Feasibility study of tri-Higgs production in the 4b2τ final state at FCC-hh

Holly Pacey, James Frost, Tom Dingley

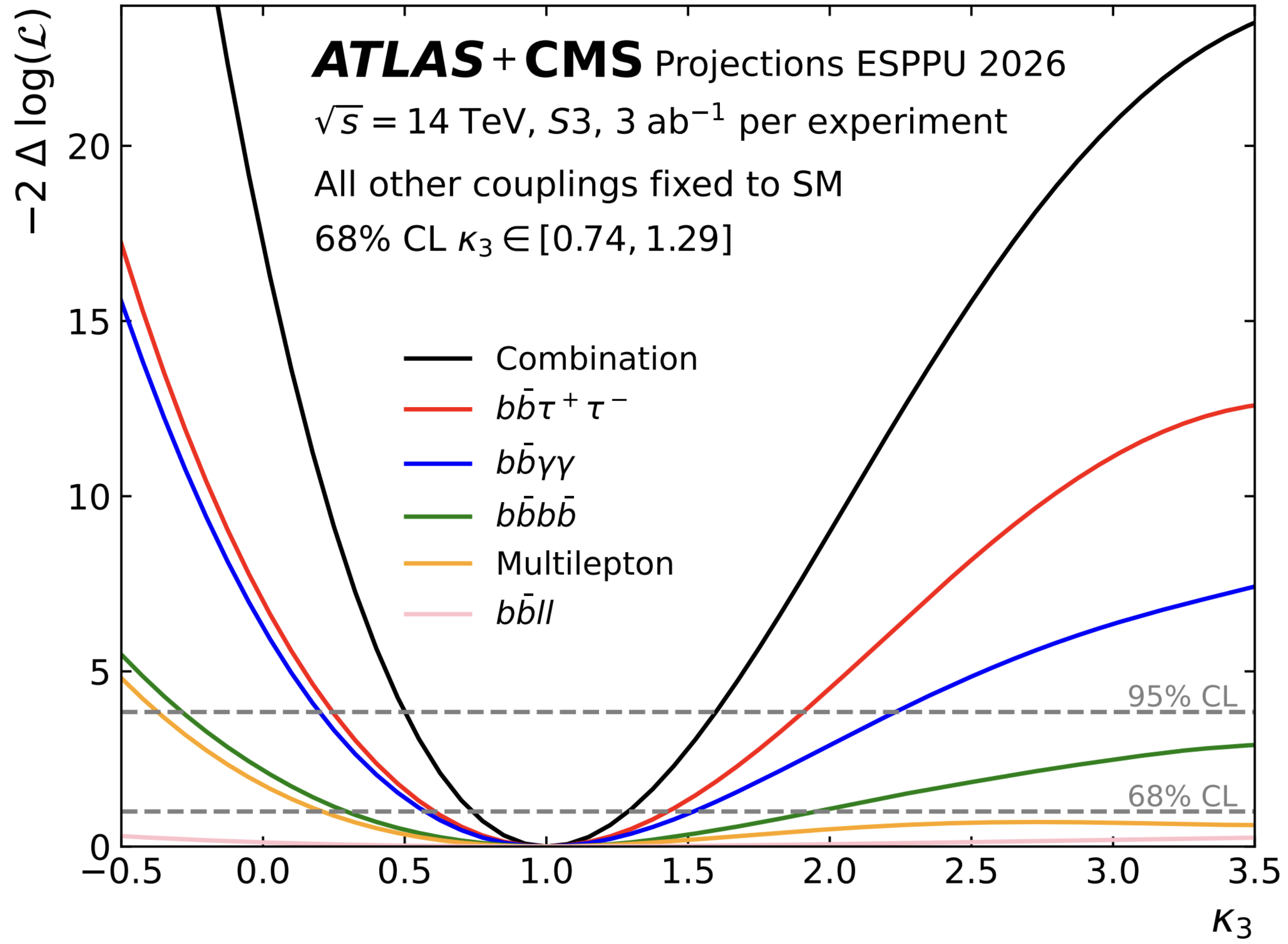
FCC UK Annual Meeting
Durham, 12th November 2025

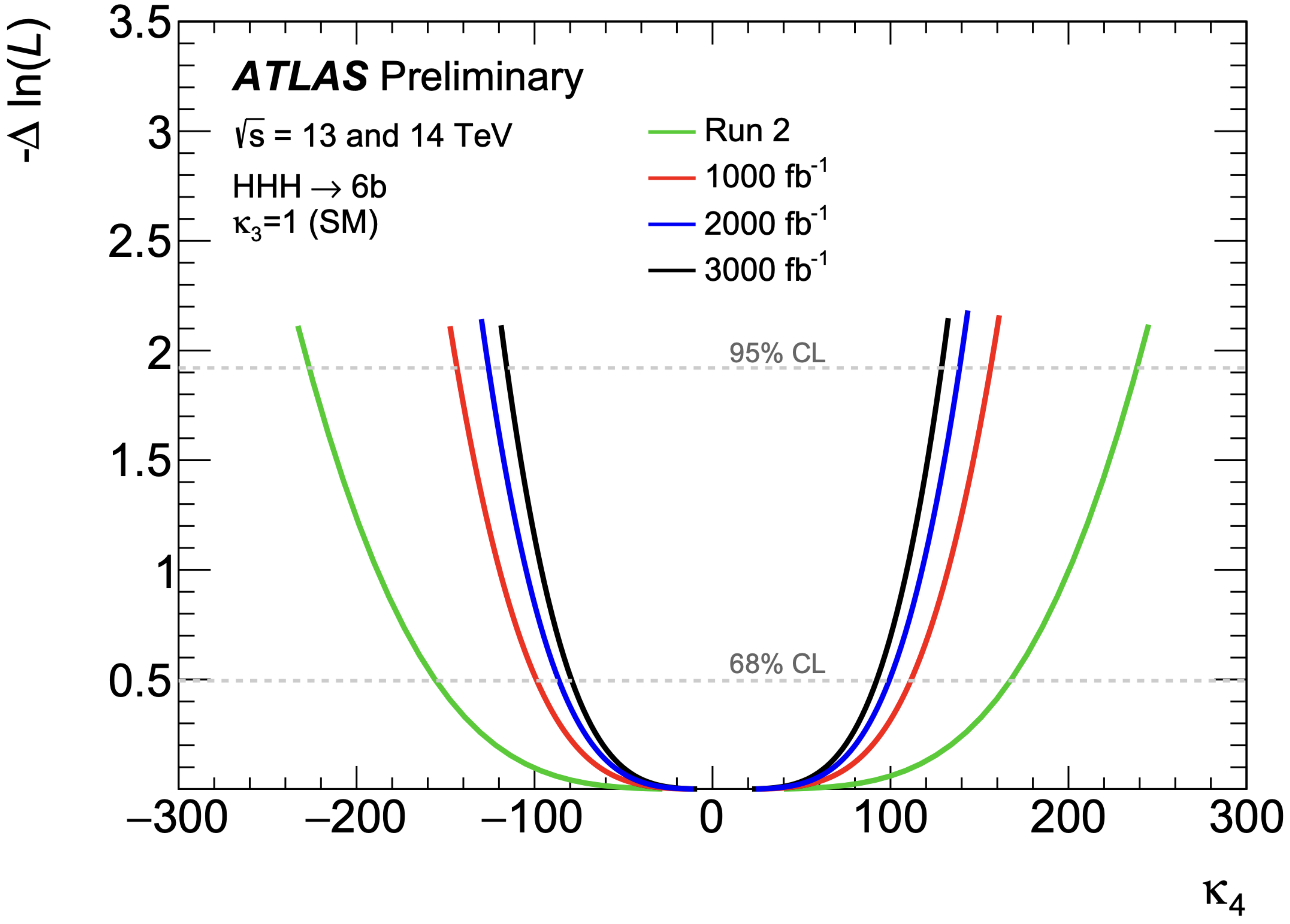
Quartic Higgs self-coupling

Higgs potential

$$V(h) = \frac{1}{2} m_h^2 h^2 + \boxed{\kappa_3} \lambda v h^3 + \frac{\boxed{\kappa_4}}{4} \lambda h^4$$

What we expect after high-lumi:

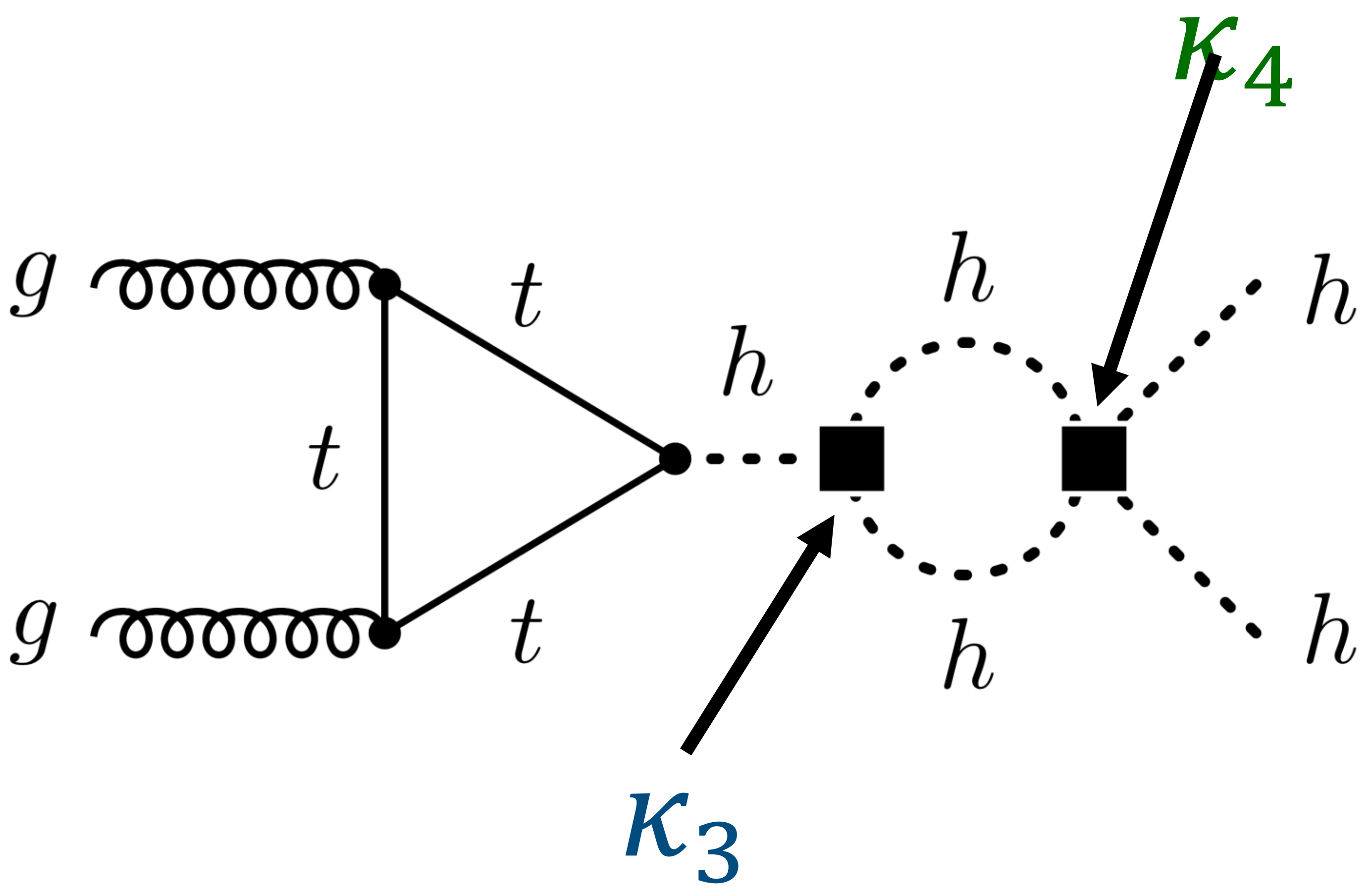


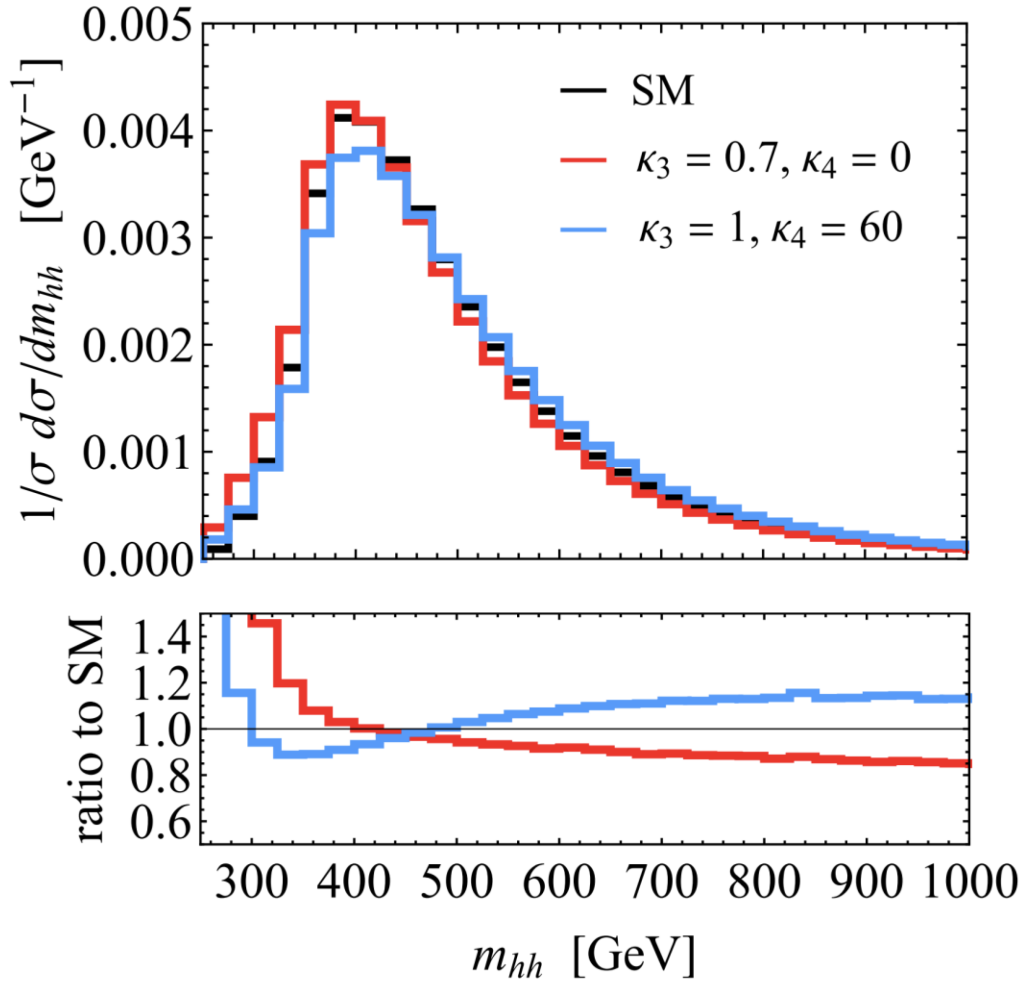


Probes of κ_4

Indirect

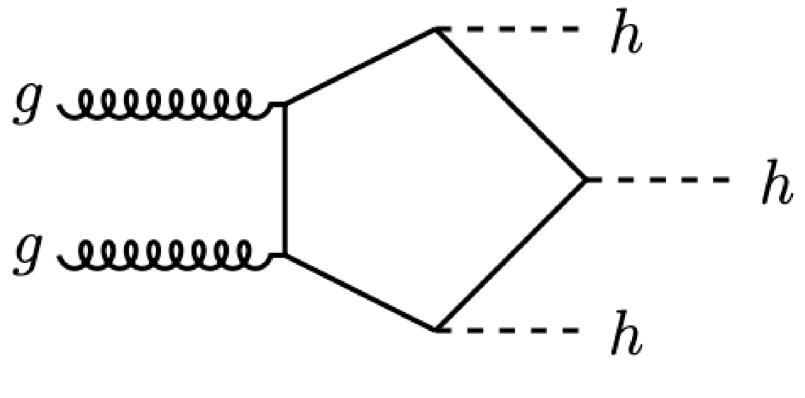
- Constraints via loop-corrections to di-Higgs [Figures from JHEP10(2019)267]
- Similar ideas explored for ggH trilinear self-couplings [Figures from arXiv:2507.02032]



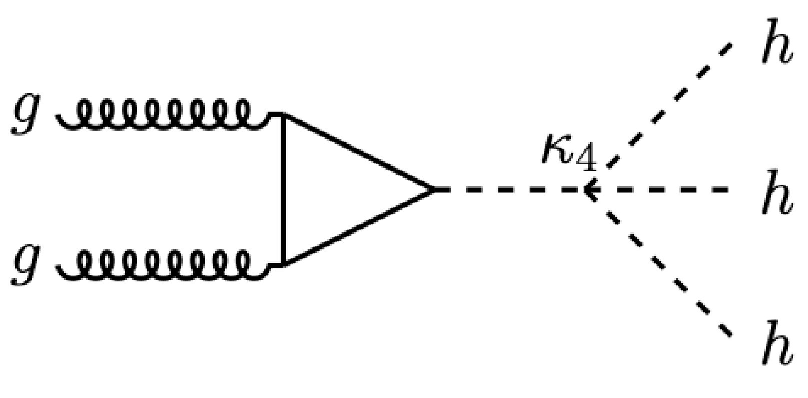


Direct

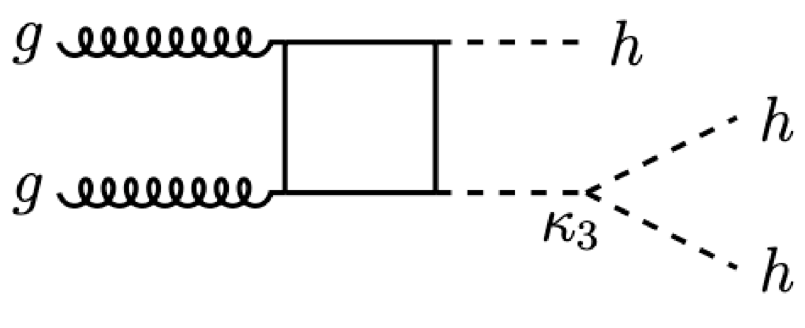
- Measuring HHH directly constrains κ_3, κ_4!



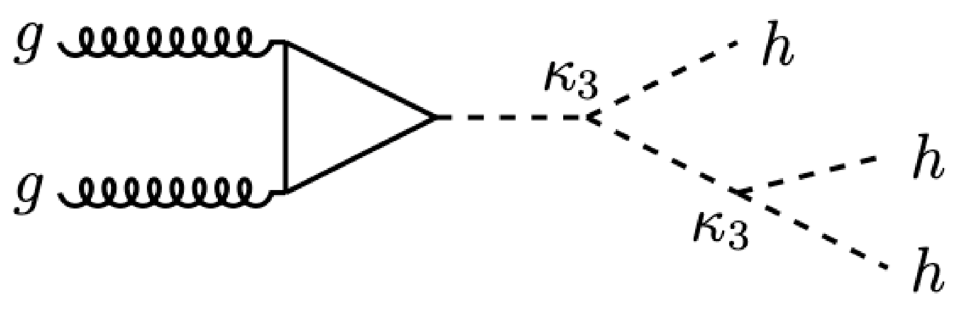
(a)



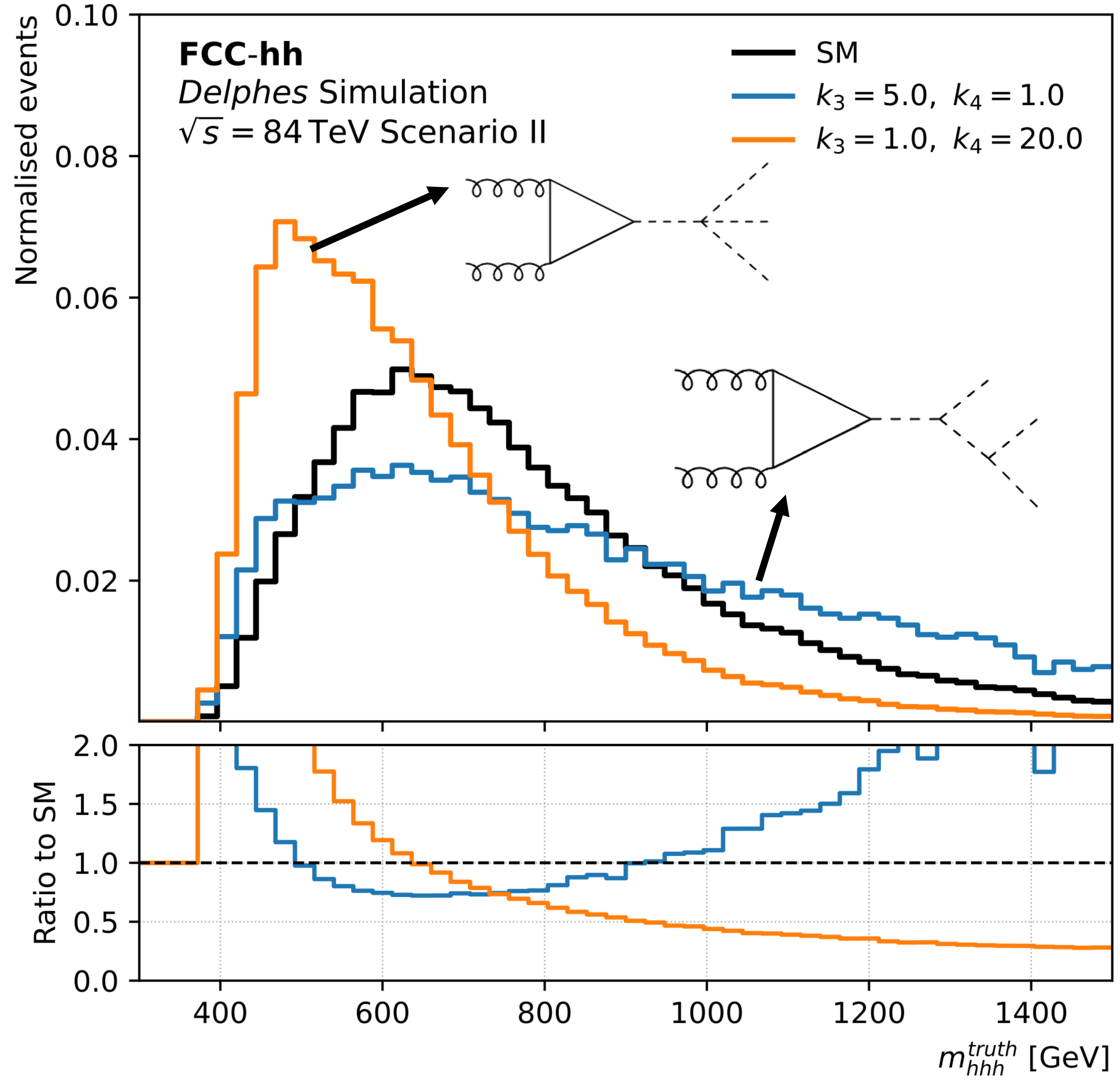
(b)



(c)



(d)



Analysis outline

- Determine sensitivity to SM-like non-resonant HHH production across (κ_3, κ_4) plane.

