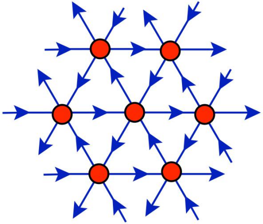
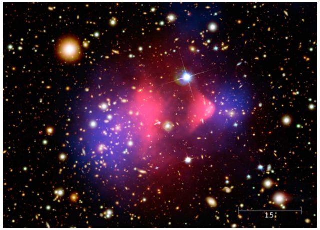
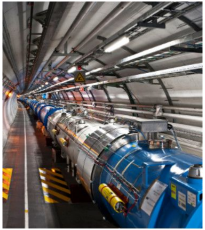
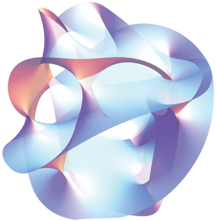
BSM Physics on the Lattice

David Schaich (University of Liverpool)









Annual Theory Meeting, IPPP Durham, 16 December 2025

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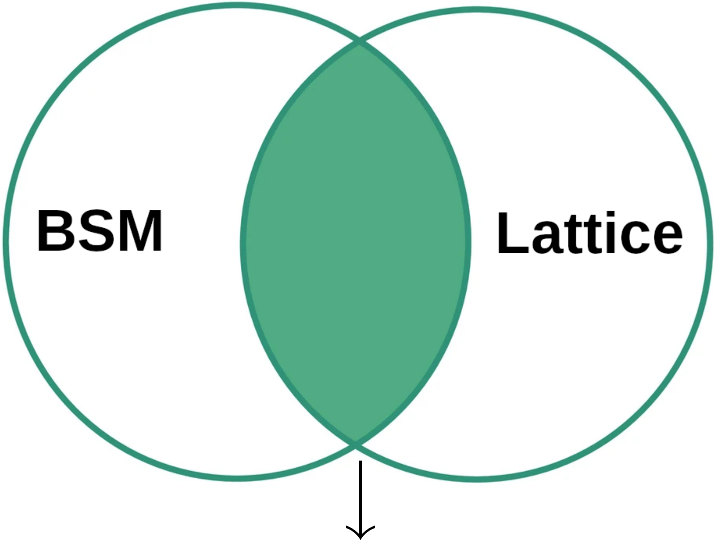
Big picture



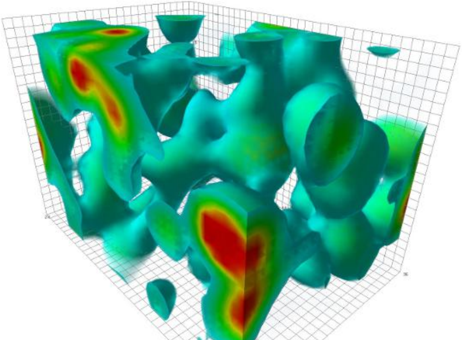
Strong dynamics unlike QCD

Big picture

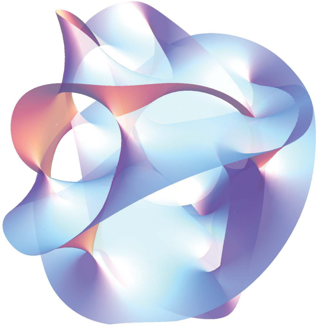
Strong dynamics unlike QCD

Both concrete calculations for phenomenology

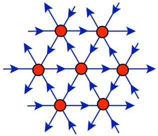
and non-perturbative explorations for more formal theory



(Derek Leinweber)



Overview and plan

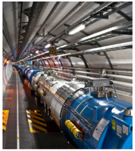


Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

Especially important when QCD-based intuition unreliable

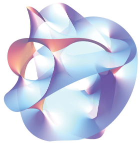
Near-conformal strong dynamics

composite Higgs; partial compositeness; symmetric mass generation



Composite dark matter

direct & collider searches; self-interactions; gravitational waves



Lattice supersymmetry

challenges; holographic duality; scaling dimensions

But wait, there's more…

Dynamical Triangulations ['lattice quantum gravity', cf. arXiv:2209.06555]

Radial quantization on $S^{d-1} \times \mathbb{R}$ ['Quantum Finite Elements', cf. arXiv:2510.03085]

3d QFTs dual to 4d de Sitter ['lattice cosmology', cf. arXiv:2009.14768]

Etc.

Annual Lattice conference is great resource to go beyond this talk

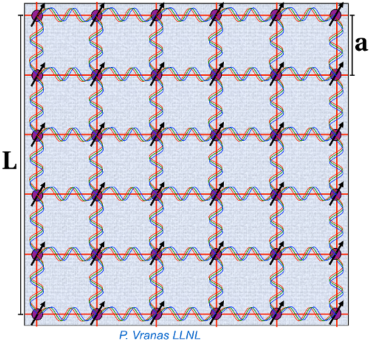
Plenary reviews — Jong-Wan Lee '23; Georg Bergner '24; Biagio Lucini '25

Parallel tracks including Particle physics beyond the Standard Model
Applications outside particle physics
Theoretical developments
Quantum Computing and Quantum Information

Lattice quantum field theory in a nutshell

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
$\nwarrow$Gauge invariant, non-perturbative, d-dimensional



Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $a \to 0$ $(L/a \to \infty)$

Discrete $\longrightarrow$ continuous symmetries ✓

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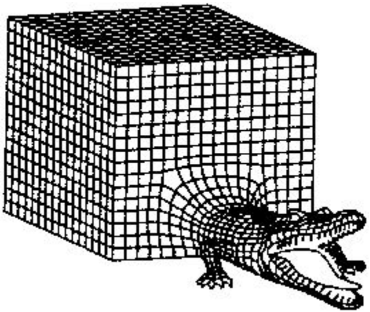


Caveats

Need UV-complete theory
[usually strong sector in isolation]

Physics needs to 'fit' in finite volume

Obstructions to chiral gauge theories,
real-time dynamics, supersymmetry

Numerical lattice field theory calculations

High-performance computing $\longrightarrow$ evaluate up to $\sim$billion-dimensional integrals
(Dirac operator as $\sim 10^9 \times 10^9$ matrix)

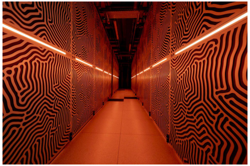
Community relies on digital research infrastructure such as STFC-DiRAC

Science only possible thanks to support from DSIT, UKRI,
universities and international partners!

