B decays at two loops

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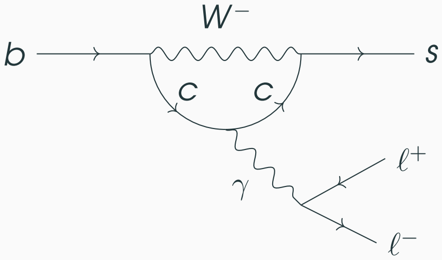
Internal Seminar, Oct 24th 2025

What are we doing?



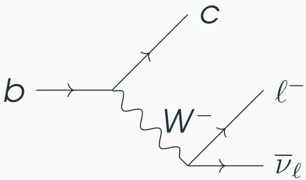
Flavour changing neutral currents only emerge at **1 loop level**

CKM supression $\sim \lambda^4$

Branching ratio $b \to s\ell^+\ell^- \sim \mathbf{10^{-6}}$

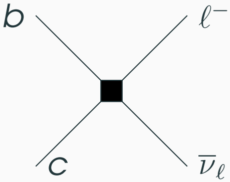
Effective Field Theory tangent

Four Fermi theory



An electroweak interaction

$\rightarrow$



A Four Fermi theory interaction

Operators

$$\mathcal{L}_{WET} \supset \mathcal{L}_{QCD+QED}^{\{\text{all fermions},\cancel{top}\}} + \frac{1}{\Lambda^2}\sum_i \mathcal{C}_i^{(6)}\,\mathcal{O}_i^{(6)}$$

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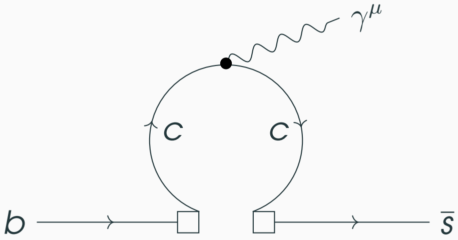
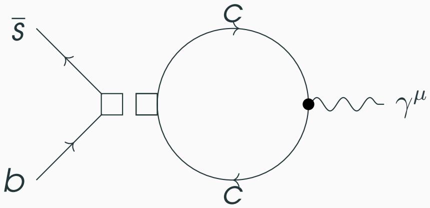
$$\mathcal{L}_{WET} \supset \mathcal{L}_{QCD+QED}^{\{\text{all fermions}, \cancel{top}\}} + \frac{1}{\Lambda^2} \sum_i \mathcal{C}_i^{(6)} \, \mathcal{O}_i^{(6)}$$

$$\mathcal{O}_i^{(6)} \supset [\overline{c}\gamma_\mu P_L b][\overline{\nu}_\ell \gamma^\mu P_L \ell^-]$$

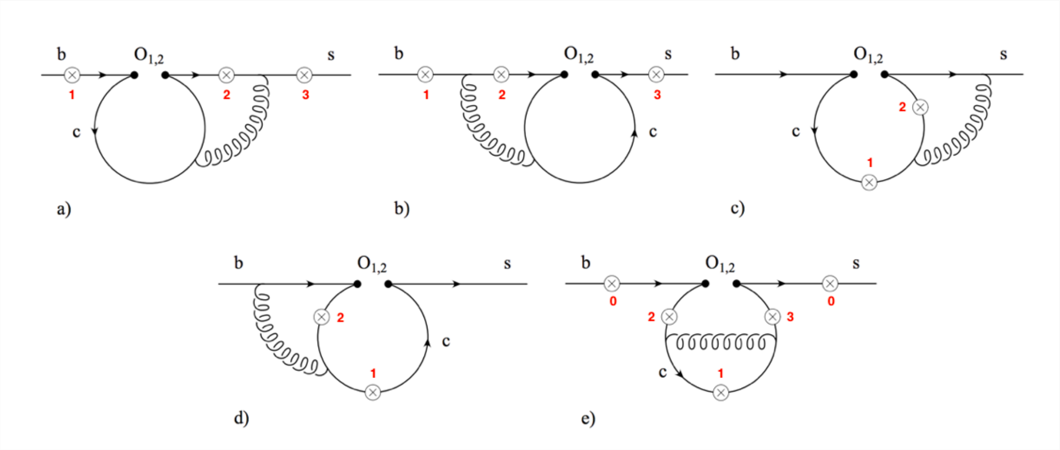
1 loop diagrams





2 loop diagrams

[JHEP 04 (2020) 012]



