lightest}, \mu_1 = \frac{1}{R}, M_D, M_J, \text{B}}$

Collecting the ingredients

Parameters: $m_D^{lightest}, \mu_1 = \frac{1}{R'}, M_D, M_J,$ B

Bulk Dirac
Mass

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Collecting the ingredients

Parameters: $m_D^{lightest}, \mu_1 = \frac{1}{R}, M_D, M_J$, B

Bulk Dirac Mass

Bulk Majorana Mass

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Collecting the ingredients

$\boxed{\text{Parameters: } m_D^{lightest}, \mu_1 = \frac{1}{R}, M_D, M_J, \mathrm{B}}$

Bulk Dirac Mass

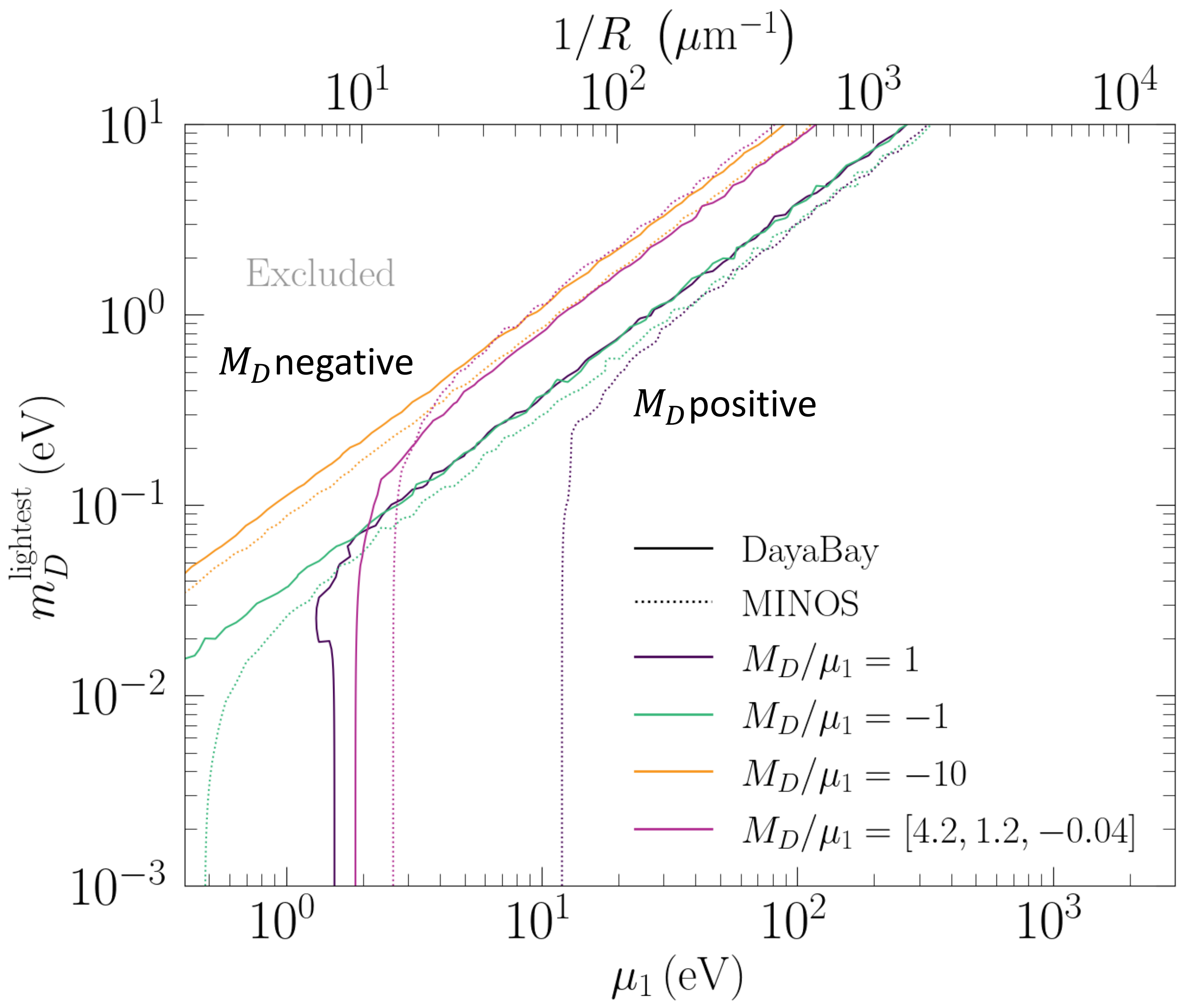
Bulk Majorana Mass

Brane Majorana Mass

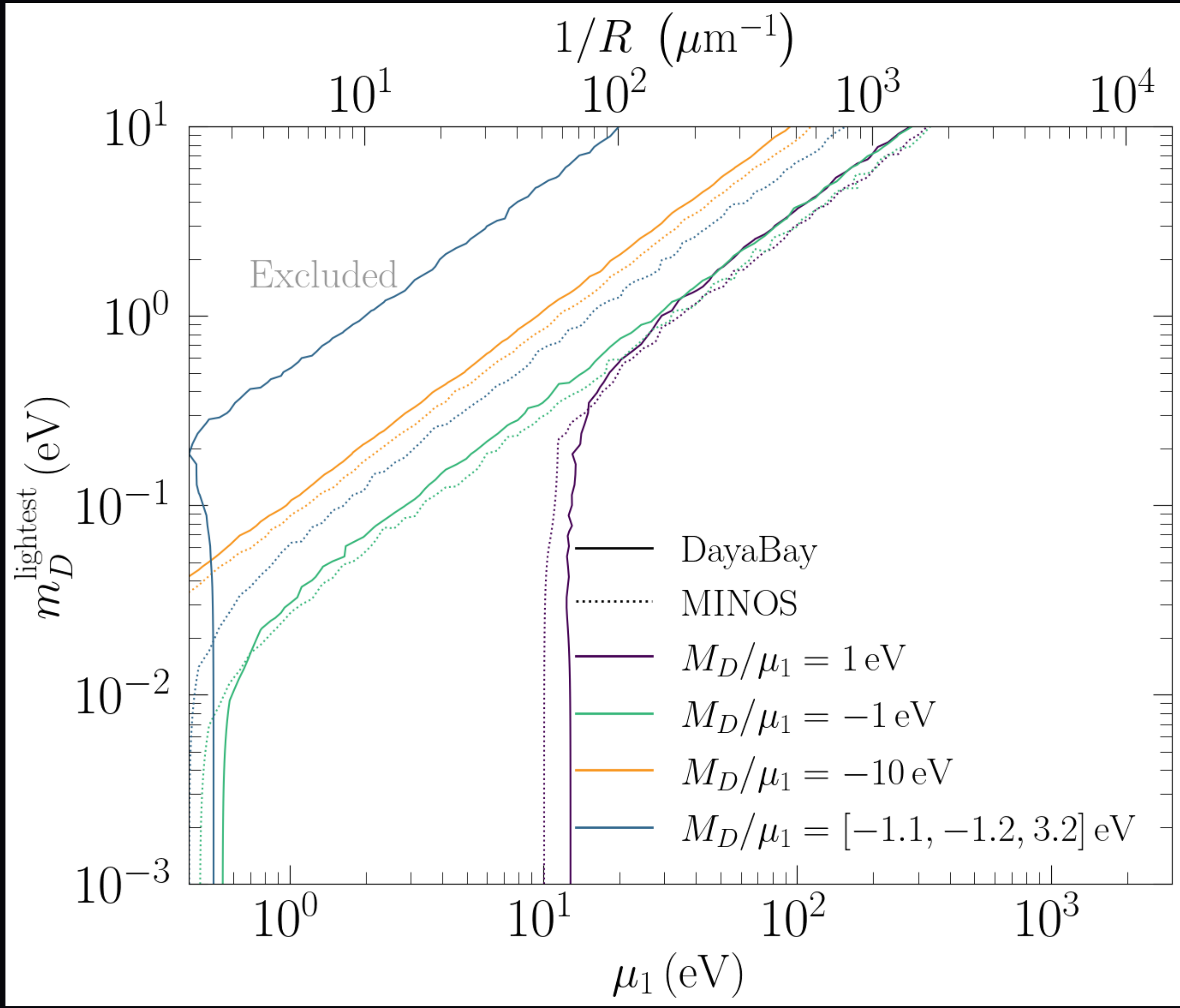
Phenomenology of oscillation in presence of extra dimension