COSMA @Durham
[STFC-DiRAC]

Tursa @Edinburgh
[STFC-DiRAC]

ARCHER @Edinburgh
[EPSRC/NERC]



JUPITER @Jülich
[EuroHPC JU]

Composite Higgs

Composite Higgs sector
can stabilize electroweak scale

QCD-like composite Higgs ruled out

New strong dynamics must differ from QCD

Typically analysed as EFT based on
spontaneous symmetry breaking
$G \to H \supset SU(2)_L \times SU(2)_R$

Must end up with 3 NGBs, 1 PNGB-Higgs,
all else much heavier (or dark)

Pathways to Innovation
and Discovery
in Particle Physics

A Strategic Plan for US Particle Physics

Decipher
the
Quantum
Realm

Elucidate the Mysteries
of Neutrinos

Reveal the Secrets of
the Higgs Boson

2023p5report.org

See also ESPPU Physics Briefing Book
[arXiv:2511.03883]

Composite Higgs — simple example

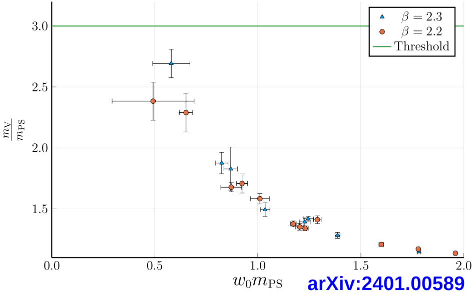
SU(2) gauge theory with $N_F = 2$ fundamental fermions

as UV completion of SU(4) $\to$ Sp(4) $\sim$ SO(6) $\to$ SO(5) with five NGBs

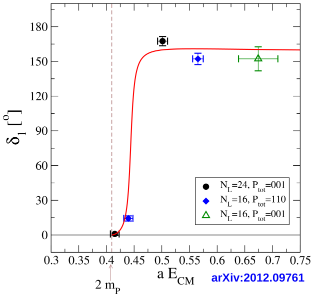
[Cacciapaglia–Sannino, arXiv:1402.0233]

Lattice studies by Plymouth–Dublin–Odense–Bern collab. [cf. arXiv:2502.07163]

$\longrightarrow$ mass & width of vector resonance, EFT params., etc.



arXiv:2401.00589



arXiv:2012.09761

Dim'less ratios

Non-zero PNGB mass to fit in lattice volume

Extrapolate by fitting to EFT

Near-conformal composite Higgs

New strong composite sector also responsible for SM fermion masses

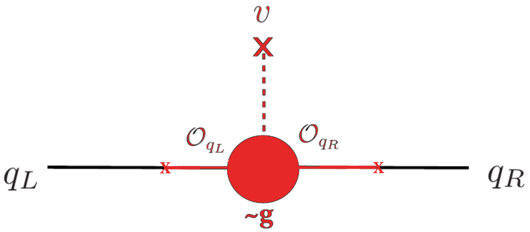
Partial compositeness

$$\lambda_q \frac{\overline{q}\mathcal{O}_q}{\Lambda_{UV}^2} \longrightarrow m_q \sim v_{\rm EW} \left(\frac{\rm TeV}{\Lambda_{UV}}\right)^{4-2\gamma_q}$$

With $\Lambda_{UV} \sim 10^{10}$ TeV,

$m_{u/d} \sim$ MeV needs $\gamma_{u/d} \simeq 1.75$

$m_b \sim$ GeV needs $\gamma_b \simeq 1.9$



Need large anomalous dimensions γ across large range of scales

Near-conformal composite Higgs

New strong composite sector also responsible for SM fermion masses

Need large anomalous dimensions γ across large range of scales
$\longrightarrow$ near-conformal strong dynamics

Partial compositeness UV completions

Composite operators $\mathcal{O}_q$ must have same quantum numbers as SM fermions q

For example, $\mathcal{O}_q \sim \psi_1\psi_2\psi_3$ baryons in SU(3) gauge theories with $N_F \geq 7$

[Vecchi, arXiv:1506.00623]

$N_F = 12$ gradient flow renormalization

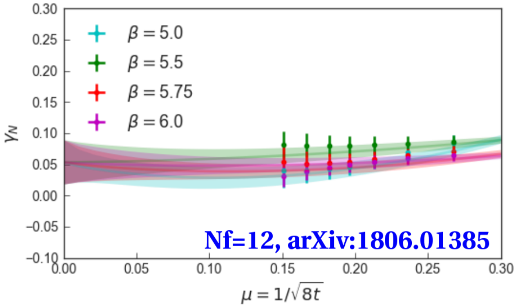
$\longrightarrow \gamma_q = 0.05(5)$

For comparison, $\gamma_m = 0.23(6)$

[Carosso–Hasenfratz–Neil, arXiv:1806.01385]

Appears perturbative

$\longrightarrow 8 \leq N_F \leq 10$ more interesting



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Alternative: 'multi-rep' theories

N_{EW} fermions ψ in one rep $\longrightarrow$ NGBs

N_{QCD} fermions χ in another $\longrightarrow$ $\mathcal{O}_q \sim \psi_1 \psi_2 \chi$ 'chimera' baryons

[Ferretti–Karateev, arXiv:1312.5330]

Lattice studies of multi-rep theories (I)