- Investigate $HHH \to 4b2\tau$ final state
  - Balance (reasonably) high BF with reduced background
  - anticipate better m($\tau\tau$) resolution.

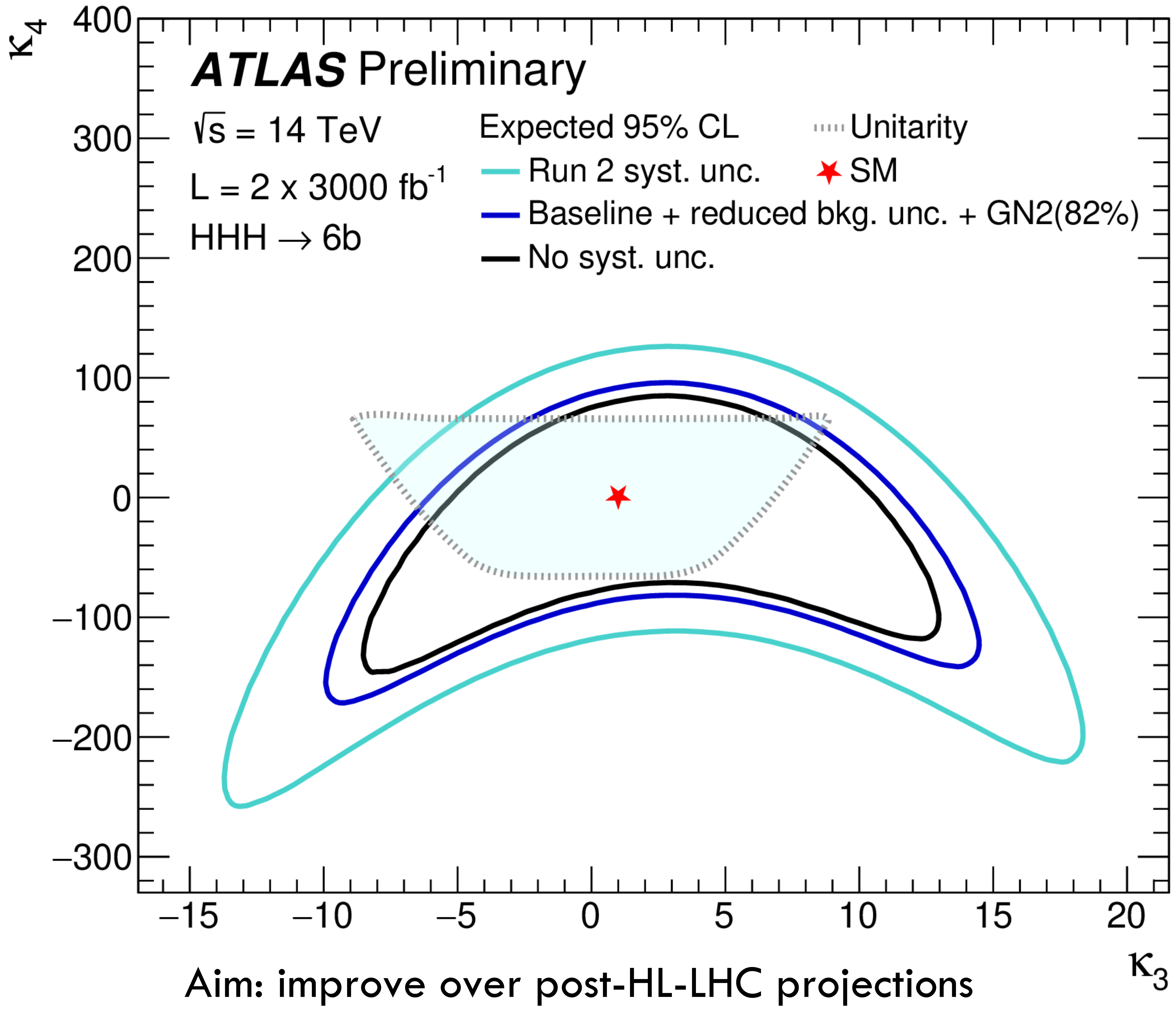
- Use both fully hadronic and mixed semi-leptonic τ final states ($\tau_{had}\tau_{had}$, $\tau_{lep}\tau_{had}$)

- Main background $t\bar{t}b\bar{b}$ (together with $t\bar{t}h/Z$)

- Pair b-jets/τs to reconstruct Higgs candidates and kinematics.

- Train NN classifiers to discriminate signal from backgrounds in each final state. Cut on to define SRs.

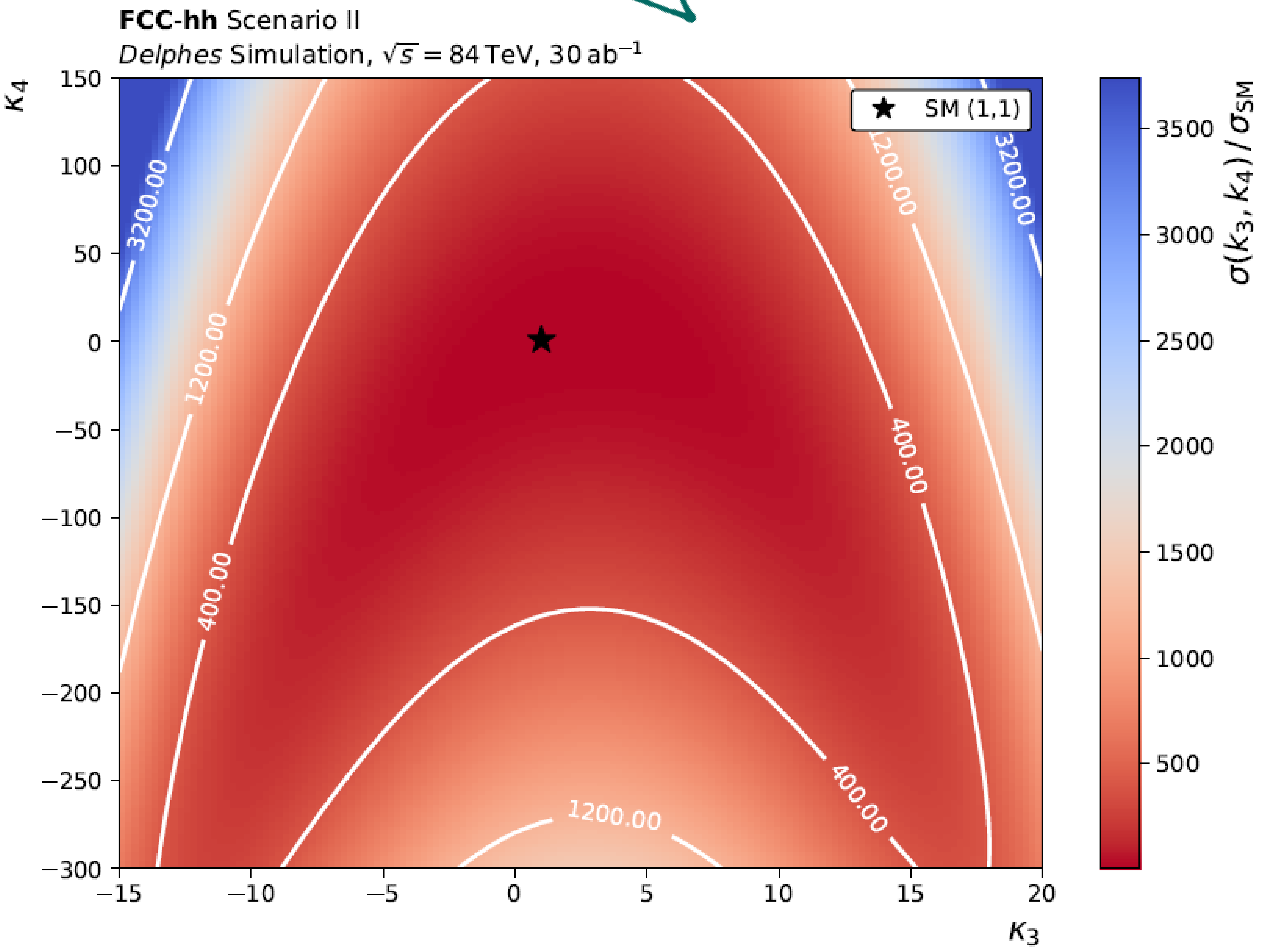
- Cut-and-count signal regions to provide simultaneous limits on (κ_3, κ_4) or $\kappa_{3/4}$ individually.



Aim: improve over post-HL-LHC projections for kappa constraints

hhh signal generation

EventProducer → k4SimDelphes → FCCAnalyses

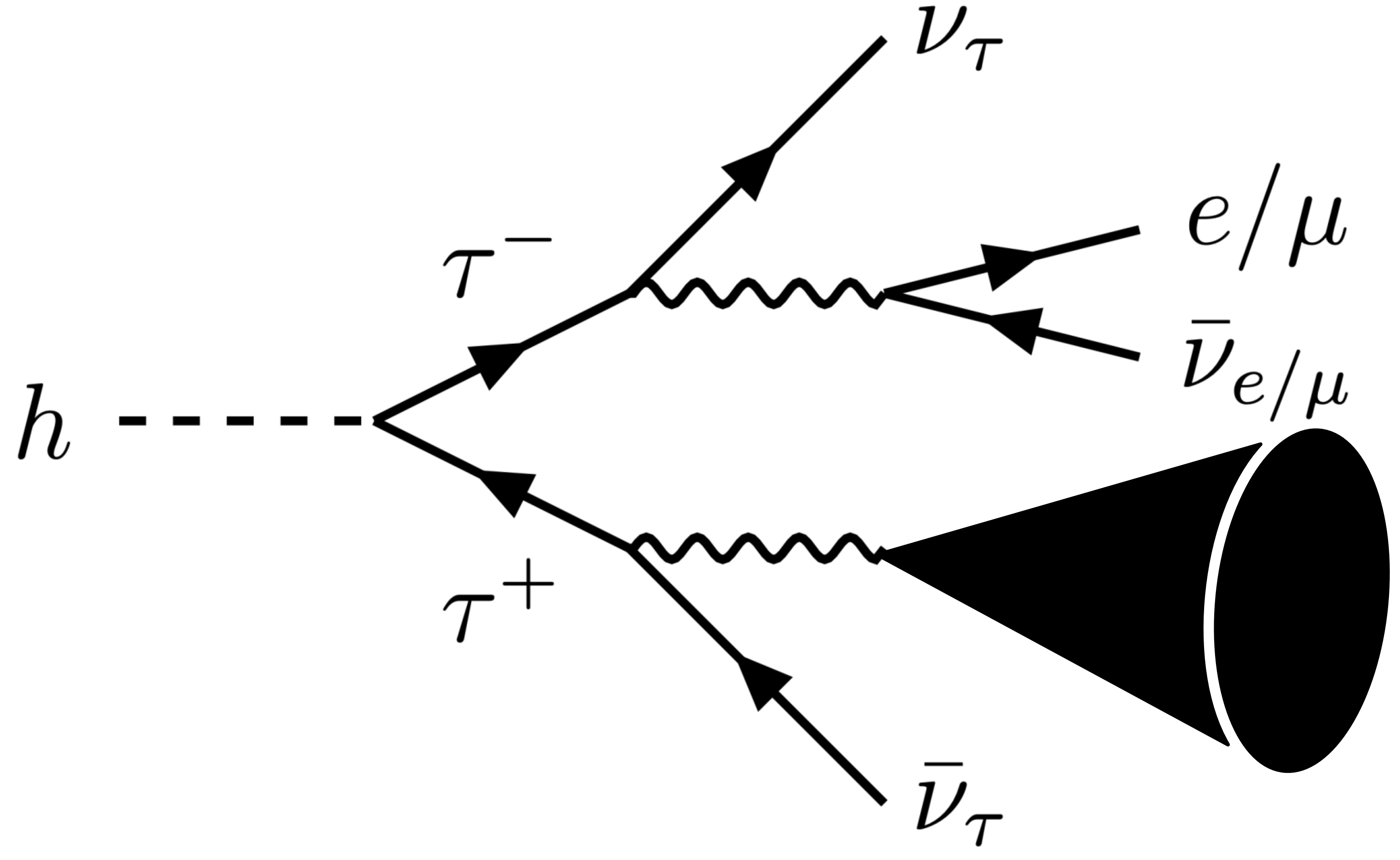


- Generate a single SM sample which can be 'morphed' to any point in (κ_3, κ_4) space (same as LHC analysis approach).
- Implemented signal and (κ_3, κ_4) Madgraph reweighting tools into FCC software.
- Strong interference near SM point – cross-section increased for high $|\kappa_3, \kappa_4|$ values, as well as aforementioned kinematic changes.

Channels: $\tau_{lep}\tau_{had} + \tau_{had}\tau_{had}$

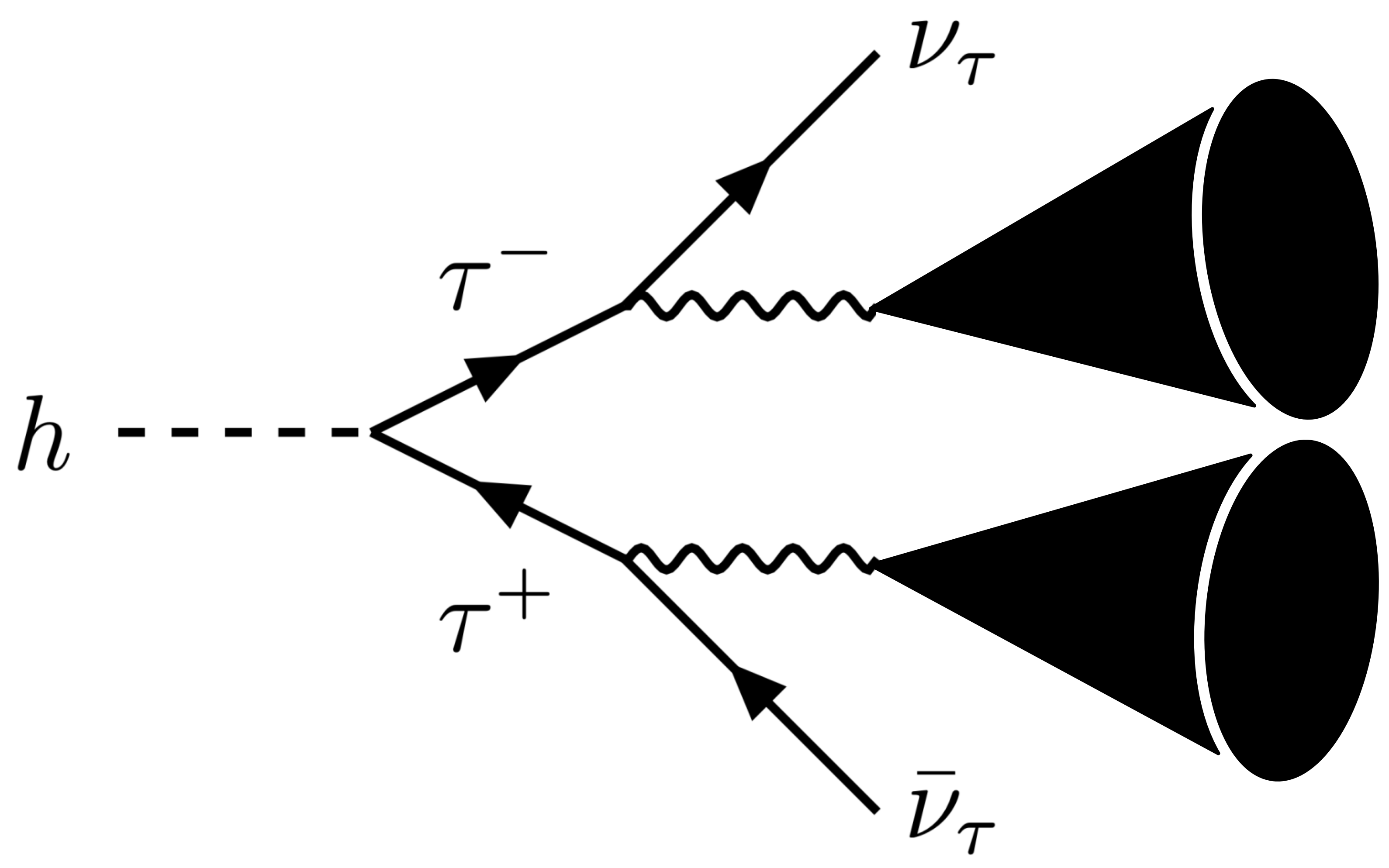
Selection

$\tau_{lep}\tau_{had}$



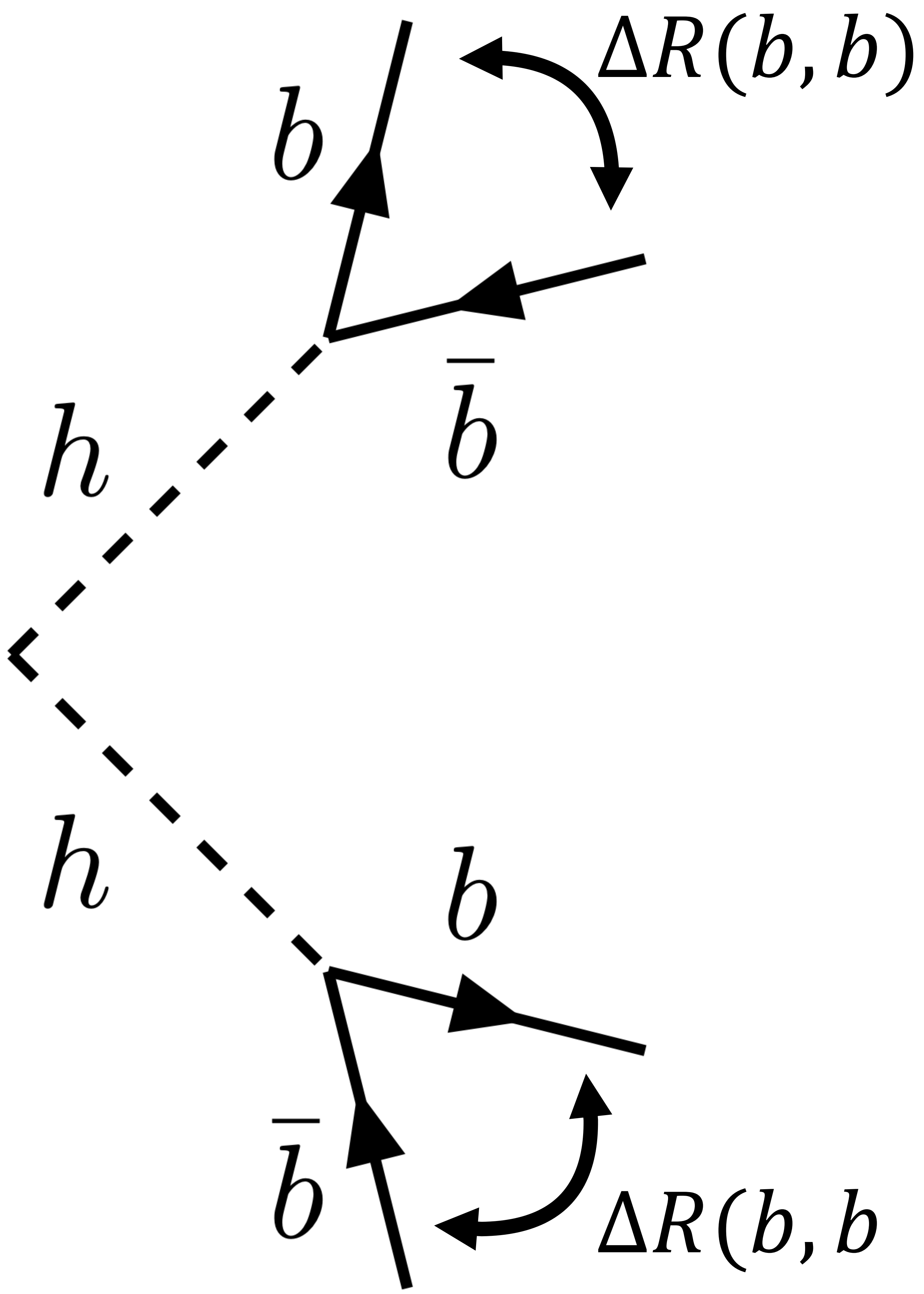
- 4 b-jets, medium operating point (p_T>25):
- Require: =1 electron/muon (pT>20);
  =1 hadronic tau (p_T>25):
- Opposite charge $e(\mu)/\tau_{had}$ candidates