Dirac Bulk

Normal Ordering

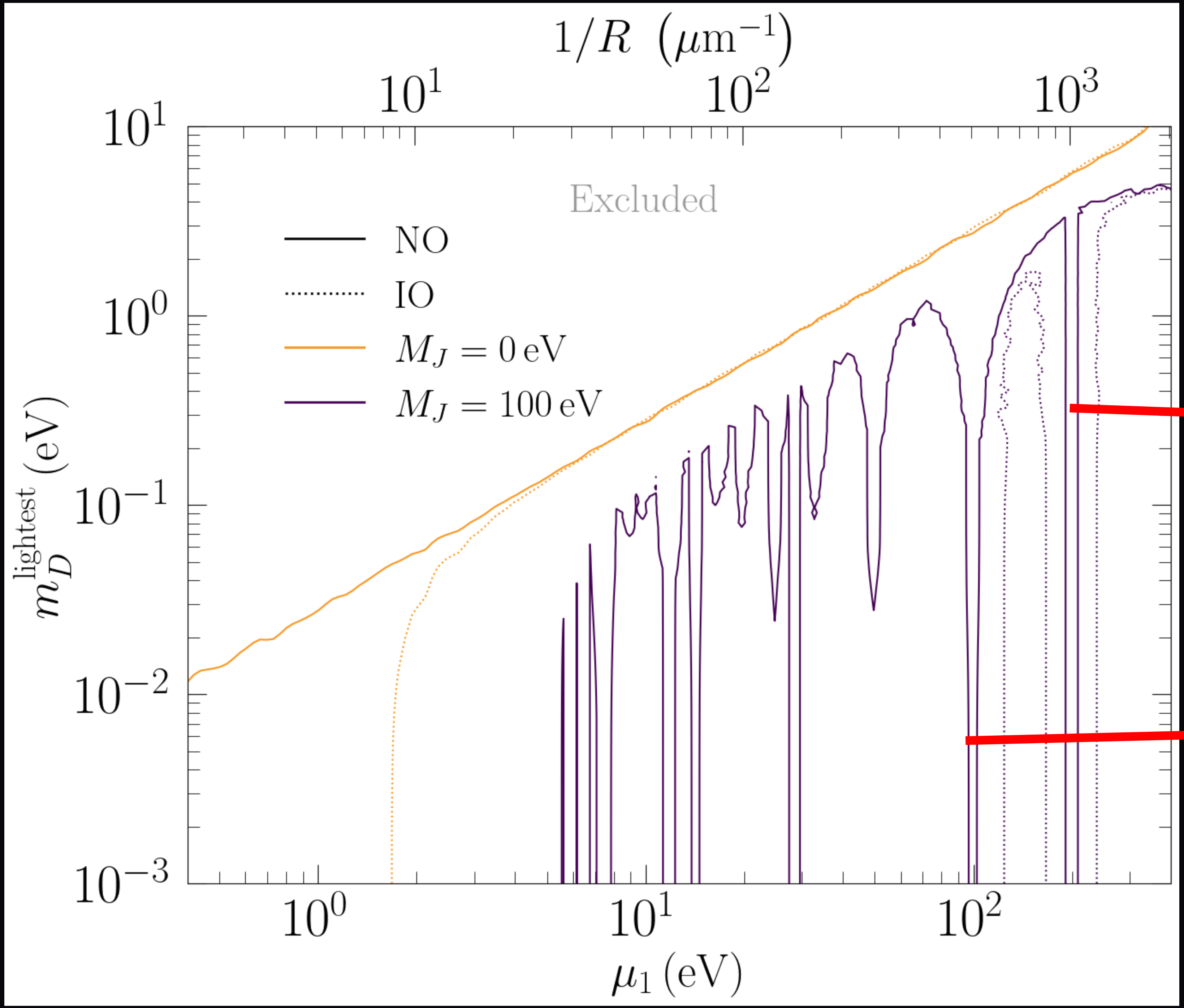


Inverted Ordering



Phenomenology of oscillation in presence of extra dimension

Majorana Bulk



$$M_J = \frac{n_{\text{odd}}}{2}\mu_1$$

$$M_J = \mu_n$$

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Conclusion: Higher Dimensions, Continued Dreams

- Neutrinos, an incredible probe to extra dimensional scenarios
- Rich phenomenology, at current as well as future detectors
- Oscillation simply one observable, could extend to various other observables
- Ex. Presence of Lepton Number Violation, Neutrinoless Double Beta Decay
- However, must motivate assumptions; very different bounds upon different assumptions

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Thank you

Backup:

Dirac brane

$$\pi \cot\left(\frac{\pi m_\lambda}{\mu_1}\right) = \frac{\mu_1 m_\lambda}{m_D^2}$$

$$\cot(x)\,, \quad x \in [0, \pi/2]:$$

$$0 < m_0 < \frac{\mu_1}{2}\,,$$

$$\Delta m_{3l}^2 \approx 2.5 \times 10^{-3}\mathrm{eV}^2$$

$$\mu_1 > 0.1\mathrm{eV}$$

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Backup:

γ_5 already in use; no notion of chirality

Abuse Notation

$$\Psi(x,y) = \begin{pmatrix} \psi_L(x,y) \\ \psi_R(x,y) \end{pmatrix},$$

$$P_{L,R} \equiv \frac{1 \mp \gamma_5}{2}, \qquad \Psi_L = \begin{pmatrix} \psi_L(x,y) \\ 0 \end{pmatrix}, \qquad \Psi_R = \begin{pmatrix} 0 \\ \psi_R(x,y) \end{pmatrix}$$

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Ingredients for particle in extra dimension

$$\psi_L(x,y) = \frac{1}{\sqrt{V}} \sum_{n=-\infty}^{\infty} \psi_{L,n}(x)\xi_n(y)\,, \qquad \psi_R(x,y) = \frac{1}{\sqrt{V}} \sum_{n=-\infty}^{\infty} \psi_{R,n}(x)\chi_n(y)$$

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Backup:

Case 1 : $M_D = M_J = 0$

$$\xi_0(y) = 0\,, \quad \xi_{n>0}(y) = \sqrt{2}\,\sin(\mu_n y)\,, \quad \chi_0(y) = 1\,, \quad \chi_{n>0}(y) = \sqrt{2}\,\cos(\mu_n y)$$