SU(4) gauge theory with $N_F = 4$ fundamental and $N_{AS} = 3$ antisymmetric

$$SU(4) \times SU(4) \to SU(4)$$

TACoS collaboration

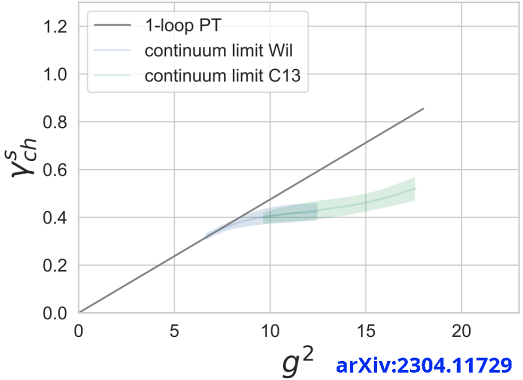
$N_{AS} = 4$ for technical reasons

Three independent chimeras

$$\longrightarrow \gamma_q \simeq \{0.25, 0.25, 0.5\}$$

For comparison, $\gamma_m^{(F)} \simeq 0.75$, $\gamma_m^{(A)} \simeq 1$

[arXiv:2304.11729]



arXiv:2304.11729

Lattice studies of multi-rep theories (II)

Sp(4) gauge theory with $N_F = 2$ fundamental and $N_{AS} = 3$ antisymmetric

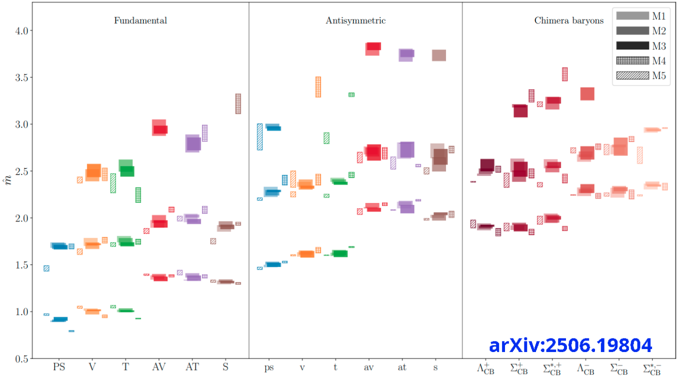
$SU(4) \to Sp(4)$

TELOS Collaboration

Masses of mesons and chimera baryons

Five independent calculations using spectral density method [arXiv:2506.19804]

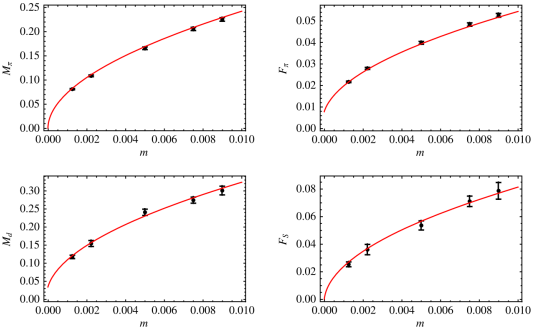
telos-collaboration.github.io

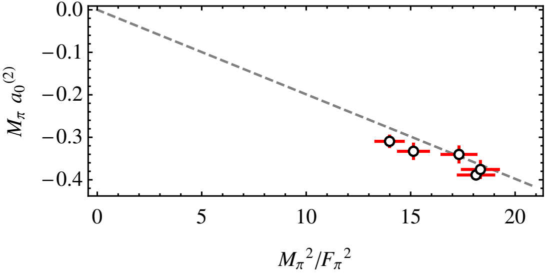


Pseudo-dilaton from near-conformality

Near-conformal lattice studies generically observe light scalar meson roughly degenerate with PNGBs

Test pseudo-dilaton interpretation by formulating dilaton-EFT [arXiv:2305.03665]
$\longrightarrow$ good description of SU(3) $N_F = 8$ spectrum and PNGB scattering





Symmetric mass generation

strong coupling

- SMG phase
- Chirally symmetric
- Confining, gapped spectrum

continuous phase transition

continuum limit exist
RG β function

weak coupling

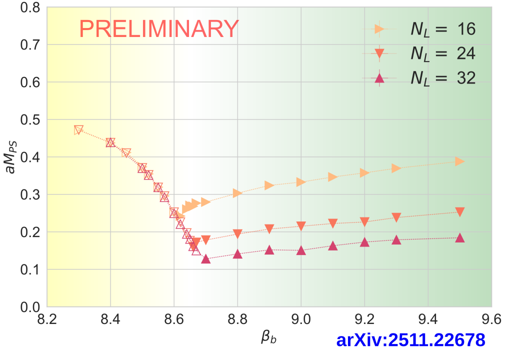
- Appears conformal
- Chirally symmetric
- Conformal hyperscaling

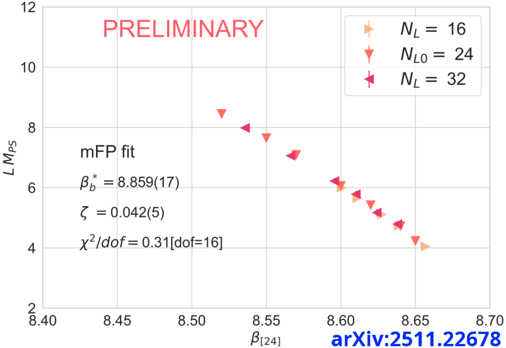
arXiv:2511.22678

SU(N) with N_F fundamental $\longrightarrow$ $N \times N_F \mod 8 = 0$ to cancel ’t Hooft anomalies

Novel strongly coupled continuum limit
where perturbatively irrelevant operators may be relevant

Symmetric mass generation





Calculations with exactly massless fermions [Hasenfratz–Witzel, arXiv:2511.22678]

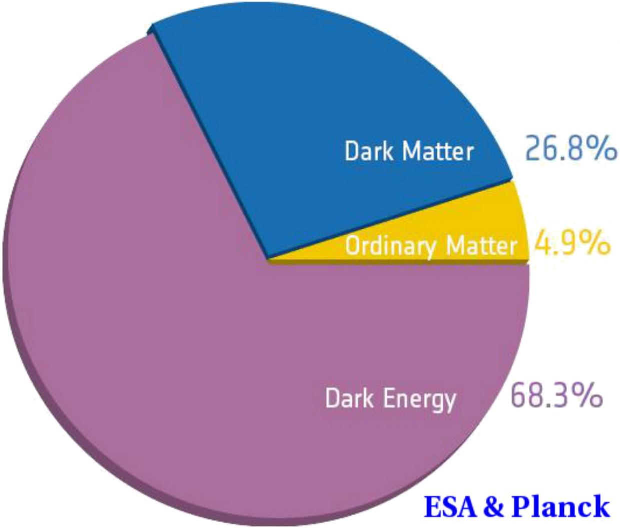
Transition continuous rather than first-order,
potentially BKT from merger of UV and IR fixed points