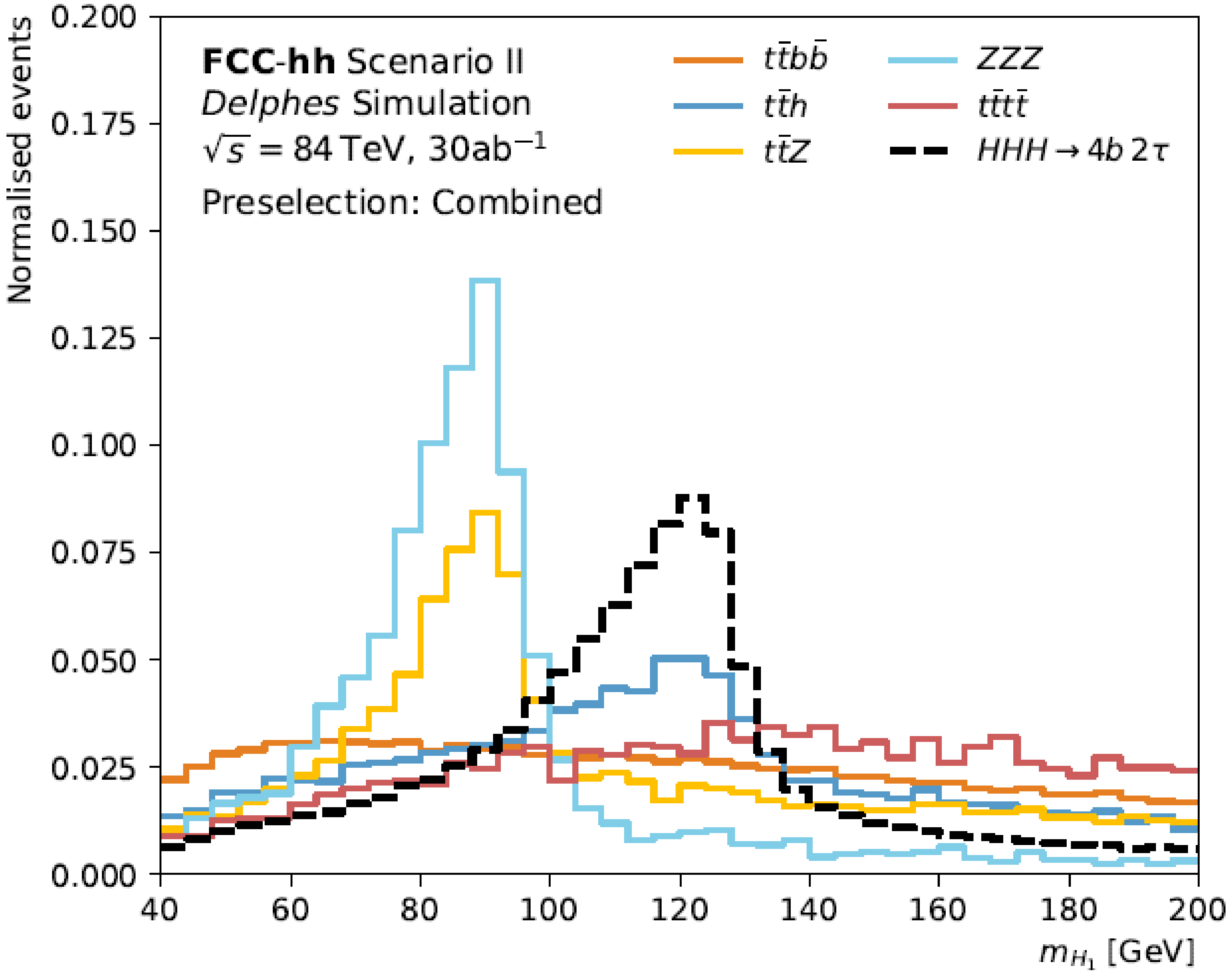
$\tau_{had}\tau_{had}$

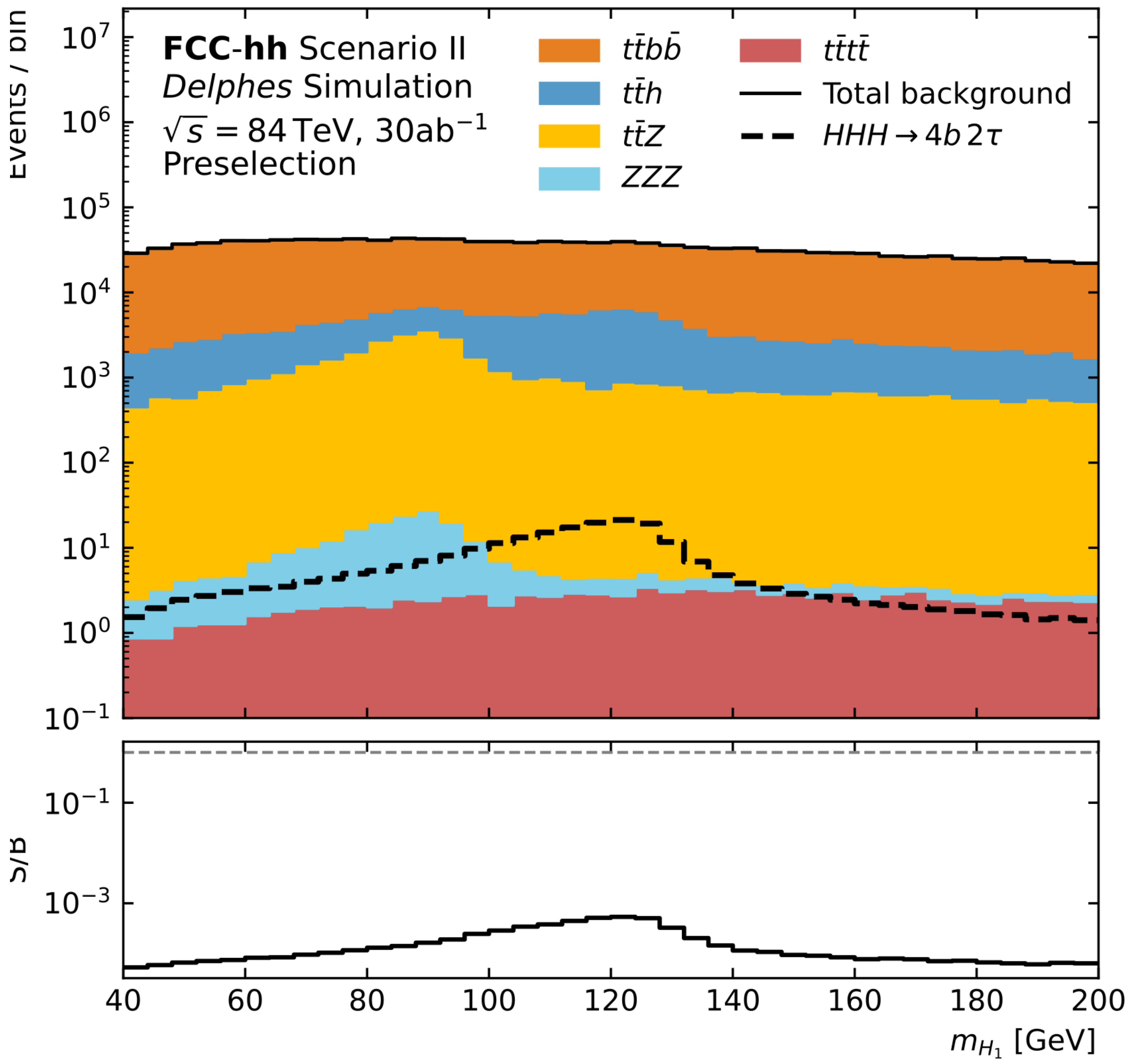


- 4 b-jets, medium operating point (p_T>25)
- Require: = 0 electrons+muons, (pT>20);
  = 2 hadronic taus (p_T>25)
- Opposite charge $\tau_{had}\tau_{had}$ candidates

Event reconstruction: pair *b*-jets to Higgs bosons





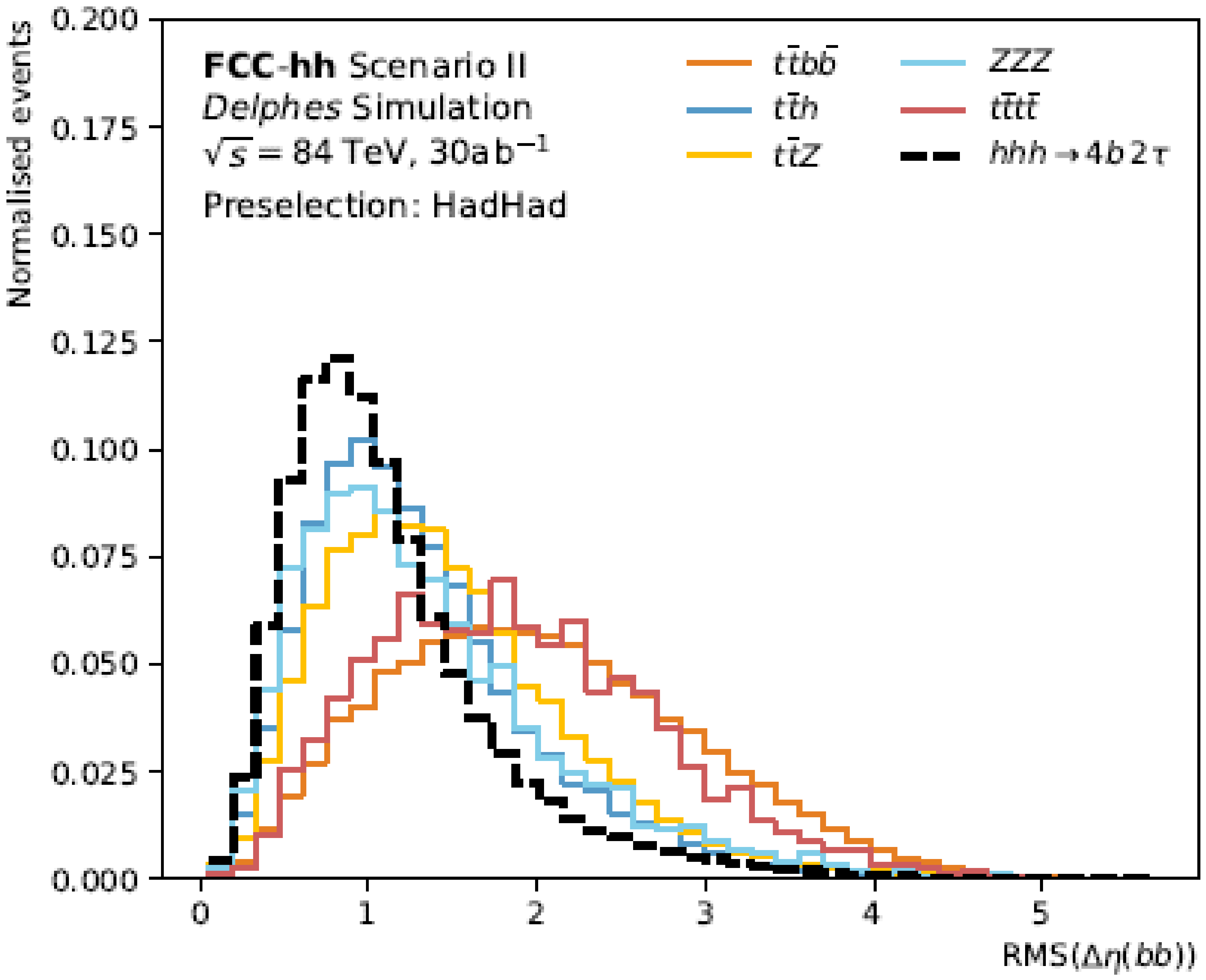


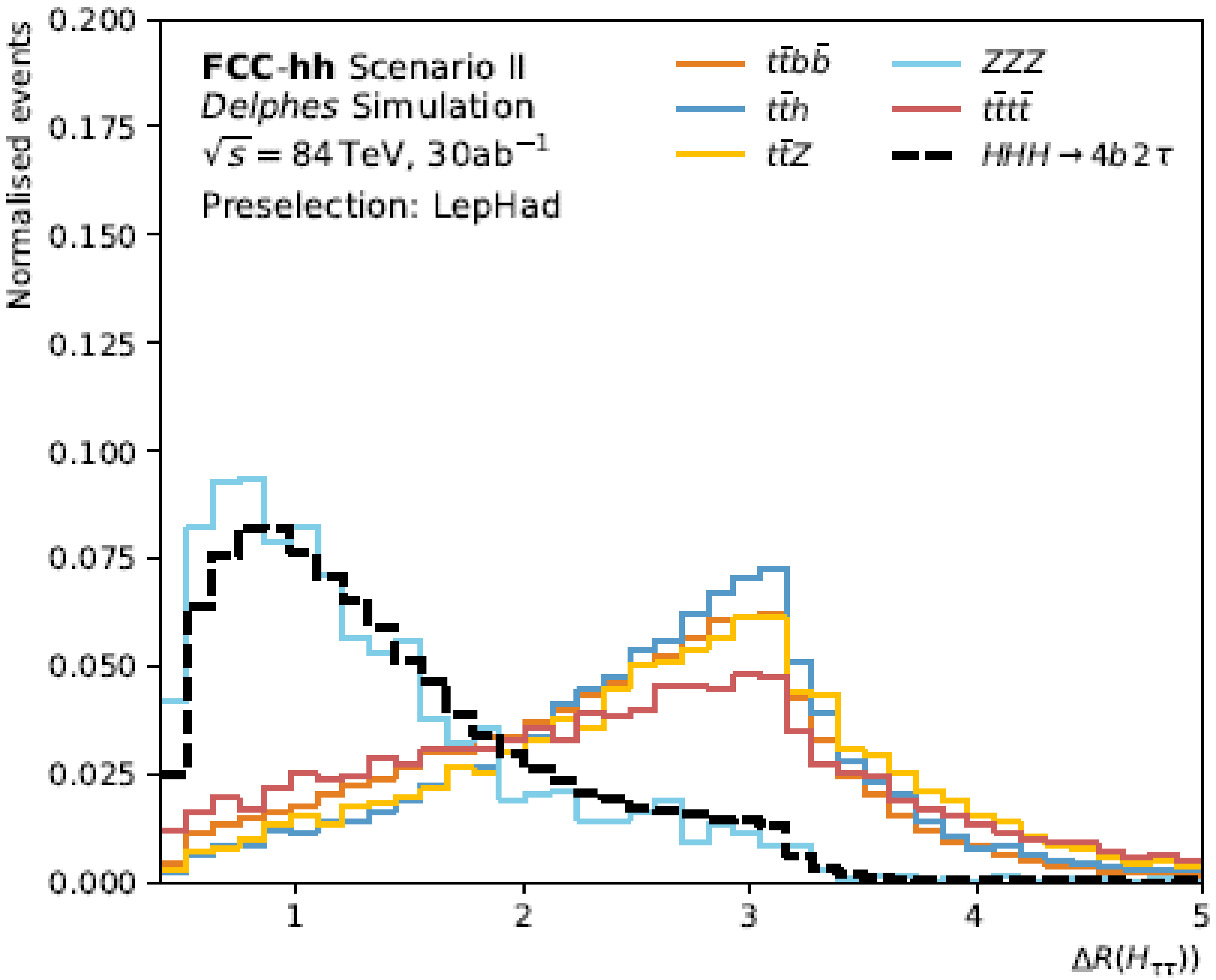
Using angular pairing to obtain $h \to bb$ candidates:
For $h \to \tau\tau$ candidate using visible tau info for now.

$$\min(\Delta R_{\max}(b,b)) = \min_{i,j,k,l}(\max(\Delta R(i,j), \Delta R(k,l)))$$

NN Training

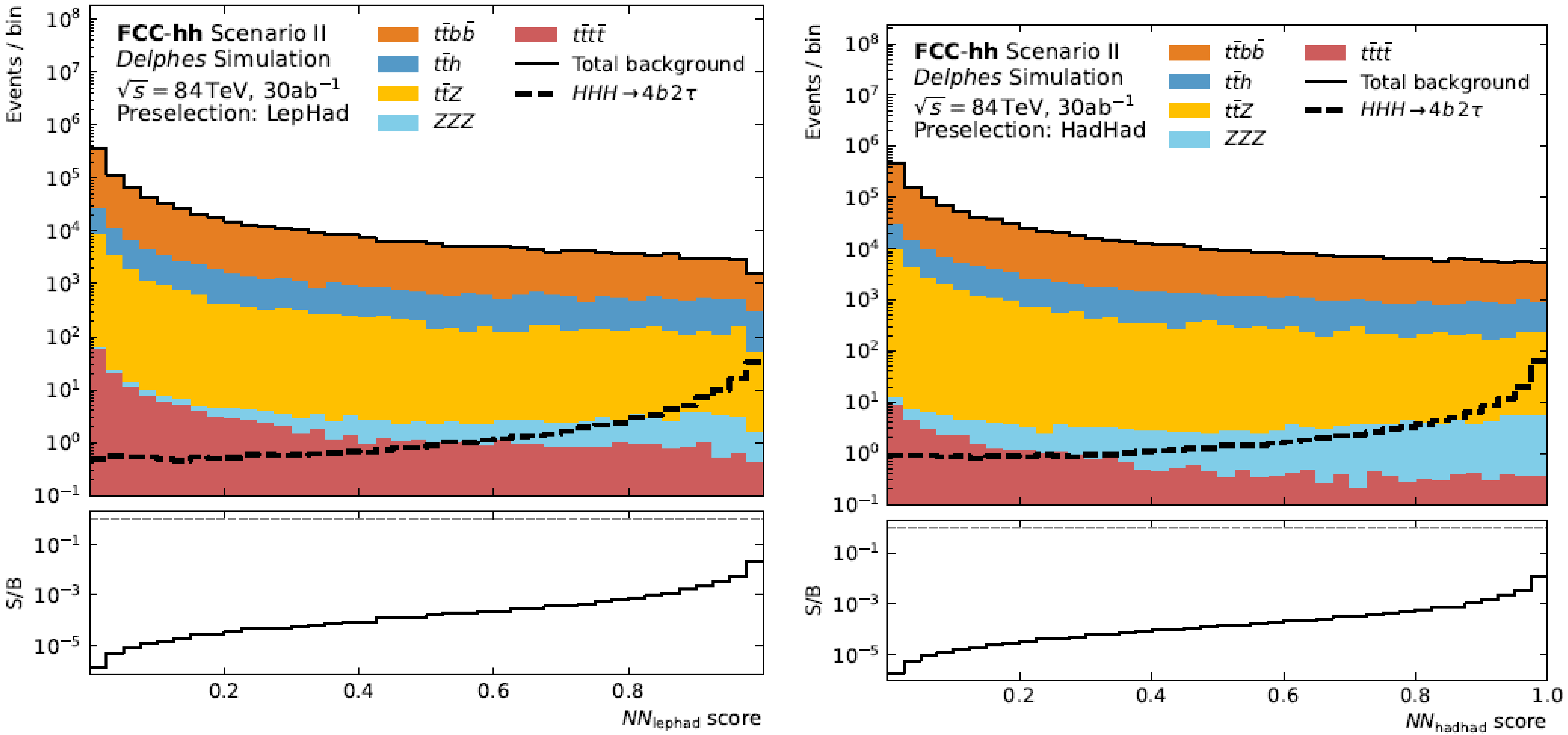
- Train a NN for each final state, to discriminate backgrounds using kinematic features of b/τ/MET/h candidates.
- Multi-layer perceptron (MLP) with hyperparameter optimisation for batch-size, number of hidden layers
- Train on SM hhh signal + all backgrounds.
- Most important features are reconstructed Higgs features (e.g h candidate masses)





NN Score

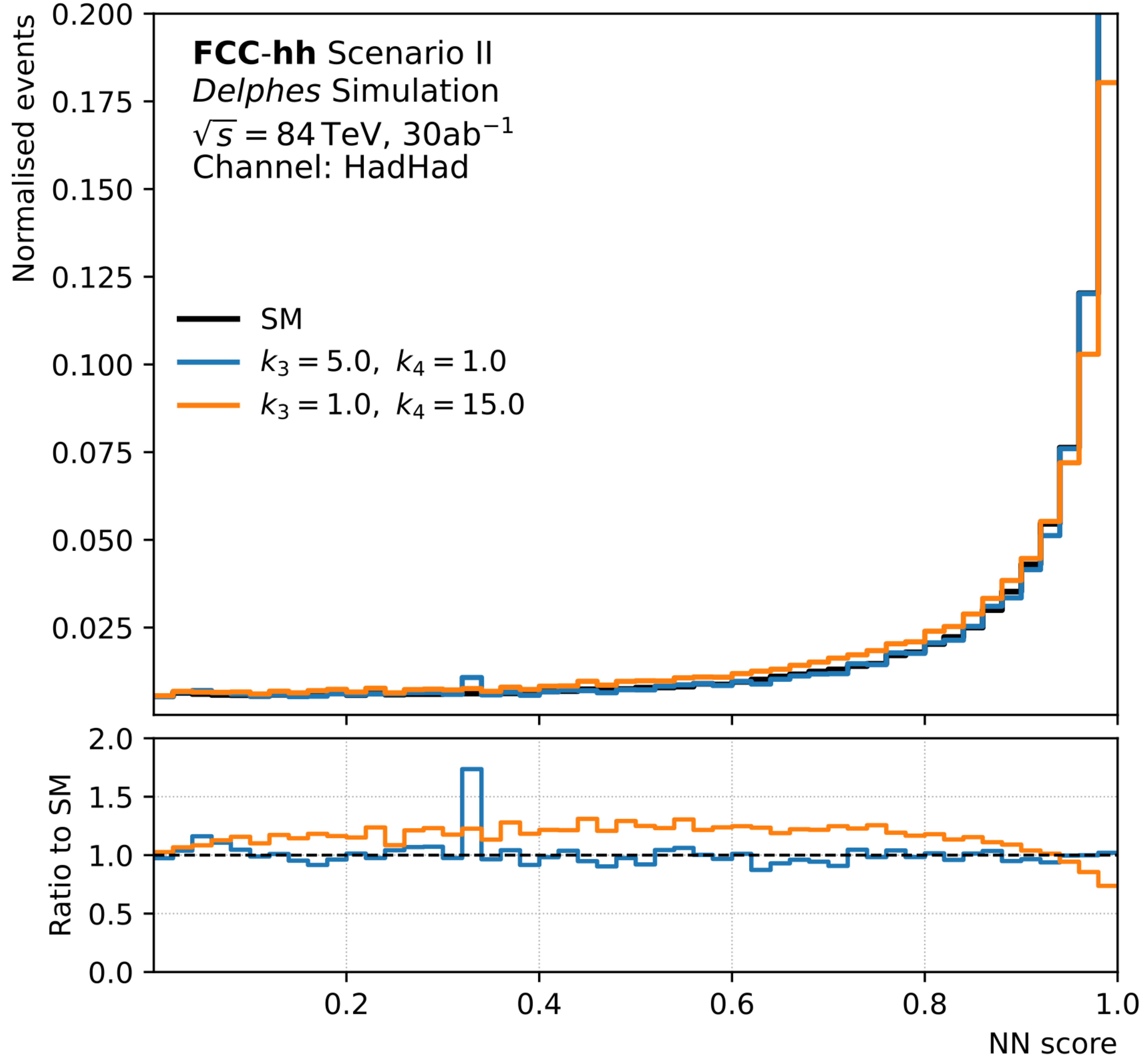
- Both NNs discriminate dominant ttbb background well.
- Perform fit and statistical analysis on single-bin NN score >0.9875 on each final state simultaneously



NN performance

Impact of training on SM-hhh only vs varied kappas?

- NN Score & sensitivity not found to degrade significantly across most of the plane.
- Conclude SM-only training ok for current input feature choices.