$$\int dy\,\overline{\Psi} i \Gamma^5 \partial_5 \Psi = -\sum_{n=1}^{N} \mu_n \left[\overline{\psi_{R,n}}\psi_{L,n} + \overline{\psi_{L,n}}\psi_{R,n}\right]$$

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Backup:

Matter effects

$$i\frac{d}{dt}|\tilde{\nu}\rangle = \mathcal{U}^\dagger\left(H_0 + \mathcal{V}_m\right)\mathcal{U}|\tilde{\nu}\rangle \approx \left(\frac{\mathrm{MM}^\dagger}{2E} + \mathcal{U}^\dagger\mathcal{V}_m\mathcal{U}\right)|\tilde{\nu}\rangle \equiv \frac{1}{2E}\left(\mathrm{MM}^\dagger + 2E\mathcal{V}\right)|\tilde{\nu}\rangle$$

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Backup:

Resonance

$$M_J = \mu_{n^\star} + \epsilon m_D = n^\star \mu_1 + \alpha m_D$$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & \alpha m_D \end{pmatrix}$$

$$m_\pm = \frac{m_D}{2}\left(\alpha \pm \sqrt{4+\alpha^2}\right) \approx \pm m_D + \frac{m_D}{2}\alpha + \mathcal{O}(\alpha^2)$$

$$P \propto \mathcal{N}_1^4 + \mathcal{N}_2^4 \approx 1 - \frac{2}{4+\alpha^2} \approx \frac{1}{2} + \mathcal{O}(\alpha^2)$$

$\alpha = 0$

$$P\left(\nu_\alpha \to \nu_\alpha\right) = \left|\sum_{in} |U_{\alpha i}|^2 \, |V_{0n}^i|^2 e^{\frac{i m_{in}^2 L}{2E}}\right|^2 \approx P\left(\nu_\alpha \to \nu_\alpha\right)_{\rm SM} \times \left(\frac{1}{2} + \frac{1}{2}\right) = P\left(\nu_\alpha \to \nu_\alpha\right)_{\rm SM}$$

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Backup:

Dirac brane

$$\pi \cot\left(\frac{\pi m_\lambda}{\mu_1}\right) = \frac{\mu_1 m_\lambda}{m_D^2}$$

Dirac Bulk

$$\pi \cot\left(\frac{\pi \tilde{m}_\lambda}{\mu_1}\right) = \frac{\mu_1}{\tilde{m}_\lambda} + \frac{\tilde{m}_\lambda \mu_1}{m_D^2}\left(1 - \frac{\chi_0^2 m_D^2}{\tilde{m}_\lambda^2 + M_D^2}\right)$$

Majorana Bulk

$$\pi \cot\left(\frac{\pi(m_\lambda - M_J)}{\mu_1}\right) = \frac{m_\lambda \mu_1}{m_D^2}$$