Composite dark matter

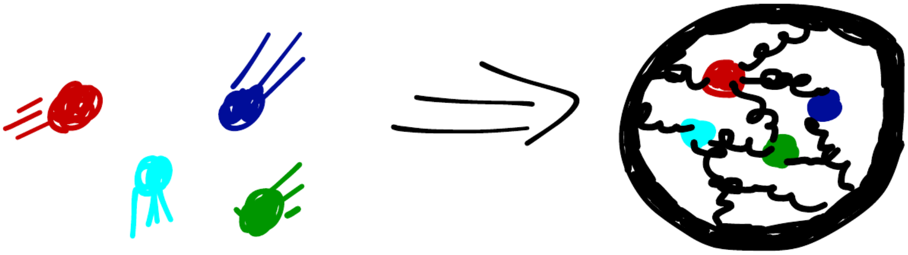
Consistent gravitational evidence
from kiloparsec to Gpc scales

$$\frac{\Omega_{\text{dark}}}{\Omega_{\text{ordinary}}} \approx 5 \qquad \ldots \text{not} \;\; 10^5 \;\; \text{or} \;\; 10^{-5}$$

Explained by non-gravitational
interactions in the early universe



Composite dark sector



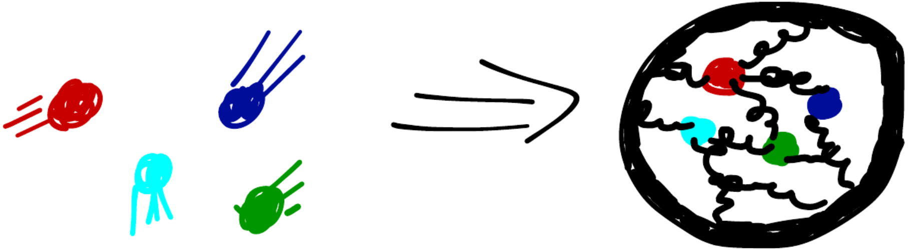
Early universe

Deconfined charged fermions $\longrightarrow$ non-gravitational interactions

Present day

Confined SM-singlet dark matter $\longrightarrow$ no experimental detections

Composite dark sector



Present day

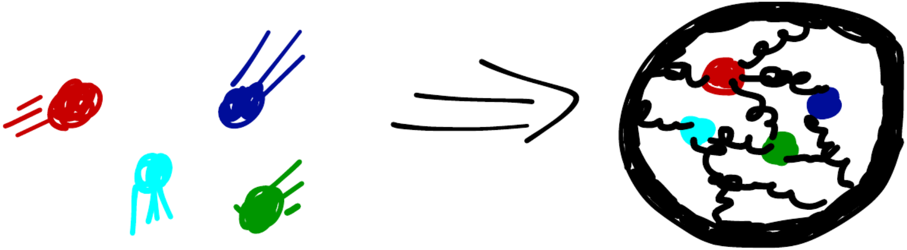
Confined SM-singlet dark matter $\longrightarrow$ no experimental detections

Could be dark baryon; dark meson; dark glueball [review: arXiv:1604.04627]

$\nwarrow$ for example 'SIMP' DM,

SU(2) or Sp(4) UV completions

Composite dark sector



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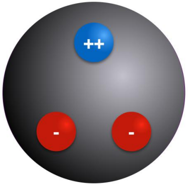
$\nwarrow$ massive and stable via dark baryon number

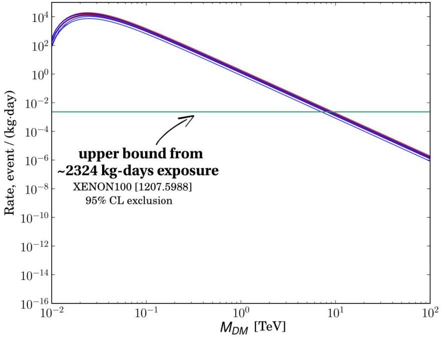
QCD-like dark baryon

Composite dark matter can be QCD-like [cf. DeGrand–Neil, arXiv:1910.08561]

Different EW charges $\longrightarrow$ SM-singlet lightest baryon

Form factors $\longrightarrow$ direct detection via Higgs or photon exchange





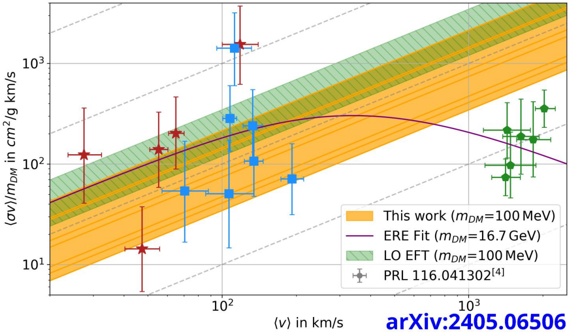
Constraint $M_B \gtrsim 30$ TeV [arXiv:1301.1693]
dominated by magnetic moment

Composite dark matter is self-interacting

GeV-scale mass $\longrightarrow$ self-interactions relevant to small-scale structure

Example cross-section
for Sp(4) dark mesons
[Dengler–Maas–Zierler, arXiv:2405.06506]

Points inferred from halo cores
[Kaplinghat–Tulin–Yu, arXiv:1508.03339]



arXiv:2405.06506

Reducing $M_B \gtrsim 30$ TeV constraint on dark baryons is interesting challenge. . .