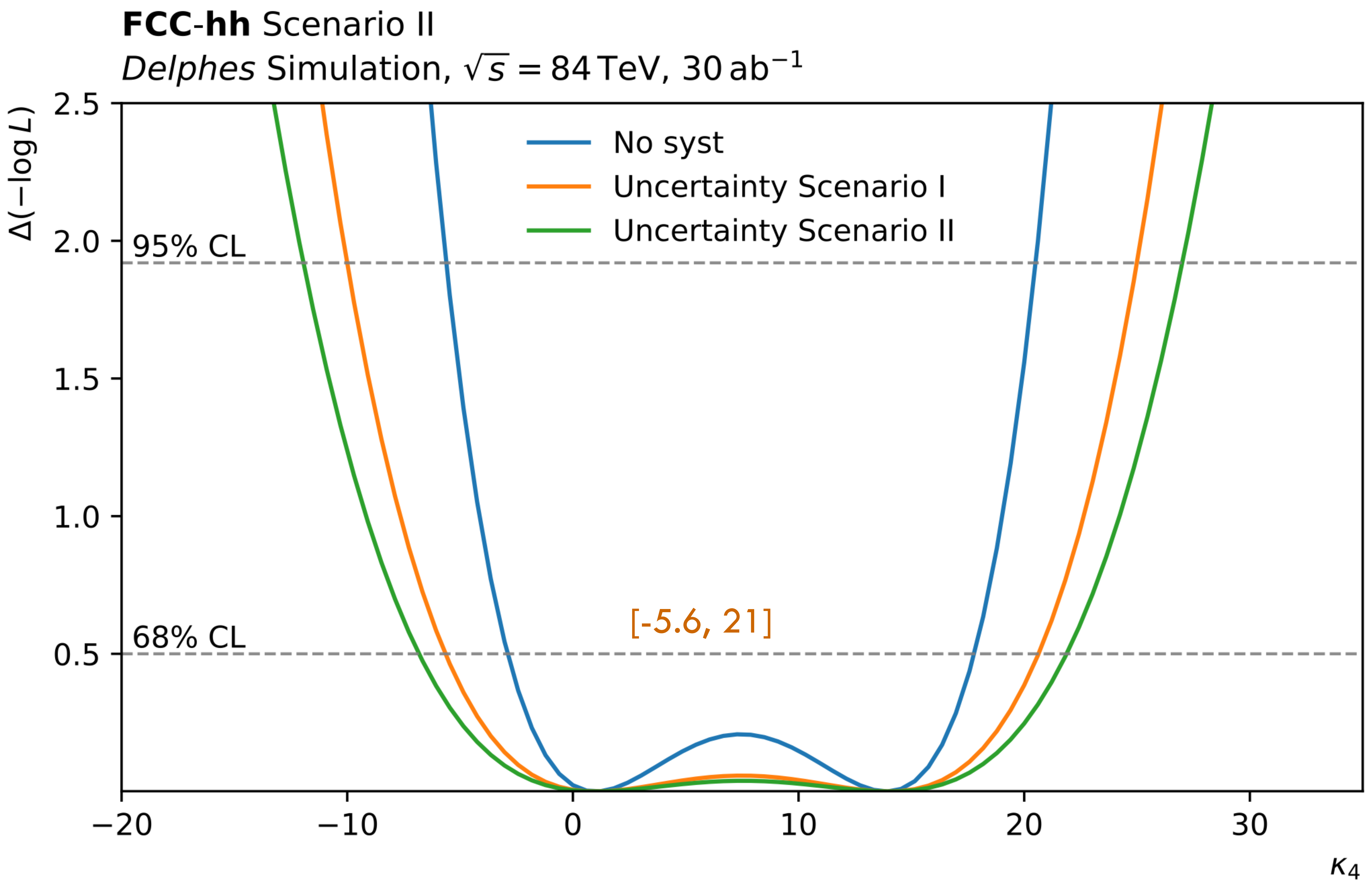


Interpretations – Profile Likelihood Fits

- Two bins fitted simultaneously: SRs for $\tau_{lep}\tau_{had} + \tau_{had}\tau_{had}$
- Systematic scenarios:
  1. Per-obj ID 1%, Bkg-norm 1%, lumi 1%
  2. Per-obj ID 2%, Bkg-norm 1%, lumi 1%

  NB. Very conservative, CR normalisation would reduce significantly (to be updated)
- Four fit configurations considered for briefing book:
  1. $\kappa_4 = 1$, κ_3 varying
  2. $\kappa_3 = 1$, κ_4 varying

Fit 2) Assume κ_3 well-constrained by HL-LHC + FCC-ee:

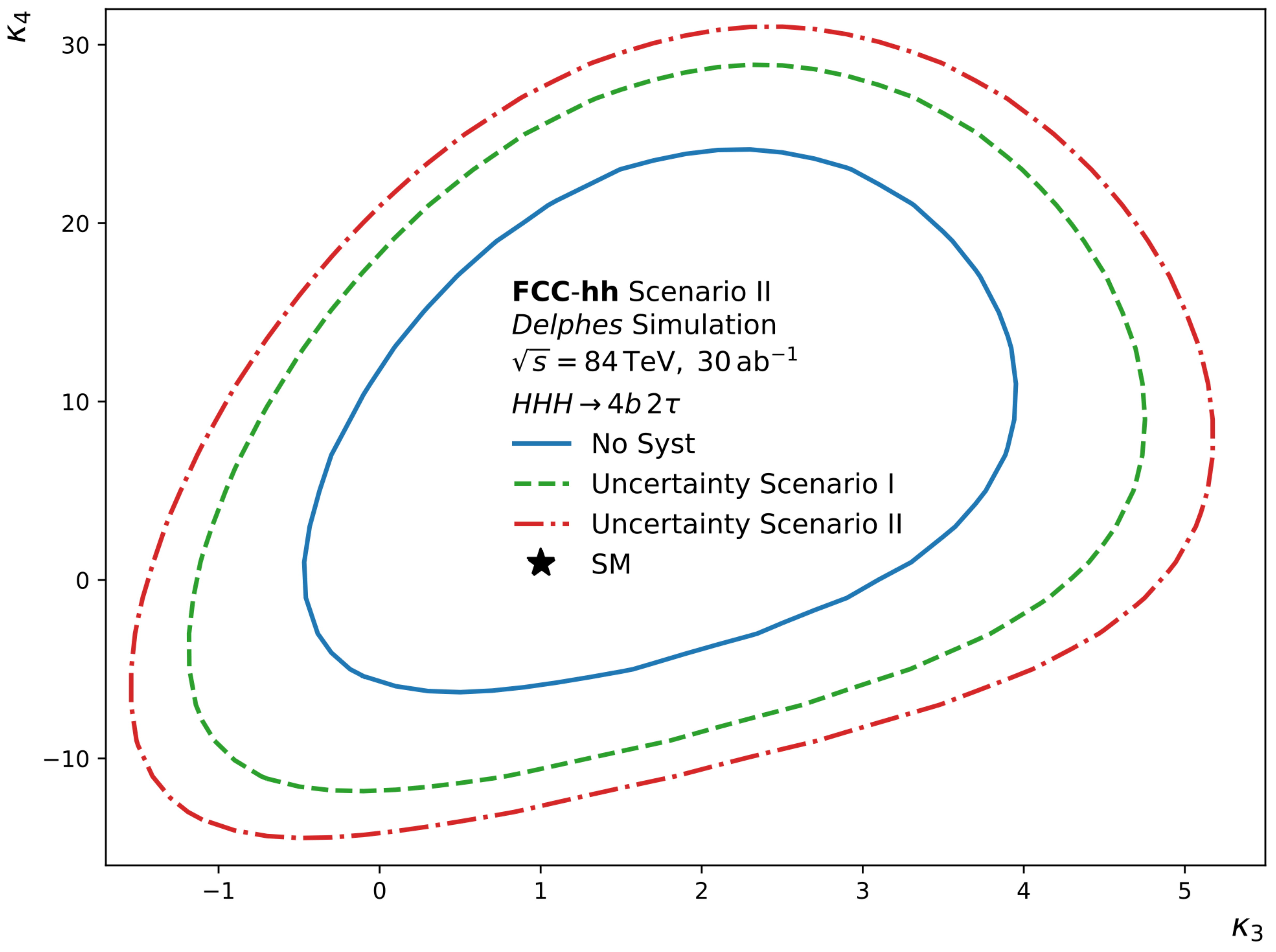


Interpretations – Profile Likelihood Fits

- Two bins fitted simultaneously: SRs for $\tau_{lep}\tau_{had} + \tau_{had}\tau_{had}$
- Systematic scenarios:
  1. Per-obj ID 1%, Bkg-norm 1%, lumi 1%
  2. Per-obj ID 2%, Bkg-norm 1%, lumi 1%

  NB. Very conservative, CR normalisation would reduce significantly (to be updated)
- Four fit configurations considered for briefing book:
  1. $\kappa_4 = 1$, κ_3 varying,
  2. $\kappa_3 = 1$, κ_4 varying
  3. κ_3, κ_4 simultaneously
  4. SM cross-section

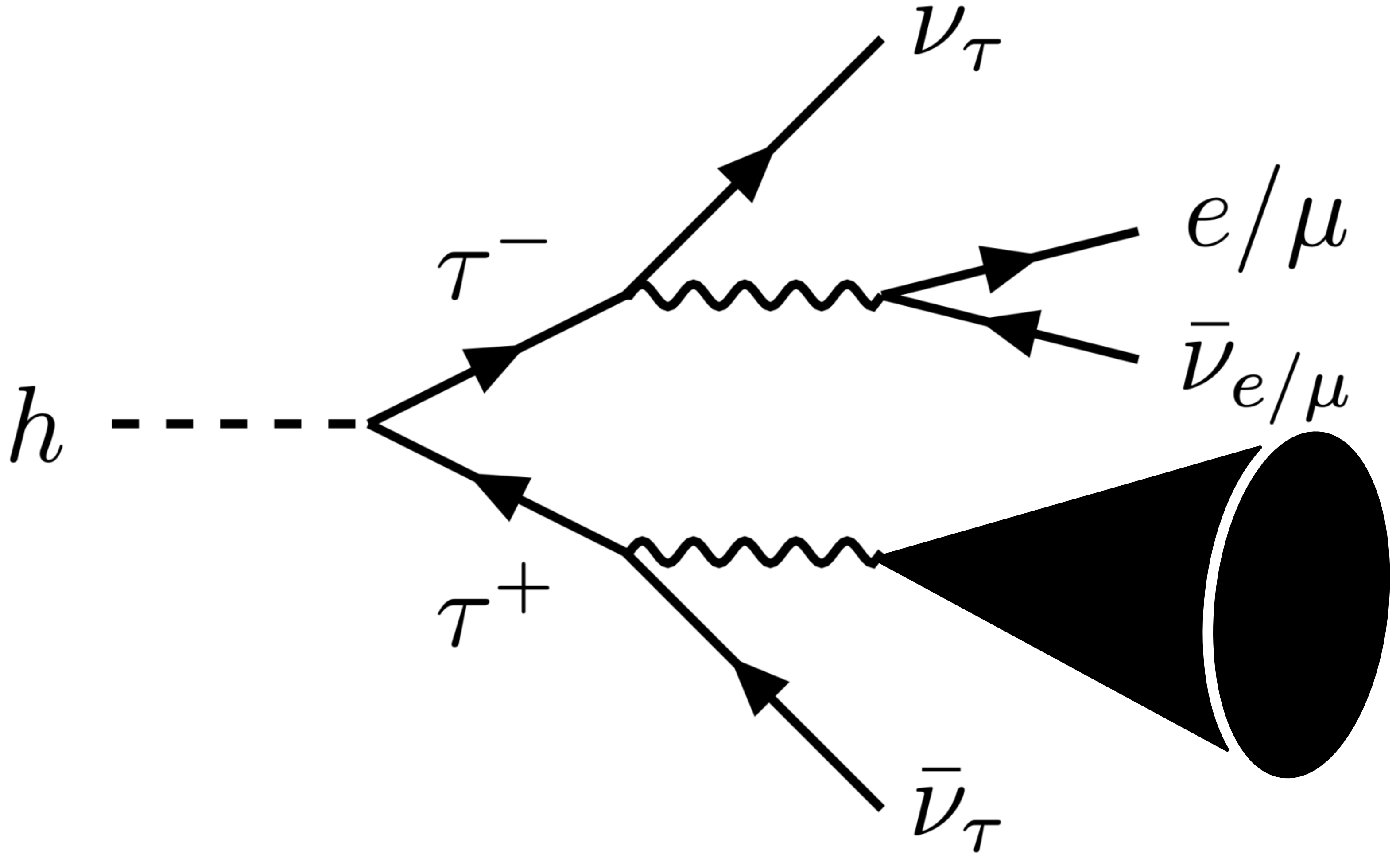
Fit 3) 95% expected CL on (κ_3, κ_4):

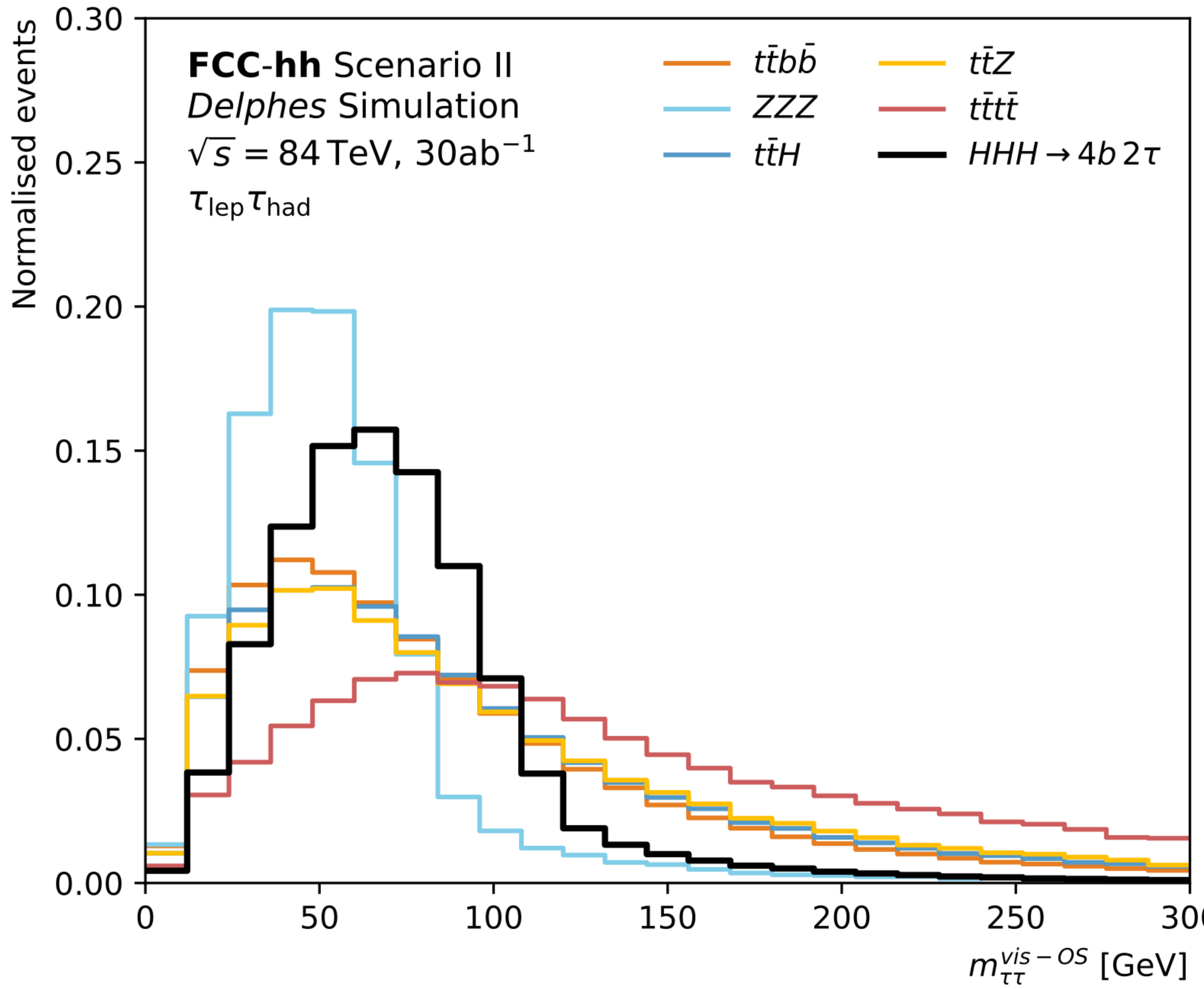


Fit 4) σ(hhh)<10.5 fb^{-1}, 3xSM (cf. 760xSM @ LHC)

Further optimisations in progress!

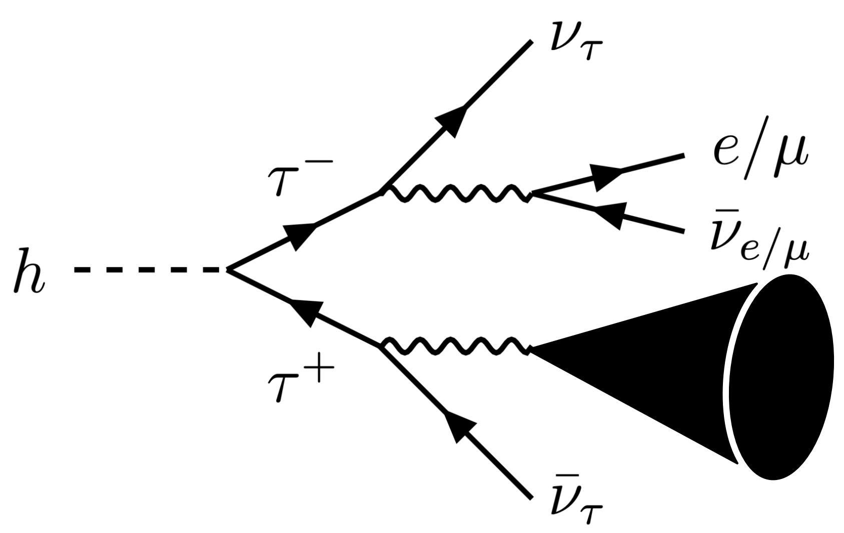
$m_{\tau\tau}$ reconstruction



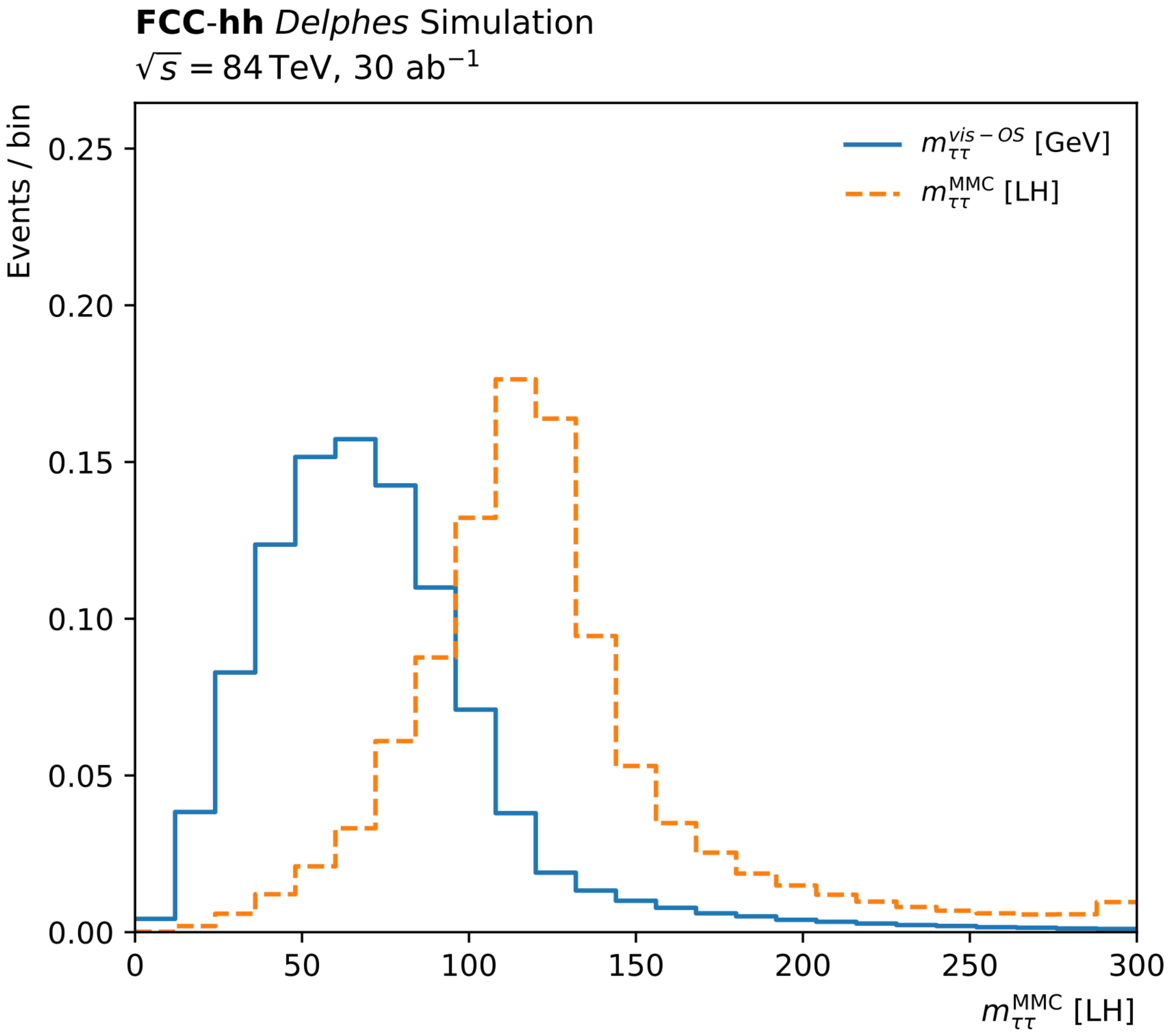


- Neutrinos from tau-decays degrade both central-value and resolution of the $m_{\tau\tau}$ distribution.
- Visible mass a poor representation of Higgs-mass.
- More powerful mass variables accounting for neutrinos under evaluation – as used in LHC analyses – expect significant improvement.