Majorana Brane

$$\pi \cot\left(\pi \frac{m_\lambda}{\mu_1}\right) = \frac{m_\lambda \mu_1}{m_D^2 + m_\lambda B}$$

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$$\begin{pmatrix} \chi_0 m_D & \chi_1 m_D & \chi_2 m_D & \dots & \chi_N m_D \\ 0 & \mu_1 & 0 & \dots & 0 \\ 0 & 0 & \mu_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \mu_N \end{pmatrix}$$

$$\chi_0(\pi R) = \left(\frac{2\pi M_D R}{e^{2\pi M_D R} - 1}\right)^{1/2}$$

Dirac

Bulk

Brane

Majorana

$M_D \neq 0$

KK mass = $\sqrt{\left(\frac{n}{R}\right)^2 + M_D^2}$

Weaker constraints at $M_D < 0$

$M_D = M_J = B = 0$

KK mass = n/R

Extensively studied

$M_J \neq 0$

KK mass = $\frac{n}{R} \pm M_J$

Resonance at $M_J = \frac{n}{R}, M_J = \frac{n}{2R}$

$B \neq 0$

KK mass = $\frac{n}{R}$

$m^{lightest} \approx 0.5\left(B - \sqrt{B^2 + 4m_D^2}\right)$

$$\begin{pmatrix} 0 & m_D & m_D & m_D & \dots & m_D & m_D \\ m_D & M_J & 0 & 0 & \dots & 0 & 0 \\ m_D & 0 & M_J + \mu_1 & 0 & \dots & 0 & 0 \\ m_D & 0 & 0 & M_J - \mu_1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ m_D & 0 & 0 & 0 & \dots & M_J + \mu_N & 0 \\ m_D & 0 & 0 & 0 & \dots & 0 & M_J - \mu_N \end{pmatrix}$$

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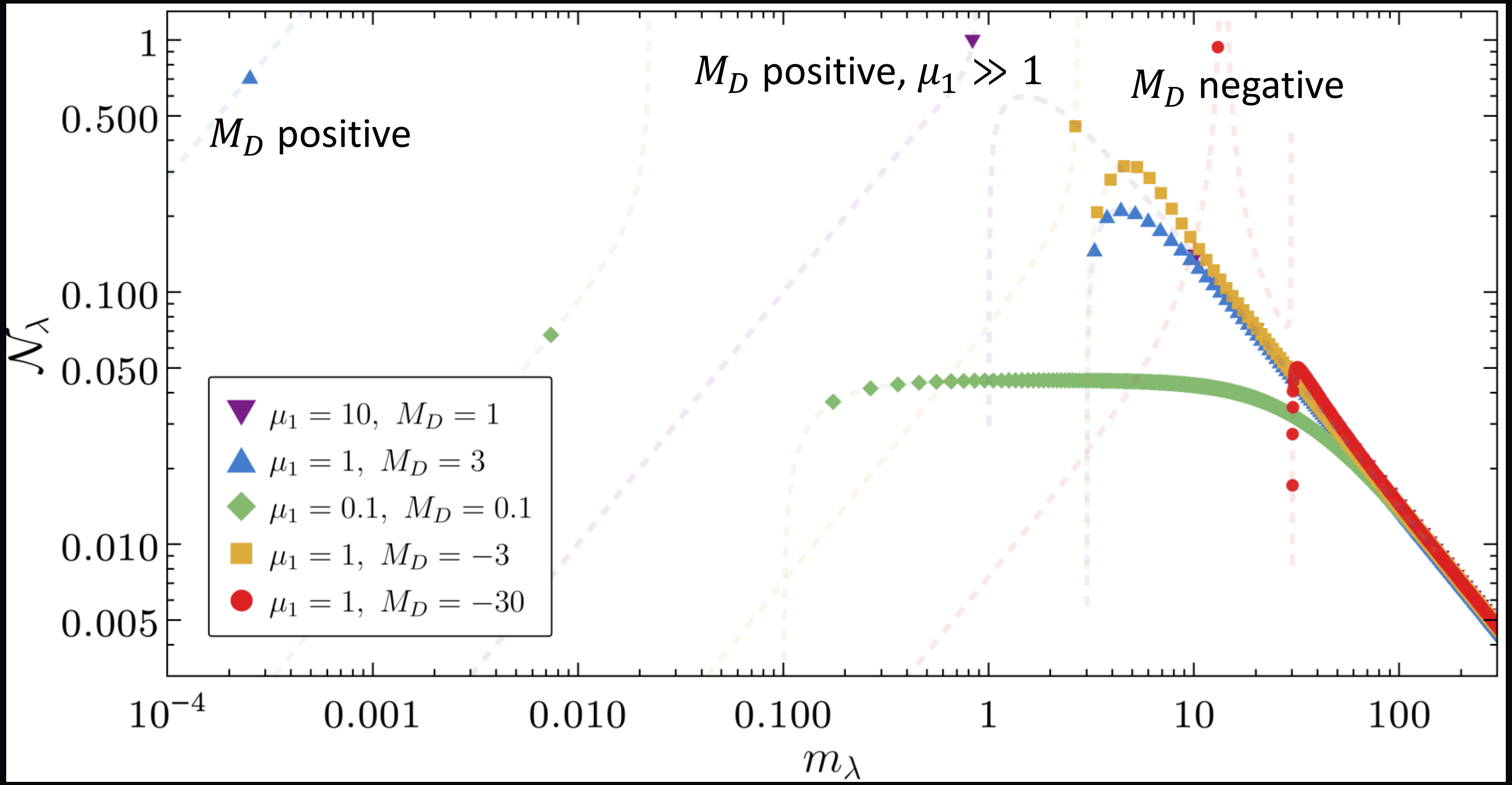
Phenomenology of oscillation in presence of extra dimension

Dirac Bulk

Mixing



Eigenvalues of mass matrix

$$\chi_0(\pi R) \approx \begin{cases} \sqrt{2\pi R M_D}\, e^{-\pi R M_D}\,, & M_D R \gg 1\,, \\ 1\,, & M_D R \sim 0\,, \\ \sqrt{2\pi R |M_D|}\,, & M_D R \ll 1\,. \end{cases}$$