Non-QCD-like bosonic baryon

Stealth Dark Matter [arXiv:1503.04205]

SU(4) gauge group, $N_F = 2 + 2$

$\longrightarrow$ scalar ‘dark baryon’

Symmetries forbid

mag. moment and charge radius

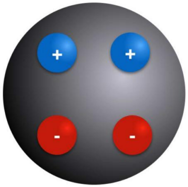
Polarizability $\longrightarrow$ $M_{DM} \gtrsim 300$ GeV,

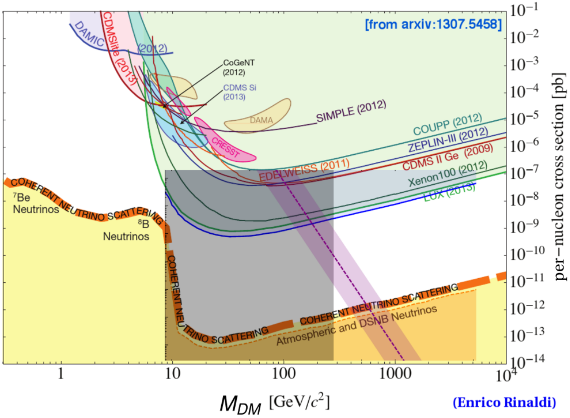
comparable collider constraint

Hyper-Stealth DM [arXiv:2412.14540]

Effectively $N_F = 1 + 2$

$\longrightarrow M_{DM} \gtrsim 3$ GeV





Gravitational waves

First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

Space-based observatories (e.g., LISA) will access relevant frequencies

Lattice tasks

Determine order of transition

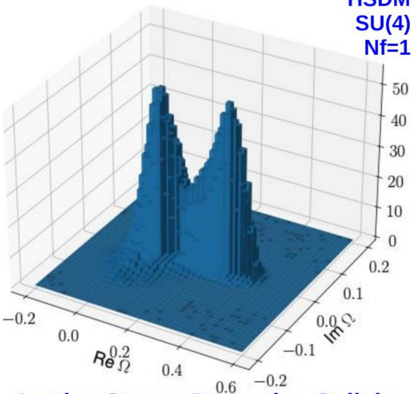
Compute latent heat and interface tension

Gravitational wave spectrum

also sensitive to supercooling,

bubble nucleation rate & wall speed

HSDM SU(4) Nf=1



Lattice Strong Dynamics Collab.

Gravitational waves

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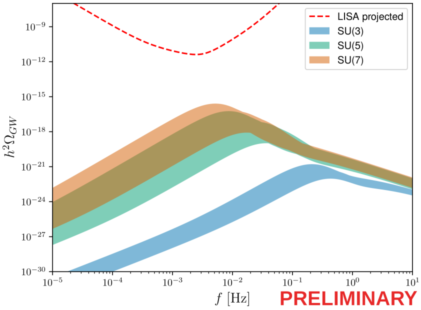


[for dark glueball]

Gravitational waves

Super-critical slowing down at first-order transition

$\longrightarrow$ avoid via density-of-states algorithm [arXiv:1204.3243]

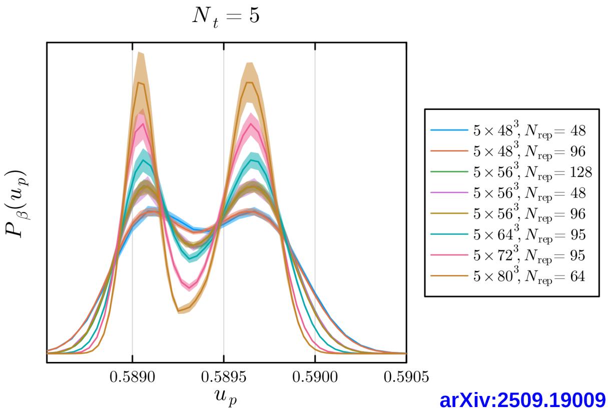
TELOS Collaboration

Pure-gauge Sp(4)

[arXiv:2509.19009]

Latent heat
from distance between peaks

Interface tension
from valley between them



arXiv:2509.19009

Supersymmetry must be broken on the lattice

Supersymmetry is a space-time symmetry, $(\mathrm{I} = 1, \cdots, \mathcal{N})$

adding spinor generators Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ to translations, rotations, boosts

$\left\{ Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}} \right\} = 2\delta^{\mathrm{IJ}} \sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$ broken in discrete space-time

$\longrightarrow$ relevant susy-violating operators

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$\longrightarrow$ relevant susy-violating operators

Significant recent progress from three simplifications [review: arXiv:2208.03580]

Reduce dimensions	**Avoid scalars**	**Maximize symmetries**

(I) Reduce dimensions

Ultimate simplification — compactify all spatial dimensions

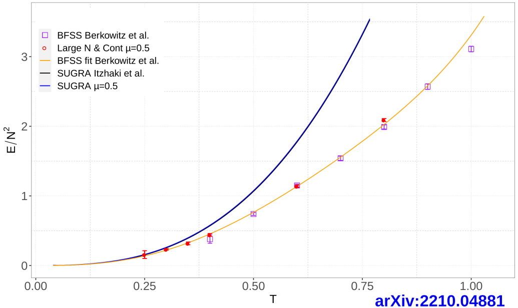
4d SU(N) super-Yang–Mills $\longrightarrow$ quantum mechanics of $N \times N$ matrices

Holographic duality conjecture continues to relate
stringy black branes $\longleftrightarrow$ lattice observables in large-N continuum limit

(I) Black hole energy [max. susy]

Match leading-order ('SUGRA') result
at low temperatures

Predict corrections at higher T
[Monte Carlo String/M-Theory Collaboration,
arXiv:2210.04881]



arXiv:2210.04881

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(II) Phase transition in BMN model

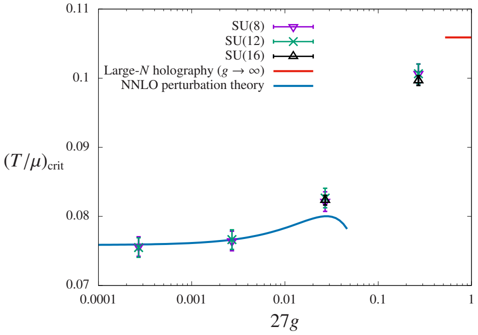
[Jha–Joseph–DS, arXiv:2412.13407]

Match pert. theory at weak coupling

Approach strong-coupling limit

from numerical solution of dual SUGRA

[Costa–Greenspan–Penedones–Santos, arXiv:1411.5541]



(II) Avoid scalars: 4d $\mathcal{N} = 1$ super-Yang–Mills

SU(N) gauge theory with single massless Majorana fermion in adjoint rep.