$m_{\tau\tau}$ reconstruction: MMC



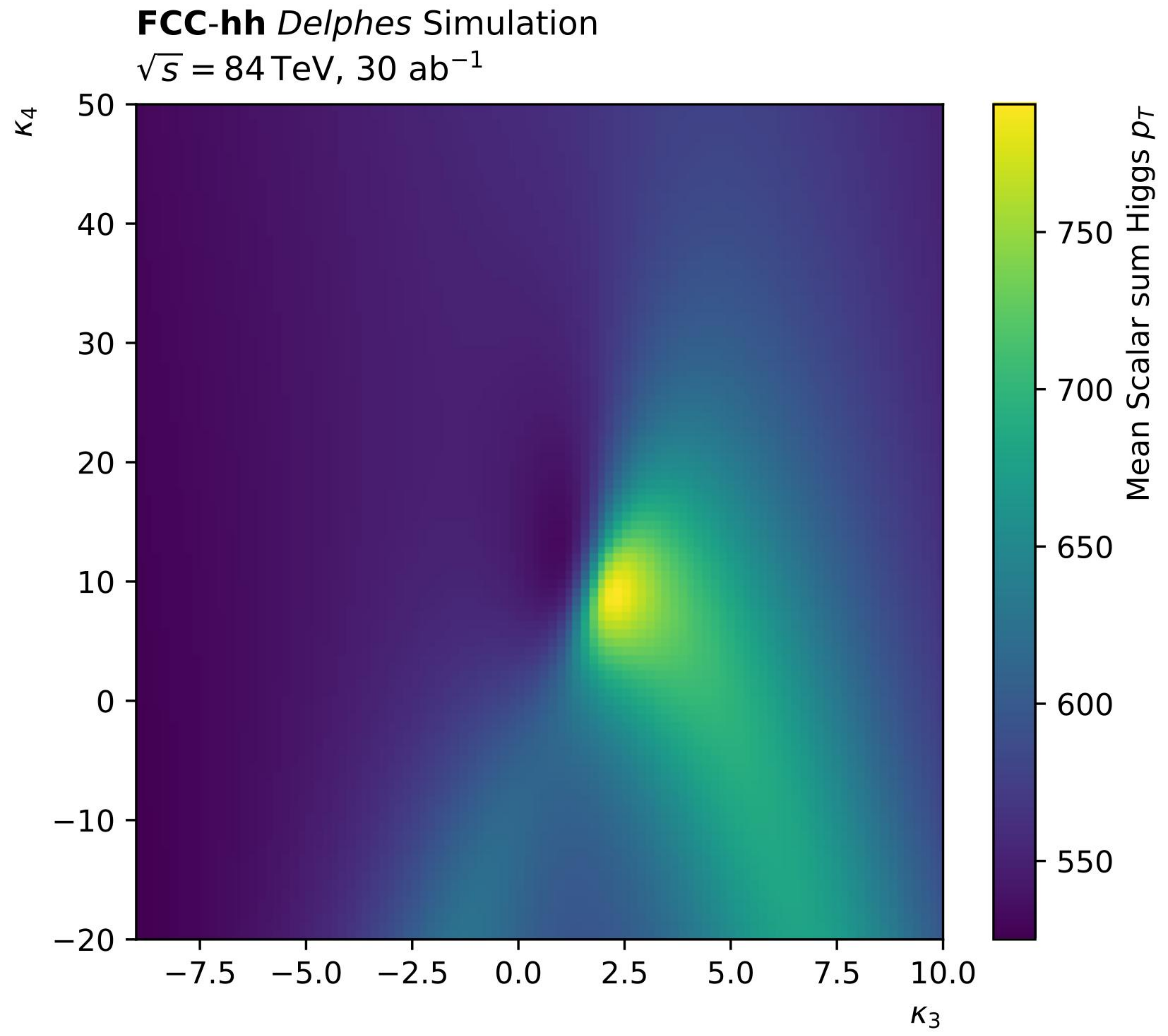
- Likelihood-based tau-reconstruction tool utilised by CDF & ATLAS experiments: Missing Mass Calculator
- Improved estimate of the true di-tau mass, using:
  i) the tau mass constraint
  ii) assuming $p_T^{miss} = p_T^{\nu_1} + p_T^{\nu_2}$.
- Scan over possible neutrino directions, considering properties of tau-lepton decays.
- Significantly improved estimate and resolution on $m_{\tau\tau}$

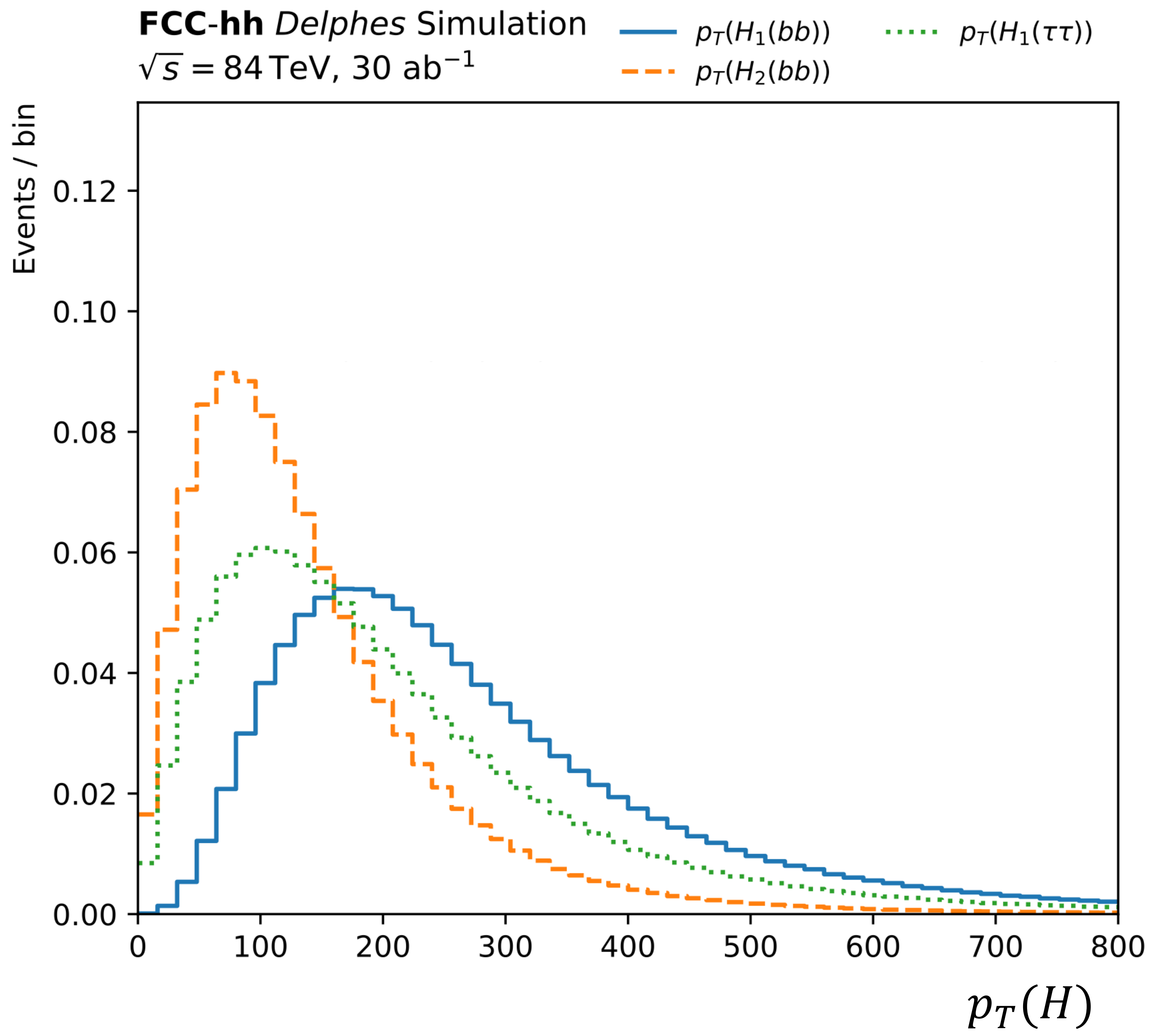


- Most probable mass over the entire event $\rightarrow m_{\tau\tau}^{MMC}$

Boosted Higgs decay products?

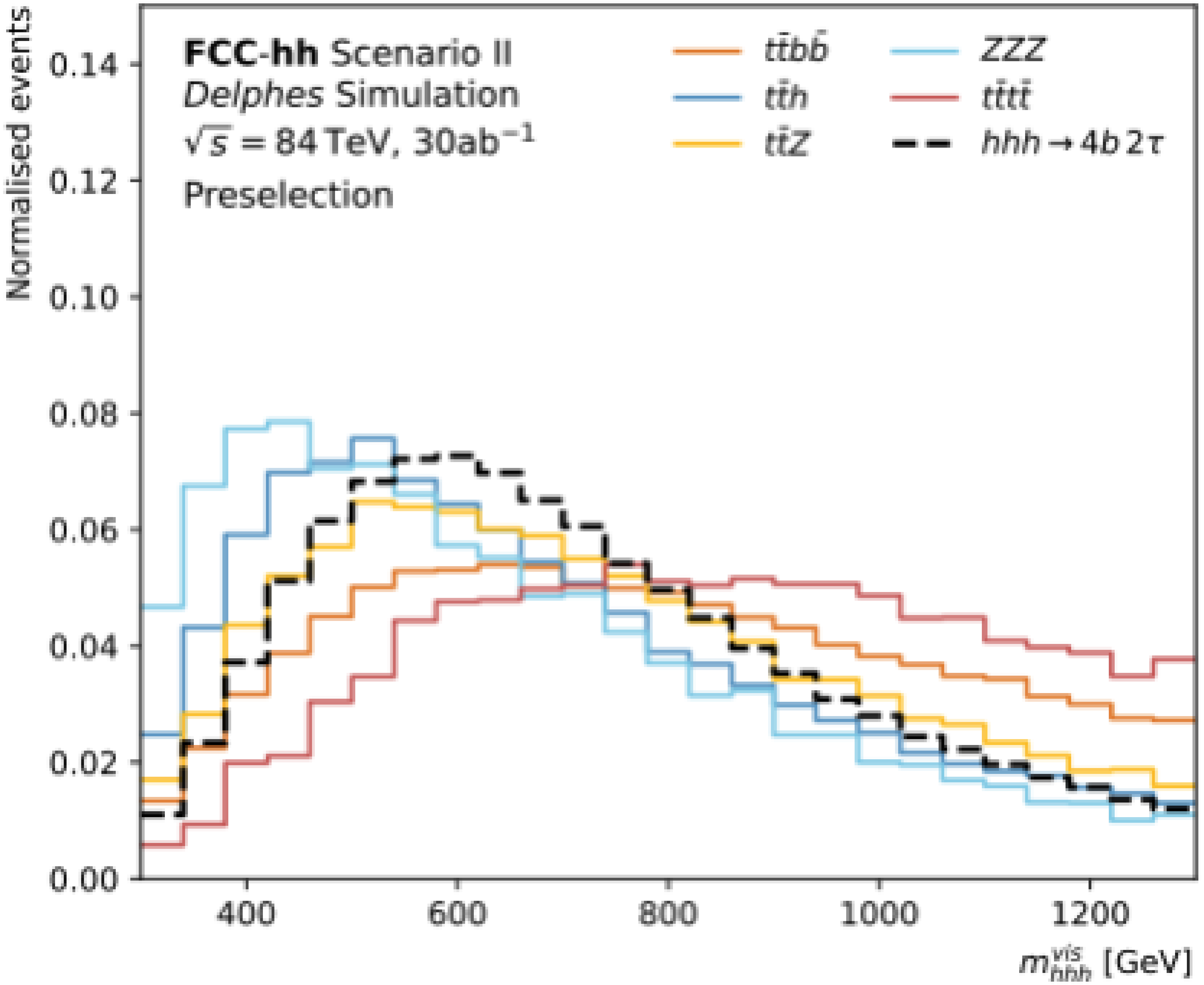
- High FCC-hh energy $\longrightarrow$ sizeable population of boosted hhh events, in much of parameter space
- 33% of SM hhh events with ≥ 1 boosted $h \rightarrow bb$ candidate (dR_{bb}<0.8)
- Potential to add boosted analysis channel

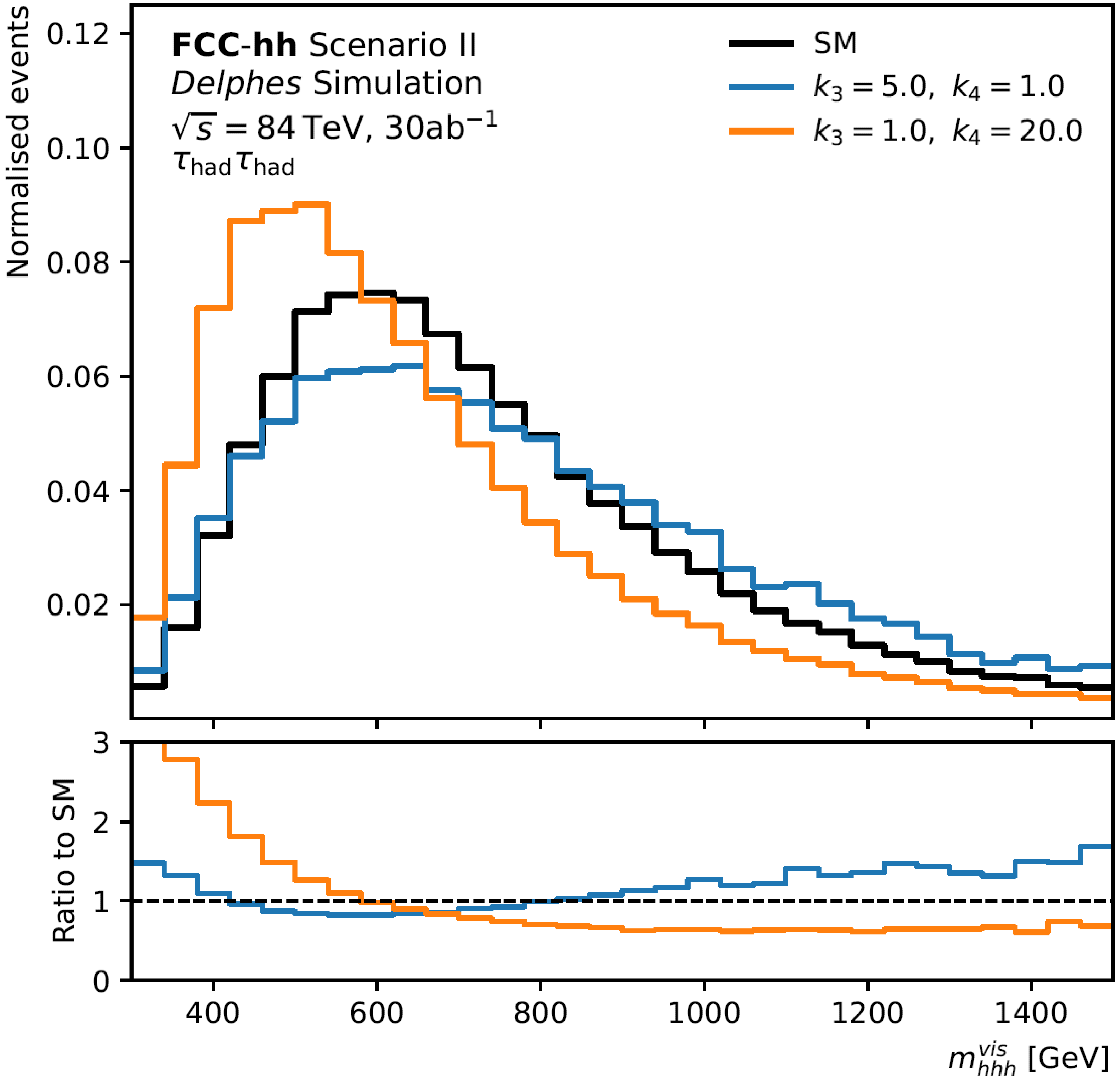




M_{hhh} shape information?

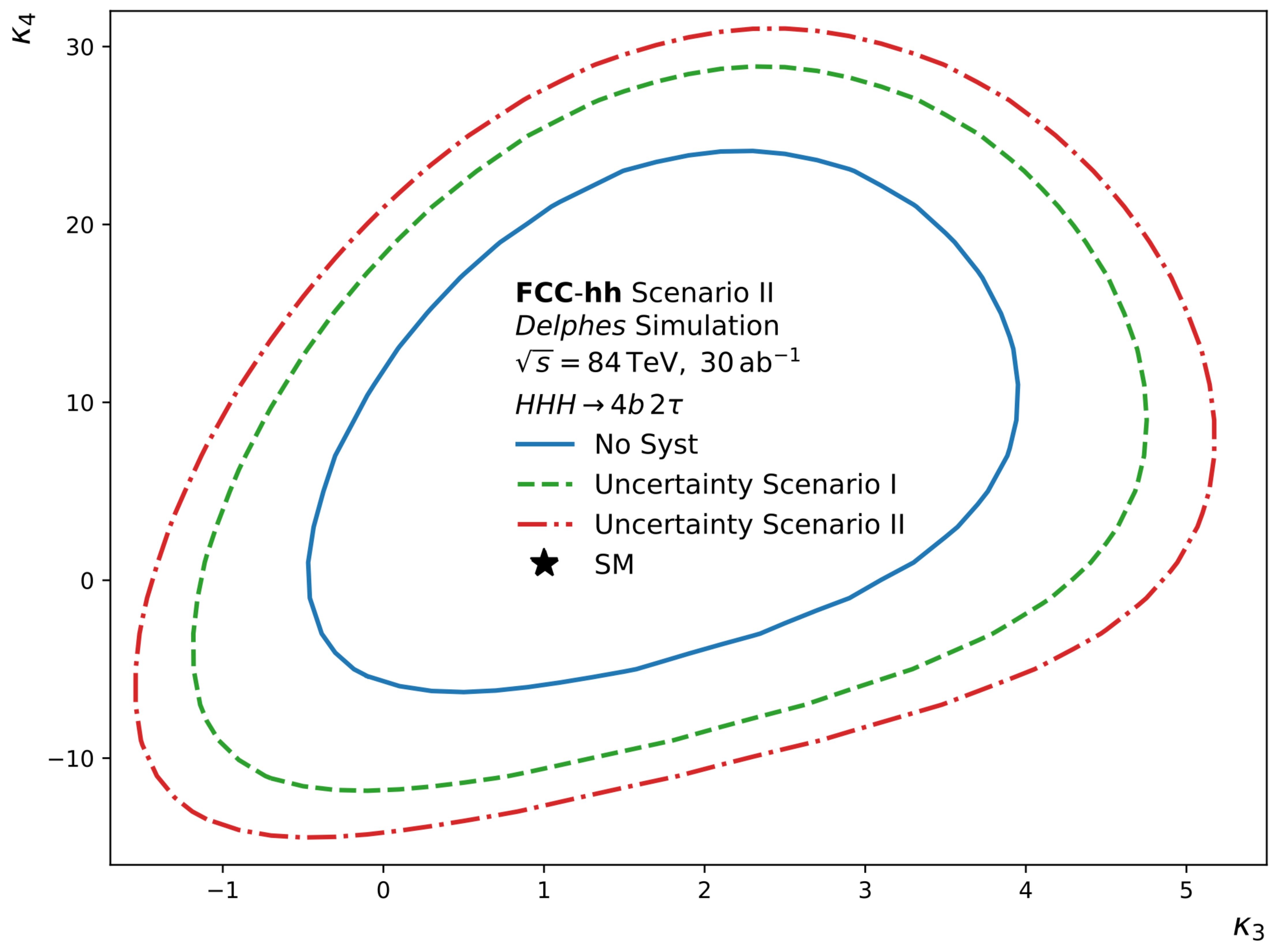
- Mass is discriminating against backgrounds and $(\kappa_3\ \kappa_4)$
- Loosen NN cut to have statistics to bin m_{hhh} and exploit shape in fit? Or use in NN.





Summary

- First hhh study within FCC framework and at 84 TeV.
- Complements di-Higgs production in constraining Higgs potential/self-coupling – quartic coupling only weakly constrainable at (HL-)LHC.
- Probe $\mathrm{hhh} \rightarrow 4\mathrm{b}2\tau$ production in $\tau_{lep}\tau_{had}$ and $\tau_{had}\tau_{had}$ final states.
- Preliminary result: 3x σ(SM), $-5.6 < \kappa_4 < 21$ ($@\ \kappa_3 = 1$).
  study documented for briefing book: [CDS]
- Optimisations ongoing to improve results further for a final paper!



Backup

Preselection

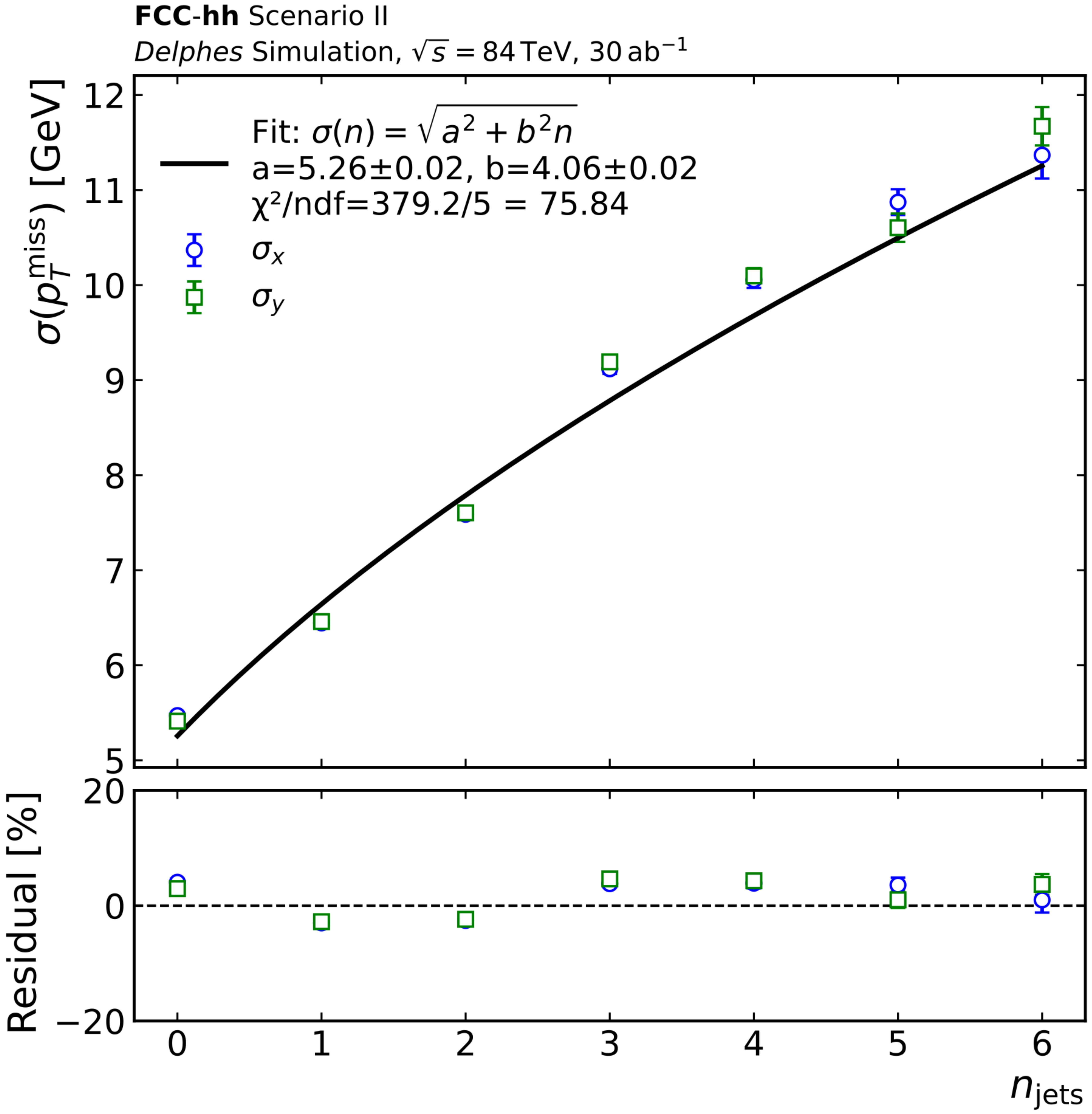
Variable	$\tau_{\mathrm{lep}}\tau_{\mathrm{had}}$	$\tau_{\mathrm{had}}\tau_{\mathrm{had}}$
	Preselection	
$n(b)$	==4	
$p_{\mathrm{T}}(b_1)$ [GeV]	> 40	
$p_{\mathrm{T}}(b_2)$ [GeV]	> 35	
$p_{\mathrm{T}}(b_3)$ [GeV]	> 30	
$p_{\mathrm{T}}(b_4)$ [GeV]	> 25	
$n(\tau_{\mathrm{had}})$	==1	==2
$n(l = e/\mu)$	==1	==0
τ charges	$OS(\tau_{\mathrm{had}}, l)$	$OS(\tau_{\mathrm{had}}, \tau_{\mathrm{had}})$
$p_{\mathrm{T}}(\tau_{\mathrm{had}\,1})$ [GeV]	> 25	
$p_{\mathrm{T}}(\tau_{\mathrm{had}\,2})$ [GeV]	-	> 25
$p_{\mathrm{T}}(l)$ [GeV]	> 20	-
	SR-$\tau_{\mathrm{lep}}\tau_{\mathrm{had}}$	SR-$\tau_{\mathrm{had}}\tau_{\mathrm{had}}$
Preselection	Pass	
NN Score	$\mathrm{NN}_{\mathrm{lephad}} > 0.9875$	$\mathrm{NN}_{\mathrm{hadhad}} > 0.9875$