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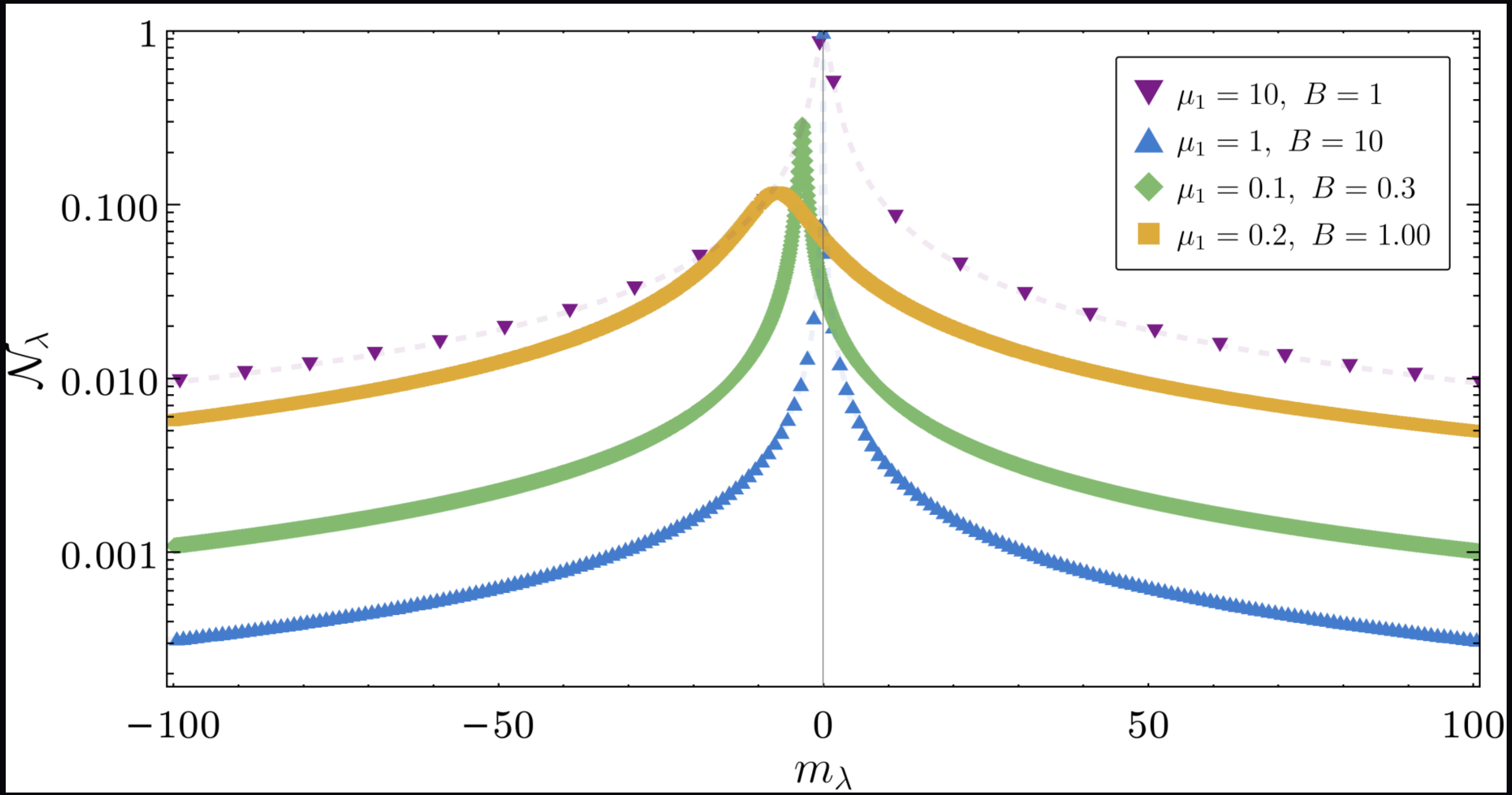
Phenomenology of oscillation in presence of extra dimension