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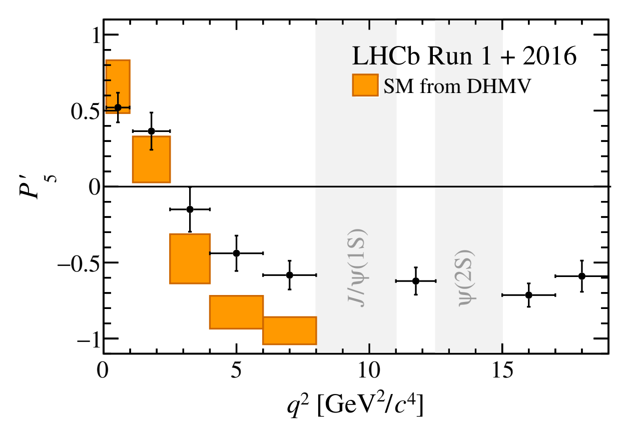
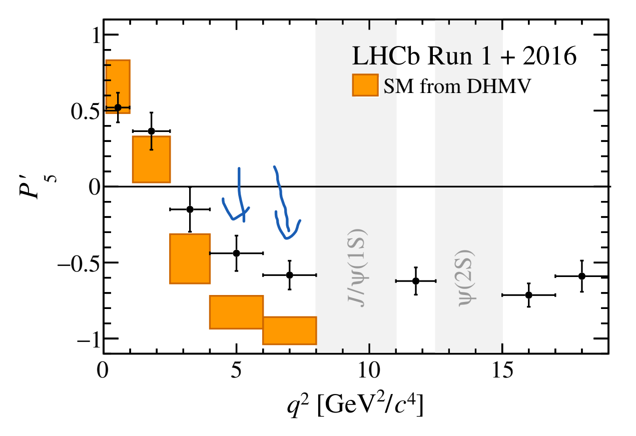

LHCb Run 1 + 2016
SM from DHMV
P_5'
$J/\psi(1S)$
$\psi(2S)$
1
0.5
0
−0.5
−1
0
5
10
15
q^2 [GeV$^2/c^4$]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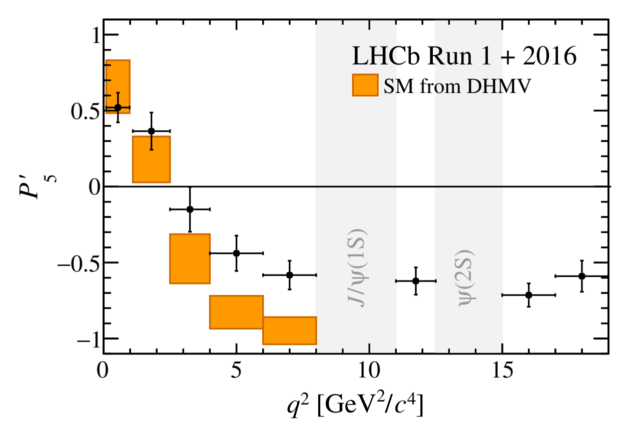
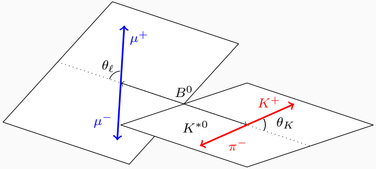
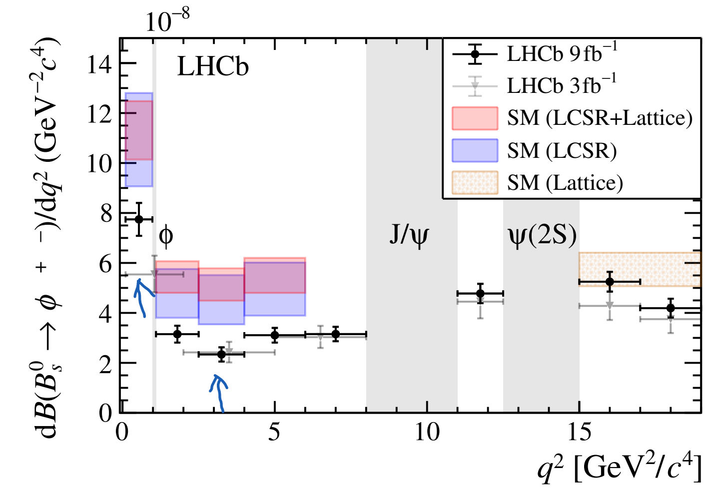