$m_{\tau\tau}$ reconstruction: MMC MET Resolution

Simplistic MET-resolution: constant + stochastic term

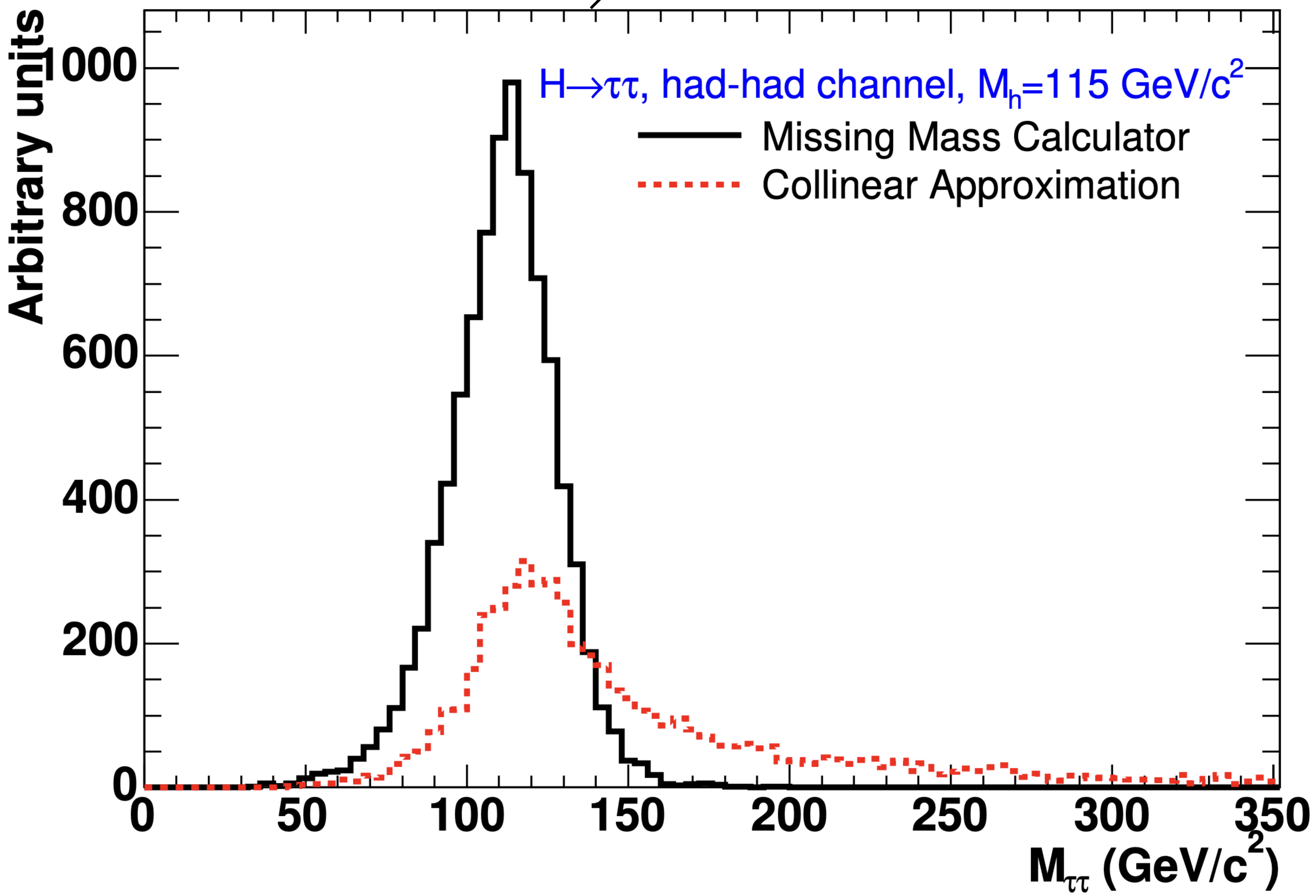
$$\sigma\left(p_T^{miss}\right) = \sqrt{a^2 + b^2 n}$$

Fit this simplistic formula as a function of n-jet



$m_{\tau\tau}$ reconstruction: MMC

MMC paper: [arXiv:1012.4686]



<u>Solving mtt:</u>

- 7 unknowns ($p_{x,y,z}^{\nu_{1/2}}, m_{miss_1,miss_2}$), with 4 equations ($E_{x,y}^{miss}, m_{\tau_1,\tau_2}$)
- Had-had ($m_{miss} = 0$) channel fully specified for given (ϕ_1, ϕ_2)
- For each real solution: store $m_{\tau\tau}$
- Use PDFs of $\mathcal{P}(\Delta\theta(p_{vis}, p_{miss})), \mathcal{P}(E_T^{miss})$ to weight the $m_{\tau\tau}$ hypothesis:

$$-\log L = -\log\left[\mathcal{P}(\theta_{3D}^1(p_\tau^1))\mathcal{P}(\theta_{3D}^2(p_\tau^2))\mathcal{P}\left(\Delta p_x^{miss}\right)\mathcal{P}\left(\Delta p_y^{miss}\right)\right]$$

- Make weighted $m_{\tau\tau}$ distribution:

$$max(\text{Hist}) = m_{\tau\tau}^{MMC}$$

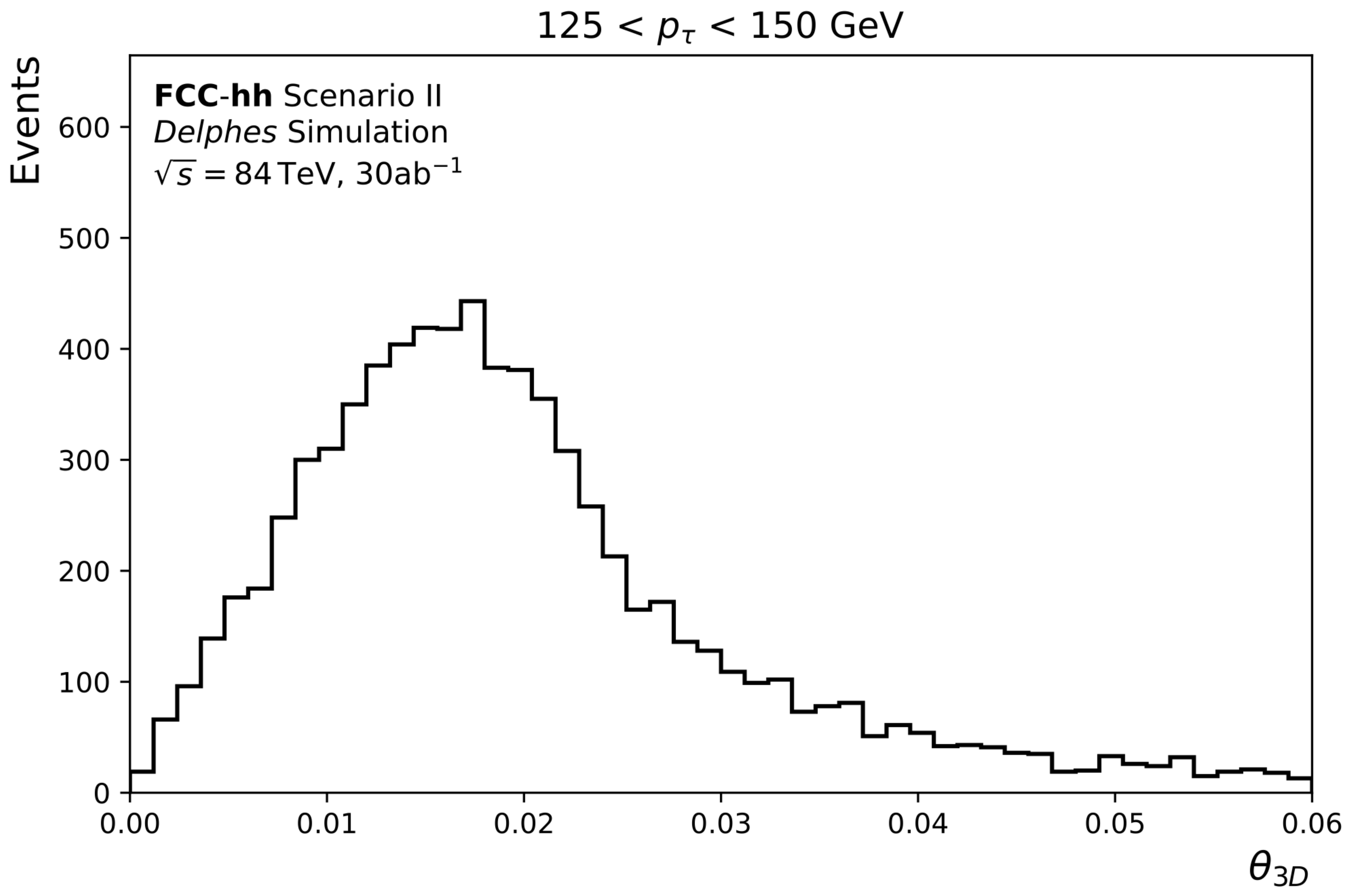
$m_{\tau\tau}$ reconstruction: MMC PDFs

"Use PDFs of $\mathcal{P}(\Delta\theta(p_{vis}, p_{miss}))$, $\mathcal{P}(E_T^{miss})$ to construct a per-event probability"

Generating PDFs for the FCC $\mathcal{P}(\Delta\theta_{3D}(p_\tau))$:

- Find PDFs $\mathcal{P}(\Delta\theta(p_{vis}, p_{miss}))$ by fitting to MC-truth
- Using $Z \rightarrow \tau^-\tau^+$+jets sample, find angle between all missing & visible τ-daughter momenta

$$\theta_{3D} = \frac{p_\tau^{vis} \cdot p_\tau^{miss}}{|p_\tau^{vis}||p_\tau^{miss}|}$$

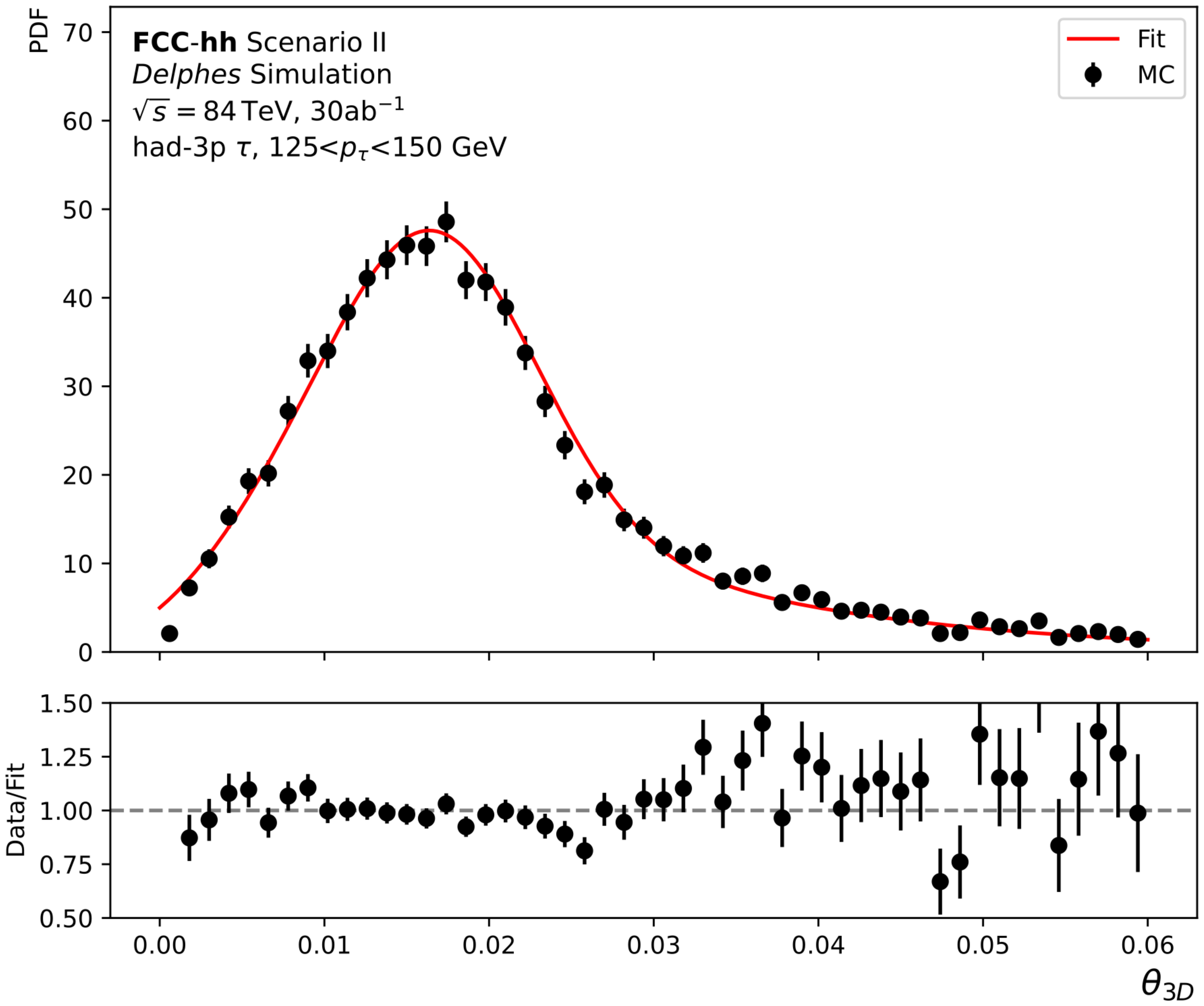


$m_{\tau\tau}$ reconstruction: MMC PDFs

$$\theta_{3D} = \frac{p_\tau^{vis} \cdot p_\tau^{miss}}{|p_\tau^{vis}||p_\tau^{miss}|}$$

- Fit the θ_{3D} bins with mixture of Gaussian + Landau
- Angular distribution depends on initial τ momentum
- Globally fit Gaussian + Landau as a function of p_τ

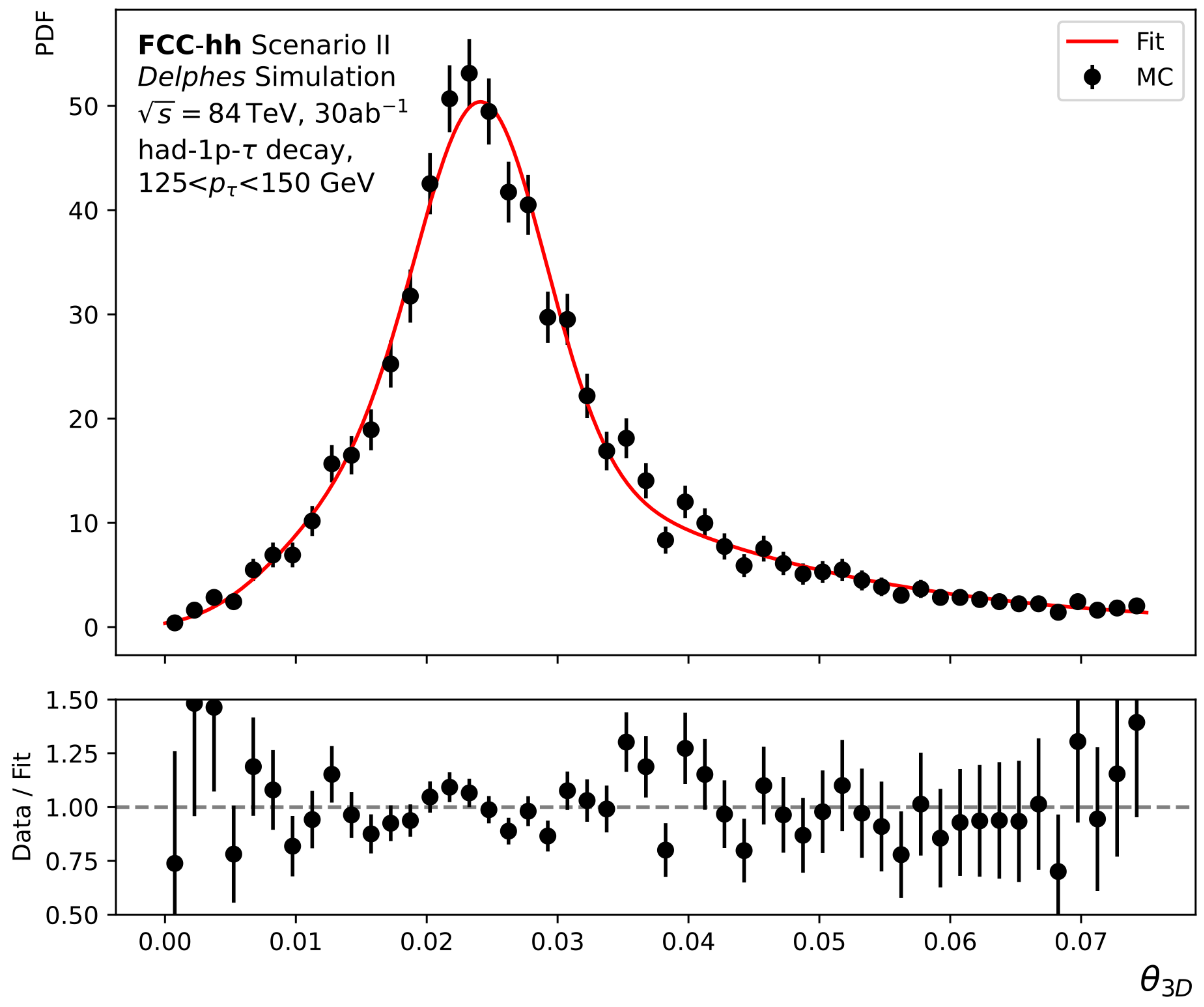
$$p_\tau \sim a_0 \left(e^{-a_1 p_\tau} - \frac{a_2}{p_\tau} \right)$$



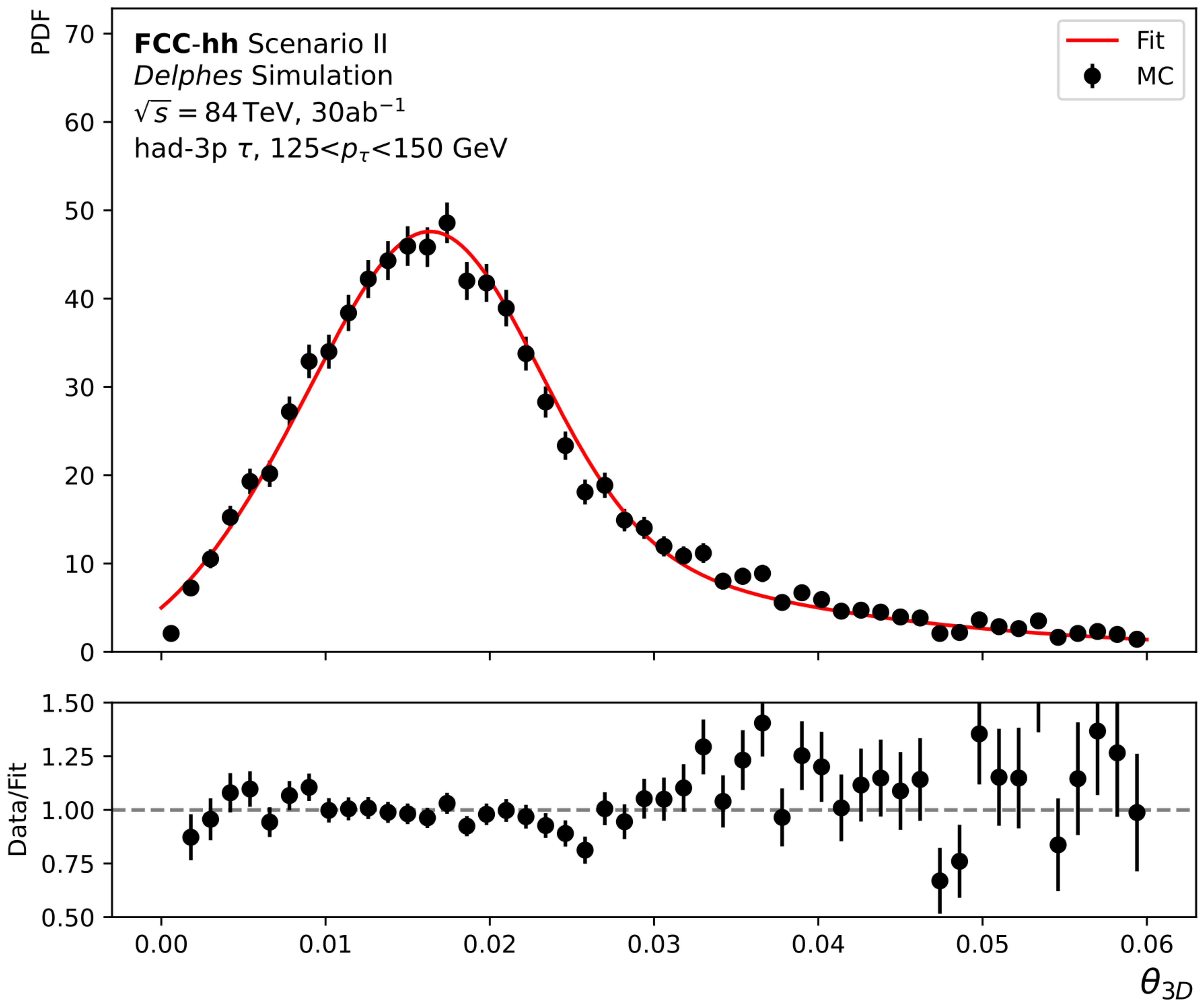
$m_{\tau\tau}$ reconstruction: 1 or 3-prong

- Likelihood function depends on τ-decay type:

1-prong

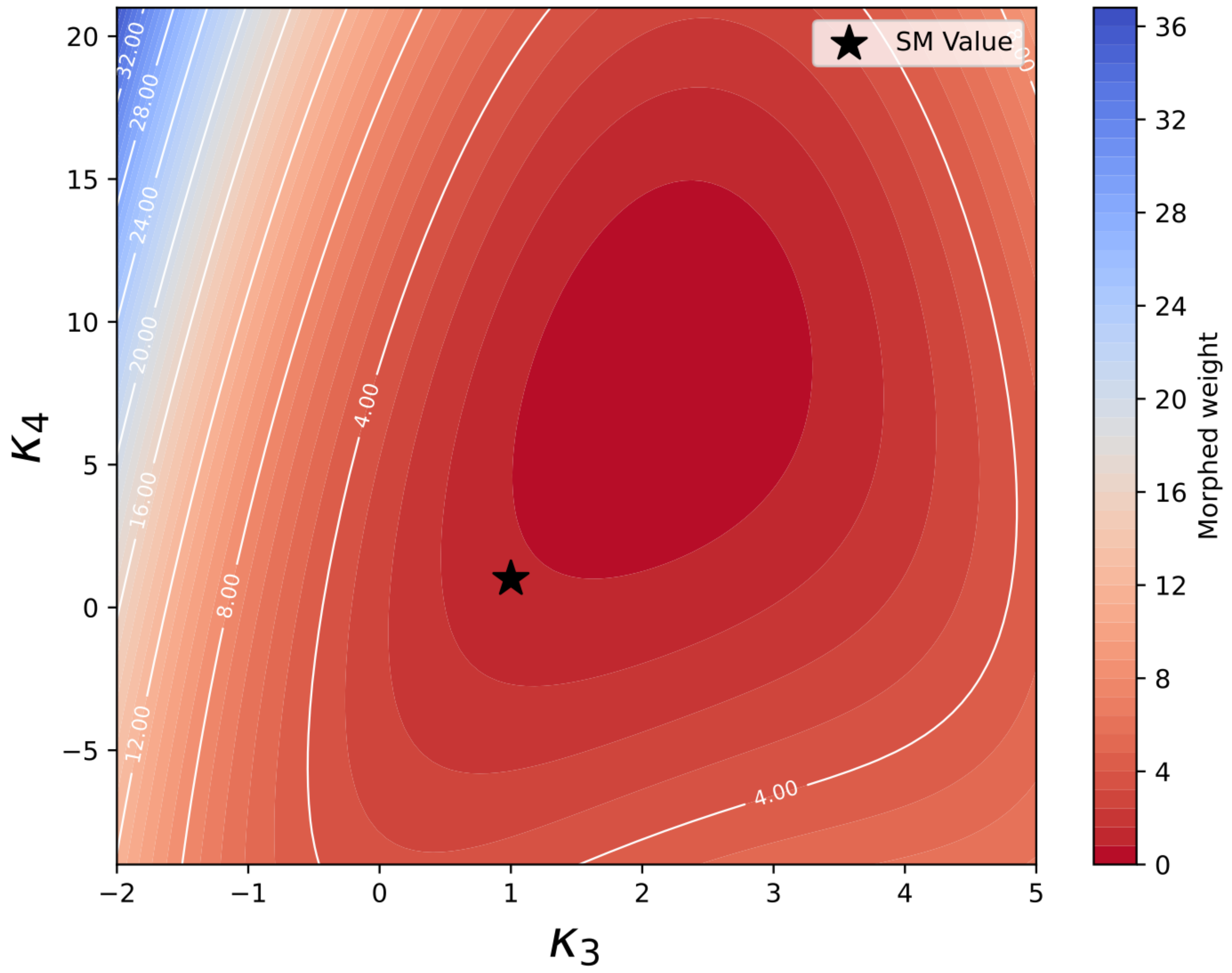


3-prong



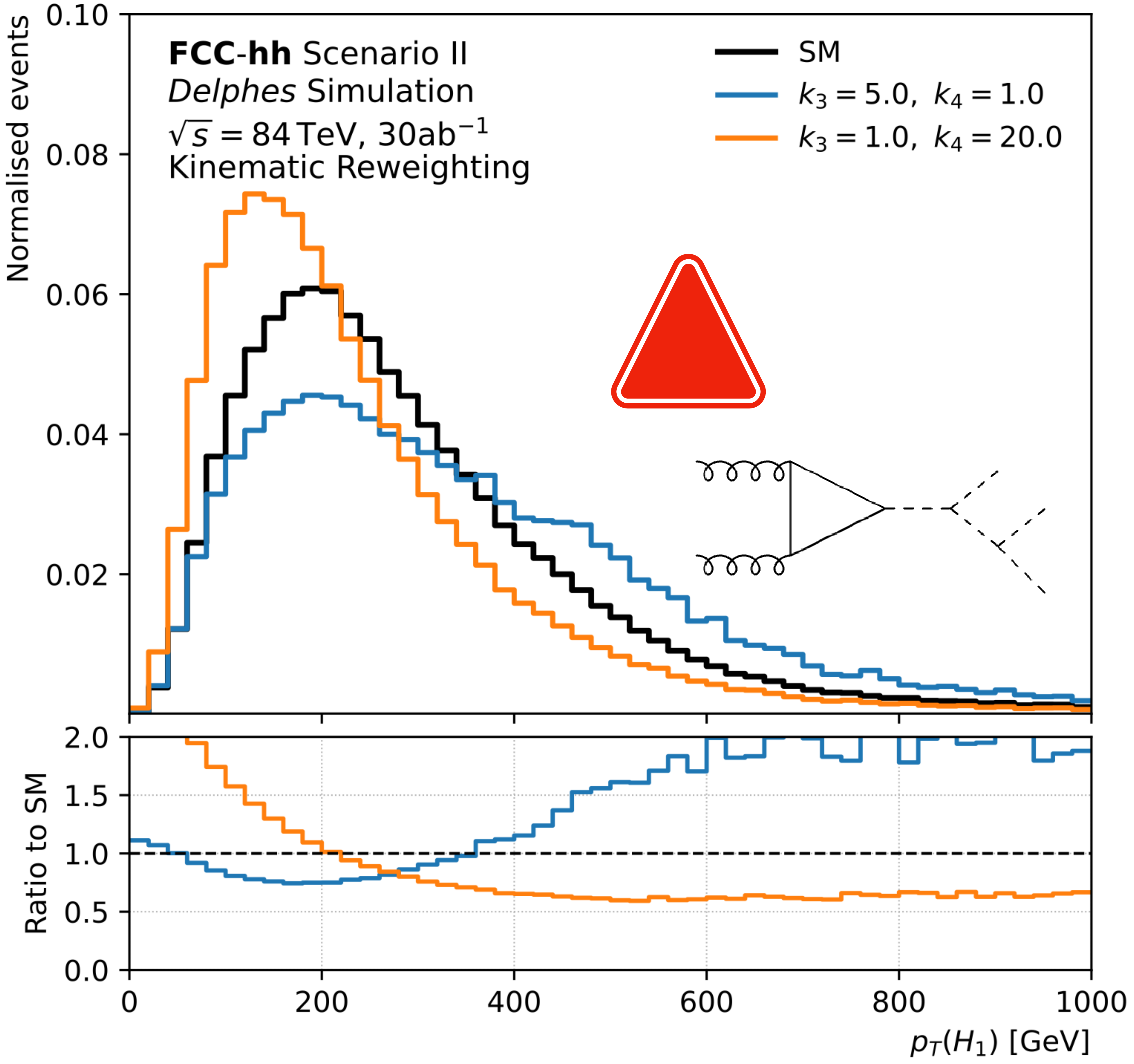
Signal implementation in FCC SW

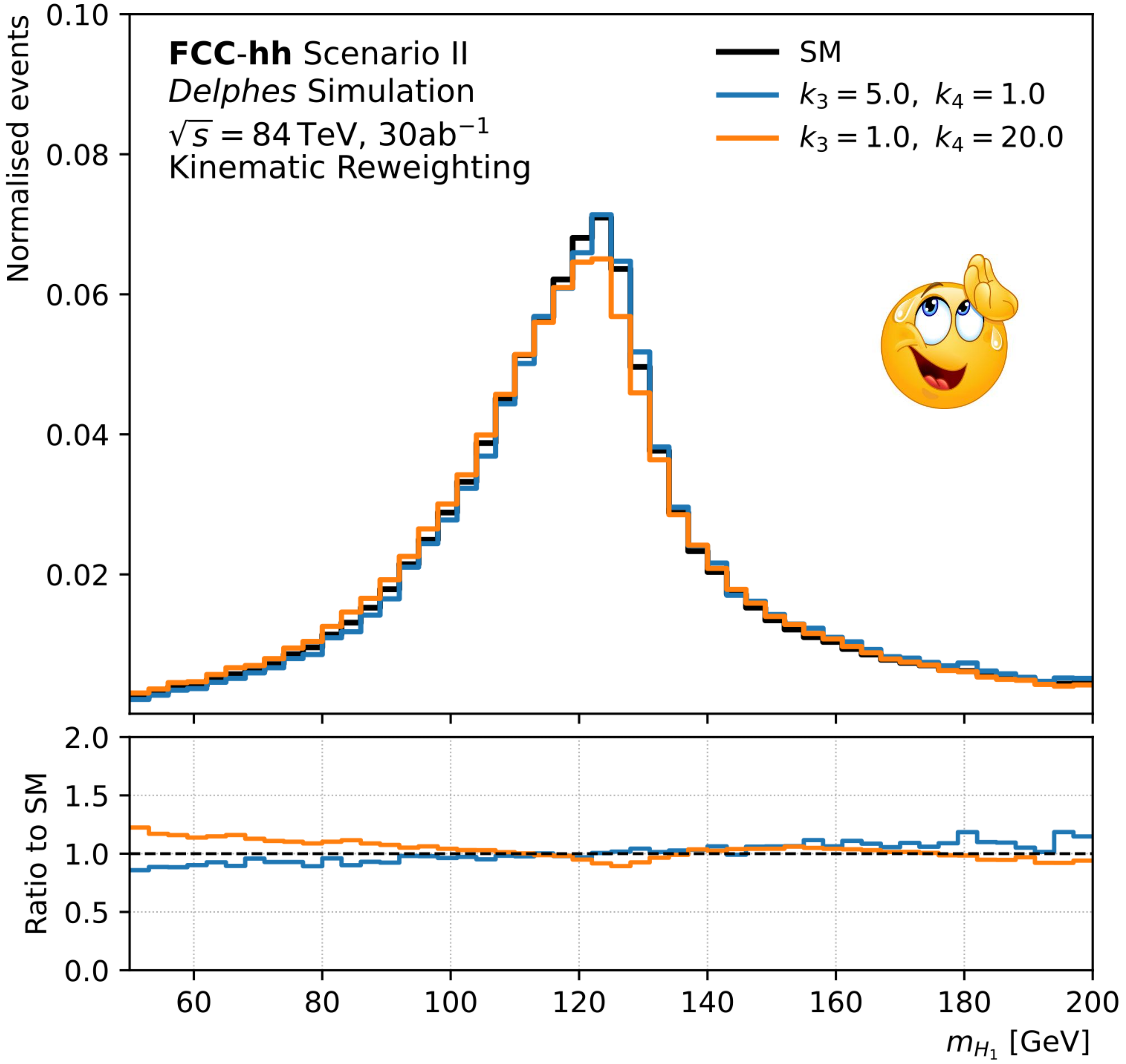
EventProducer → k4SimDelphes → FCCAnalyses



- EventProducer: MadGraph reweighting via reweighting file + MadSpin decays
- key4SimDelphes: Event weight vector stored in output root files with weight names in metadata
- FCCAnalysis: Construct per-event weight (moment-morphing) and apply for kinematic distributions

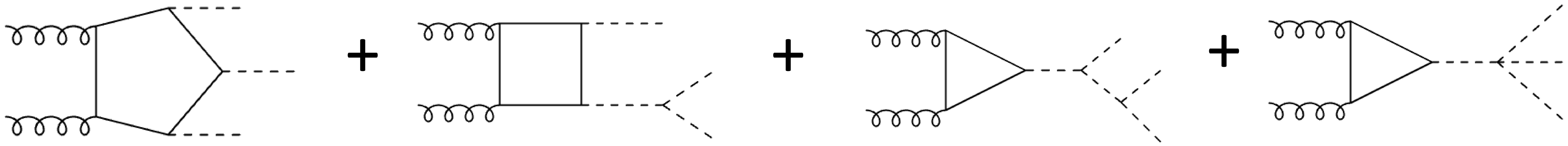
Mass variables and kappa variations





Reweighting: moment morphing

$\mathcal{M} \sim$



$a \qquad + \qquad b\kappa_3 \qquad + \qquad c\kappa_3^2 \qquad + \qquad d\kappa_4$

- We can span the space by sampling the basis-functions: $B = \{(\kappa_{3,i}, \kappa_{4,i})\}_{i=1}^{9}$

$$w(B) = \sum_{i=0}^{8} m_i f_i(B)$$ rewritten as $$w = M \cdot m$$ And $$m = M^{-1} w$$

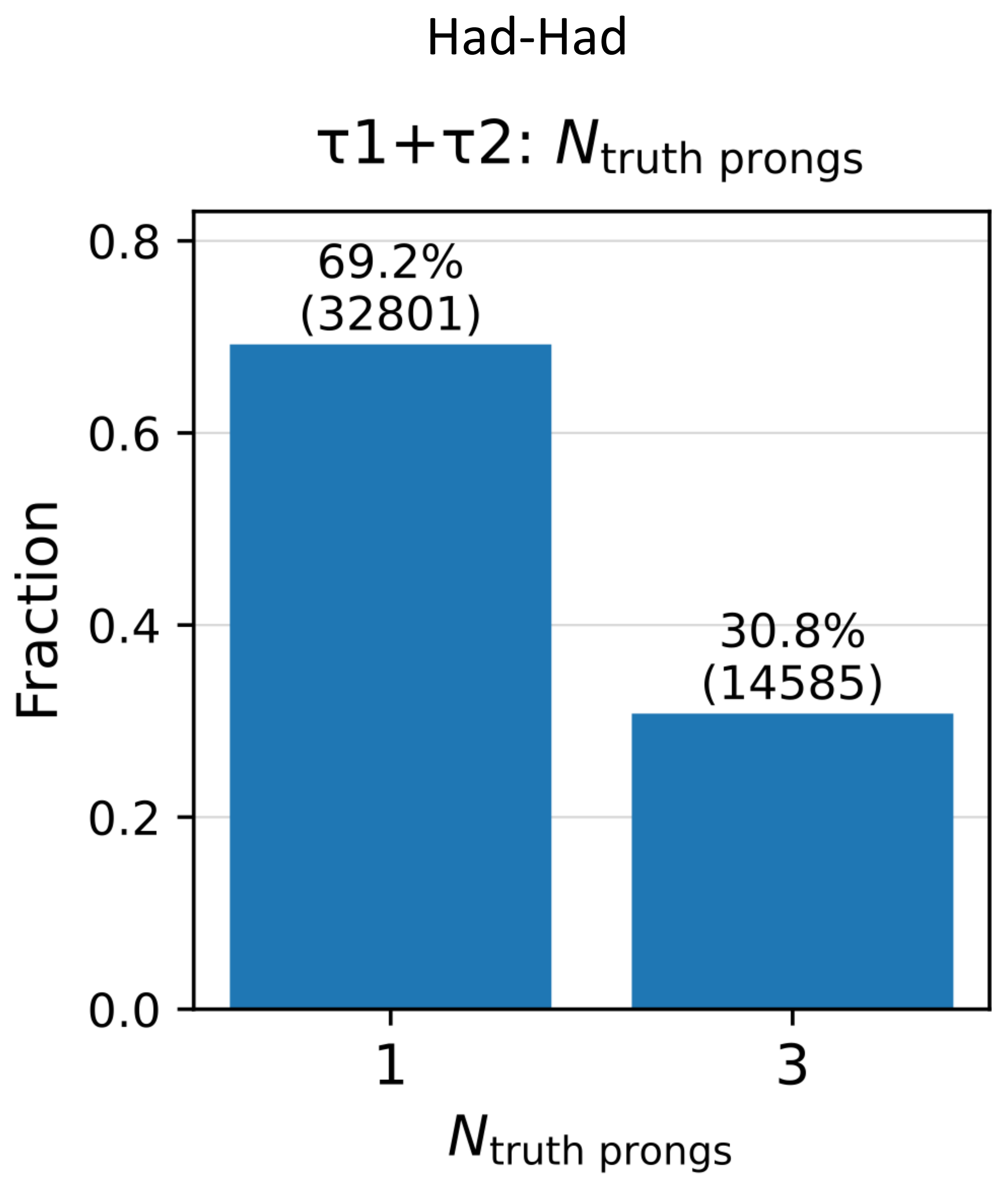
- We then have the moment required to multiply by a known basis set
- To generate the average reweighted cross-section, we first find the average vector of moments:

$$\langle \mathbf{m} \rangle = \frac{1}{N} \sum_{j=1}^{N} M^{-1} w_j$$

to allow any variation in κ_3, κ_4:

$$\boxed{w(\kappa_3, \kappa_4) = \sum_{i=0}^{8} \langle \mathbf{m} \rangle_i f_i(\kappa_3, \kappa_4)}$$

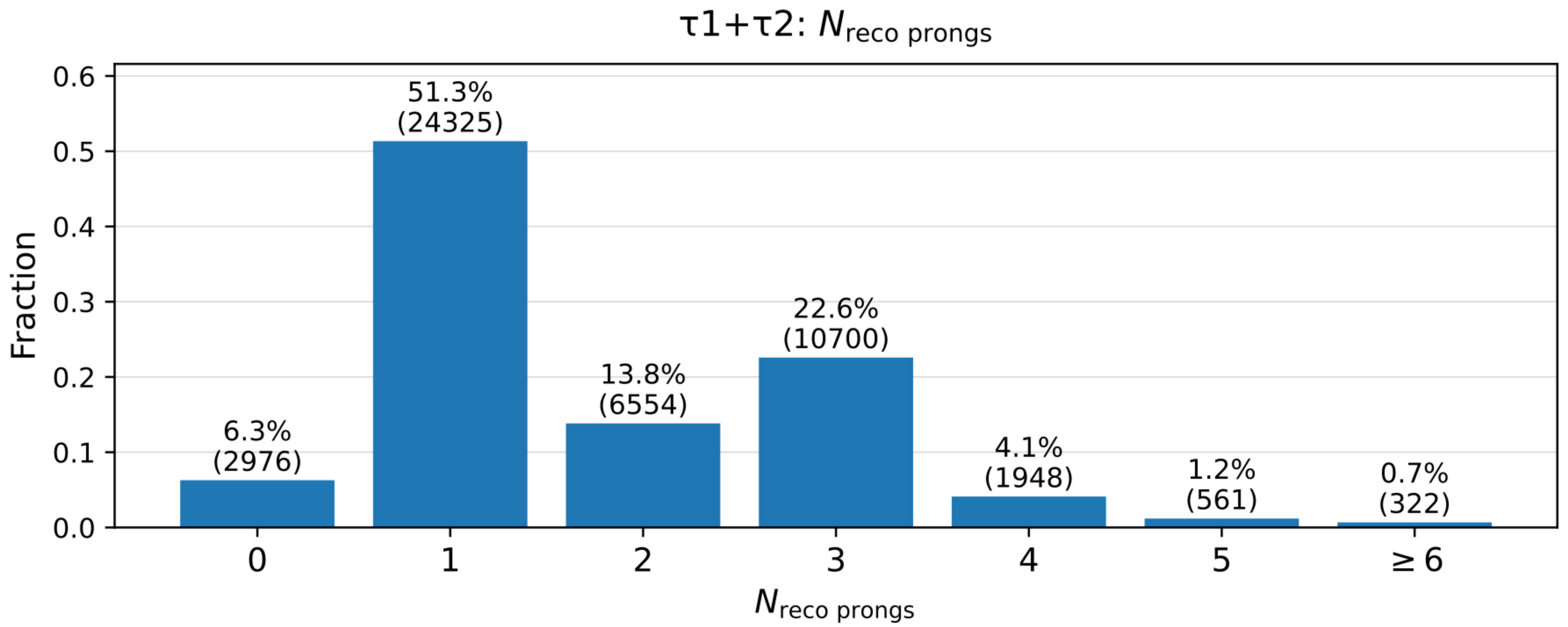
$m_{\tau\tau}$ reconstruction: 1 or 3-prong

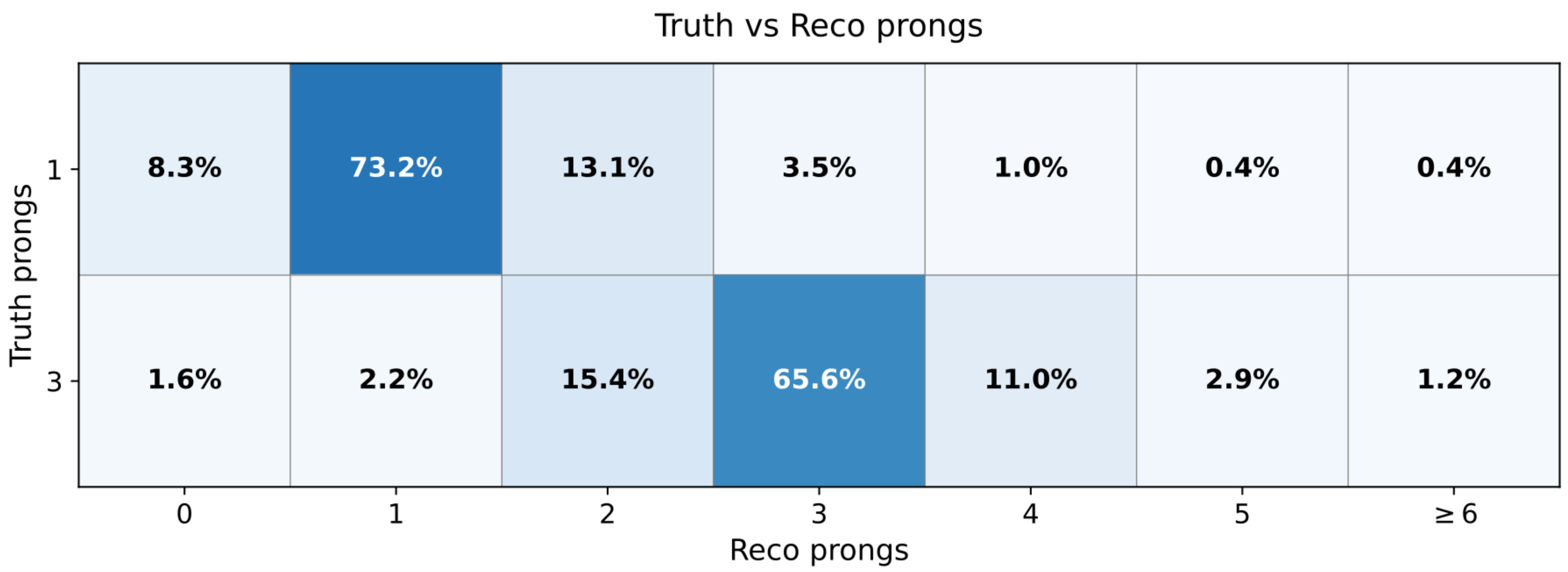


- Likelihood function depends on τ-decay type -> Need to accurately define 1 or 3-prong decays