Case 4: Majorana Brane



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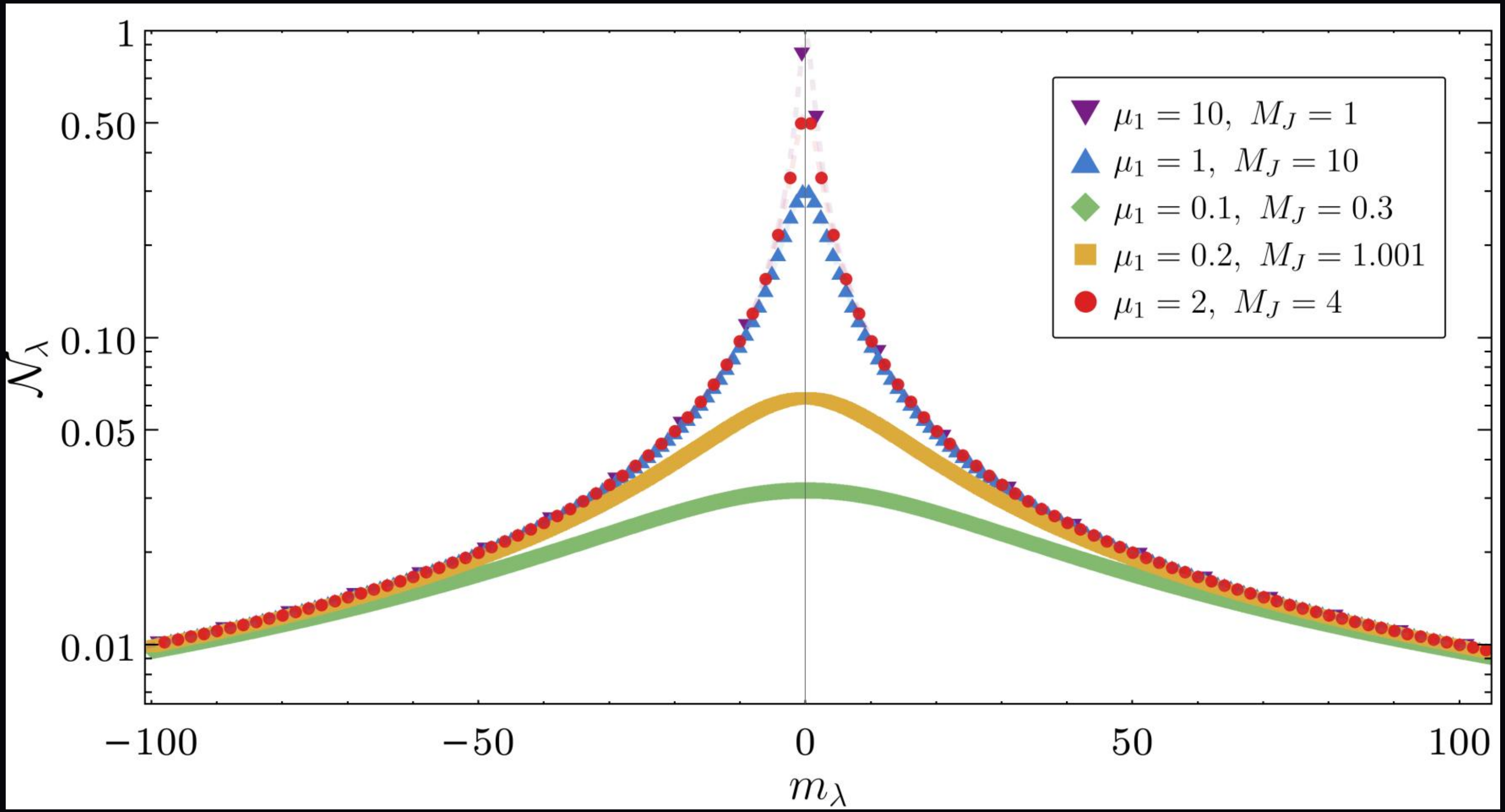
Phenomenology of oscillation in presence of extra dimension

Case 3: Majorana Bulk



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Phenomenology of oscillation in presence of extra dimension

Case 3: Majorana Bulk

$$X \equiv \left(\nu_L \ \psi^c_{R,0} \ \psi^c_{1,1} \ \psi^c_{2,1} \ \psi^c_{1,2} \ \psi^c_{2,2} \ \dots \ \psi^c_{1,N} \ \psi^c_{2,N}\right)^T$$

$$-\mathcal{L} \supset \frac{1}{2}\overline{X}\mathbf{M}X^c, \quad \mathbf{M} = \begin{pmatrix} 0 & \chi_0 m_D & \tilde{\chi}_1 m_D & \tilde{\chi}_1 m_D & \dots & \tilde{\chi}_N m_D & \tilde{\chi}_N m_D \\ \chi_0 m_D & M_J & 0 & 0 & \dots & 0 & 0 \\ \tilde{\chi}_1 m_D & 0 & M_J + \mu_1 & 0 & \dots & 0 & 0 \\ \tilde{\chi}_1 m_D & 0 & 0 & M_J - \mu_1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \tilde{\chi}_N m_D & 0 & 0 & 0 & \dots & M_J + \mu_N & 0 \\ \tilde{\chi}_N m_D & 0 & 0 & 0 & \dots & 0 & M_J - \mu_N \end{pmatrix}, \quad \tilde{\chi} \equiv \frac{\chi_n}{\sqrt{2}}$$

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Phenomenology of oscillation in presence of extra dimension

Case 4: Majorana Brane

$$-\mathcal{L} \supset \overline{L_L} Y_D \widetilde{H} \left(\sum_{n=0}^{\infty} \psi_{R,n} \chi_n \right) + \frac{1}{2} \sum_{ij=0} \overline{\psi^c_{R,i}} B \psi_{R,j} + \text{h.c.}$$

$$-\mathcal{L} \supset \frac{1}{2} \overline{X} \mathbf{M} X^c,$$

$$\mathbf{M} = m_D \times \begin{pmatrix} 0 & \chi_0 & \tilde{\chi}_1 & \tilde{\chi}_1 & \dots & \tilde{\chi}_N & \tilde{\chi}_N \\ \chi_0 & \beta\chi_0^2 & \beta\chi_0\tilde{\chi}_1 & \beta\chi_0\tilde{\chi}_1 & \dots & \beta\chi_0\tilde{\chi}_N & \beta\chi_0\tilde{\chi}_N \\ \tilde{\chi}_1 & \beta\chi_0\tilde{\chi}_1 & \beta\tilde{\chi}_1^2 + \rho_1 & \beta\tilde{\chi}_1^2 & \dots & \beta\tilde{\chi}_1\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N \\ \tilde{\chi}_1 & \beta\chi_0\tilde{\chi}_1 & \beta\tilde{\chi}_1^2 & \beta\tilde{\chi}_1^2 - \rho_1 & \dots & \beta\tilde{\chi}_1\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \tilde{\chi}_N & \beta\chi_0\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N & \dots & \beta\tilde{\chi}_N^2 + \rho_N & \beta\tilde{\chi}_N^2 \\ \tilde{\chi}_N & \beta\chi_0\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N & \beta\tilde{\chi}_1\tilde{\chi}_N & \dots & \beta\tilde{\chi}_N^2 & \beta\tilde{\chi}_N^2 - \rho_N \end{pmatrix}$$