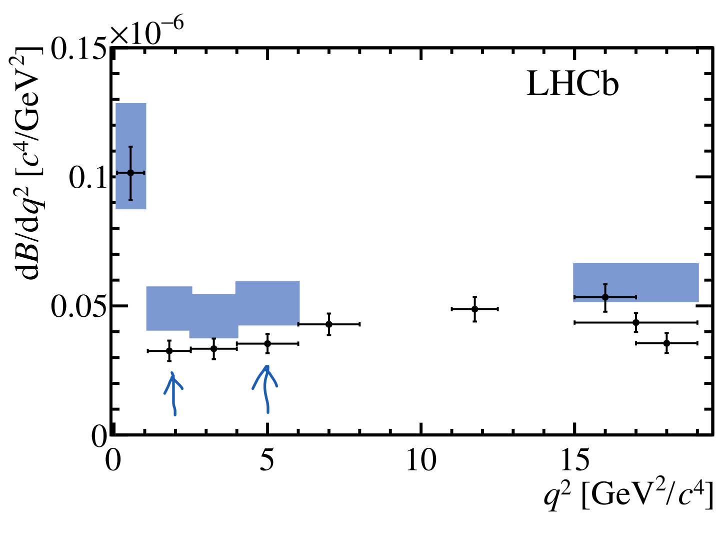
(a) θ_K and θ_ℓ definitions for the B^0 decay

$B^0 \to K^{*0}\mu^+\mu^-$

Other Observables



[PRL 127 (2021) 151801]



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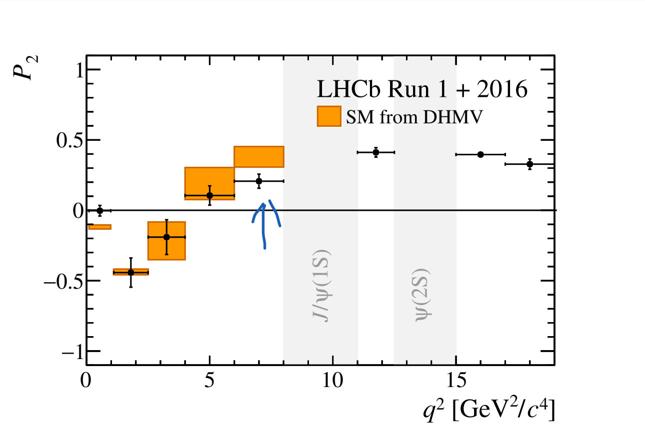
[Erratum to: JHEP11(2016)047]

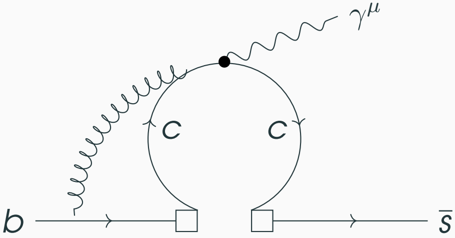
Other Observables



[CERN-EP-2020-027]

Operators

$$\mathcal{O}_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b) \ , \qquad \mathcal{O}_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b) \ ,$$



How are we doing it?