Computational challenge comparable to lattice QCD

(II) Avoid scalars: 4d $\mathcal{N} = 1$ super-Yang–Mills

SU(*N*) gauge theory with single massless Majorana fermion in adjoint rep.

Computational challenge comparable to lattice QCD

DESY–Münster–Regensburg–Jena collab.

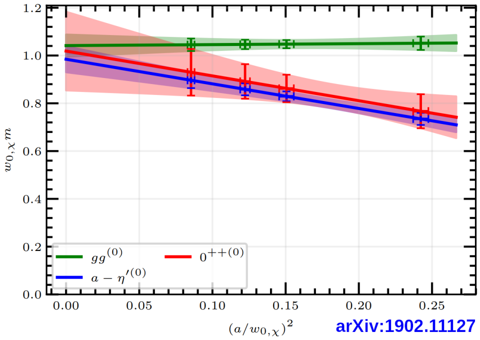
$\mathcal{N} = 1$ SYM, SU(3) gauge group

Recover degenerate supermultiplet in chiral continuum limit

[arXiv:1902.11127]

Starting point for $\mathcal{N} = 1$ superQCD

[Bergner–Piemonte, arXiv:2008.02855; Carstensen–Bergner, Lattice '24]



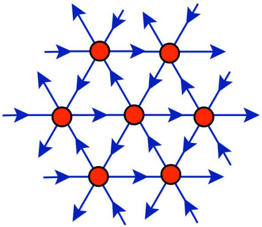
arXiv:1902.11127

(III) Maximize symmetries

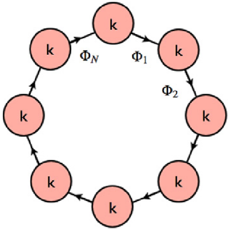
Preserve susy sub-algebra in discrete lattice space-time

$\Longrightarrow$ correct continuum limit with little or no fine tuning

Equivalent constructions from 'topological' twisting and dim'l deconstruction



Review:
Catterall–Kaplan–Ünsal,
arXiv:0903.4881



Need 2^d supersymmetries in d dimensions

$d = 4 \longrightarrow \mathcal{N} = 4$ super-Yang–Mills

Lattice $\mathcal{N} = 4$ SYM scaling dimension

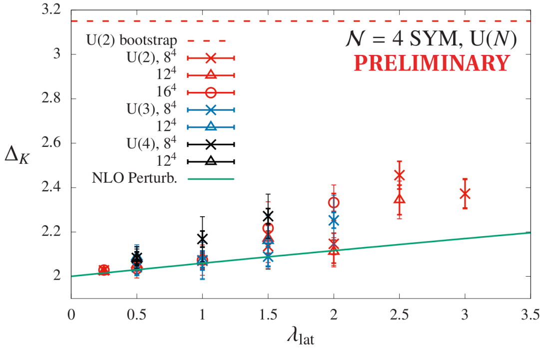
$\mathcal{O}_K(x) = \sum_I \text{Tr}\left[\Phi^I(x)\Phi^I(x)\right]$ is simplest conformal primary operator

Scaling dimension $\Delta_K(\lambda) = 2 + \gamma_K(\lambda)$ investigated through
perturbation theory (& S duality), holography, conformal bootstrap

Δ_K from Monte Carlo RG analyses
[arXiv:2304.04655]

Roughly perturbative for $\lambda_{\text{lat}} \lesssim 3$

Sign problem challenging for larger λ_{lat}

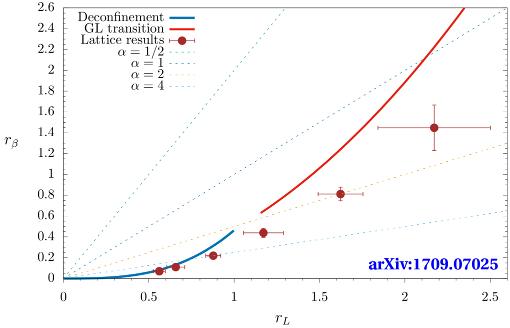


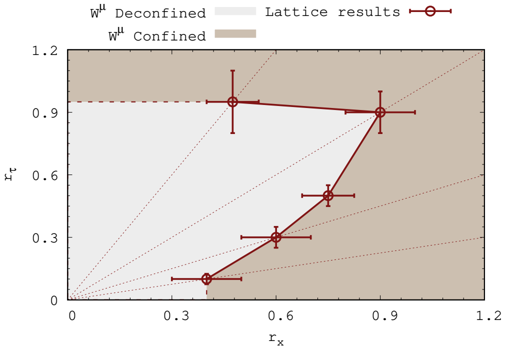
Supplement: Twisted lattice SYM in lower dimensions

Low-temperature large-N phase transition predicted by holography

Two dimensions: Can have $Q = 4$, 8 or 16 supersymmetries

Lattice results for maximal $Q = 16$ consistent with holography (left),
not the case for minimal $Q = 4$ (right, arXiv:2312.04980)





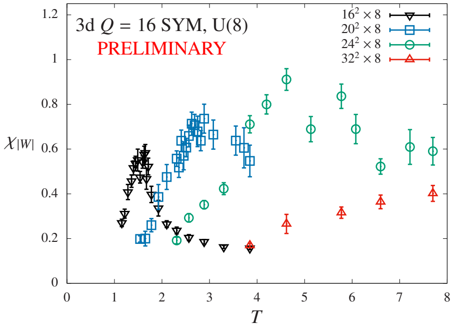
Supplement: Twisted lattice SYM in lower dimensions