$$\rho_n \equiv \frac{\mu_n}{m_D},$$

$$\beta \equiv \frac{B}{m_D}$$

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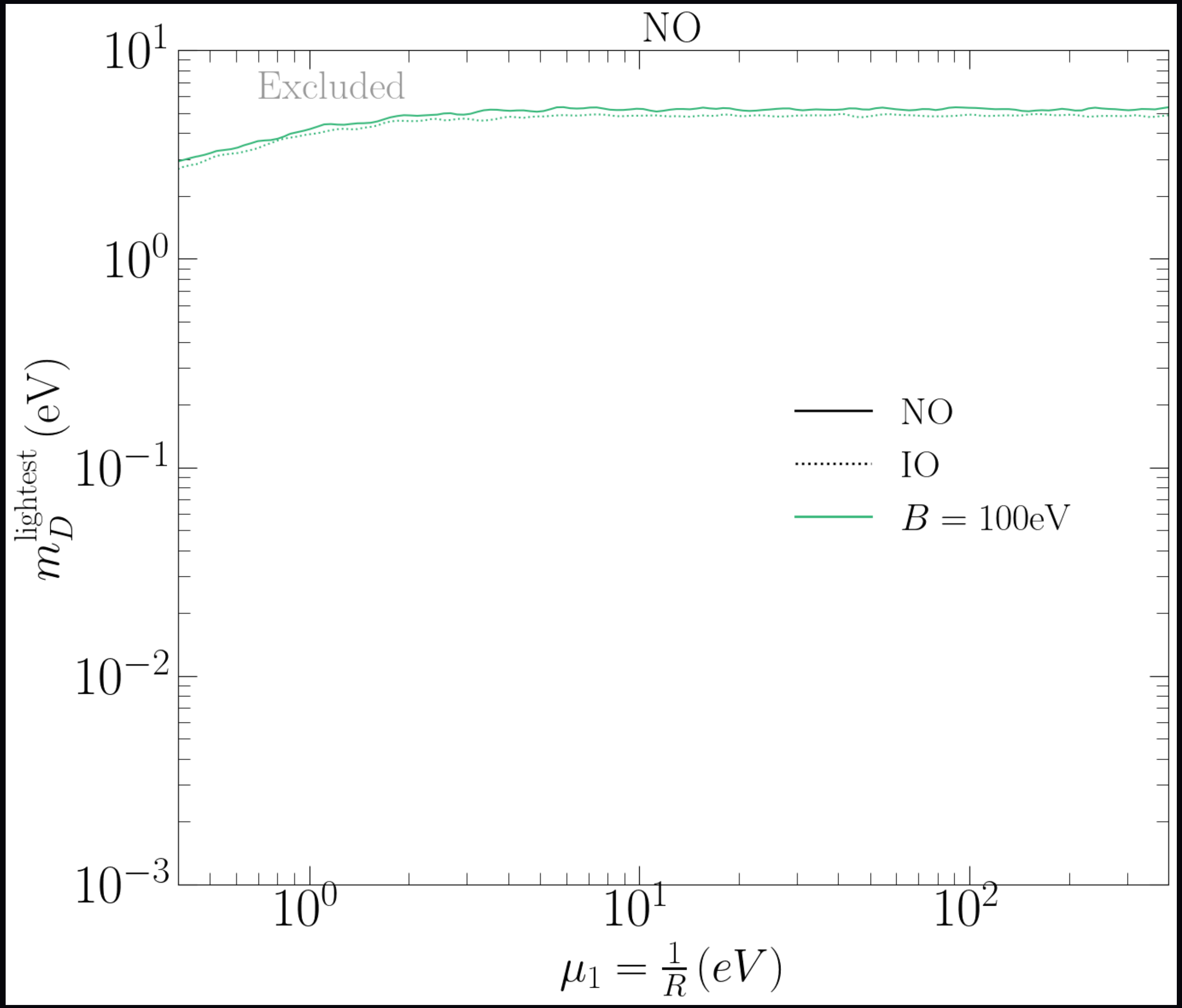
Phenomenology of oscillation in presence of extra dimension

Majorana Brane



$$m_\lambda^{\text{lightest}} \approx \frac{1}{2}\left(B - \sqrt{B^2 + 4m_D^2}\right) \overset{B \gg m_D}{\approx} -\frac{m_D^2}{B}$$