<u>Dominant modes for hadronic-taus:</u>

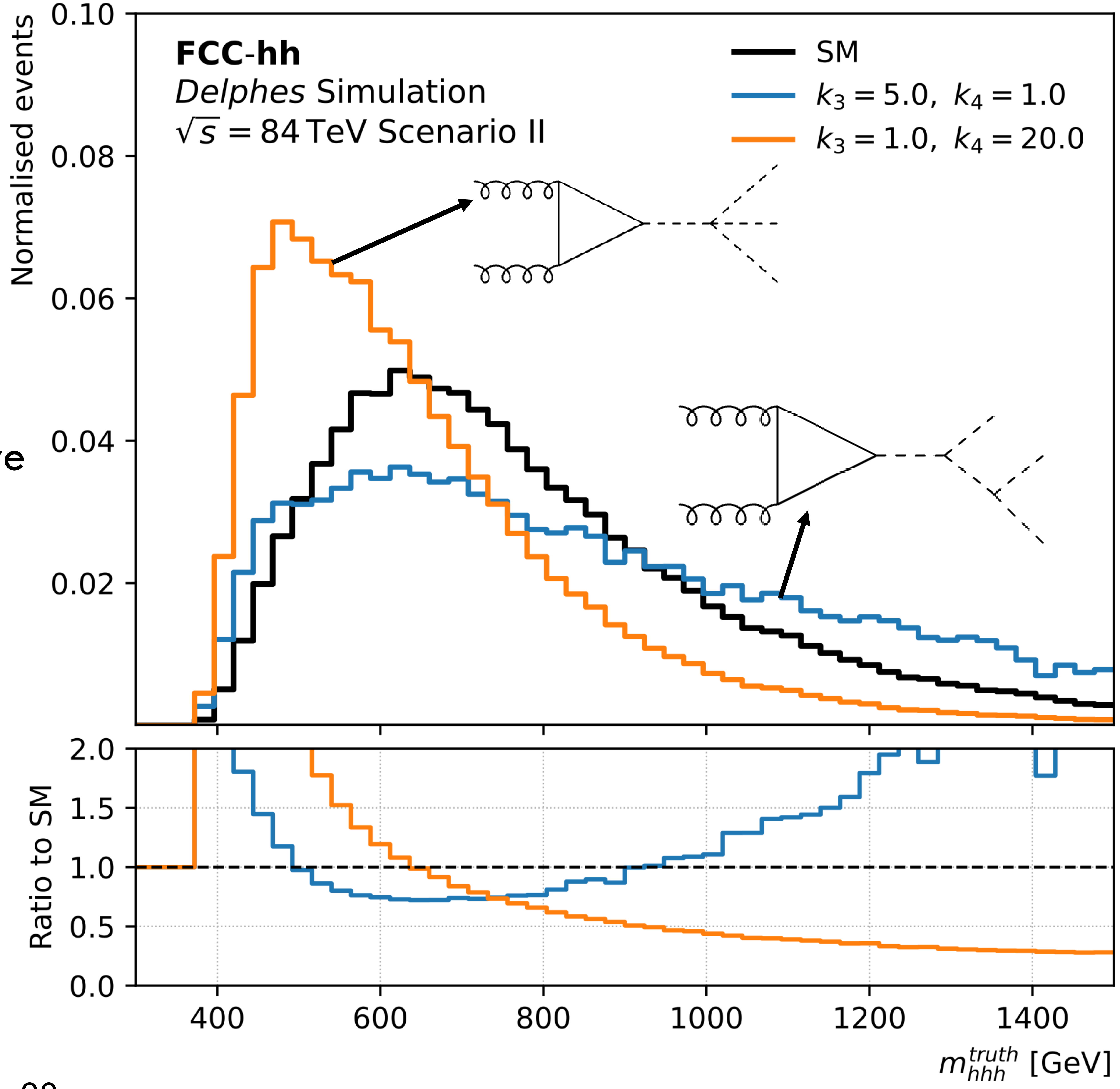
- $\tau^- \rightarrow \nu_\tau \pi^-$ (1-charged track) '1-prong'
- $\tau^- \rightarrow \nu_\tau \pi^- \pi^+ \pi^-$ (3-charged tracks) '3-prong'
- Truth $Z \rightarrow \tau\tau$: count charged hadrons
- Reco $Z \rightarrow \tau\tau$: count charged tracks within $\Delta R < 0.2$, $p_T > 1$ GeV to τ





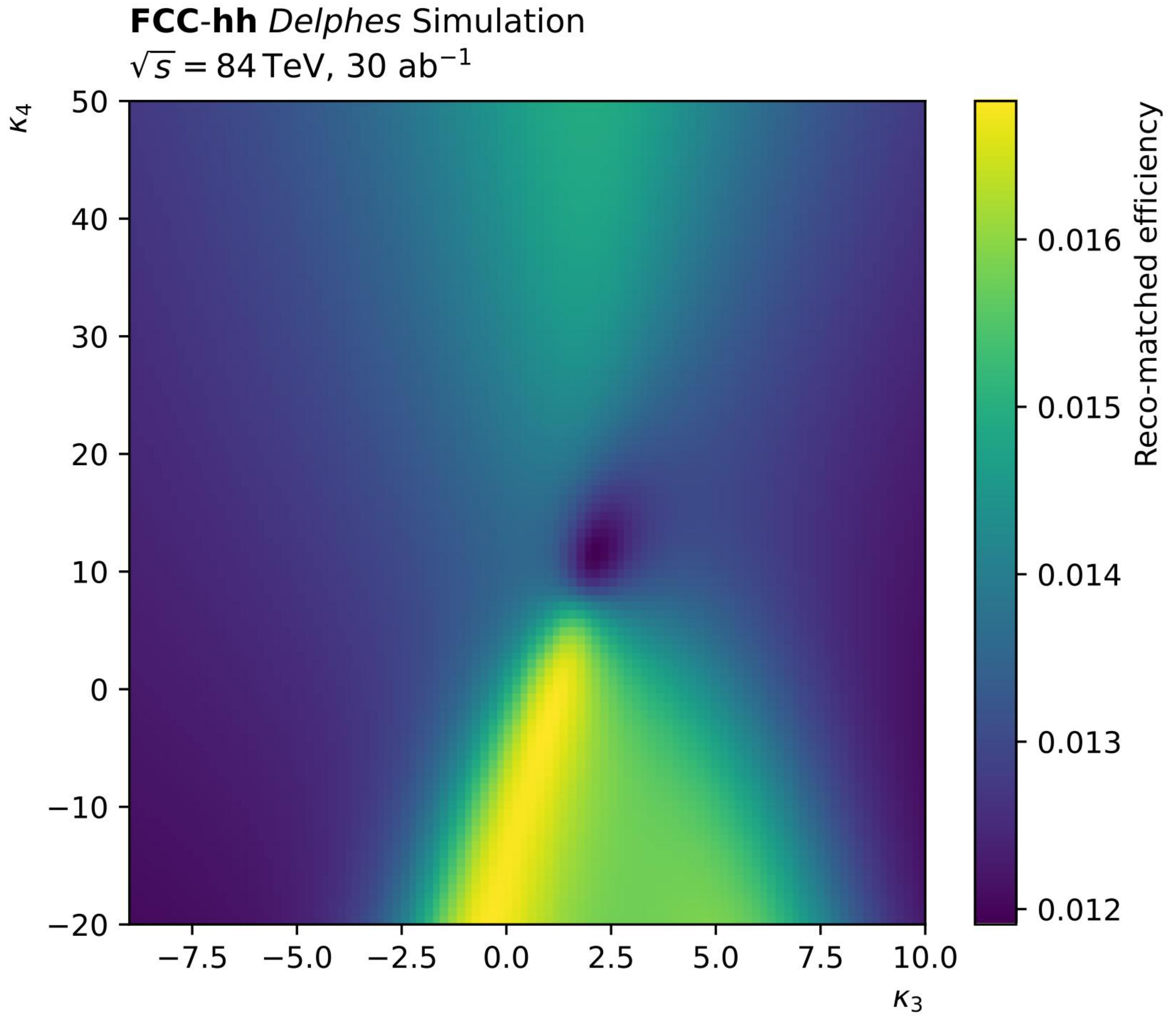
Fitting: m_{hhh} information

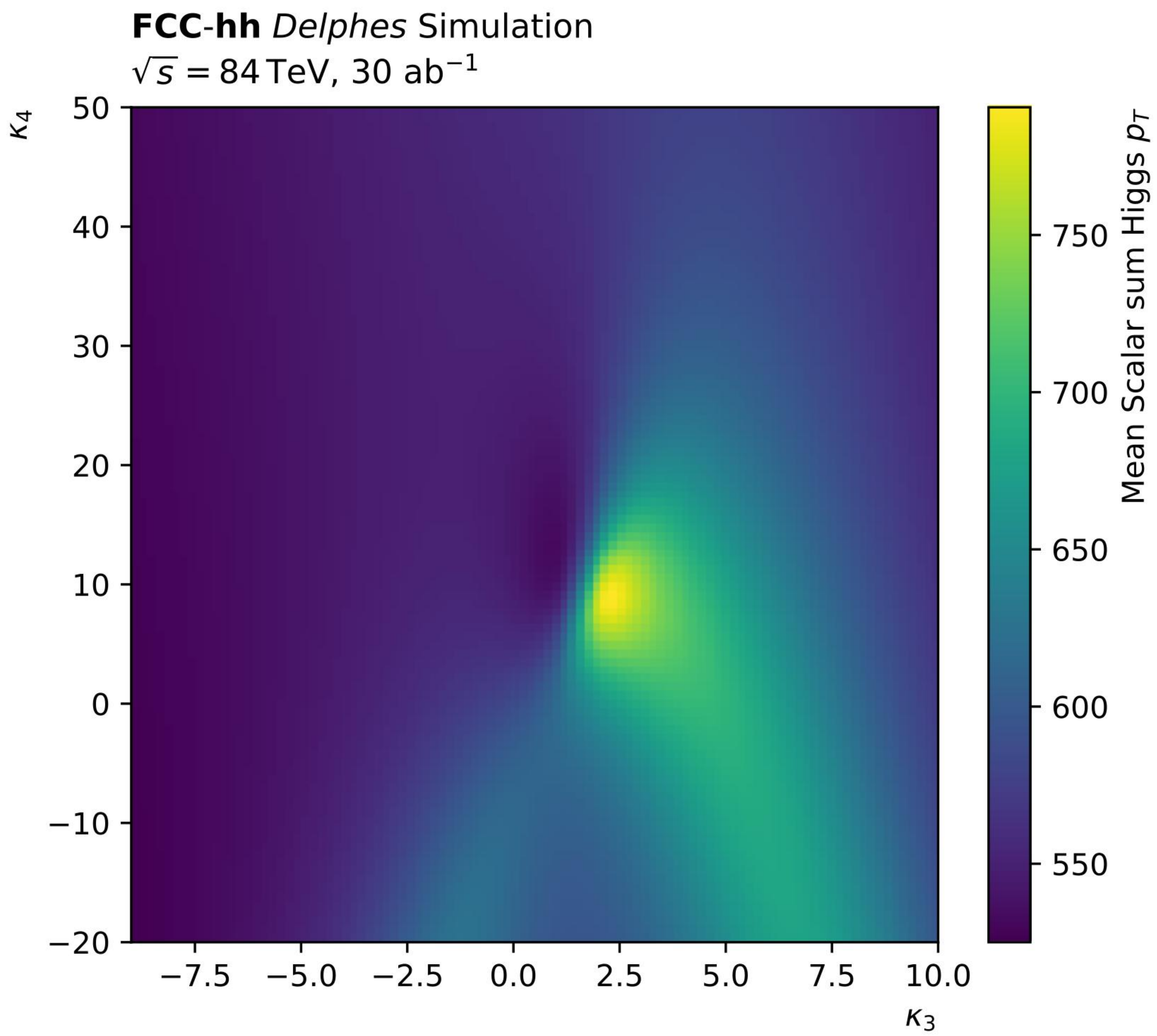
- Strong shape variations across m_{hhh} plane for positive variations!
- Perform fit on m_{hhh}?



Event reconstruction

- Scalar $\sum p_T(h)$: highest-pT region is where we're losing out the most…

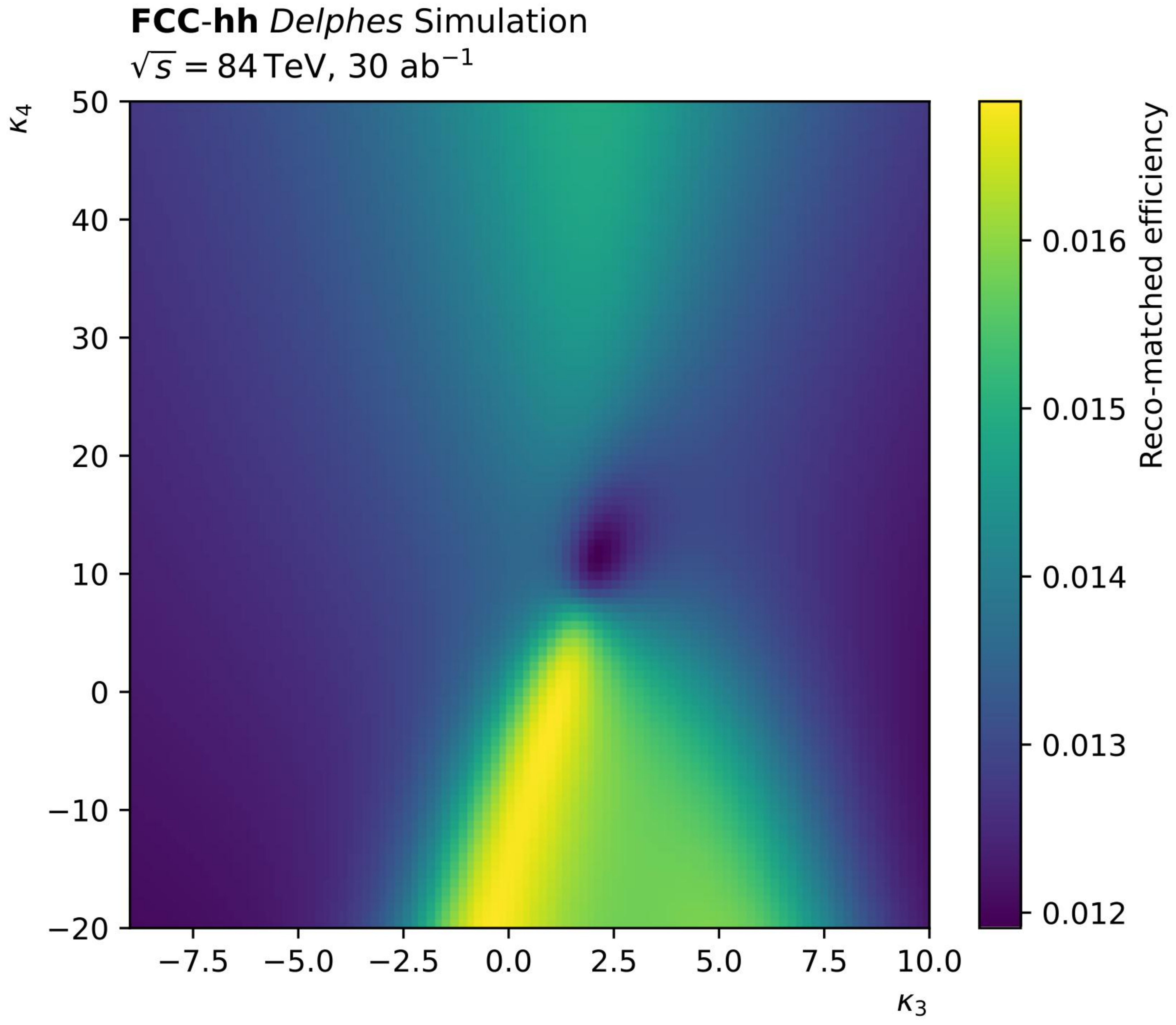




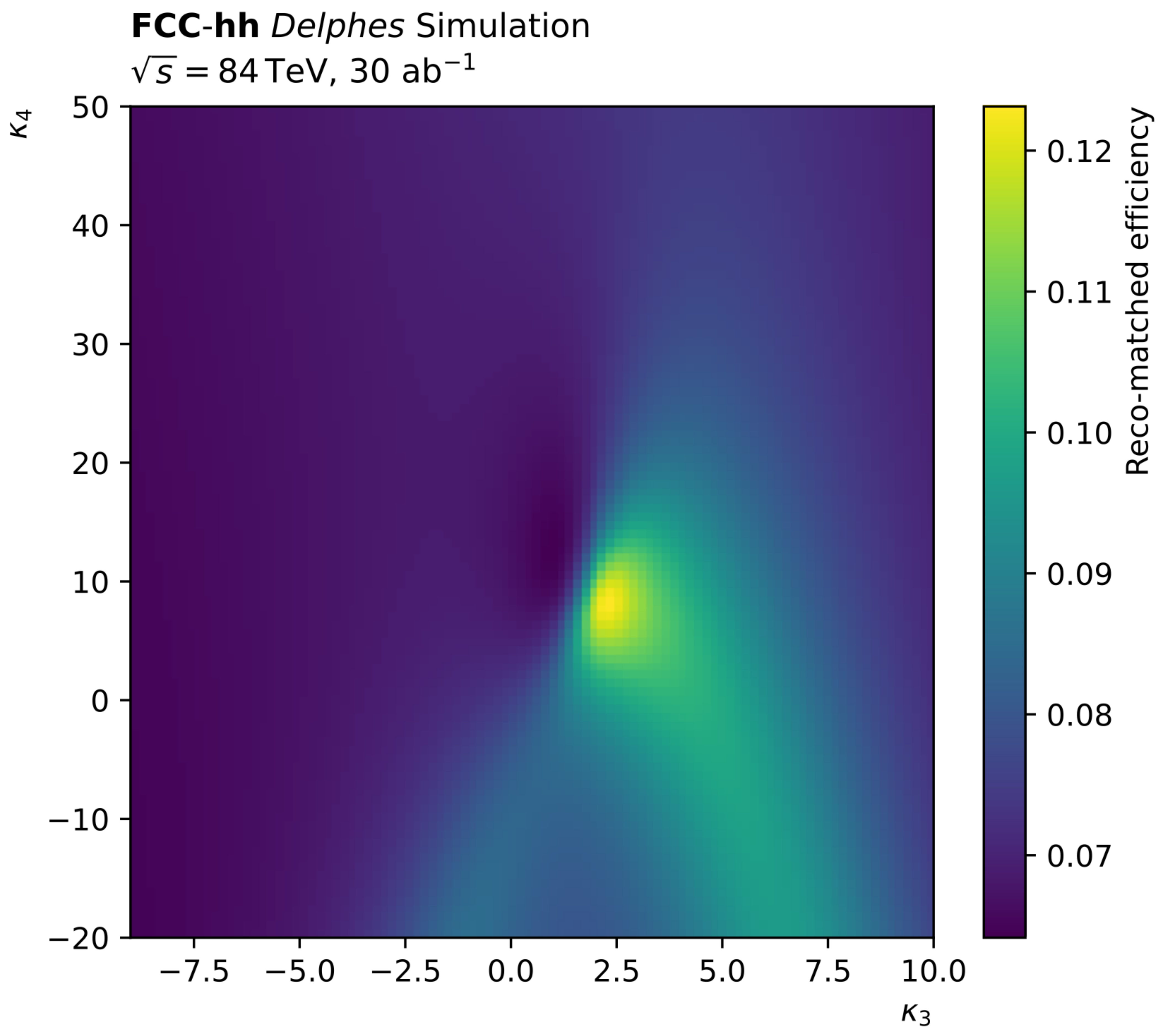
- Why do we have a low efficiency in certain phase space?
- Reconstructing events with small-radius jets not enough!

Event reconstruction

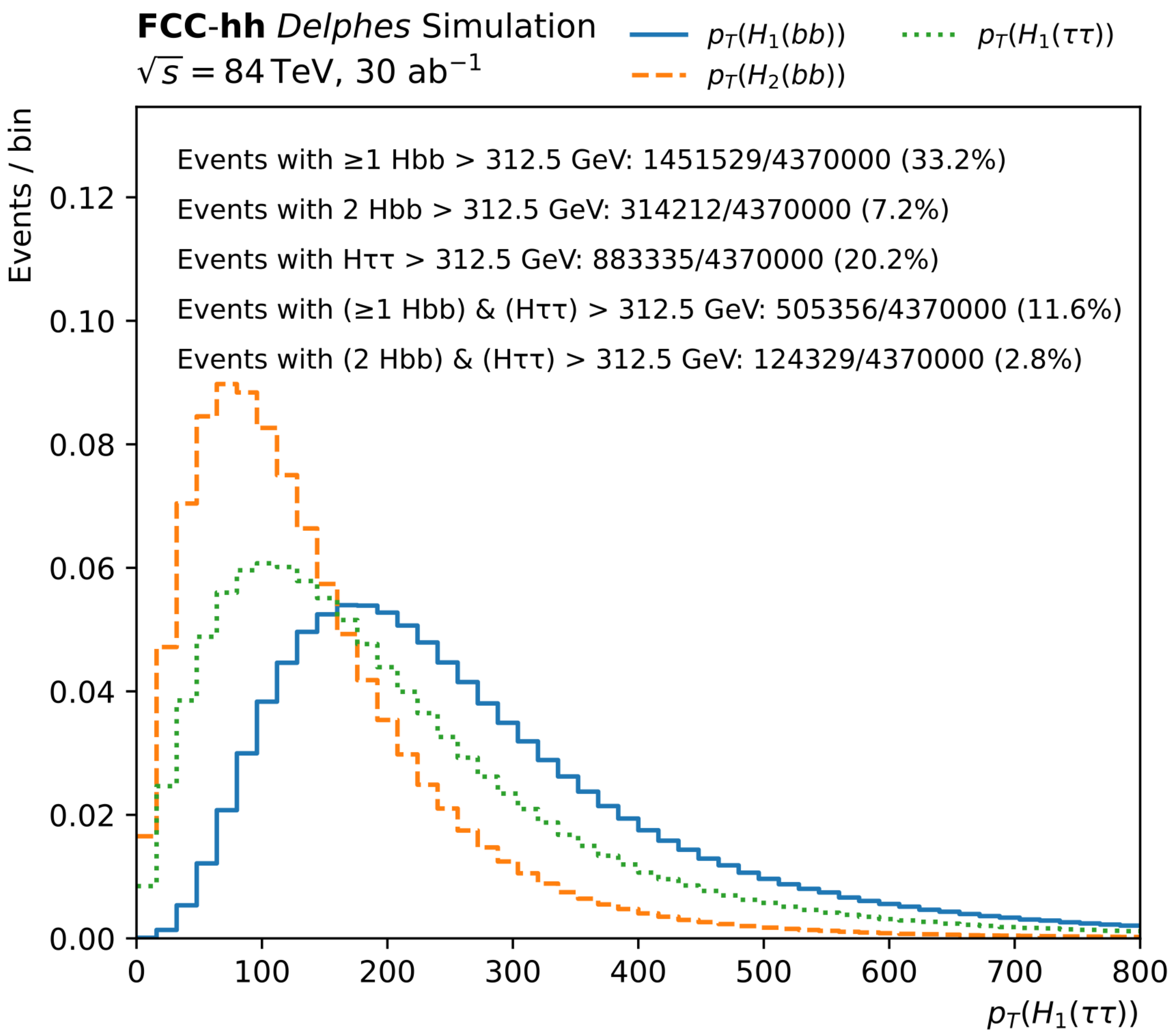
Without boosted channel:



With:



Large-radius jets within Delphes



- Method: use dR=0.2 track jets to tag flavour within the LR jets
- Use default delphes track/calo/PF jets already in configuration
- Create new pass of the flavour-association module to 0.2 track jets with truth match dR=0.2
- Store 0.8 PF jets and 0.2 track jets in root files
- bb-tagged if two b-tagged track jets are within R=0.8 of large-R jet axis

Event reconstruction

(very) preliminary test: reach with preselection only greatly enhanced by more sophisticated event reconstruction!

Cut	HHH	$t\bar{t}b\bar{b}$	$t\bar{t}H$	$t\bar{t}Z$	ZZZ	$t\bar{t}t\bar{t}$	$\frac{S}{\sqrt{B}}$
Resolved	109.15	9.82×10^5	7.23×10^4	2.60×10^4	84.57	32.47	0.10
b boosted	73.11	2.61×10^5	2.81×10^4	1.46×10^4	63.89	10.35	0.13
Tau/b Boosted	51.50	1.31×10^5	1.35×10^4	5446.47	46.69	27.05	0.13
Fully boosted	7.83	5960.00	813.42	304.63	7.25	2.18	0.09

HadHad=Combined
HadHad_resolved=4b2tau small-radius jets
HadHad_bb=At least one large-radius bb-jet, no tau boost
HadHad_tautau=at least one tautau / bb-jet boosted
HadHad_boost=All three in large-radius jets