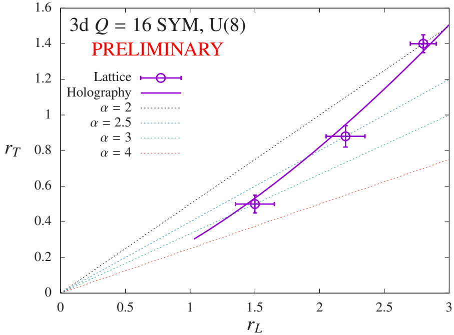
Low-temperature large-N phase transition predicted by holography

Three dimensions: Can have $Q = 8$ or 16 supersymmetries

[starting point for 2d quiver superQCD]

Preliminary $Q = 16$ lattice results consistent with holography [Joseph–DS, Lattice '25]

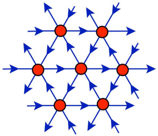




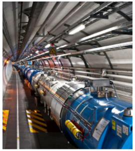
Recap: An exciting time for lattice BSM

Lattice field theory is a broadly applicable tool
to study strongly coupled quantum field theories

Especially important when QCD-based intuition unreliable

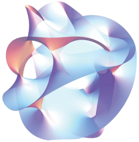


Near-conformal strong dynamics



Composite dark matter

Lattice supersymmetry



Much more to explore

Recap: An exciting time for lattice BSM

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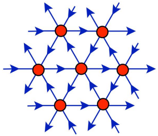
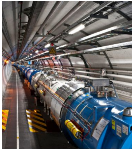
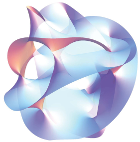






Thank you!

UK Research and Innovation

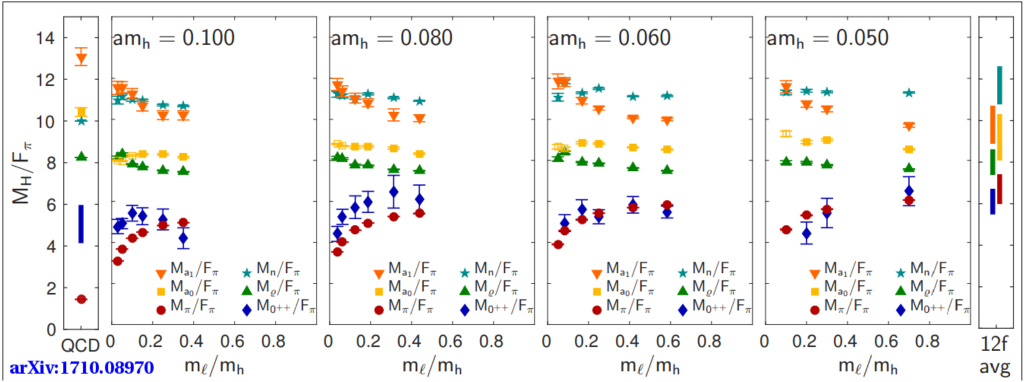
LIVERPOOL THEORETICAL PHYSICS

Backup: Light scalar in near-conformal composite Higgs models

Consistently observed by many groups considering various theories

✓ SU(3) with $N_F = 8$ fundamental

✓ SU(3) with $N_F = 12$ fundamental



Backup: Light scalar in near-conformal composite Higgs models

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✓ SU(3) with $N_F = 8$ fundamental

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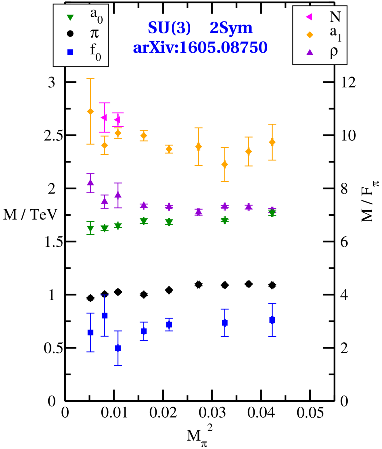
✓ SU(3) with $N_F = 2$ sextet



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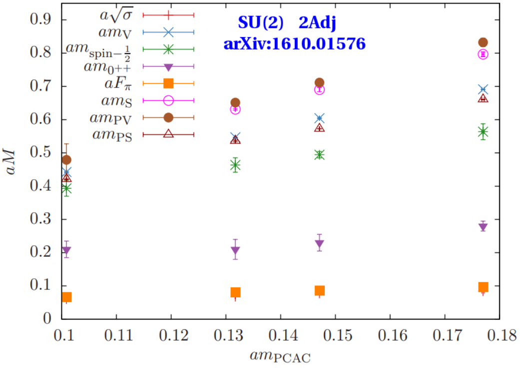
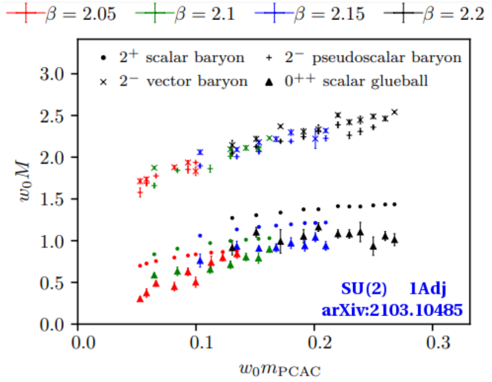
✓ SU(2) with $N_F = 2$ adjoint



Backup: Light scalar in near-conformal composite Higgs models

Consistently observed by many groups considering various theories

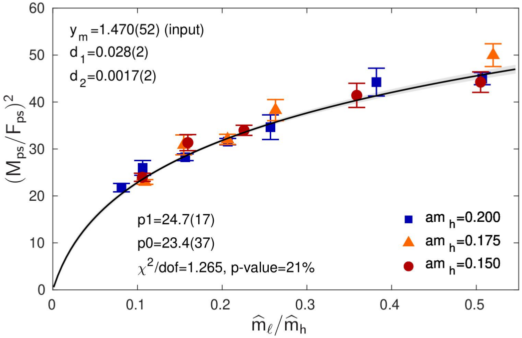
- ✓ SU(3) with $N_F = 8$ fundamental
- ✓ SU(3) with $N_F = 12$ fundamental
- ✓ SU(3) with $N_F = 2$ sextet
- ✓ SU(2) with $N_F = 2$ adjoint
- ✓ SU(2) with $N_F = 1$ adjoint