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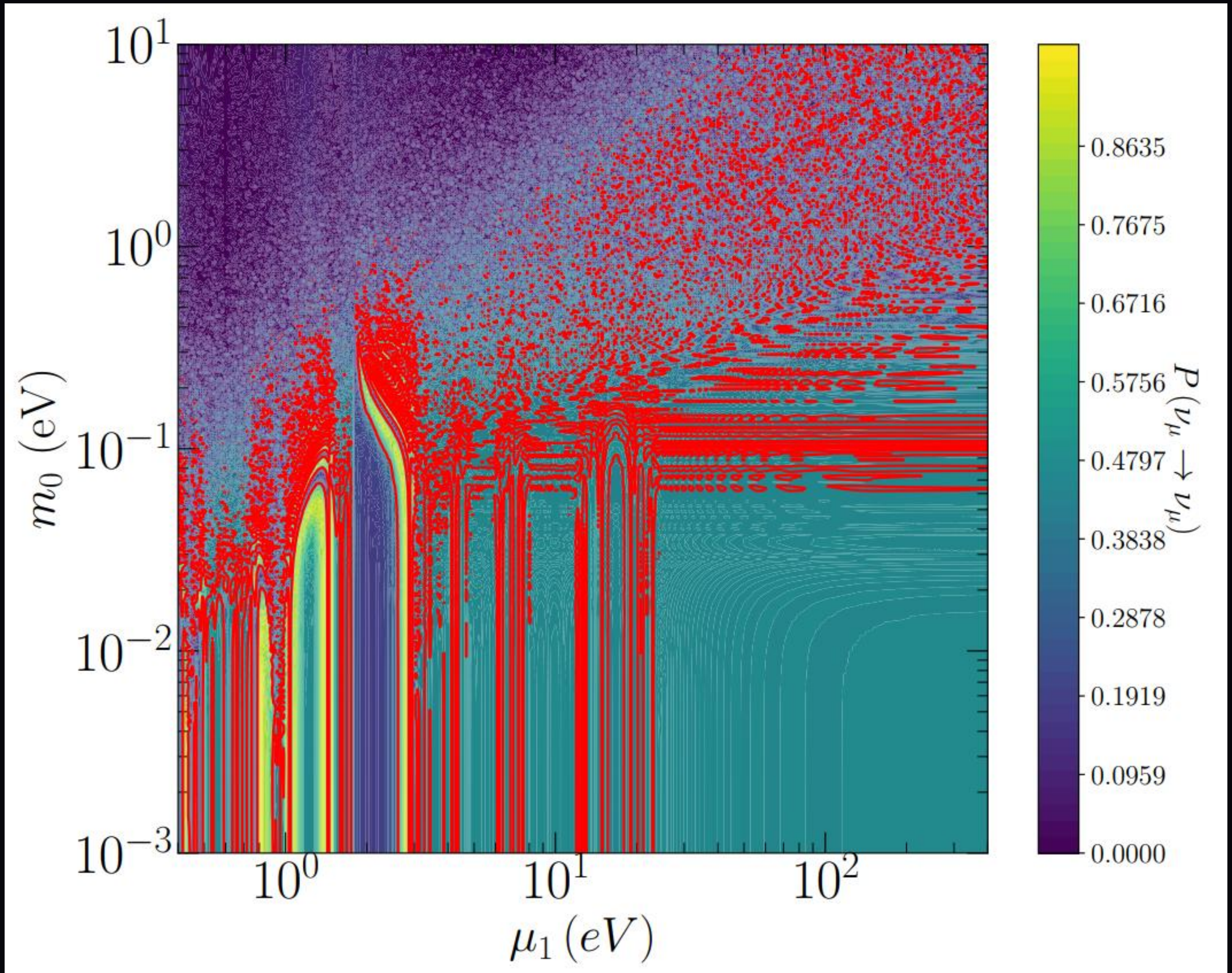
Phenomenology of oscillation in presence of extra dimension



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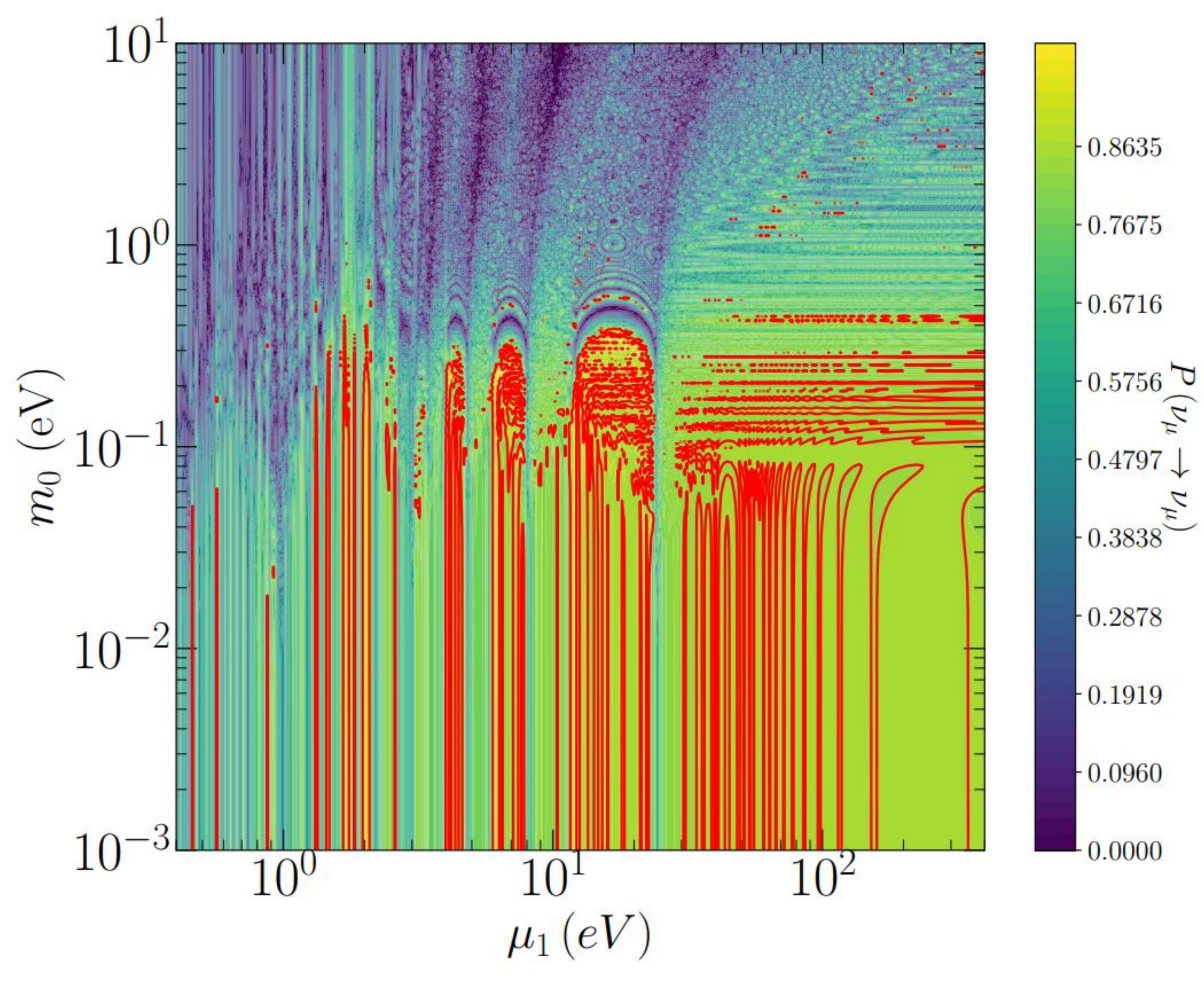

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