New Operators (Part 1) [JHEP 1709 158 (2017)]

Four-quark ($q \neq s$):

$$\mathcal{O}_1^{sbqq} = (\bar{s}\, P_R\, \gamma_\mu b)\ (\bar{q}\gamma^\mu q)\,, \qquad \mathcal{O}_2^{sbqq} = (\bar{s}\, P_R\, \gamma_\mu\, T^A b)\ (\bar{q}\gamma^\mu\, T^A q)\,,$$

$$\mathcal{O}_3^{sbqq} = (\bar{s}\, P_R\, \gamma_{\mu\nu\rho}\, b)\ (\bar{q}\gamma^{\mu\nu\rho} q)\,, \qquad \mathcal{O}_4^{sbqq} = (\bar{s}\, P_R\, \gamma_{\mu\nu\rho} T^A b)\ (\bar{q}\gamma^{\mu\nu\rho}\, T^A q)\,,$$

$$\mathcal{O}_5^{sbqq} = (\bar{s}\, P_R\, b)(\bar{q}\, q)\,, \qquad \mathcal{O}_6^{sbqq} = (\bar{s}\, P_R\, T^A b)(\bar{q}\, T^A q)\ ,$$

$$\mathcal{O}_7^{sbqq} = (\bar{s}\, P_R\, \sigma^{\mu\nu}\, b)(\bar{q}\, \sigma_{\mu\nu}\, q)\,, \qquad \mathcal{O}_8^{sbqq} = (\bar{s}\, P_R\, \sigma^{\mu\nu}\, T^A b)(\bar{q}\sigma_{\mu\nu}\, T^A q)\,,$$

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New Operators (Part 2)

Four-quark $(q = s)$:

$$\mathcal{O}_1^{sbss} = (\bar{s}\,\gamma_\mu\,P_L\,b)\;(\bar{s}\gamma^\mu\,s)\,,\qquad \mathcal{O}_{1'}^{sbss} = (\bar{s}\,\gamma_\mu\,P_R\,b)\;(\bar{s}\gamma^\mu\,s)\,,$$

$$\mathcal{O}_3^{sbss} = (\bar{s}\,\gamma_{\mu\nu\rho}\,P_L\,b)\;(\bar{s}\gamma^{\mu\nu\rho}s)\,,\qquad \mathcal{O}_{3'}^{sbss} = (\bar{s}\;\gamma_{\mu\nu\rho}\,P_R\,b)\;(\bar{s}\gamma^{\mu\nu\rho}\,s)\,,$$

$$\mathcal{O}_5^{sbss} = (\bar{s}\,P_L\,b)(\bar{s}\,s)\,,\qquad \mathcal{O}_{5'}^{sbss} = (\bar{s}\,P_R\,b)(\bar{s}\,s)\,,$$

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- **Reduce** number of **integrals** that need to be computed

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- **Simplify** integral **process**

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- Reduce to **scalar integrals** (Passarino-Veltman or $\boxed{\text{projector method}}$)

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Laporta Algorithm [hep-ph 1705.05610]

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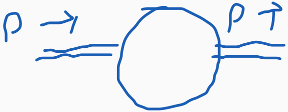
- Write complex integrals in terms of **simpler ones**
- For example, integrals with higher propagator powers are more complex

For example

$$T(d, a_1, a_2) = \int d^d\ell \frac{1}{(\ell^2 - m_1^2)^{a_1}((\ell + p)^2 - m_1^2)^{a_2}}$$



For example

$$T(d, a_1, a_2) = \int d^d\ell \frac{1}{(\ell^2 - m_1^2)^{a_1}((\ell + p)^2 - m_1^2)^{a_2}}$$

obeys

$$T(d, a_1, a_2) = T(d, a_2, a_1)$$

under

$$\ell \to -\ell - p$$

Our integrals are invariant under **Lorentz transformations**, so under

$$p^\mu \to p^\mu + \delta p^\mu = p^\mu + \delta\epsilon^\mu_\nu p^\nu \text{ with } \delta\epsilon^\mu_\nu = -\delta\epsilon^\nu_\mu$$