Backup: Dilaton-EFT test from SU(3) with $N_F = 4 + 6$

Dilaton-EFT includes light pseudo-Nambu–Goldstone boson (PNGB) of broken scale invariance, in addition to usual PNGBs

Good description of spectrum for $N_F = 4 + 6$ [arXiv:2007.01810]

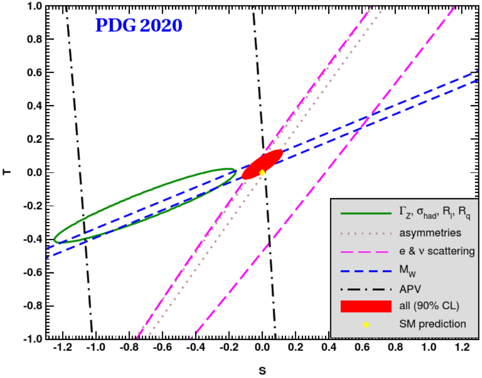


Analyze

- PNGB scattering
- Electroweak S parameter
  (EFT coeff. related to W mass)

Backup: The S parameter

Constrain Higgs sector from vector-minus-axial vacuum polarization $\Pi_{V-A}(Q)$



Experimental $S = -0.01 \pm 0.10$

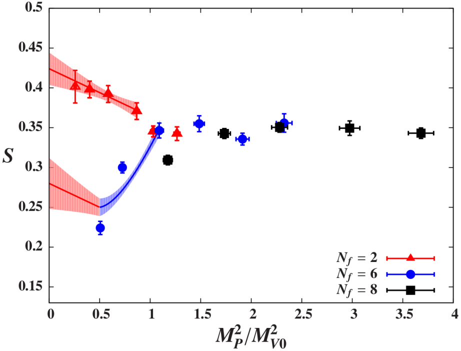
vs. QCD-like $S \approx 0.43\sqrt{\xi}$

Related to W boson mass [arXiv:2204.03796]

Domain-wall fermion symmetries important

Backup: *S* parameter on the lattice

$$\mathcal{L}_\chi \supset -\frac{S}{32\pi^2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \tau_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$



Prior LSD study of $N_F = 2, 6, 8$ [arXiv:1405.4752]

$N_F = 4 + 6$ planned for the near future

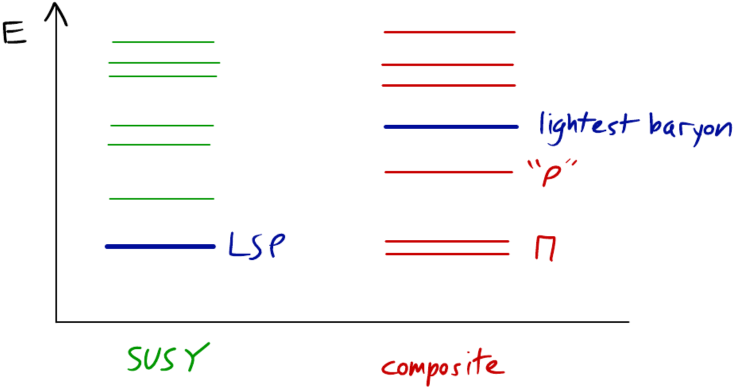
$S/\sqrt{\xi} = 0.42(2)$ for $N_F = 2$ matches QCD ✓

Significant reduction from larger N_F,

chiral extrapolation again challenging

V–A vacuum polarization also contributes to Higgs potential [arXiv:1903.02535]

Backup: Collider constraints on composite dark matter



Dark baryon not lightest composite particle

'Missing energy' searches inefficient

Collider constraints from lighter **charged** 'Π' plus lattice calculation of M_{DM}/M_{Π}

[cf. ATLAS, arXiv:2405.20061]

Backup: More about holographic duality

Holographic duality conjecture

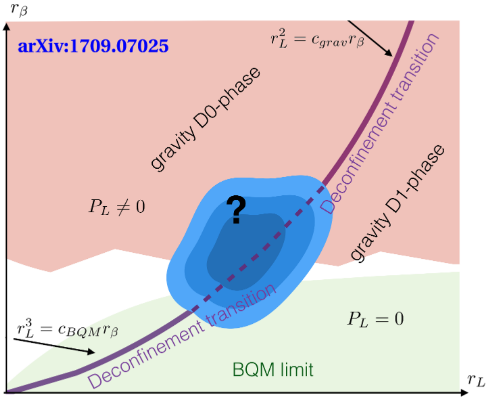
Thermodynamics of supersymmetric QFT $\longleftrightarrow$ black holes in dual supergravity

2d example: For decreasing r_L
at low $t = 1/r_\beta$ and large N

homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

$\updownarrow$

"spatial deconfinement"
signalled by Wilson line



Backup: Quiver superQCD from twisted SYM

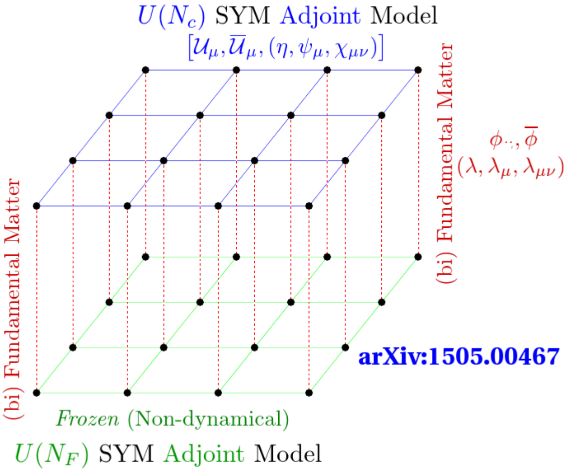
2-slice lattice SYM
with U(N) $\times$ U(F) gauge group

Adj. fields on each slice

Bi-fundamental in between

Decouple U(F) slice

$\longrightarrow$ U(N) SQCD in ($d-1$) dims.
with F fund. hypermultiplets



Backup: Dynamical susy breaking in 2d lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Observe spontaneous susy breaking only for $N > F$, as expected

Catterall–Veernala, arXiv:1505.00467