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we get

$$\sum_{i=1}^{E} \left(p_i^\nu \frac{\partial}{\partial p_{i,\mu}} - p_i^\mu \frac{\partial}{\partial p_{i,\nu}} \right) I(p_1, \ldots, p_n) = 0$$

Lorentz Invariant Identities

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which we contract with all **antisymmetric** combinations of

$$p_{r,\mu}p_{s,\nu} - p_{s,\mu}p_{r,\nu}$$

Integration By Parts

$$\int d^d\ell \frac{\partial}{\partial \ell^\mu}\left[\left(\frac{\ell^\mu}{(\ell^2 - m_1^2)^n}\right)\right] = 0$$

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$$(d - 2n) \int d^d\ell \left(\frac{1}{(\ell^2 - m_1^2)^n}\right) - 2nm^2 \int d^d\ell \left(\frac{1}{(\ell^2 - m_1^2)^{n+1}}\right) = 0$$

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$$(d - 2n) \underbrace{\int d^d\ell \left(\frac{1}{(\ell^2 - m_1^2)^n}\right)}_{F(n)} - 2nm^2 \underbrace{\int d^d\ell \left(\frac{1}{(\ell^2 - m_1^2)^{n+1}}\right)}_{F(n+1)} = 0$$

$$\int \prod_{j=1}^{L} d^d \ell_j \frac{\partial}{\partial \ell_f^\mu} \left[\left(\frac{q_l^\mu}{P_1^{a_1} \dots P_N^{a_N}} \right) \right] = 0$$

With $f = 1, \dots, L$ and $l = 1, \dots, L + E$

Integration By Parts Identities

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With $f = 1, \dots, L$ and $l = 1, \dots, L + E$

giving $L(L + E)$ identities for fixed **a**

Simplify the integral process

- Compare to known integrals

Simplify the integral process

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or

- Differential equations and **canonical form**

Canonical Form

[hep-ph 1611.01087]

For a vector of master integrals $\vec{M}(\epsilon, \{x\})$, the aim is to get it in a form

$$\partial \vec{M}(\epsilon, \{x\}) = \epsilon a(x) \vec{M}(\epsilon, \{x\})$$

which allows us to solve for $\vec{M}$ **order by order** in ϵ.

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which allows us to solve for $\vec{M}$ **order by order** in ϵ. Even better, is to get it in **dlog form**

$$\partial \vec{M}(\epsilon, \{x\}) = \epsilon A_l dlog(L_l(\{x\})) \vec{M}(\epsilon, \{x\})$$

Example

Assume the only master integral M has no poles (or multiply by ϵ^n)

$$M = \sum_{n=0}^{N} \epsilon^n M^{(n)} \qquad \text{and} \qquad M = \epsilon A(x) M$$

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$$M = \sum_{n=0}^{N} \epsilon^n M^{(n)} \qquad \text{and} \qquad M = \epsilon A(x) M$$

Then we have

$$dM^{(0)} = 0, \qquad \text{so } M^{(0)} = C$$

$$dM^{(1)} = CA(x), \text{ so } M^{(1)} = C \int dx A(x) \qquad (\text{ etc.})$$

Conclusions

What are we doing (again)?

- For the $40 + 40$ operators, **all integrals** reduced to master integrals

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- For the $40+40$ operators, **all integrals** reduced to master integrals
- For one family in the first $10+10$ operators, we have integrals in **dlog form**
- For remaining families, we have compared to known integrals using **Laporta algorithm**

What are we doing next?

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- Express the full amplitude in terms of **polylogs** of invariants

Conclusions

- B decays are an important and interesting area of work
- The multi-loop toolkit is invaluable (Laporta algorithm and canonical form)
- I'm looking at 2 loop calculations (where SM is not assumed)

Backup Slides

Laporta Complexity

Let our integral be

$$\frac{1}{D_1^{\alpha_1} D_2^{\alpha_2} \dots D_n^{\alpha_n}}$$

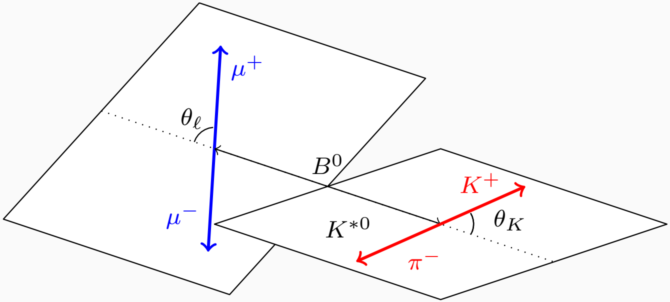
with

$$\alpha_i = \begin{cases} a_i, & \alpha_i \geq 0, \\ b_i, & \alpha_i < 0. \end{cases}$$

The most complicated integrals have

1. Largest n
2. Largest $\sum_i a_i$
3. Largest $\sum_i b_i$
4. Largest i_1, largest $i_2, \dots$, largest i_n
5. Largest a_1, largest $a_2, \dots$, largest a_n
6. Largest $-b_1$, largest $-b_2, \dots$, largest $-b_n$

Explain P_5'



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P_5' and P_2

The angular distribution of $B^0 \to K^{*0}\mu^+\mu^-$ decay is given by

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})\,\mathrm{d}q}\frac{\mathrm{d}^4(\Gamma+\bar{\Gamma})}{\mathrm{d}q^2\mathrm{d}\vec{\Omega}}\bigg|_P = \frac{9}{32\pi}\bigg[\frac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K$$
$$+\frac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_\ell - F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + \frac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_\ell$$
$$+S_7\sin 2\theta_K\sin\theta_\ell\sin\phi + S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\bigg]$$

F_L is a fraction of longitudinal polarization of K^{*0};

$$P_5' \frac{S_5}{\sqrt{F_L(1-F_L)}} \qquad \text{and} \qquad P_2 = \frac{2}{3}\frac{A_{FB}}{(1-F_L)}$$

$sb\{\gamma, \ell\ell\}$ basis

$$J_7 = 2im_b q_\nu [\bar{s}\sigma^{\mu\nu} P_R b]$$

$$J_9 = (q^\mu q^\nu - q^2 g^{\mu\nu})[\bar{s}\gamma_\nu P_L b]$$

sbcc operators

$$
\begin{aligned}
\mathcal{O}_1^{\mathrm{sbcc}} &= [\bar{s}\gamma_\mu t^a P_L c][\bar{c}\gamma^\mu t^a P_L b] & \mathcal{O}_6^{\mathrm{sbcc}} &= [\bar{s}\gamma_{\mu\nu\rho} t^a P_L b][\bar{c}\gamma^{\mu\nu\rho} t^a c] \\
\mathcal{O}_2^{\mathrm{sbcc}} &= [\bar{s}\gamma_\mu P_L c][\bar{c}\gamma^\mu P_L b] & \mathcal{O}_7^{\mathrm{sbcc}} &= [\bar{s}\gamma_\mu P_L b] Q_c [\bar{c}\gamma^\mu c] \\
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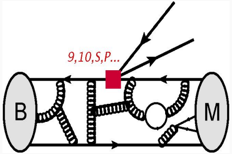
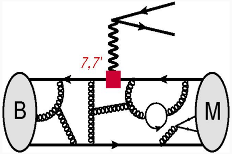
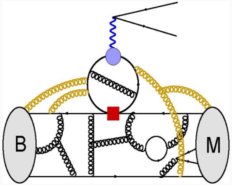
\mathcal{O}_5^{\mathrm{sbcc}} &= [\bar{s}\gamma_{\mu\nu\rho} P_L b][\bar{c}\gamma^{\mu\nu\rho} c] & \mathcal{O}_{10}^{\mathrm{sbcc}} &= [\bar{s}\gamma_{\mu\nu\rho} t^a P_L b] Q_c [\bar{q}\gamma^{\mu\nu\rho} t^a q].
\end{aligned}
$$

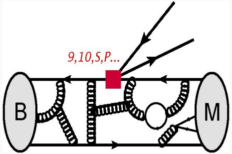
Form Factors

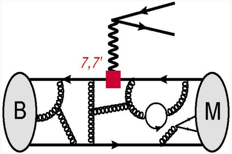






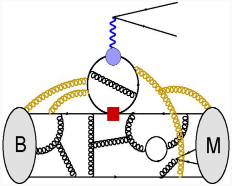
Form Factors





- Local $\mathcal{P}_\mu \langle \overline{M}(k)|\bar{s}\Gamma^\mu b|\overline{B}(k+q)\rangle$

Form Factors



- Non-local $i\mathcal{P}_\mu \int d^4x e^{iq.x} \langle \overline{M}(k)| T\{J^\mu_{em}(x), \mathcal{C}_i^{sbcc}\mathcal{O}_i^{sbcc}\}|\overline{B}(k+q)\rangle$

Weak Effective Theory Sectors

Sector: Complete set of operators so that at leading order in G_F they can only mix into each other

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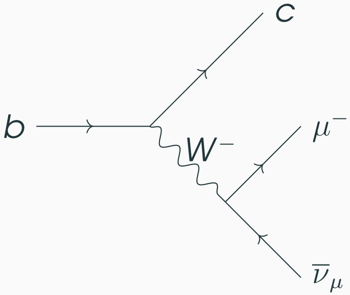
e.g. $cb\mu\nu_\mu$.



Mixing

$sbqq$ mixes into $sb\ell\ell$ through non-local operator.

$$\int d^4x e^{iq.x} T\{J^{\mu}_{em}(x-y), \mathcal{C}_i^{\mathrm{sbcc}} \mathcal{O}_i^{\mathrm{sbcc}}(y)\}$$

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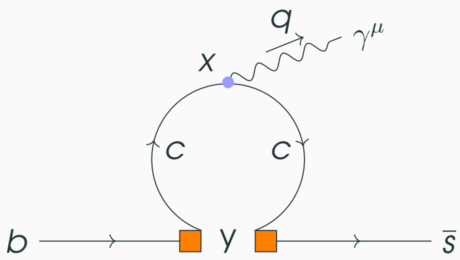
for example:

- $J^{\mu} = Q_c \overline{c} \gamma^{\mu} c$
- $\mathcal{O}_2^{\text{sbcc}} = [\overline{s}\gamma_{\nu} P_L c][\overline{c}\gamma^{\nu} P_L b]$



Operator Product Expansion

Technical point: **Pertubation constraints:** $|q^2 - 4m_c^2| > \Lambda^2_{hadronic}$

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$$\int d^4x e^{iq.x} T\{J^\mu_{em}(x), \mathcal{C}_i^{sbcc} \mathcal{O}_i^{sbcc}\}$$

$$= \sum F_{ij}\, \mathcal{C}_i^{sbcc}\, [\bar{s}\Gamma_j^\mu b]$$

Example

$$F_{29}\, \mathcal{C}_2^{\text{sbcc}}\, [\bar{s}\Gamma_9^{\mu} b]$$

$$= \frac{2}{9}(4\pi e^{-\gamma})^{\epsilon}\left[\frac{12 m_c^2}{q^2} + \left(2 + \frac{3}{\epsilon} + 3\log\frac{\mu^2}{m_c^2}\right) + 3\text{DiscB}(q^2, m_c, m_c)\frac{(2m_c^2 + q^2)}{q^2}\right]$$
$$\times\, \mathcal{C}_2^{\text{sbcc}} (q^2 g^{\mu\nu} - q^{\mu} q^{\nu})[\bar{s}\gamma_{\nu} P_L b]$$

DiscB

$$\mathrm{DiscB}(q^2, m_c, m_c) = \frac{\sqrt{q^2(q^2 - 4m_c^2)}}{q^2} + \log\left(\frac{2m_c^2 - q^2 + \sqrt{q^2(q^2 - 4m_c^2)}}{2m_c^2}\right)$$